Cellular automata as universal computers

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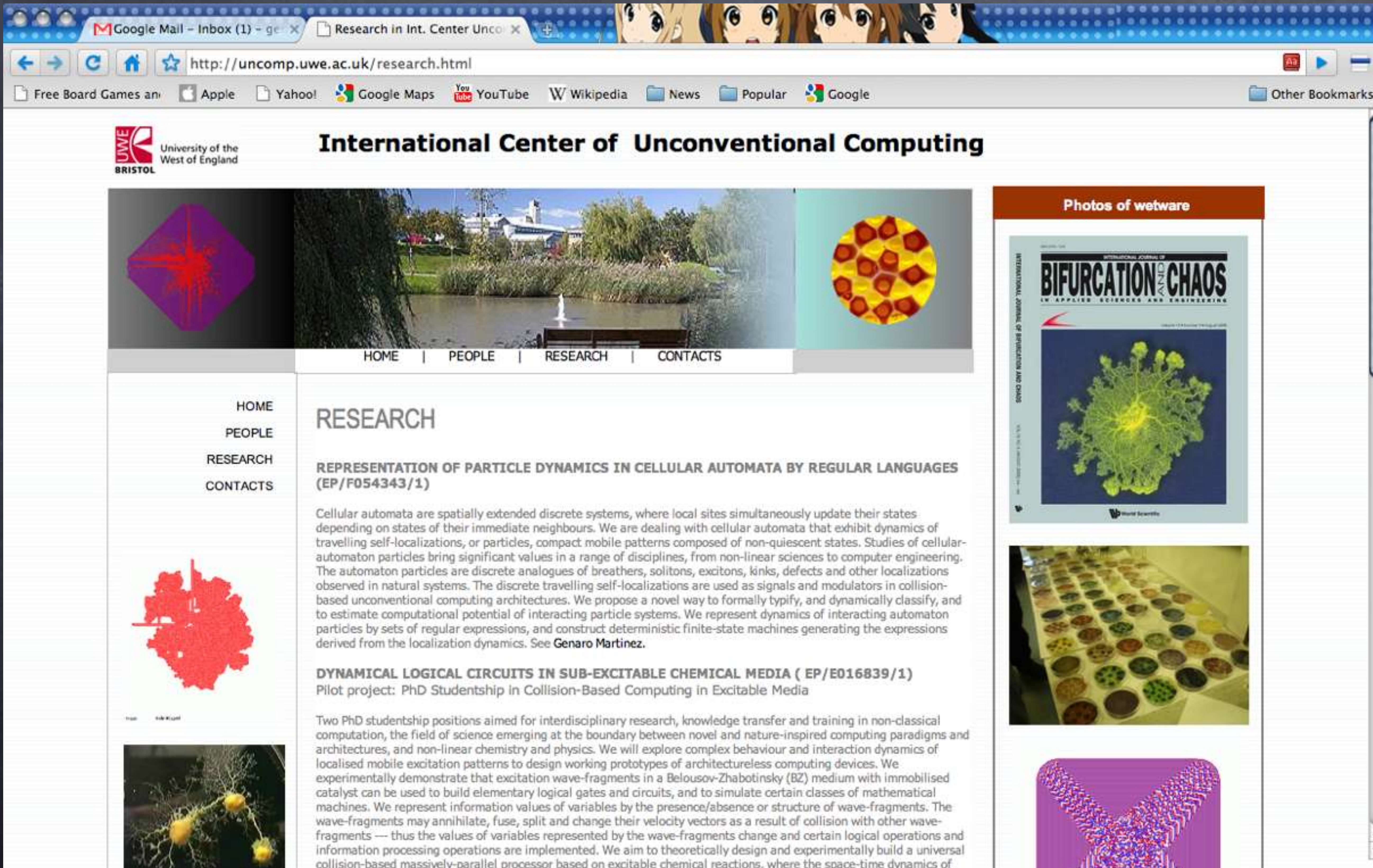

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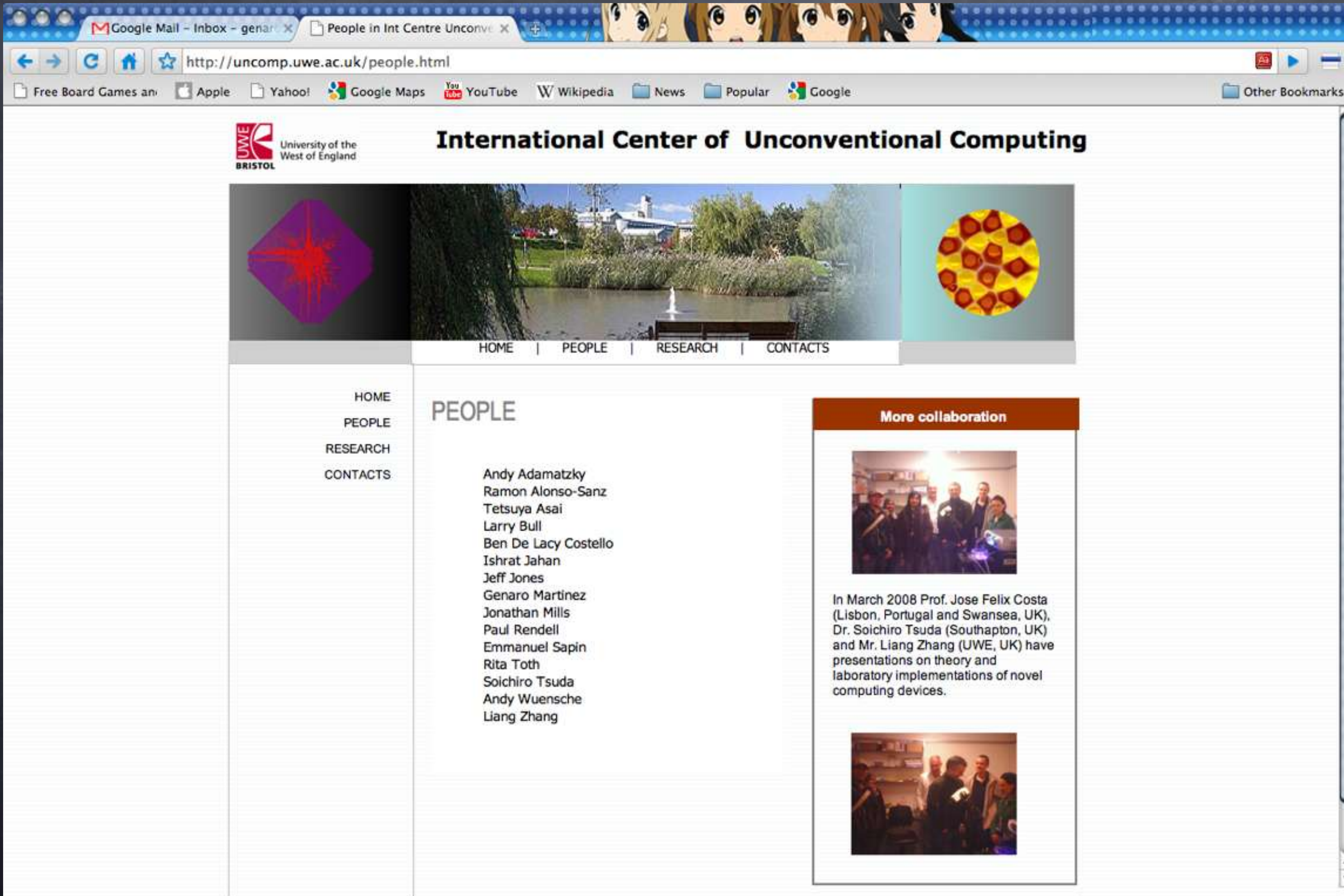
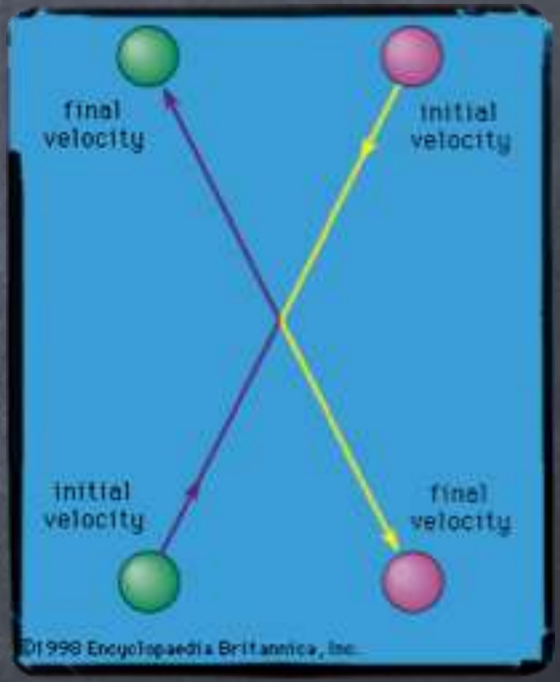


Motivations

- We can see a number of efforts to underst particle physics, solitons, molecular codes, cellular process, complex systems, self-organisation, artificial life, nano-technology, non-linear phenomena, ...



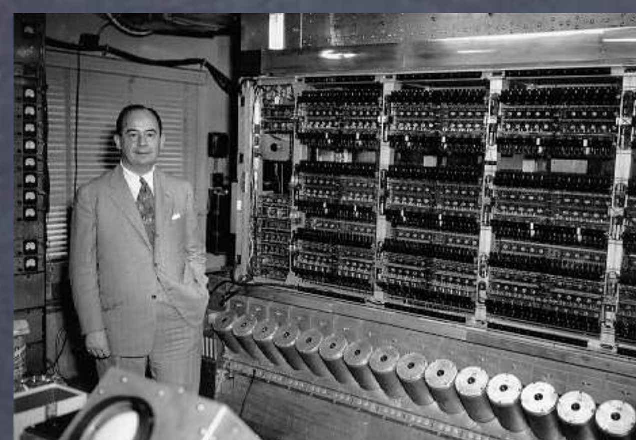
- Abstract discrete models help to describe and design specific local conditions to project and simulate some process from complex behavior.

- As additional stimulant bonus, we intent controller such artificial organism to implement “computations” and to get powerful sources of massive calculus!

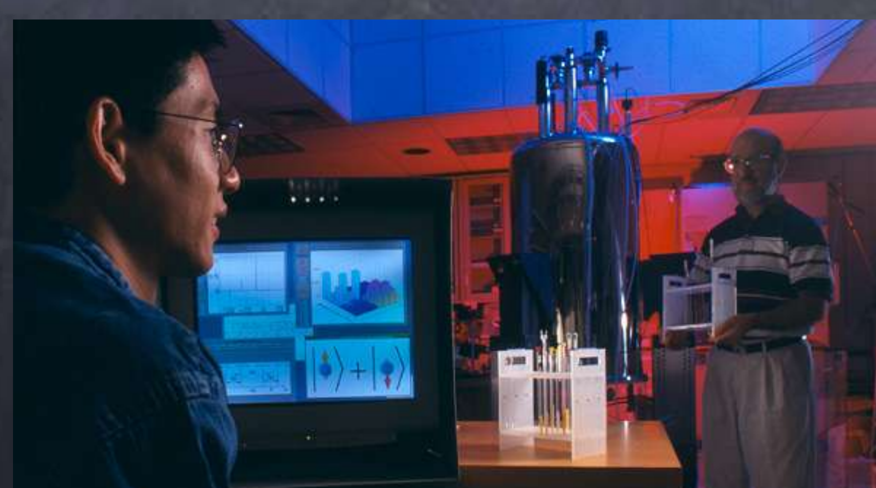
Needs of information systems ...



automatizations as a consequence of quantities of information,

future? How many information we will process?

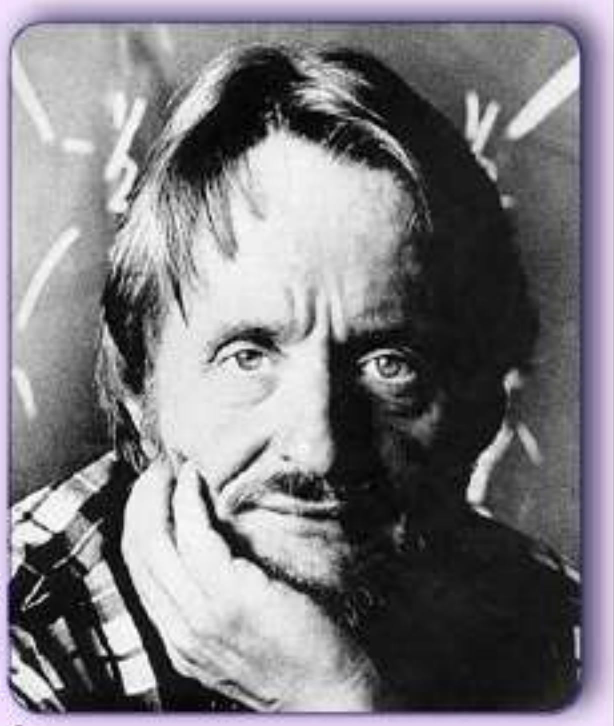




nature computing?

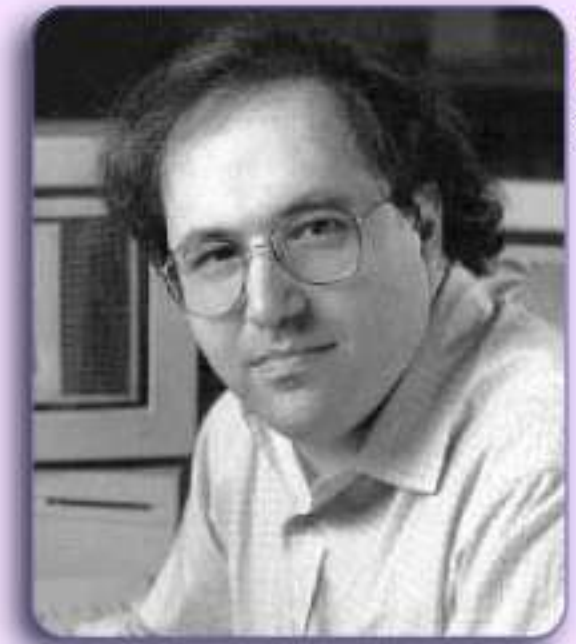
machines assembling machines?

most popular cellular automata stages

John von Neumann (December 28, 1903 - February 8, 1957)
Precursor of cellular automata, universal constructor, self-reproduction, universality



John Horton Conway (December 26, 1937 - ?)
The Game of Life, system of gliders, spatial universality



Stephen Wolfram (August 29, 1959 - ?)
One-dimensional CA, classes, complexity, languages

Matthew Cook (February 7, 1970 - ?)
Minimum universality in CA, Rule 110, cyclic tag systems

Cellular automata are discrete dynamical systems evolving on an infinite regular lattice.

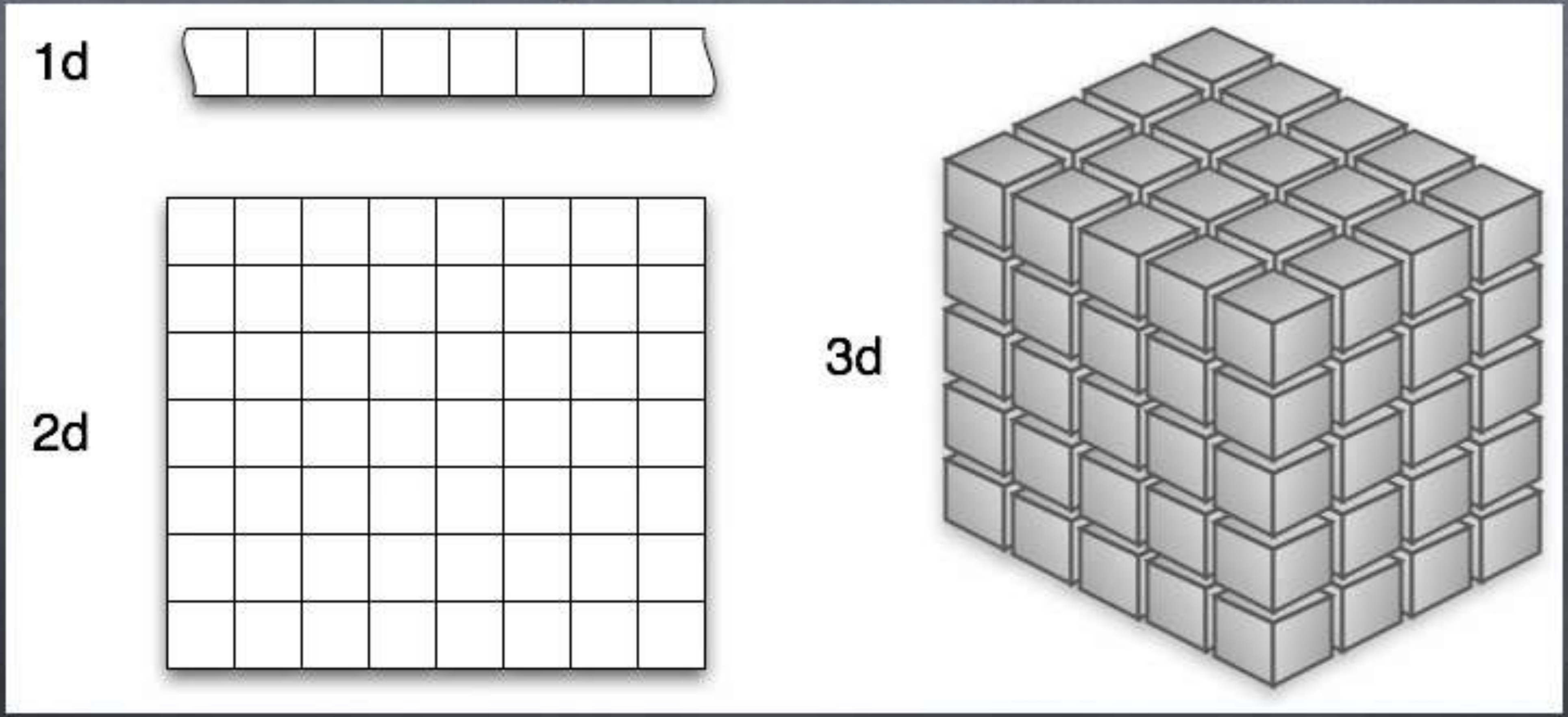
Definition.
A CA can be defined as a 4-tuple:

$$< \Sigma, u, \phi, c_0 >$$

evolving in d-dimensional lattice, where $d \in Z^+$. Such that:

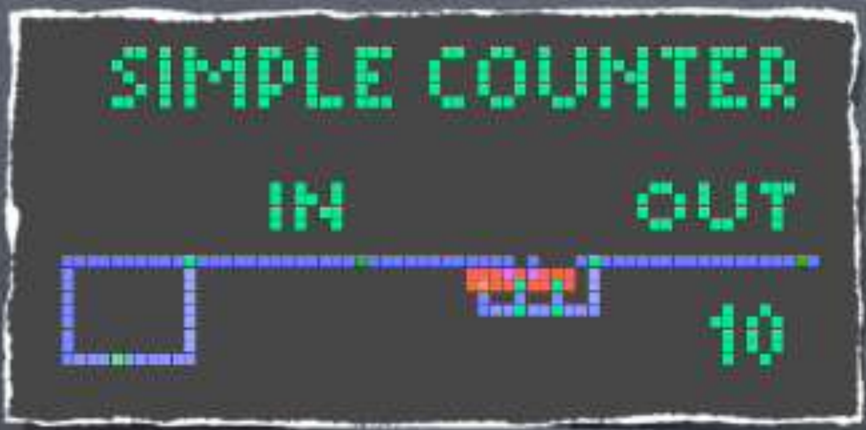
- Σ represents the finite alphabet
- u the local connection, where, $u = \{x_{0,1,\dots,n-1:d} \mid x \in \Sigma\}$
- ϕ the local function, such that, $\phi : \Sigma^u \to \Sigma$
- c_0 the initial condition, such that, $c_0 \in \Sigma^Z$

Also, the local function induces a global transition between configurations $\Sigma^Z \to \Sigma^Z$.

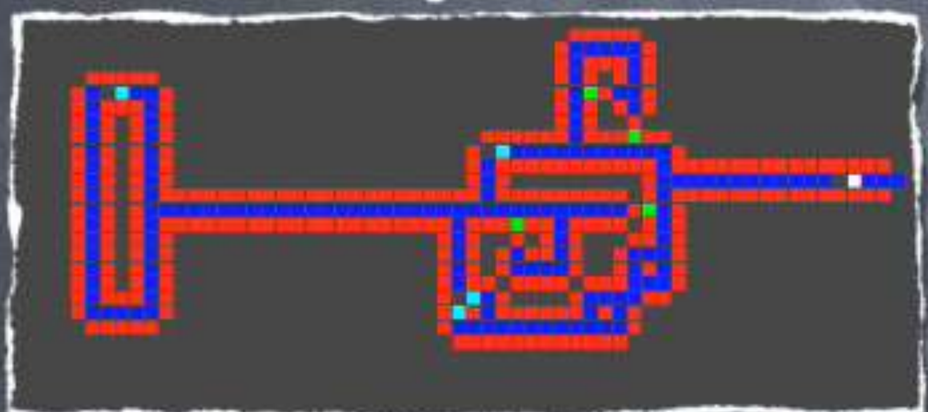


CA samples

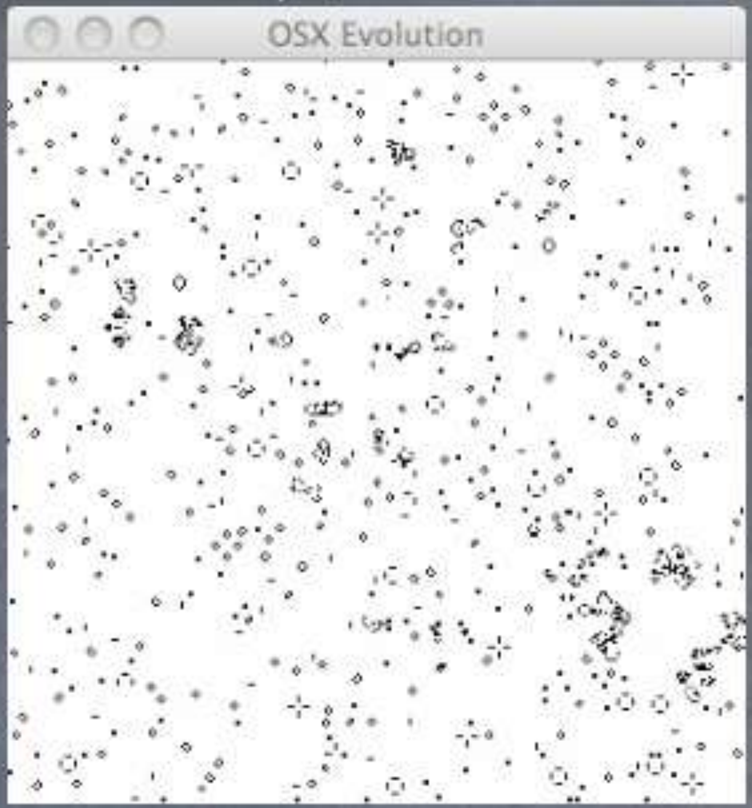
John von Neumann (signal transmission)



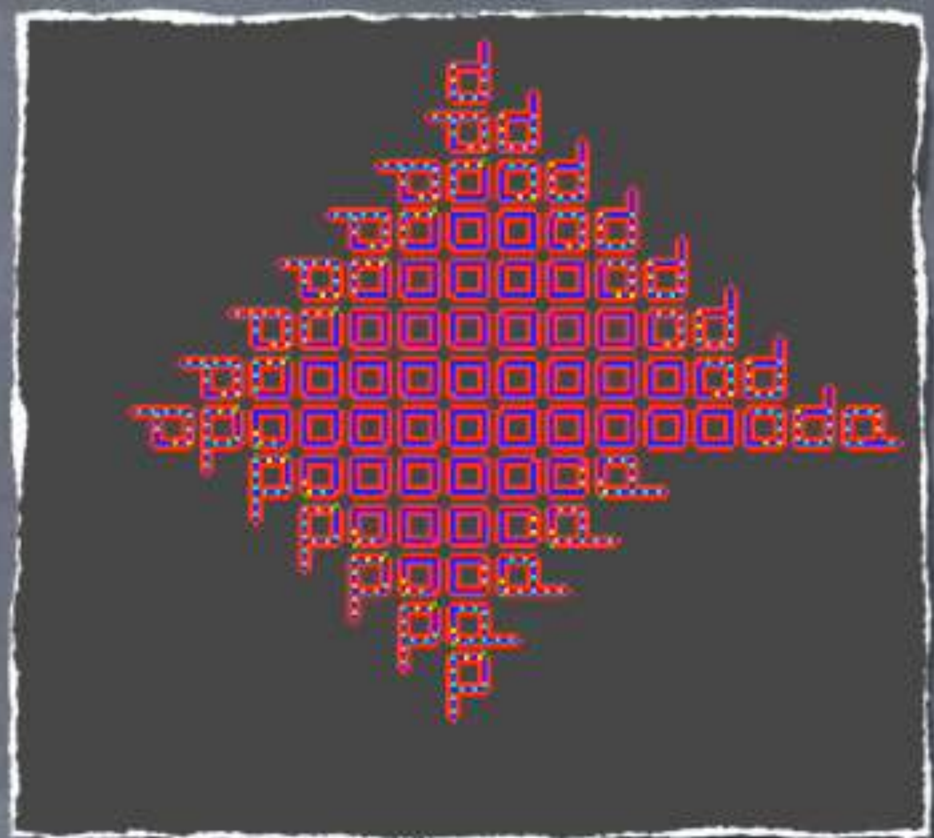
Edward Cood (signal transmission)



John Conway (the Game of Life)

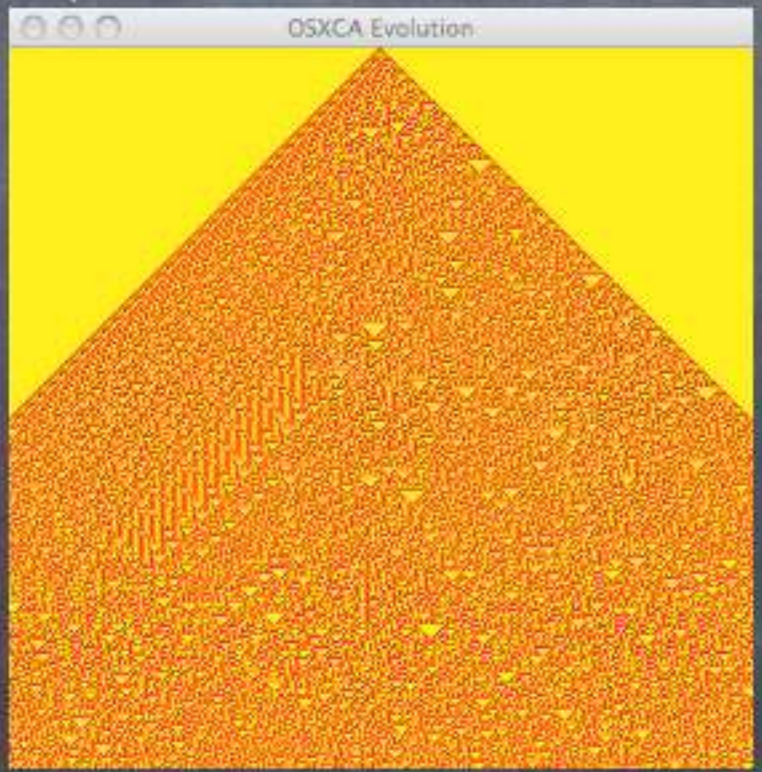


Chris Langton (self-reproduction)

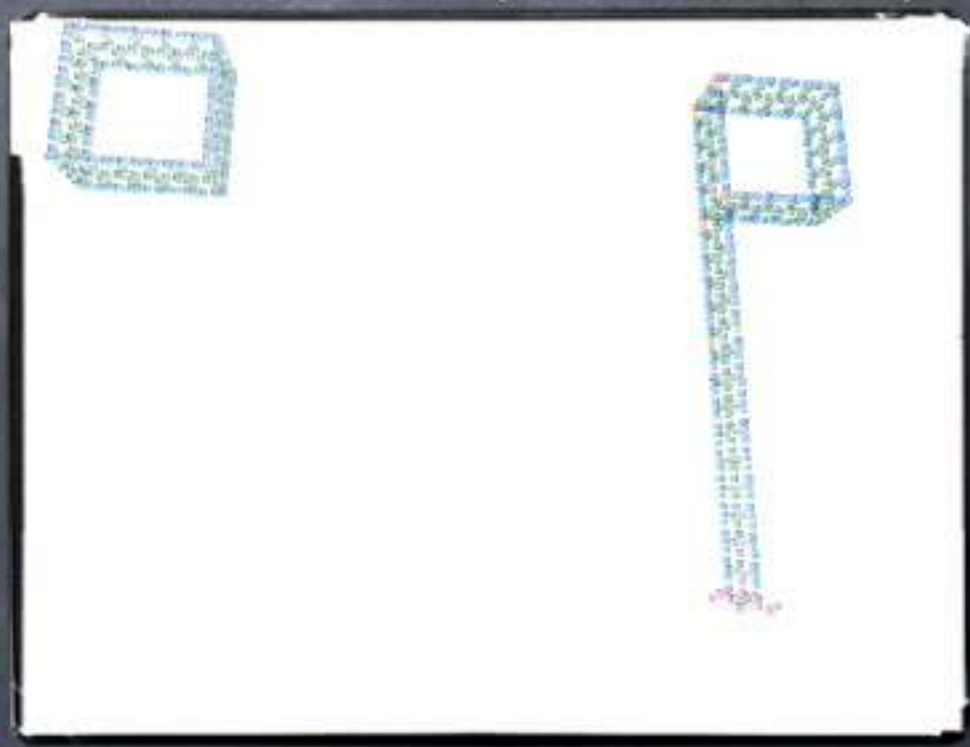


Norman Margolus (partitioned)

Stephen Wolfram (one dimension)



Katsunobu Imai (three dimensions)



Approaching Complex CA

- Mean field analysis (probabilistic polynomial) [Gutowitz]
- λ-calculus [Langton]
- de Bruijn diagrams (formal languages) [McIntosh]
- genetic algorithms [Mitchel]
- basins analysis (attractors) and Z -function [Wuensche]
- differential equations [Chua]

(1) Karel Culik II and Sheng Yu, "Undecidability of CA Classification Schemes," *Complex Systems* 2, 177-190, 1988.
(2) Chris G. Langton, "Computation at the edge of chaos: Phase transitions and emergent computation," *Physica D* 42, 12-37, 1990.
(3) Howard A. Gutowitz and Jonathan D. Victor, "Local structure theory in more that one dimension," *Complex Systems* 1, 57-68, 1987.
(4) Harold V. McIntosh, "Wolfram's Class IV and a Good Life," *Physica D* 45, 105-121, 1990.
(5) Melanie Mitchel *et al.*, "Evolving cellular automata to perform computations: mechanisms and impediments," *Physica D* 75, 361-391, 1994.
(6) Andrew Wuensche, "Classifying Cellular Automata Automatically," *Complexity* 4(3), 47-66, 1999.
(7) Leon O. Chua, *A Nonlinear Dynamics Perspective of Wolfram's New Kind of Science*, World Scientific Series on Nonlinear Science Series A, Vol. 57, 2007.

General interest

Our interest is to find and understand *particle dynamics* in abstract models as cellular automata, and project such results on real implementation as unconventional computing models.

What means universal computer? and, unconventional computing?

Let us now return to the analogy of the theoretical computing machines ... It can be shown that a single special machine of that type can be made to do the work of all. It could in fact be made as a model of any other machine. **The special machine may be called the universal machine.**

- Alan Turing, 1947

Today, a "computer", without further qualifications, denotes a rather well-specified kind of object; we'll consider a computer "non-conventional" if its physical substrate or its organization significantly depart from this de facto norm.

- Tommaso Toffoli, 1998

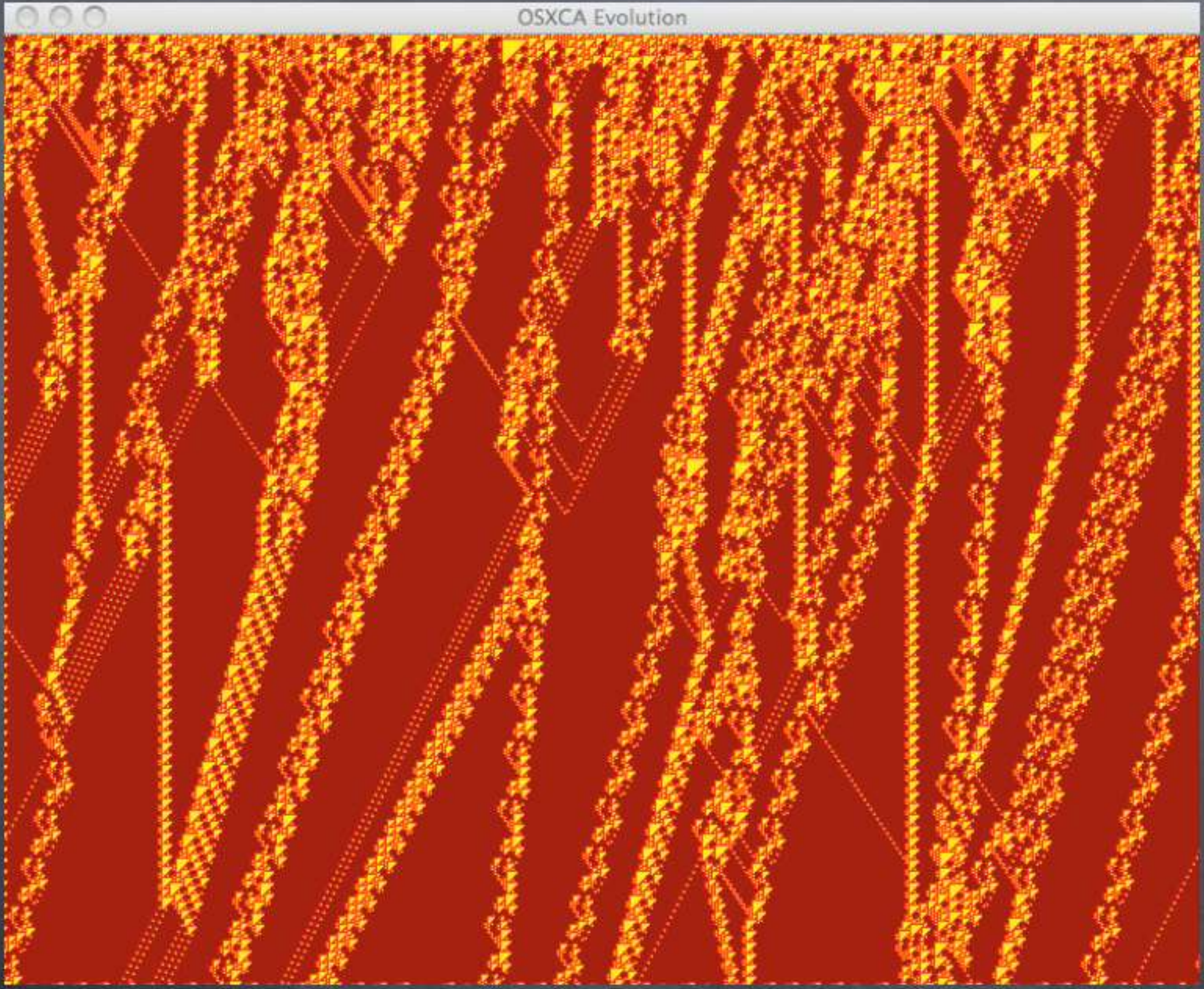
(1) Martin Davis, *The Universal Machine: the road from Leibniz to Turing*, W. W. Norton and Company, 2000.
(2) Tommaso Toffoli, "Non-Conventional Computers," *Encyclopedia of Electrical and Electronics Engineering* 14 (John Webster ed.), Wiley & Sons, 455-471, 1998.
(3) *International Journal of Unconventional Computing* http://www.oldcitypublishing.com/IJUC/IJUC.html

One-dimensional CA

A nice example was proofed by Matthew Cook showing that most simple *ECA Rule 110 can be universal*. Developing a novel *cyclic tag system* by glider collisions.

(1) Matthew Cook, "Universality in Elementary Cellular Automata," *Complex Systems* 15(1), 1-40, 2004.
(2) Stephen Wolfram, A New Kind of Science, Wolfram Media, Ins., Champaign, Illinois, 2002.
(3) Harold V. McIntosh, "Rule 110 Is Universal," http://delta.cs.cinvestav.mx/~mcintosh/oldweb/pautomata.html, June 30, 2002.
(4) Genaro J. Martínez, Harold V. McIntosh, Juan C. S. T. Mora and Sergio V. C. Vergara, "Reproducing the cyclic tag systems developed by Matthew Cook with Rule 110 using phases fi_1," *Journal of Cellular Automata*, in press, 2010.
(5) Rule 110 repository at: http://uncomp.uwe.ac.uk/genaro/Rule110.html

ECA Rule 110 complex dynamics (filtered also)



understanding particles in Rule 110 by de Bruijn diagrams

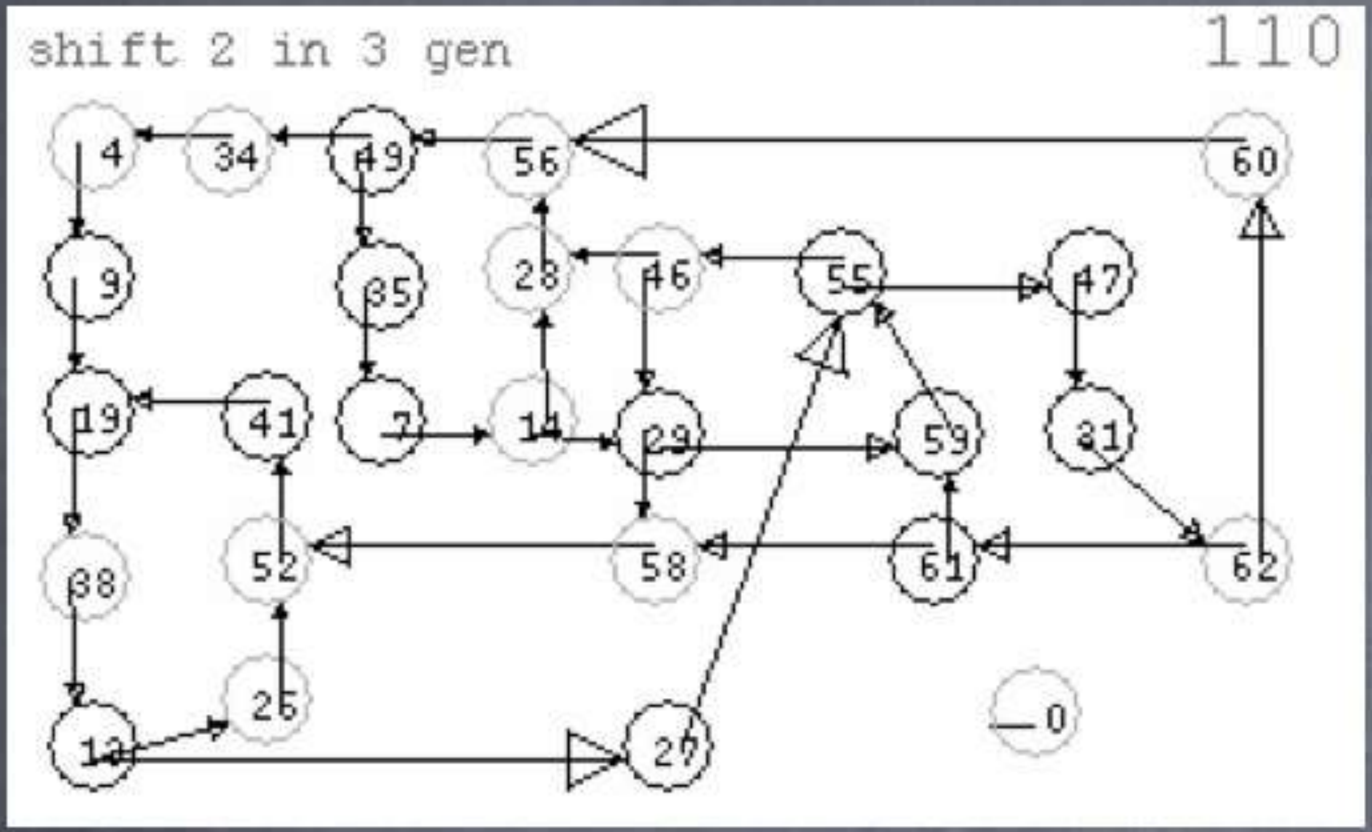
For an one-dimensional cellular automaton of order (k,r), the de Bruijn diagram is defined as a directed graph with k^{2r} vertices and k^{2r+1} edges. The vertices are labeled with the elements of the alphabet of length $2r$. An edge is directed from vertex i to vertex j, if and only if, the $2r-1$ final symbols of i are the same that the $2r-1$ initial ones in j forming a neighborhood of $2r+1$ states represented by $i{\cdot}j$. In this case, the edge connecting i to j is labeled with $\phi(i{\cdot}j)$.

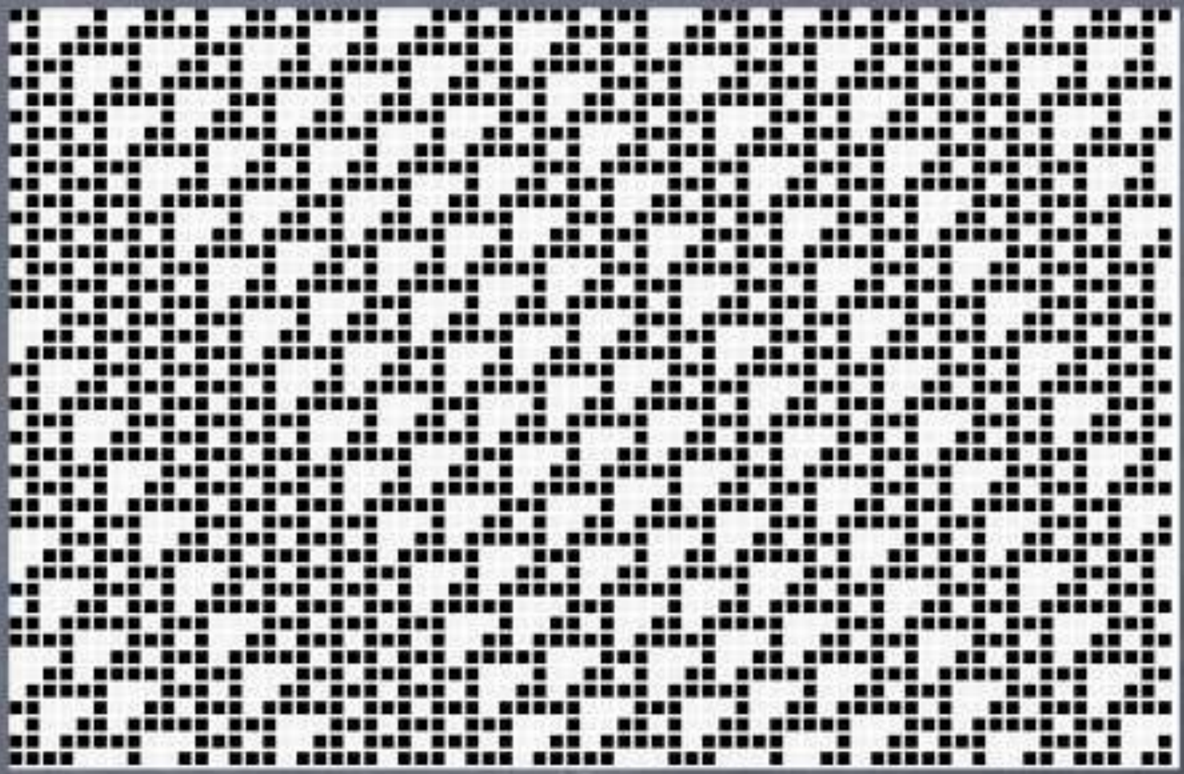
The connection matrix M corresponding with the de Bruijn diagram is as follows:

$$M_{i,j} = \begin{cases} 1 & \text{if } j = ki, ki+1, \ldots, ki+k-1 \pmod{k^{2r}} \\ 0 & \text{in other case} \end{cases}$$

(1) Harold V. McIntosh, *One-Dimensional Cellular Automata*, Luniver Press, 2009.
(2) Burton H. Voorhees, *Computational Analysis of One-Dimensional Cellular Automata*, World Scientific Series on Nonlinear Science, Series A, Vol. 15, 1996.

particles by regular expressions



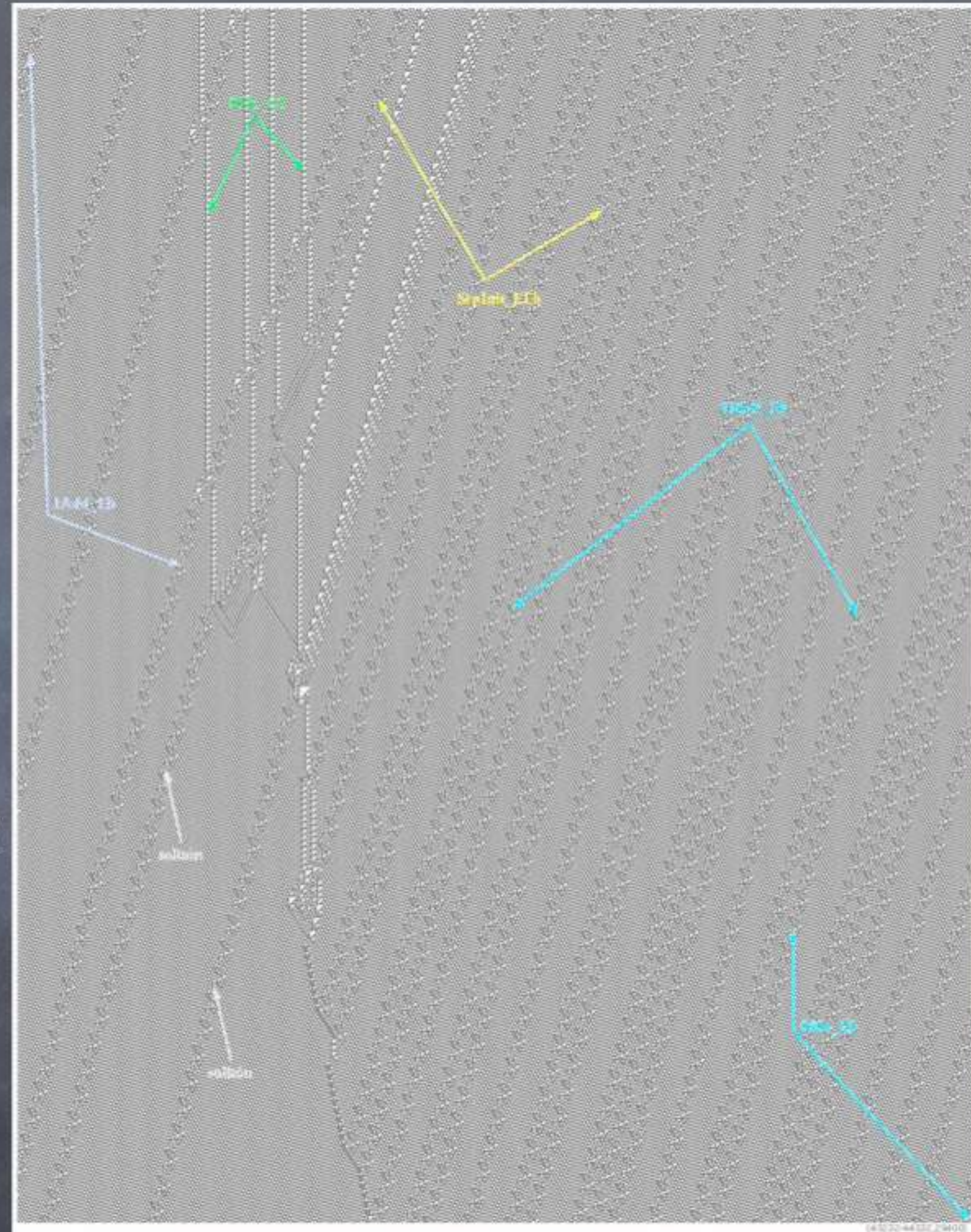


(1) Genaro J. Martínez, Harold V. McIntosh, Juan C. S. T. Mora and Sergio V. C. Vergara, Determining a regular language by glider-based structures called phases f_i_1 in Rule 110, *Journal of Cellular Automata* 3(3), 231-270, 2008.

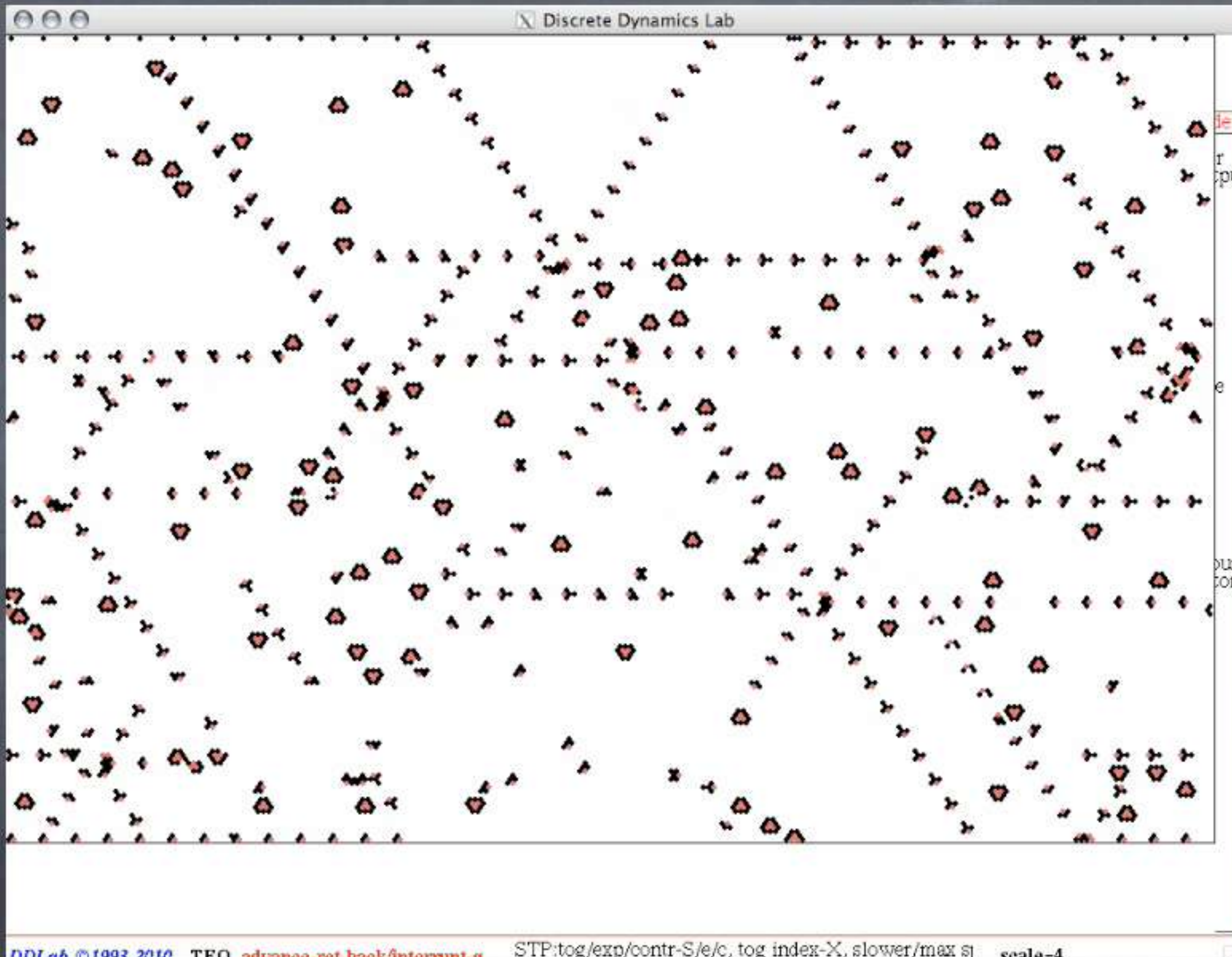
(2) Set of regular expressions based particles in Rule 110 available at: http://uncomp.uwe.ac.uk/genaro/rule110/listPhasesR110.txt

coding particles to get computations in Rule 110

Cyclic tag system working in rule 110 by particle collisions: **http://uncomp.uwe.ac.uk/genaro/rule110/ctsRule110.html**



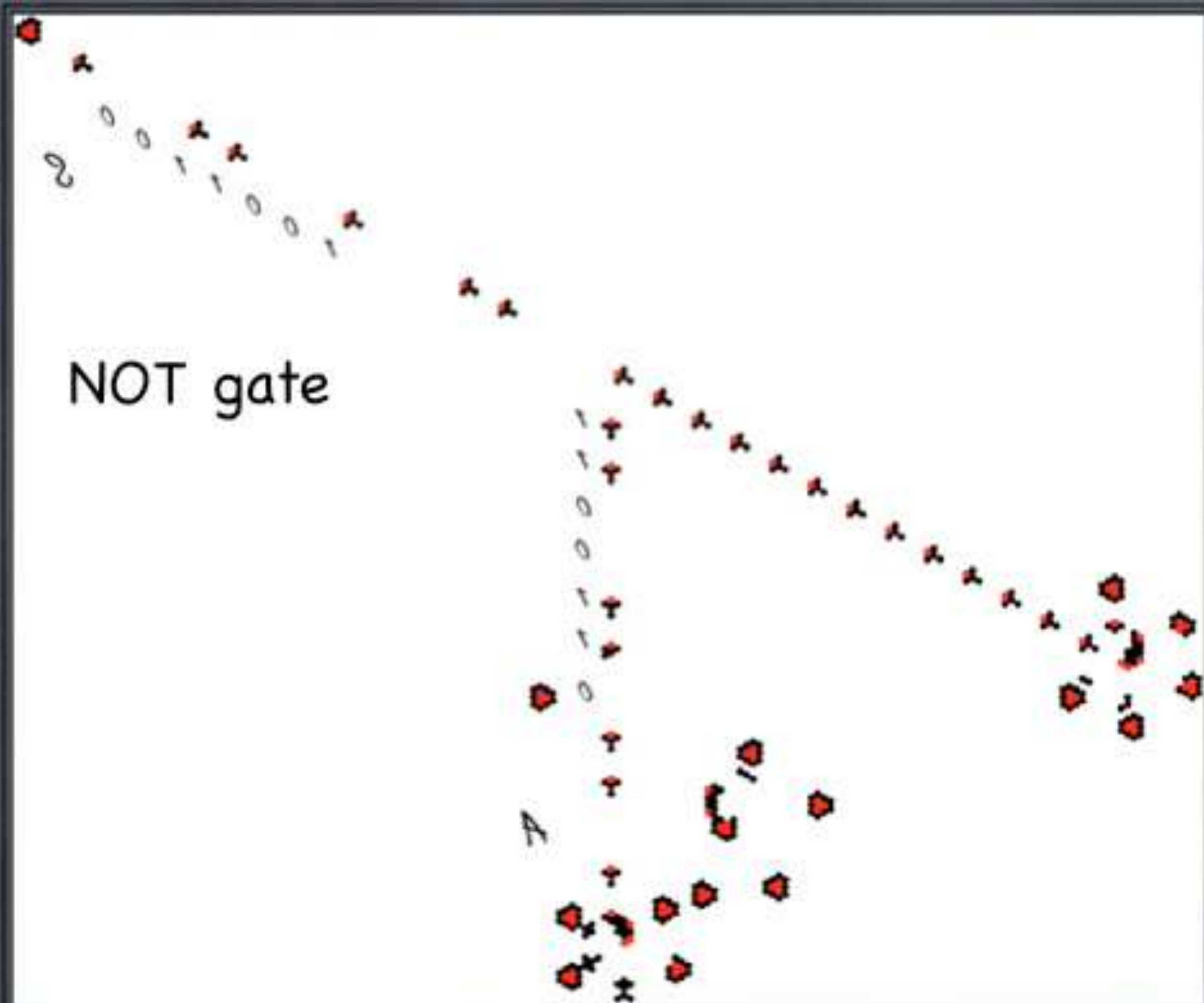
2D hexagonal CA case: the Spiral rule

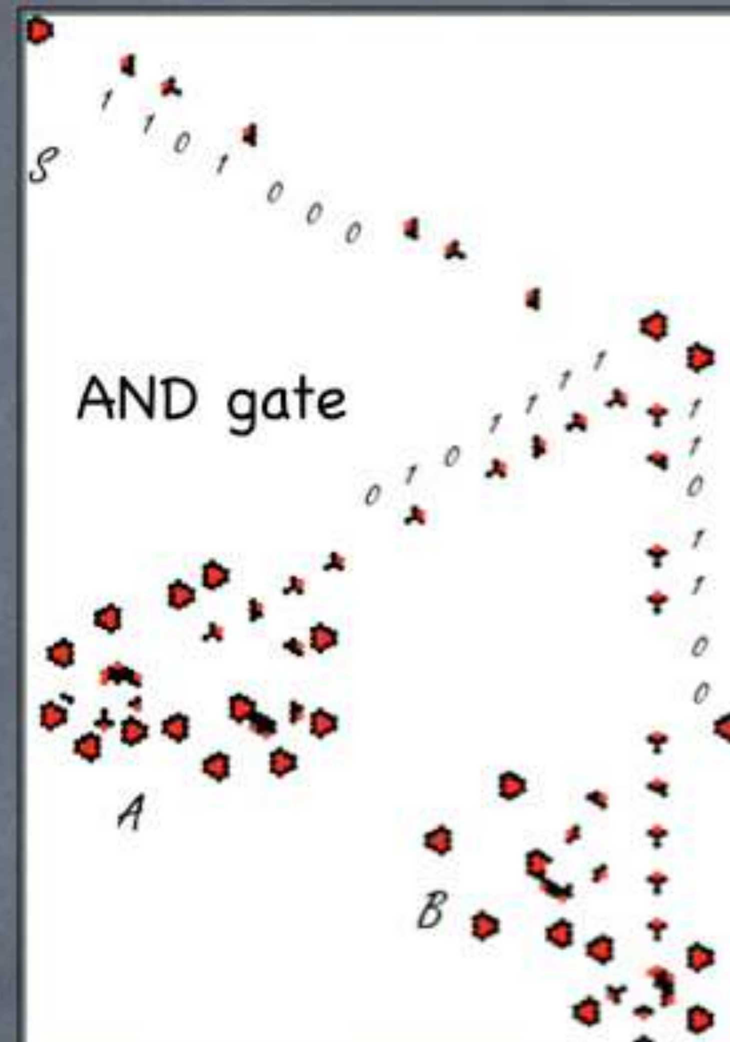


(1) Andrew Adamatzky and Andrew Wuensche, "Computing in spiral rule reaction-diffusion totalistic cellular automata," *Complex Systems* 16(4), 277-297, 2006.

(2) Andrew Wuensche and Andrew Adamatzky, "On spiral glider guns in hexagonal cellular automata: activator-inhibitor paradigm," *Rev. Modern Physics* 55, 601-644, 2006

selecting a kind of engineering hence we can develop logic gates also in CA





potential applications of such results are focused to reaction-diffusion computers

(1) Paulina A. L. Hernández, Rogelio B. Flores, Genaro J. Martínez and Juan C. S. T. Mora, "Logic gates and complex dynamics at the hexagonal cellular automaton: the Spiral rule," *by publish*.
(2) Andrew Adamatzky, Ben D. L. Costello and Tetsuya Asai, *Reaction-Diffusion Computers*, Elsevier, 2005.

The End

Thank you for the attention!

http://uncomp.uwe.ac.uk/genaro/