Introduction to Finite Element Simulation

Presented in the Embryo Physics Course http://www.embryophysics.org
September 22, 2010
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Introduction into FEM

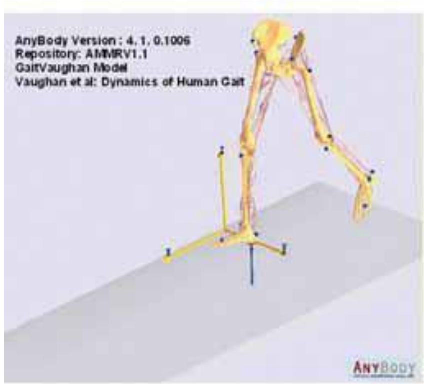
Computer Aided Engineering (CAE)

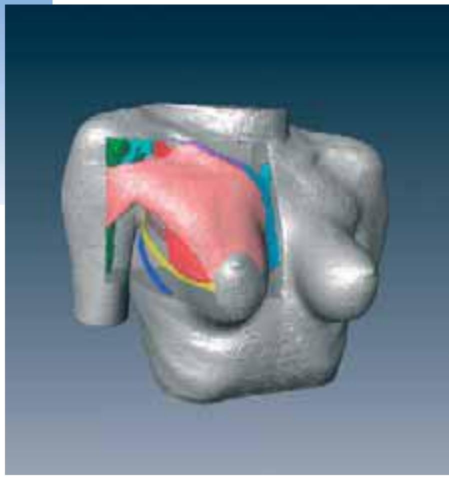
Outline

- PDE, numerical solution
- Discretization
- Meshing
- Numerical errors
- Typical simulation process
- System simulation

Infoplaner 2-2010

- http://www.cadfem.de/unternehmen/fem-magazin-infoplaner.html





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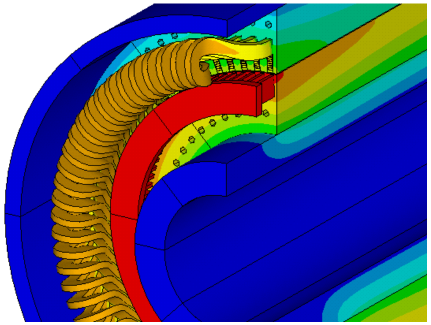
Partial Differential Equations

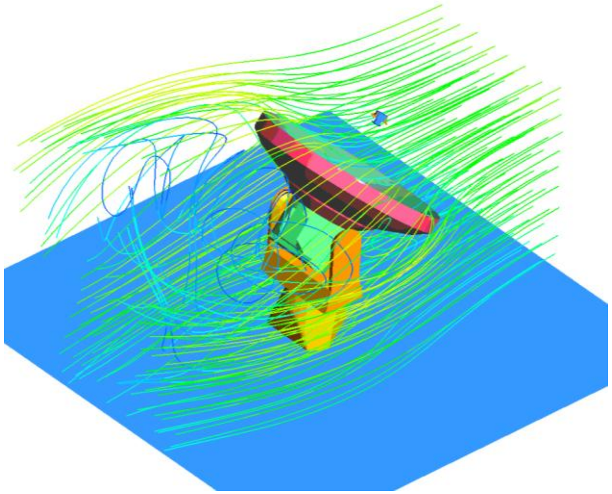
- Heat Transfer Equation

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{Q}$$

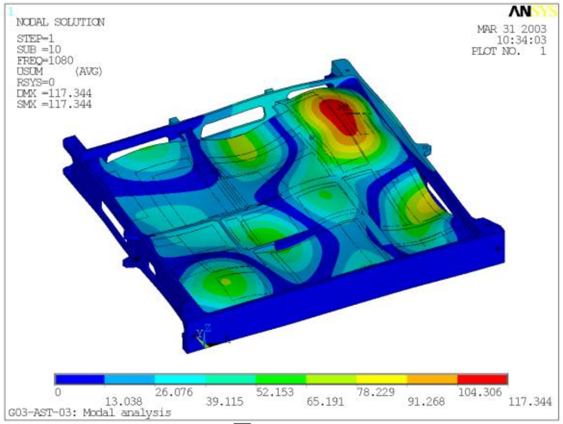
- Goal is to find the temperature field (scalar field)
  - Stationary *T(x, y, z)*
  - Transient *T(t, x, y, z)*



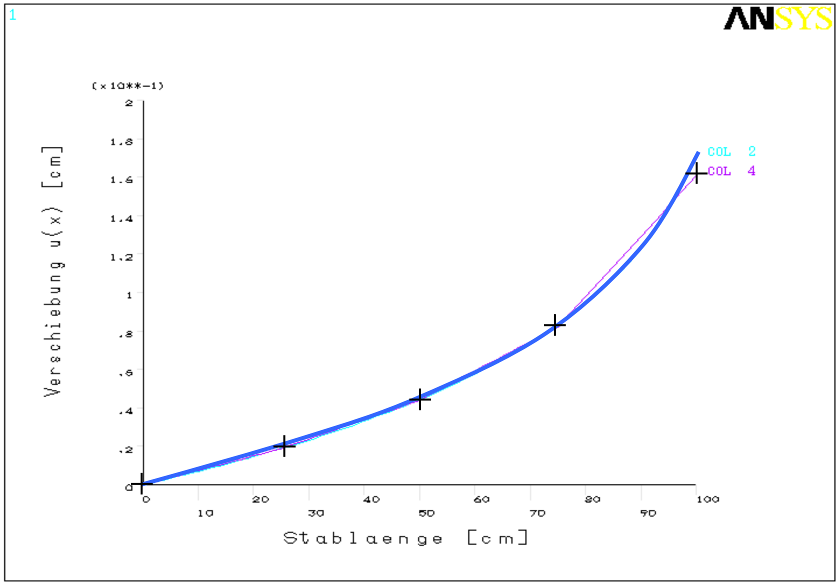
- Vector field

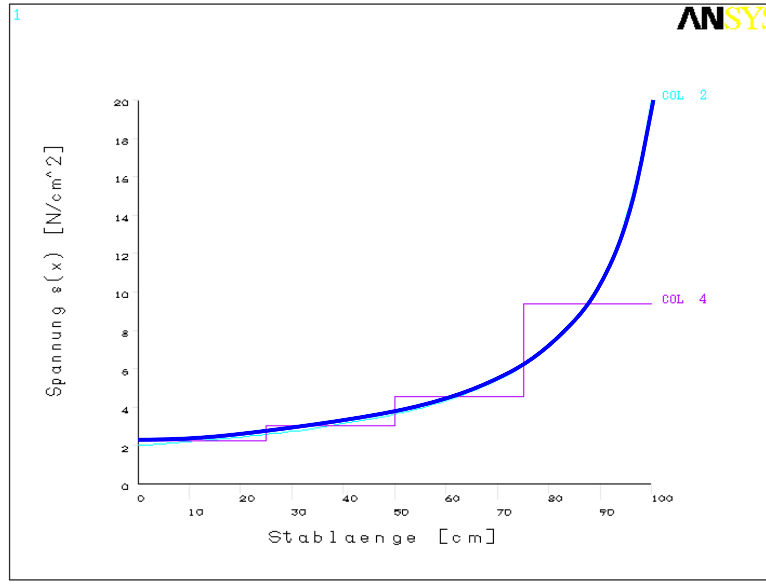


- Structural Mechanics
- Fluid Dynamics
- Electromagnetics



Numerical Solution – Discretization





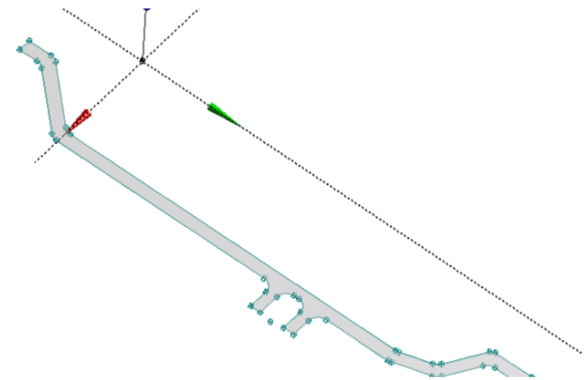
- From PDE to:
  - Stationary problem: a system of algebraic equations
  - Transient problem: ODEs

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{Q}$$

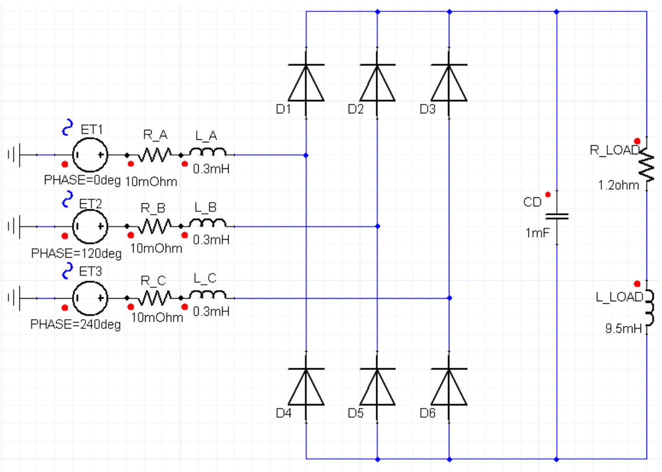
$$E \frac{\partial T}{\partial t} + KT = F$$

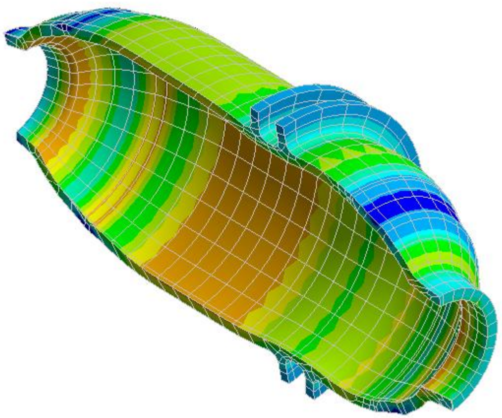
Space Simplifications

- 3D simulation: Mesh 10x10x10 = 1000
  - How to reduce complexity?
- 2D simulation *T(z) = const => T(x, y)*
- 2D-axisymmetrical simulation



- 1D simulation
- 0D: a lumped model





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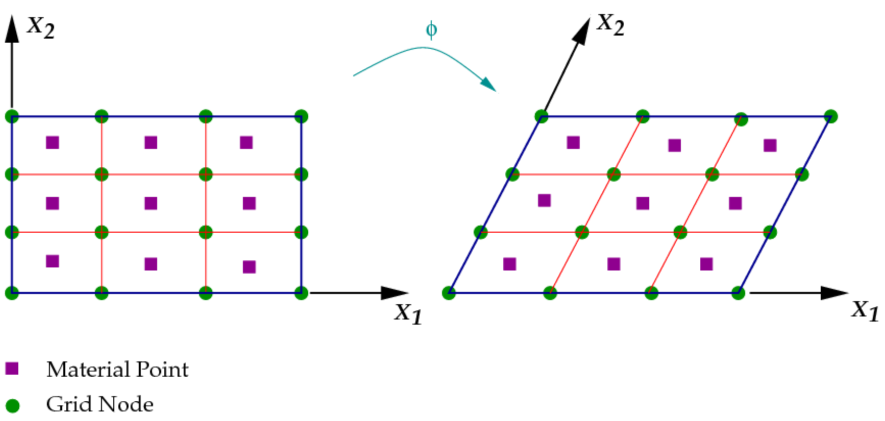
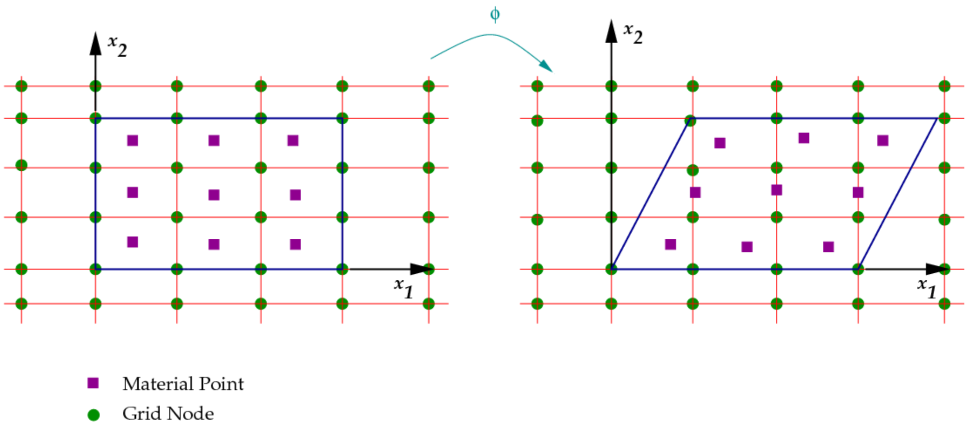
Lagrangian and Eulerian Formulations





- http://en.wikiversity.org/wiki/Nonlinear_finite_elements/Lagrangian_and_Eulerian_descriptions

Finite Differences

- Derivative is approximated through finite differences;
- Difficult to use on not Cartesian mesh;
- Difficult to apply boundary conditions.

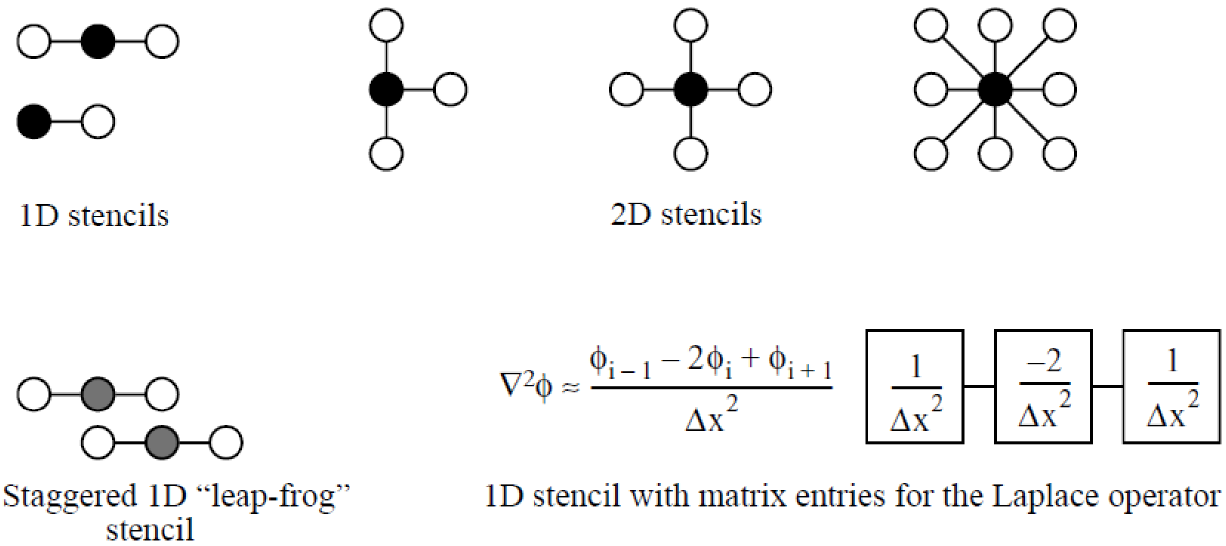


Figure 22. Finite difference nodal equation structure, showing typical stencil diagrams. Special stencils are required at the mesh boundaries, and multiple stencils are needed for some coupled PDEs (e.g. leap-frog schemes such as the 1D Yee stencil.) For a particular operator, a stencil may be drawn with its matrix entry values.

Figure from "MEMS and NEMS Simulation". In MEMS: A Practical Guide to Design, Analysis, and Applications, William Andrew Publishing, 2005.

Finite Volumes

- CFD world
- Finite volume schemes are conservative
- Needs dual meshes

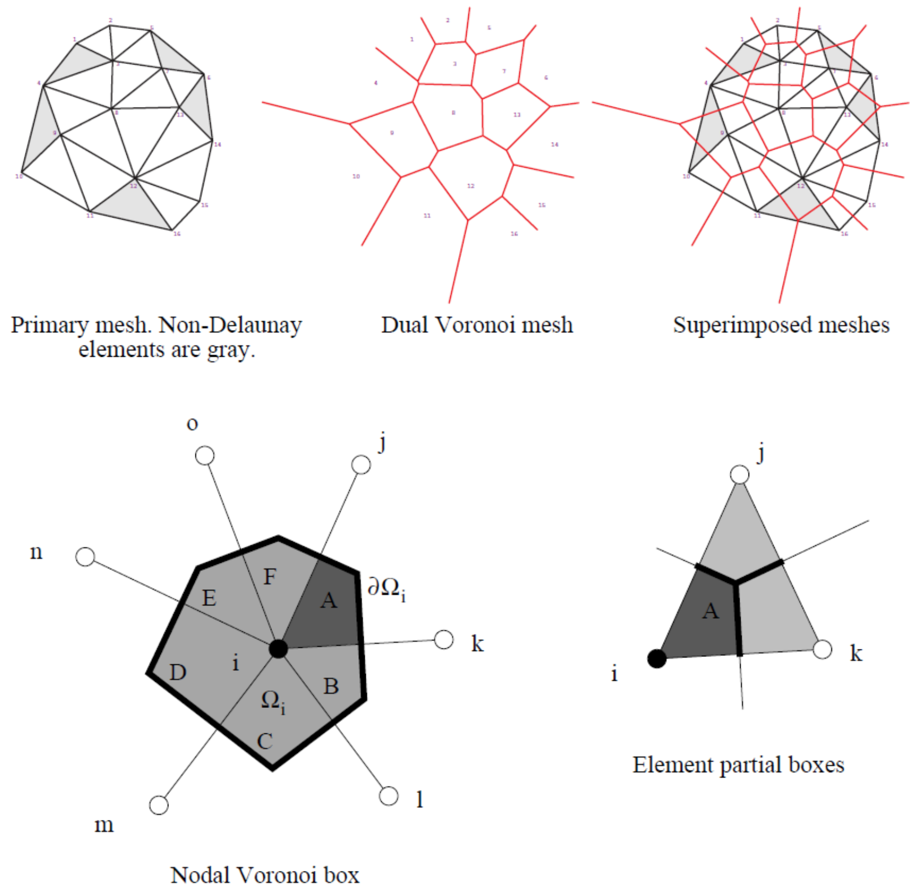


Figure 19. Finite volume mesh concepts. Above: general mesh constructions. Below: nodal and element view of the mesh. Adapted from [3].

Figure from "MEMS and NEMS Simulation". In MEMS: A Practical Guide to Design, Analysis, and Applications, William Andrew Publishing, 2005.

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Finite Elements

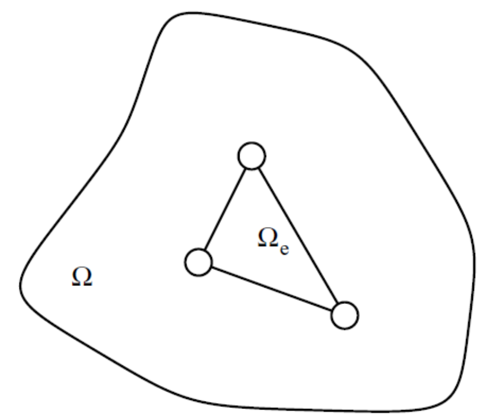
- Variational formulation
- Virtual work principle
- Very general

$$a(u, v) + b(v) = 0 \qquad a(u, v) \Rightarrow \int_{\Omega} g(u, v)\, d\Omega \Rightarrow [A]\{u\}$$

$$b(v) \Rightarrow \int_{\Omega} h(v)\, d\Omega \Rightarrow \{F\}$$

Typical form of the finite element equations

$$\int_{\Omega} \ldots d\Omega = \sum_{e=1}^{ne} \int_{\Omega_e} \ldots d\Omega_e$$



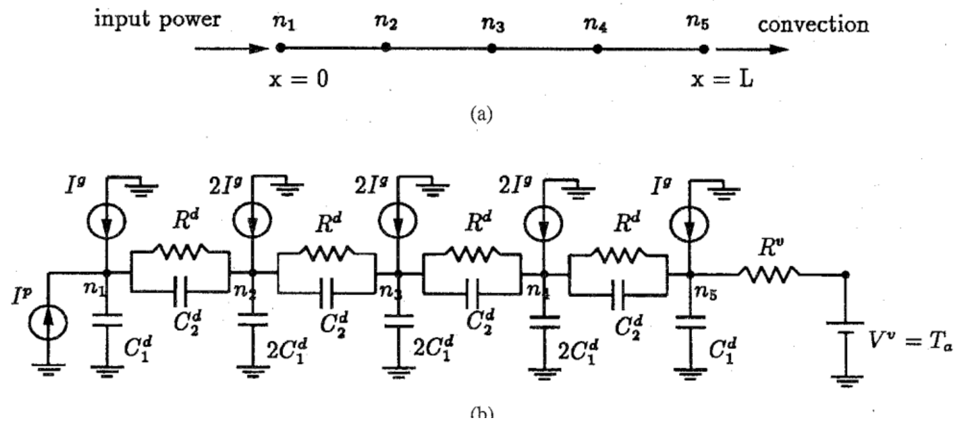
Decomposing domain integrals into a sum of element integrals

Figure 17. Finite element matrix equations typically arise from a bilinear and a linear form that are domain integrals (top). Here u is the unknown field (i.e., the temperature) and v is a known test function. Since the integrals are linear over their domain, we can perform all integrations element-by-element (bottom). Once the integrals are evaluated by the finite element software, we obtain a simultaneous equation system $[A]\{u\} = \{F\}$ that is solved for the nodal unknowns (the values of field u at the finite element nodes).

Thermal Networks in Finite Elements

- J. T. Hsu, L. Vu-Quoc, “A Rational Formualation of Thermal Circuit Models for Electrothermal Simulation - Part I: Finite element Method”, *IEEE Transactions on Circuits and Systems,* 43(9), pp. 721-732, (1996).

$$E\frac{\partial T}{\partial t} + KT = F$$



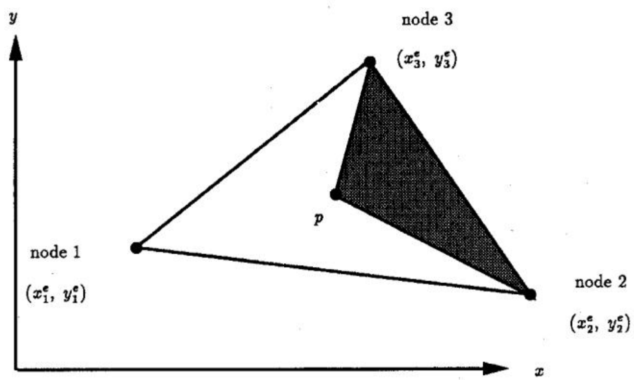
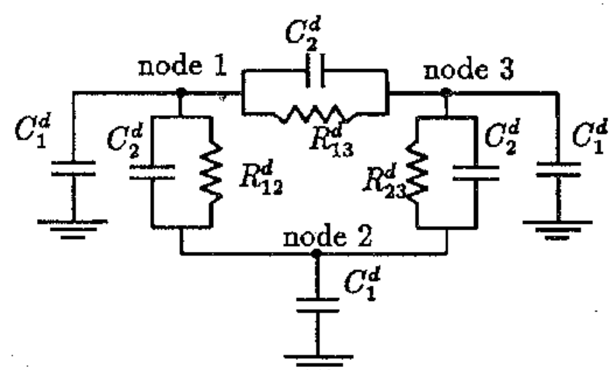
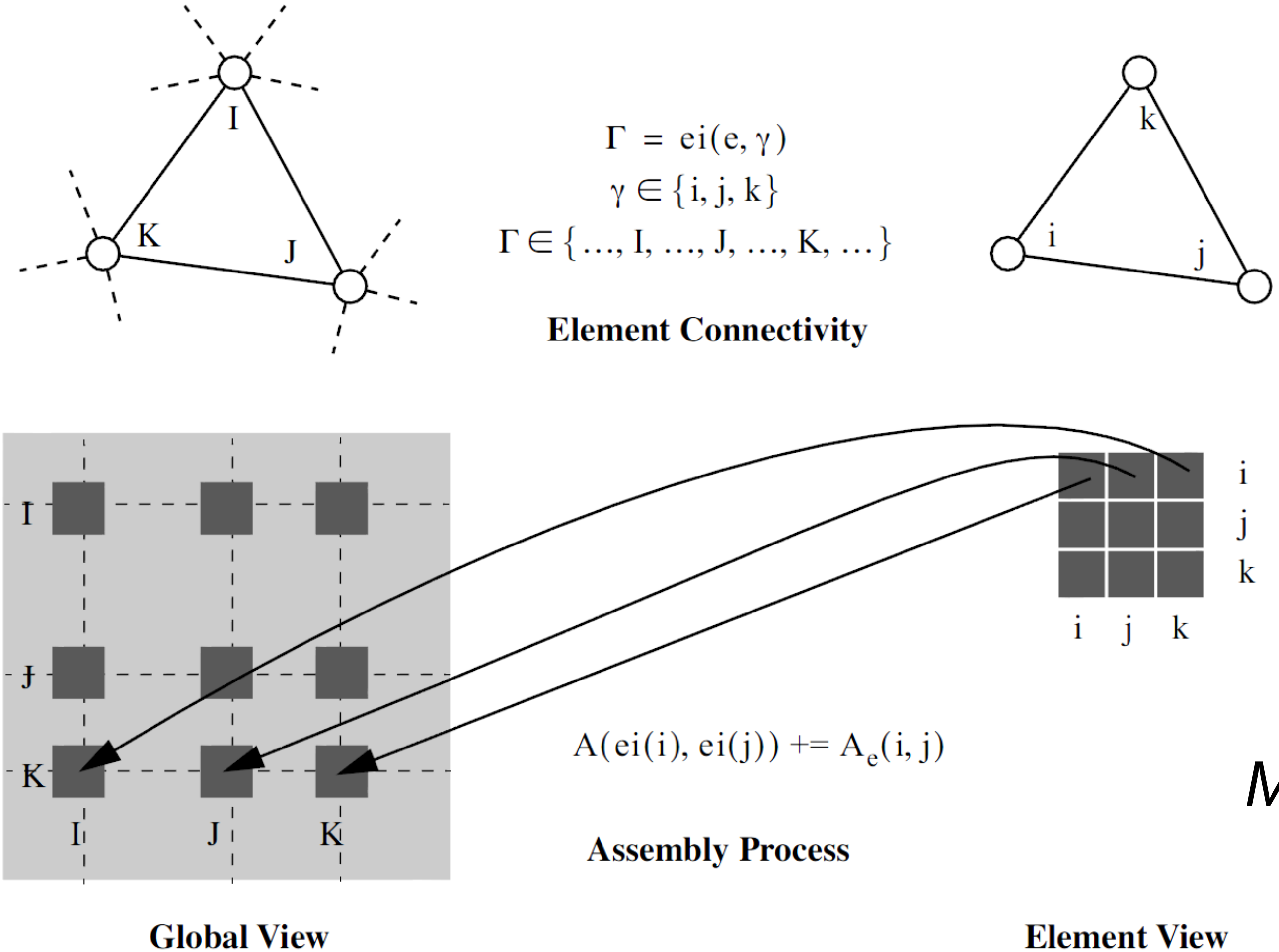


Fig. 13. Area coordinate.



Element and Global Matrices: Assembly

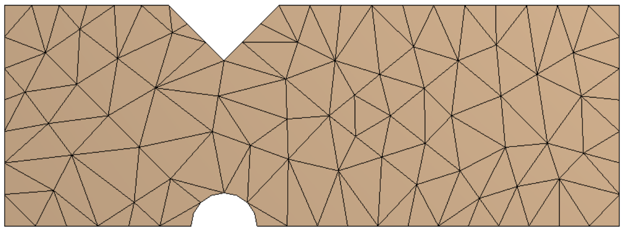


$$E\frac{dx}{dt} + Kx = f$$

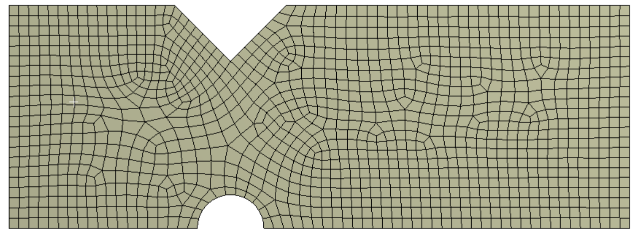
$$M\frac{d^2x}{dt^2} + E\frac{dx}{dt} + Kx = f$$

Figure from “MEMS and NEMS Simulation”. In MEMS: A Practical Guide to Design, Analysis, and Applications, William Andrew Publishing, 2005.

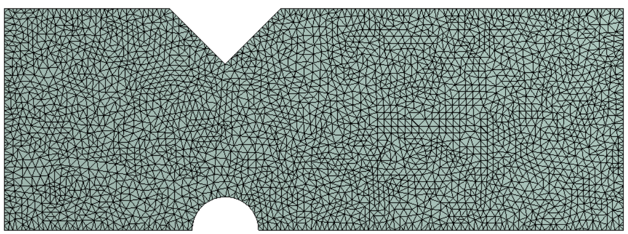
Meshing and Accuracy – Structural Mechanics



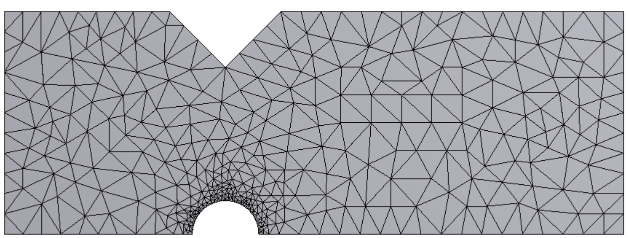
Coarse Tet Mesh



Moderately Fine Hex Mesh



Fine Tet Mesh



Locally Refined Tet Mesh

	Max Deformation	**Max Principle Stress**	**No of Nodes**
Coarse Tet	0,192	95,4	1086
Fine Tet	0,2	125	38950
Locally Refined Tet	0,198	126	7702
Hex	0,199	125	14511

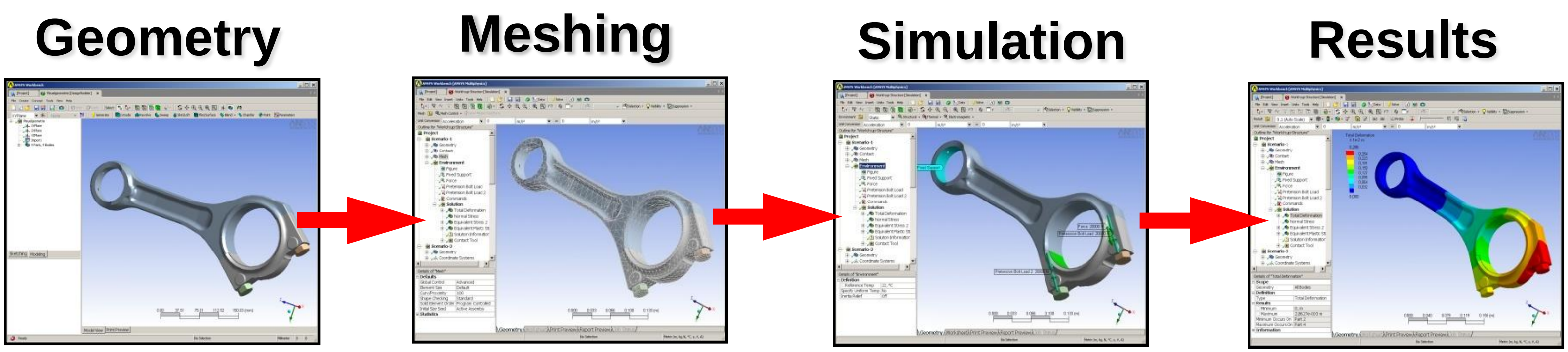
Workflow

- Preprocessing
  - Import geometry and simplify if possible
  - Material properties
  - Boundary conditions
  - Loads
  - Meshing

- Solution

- Postprocessing
  - Visualization
  - Reports

The Simulation Workflow



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What is the Analysis Goal

- Structural deformation
- Structural stresses
- Temperatures
- Fluid Velocities
- Fluid Pressures
- Electric Current
- Electric Voltage
- Magnetic field
- …..

Geometry imported from
CAD-system

➡ The simulation is done with a mathematical model derived from the CAD-geometry

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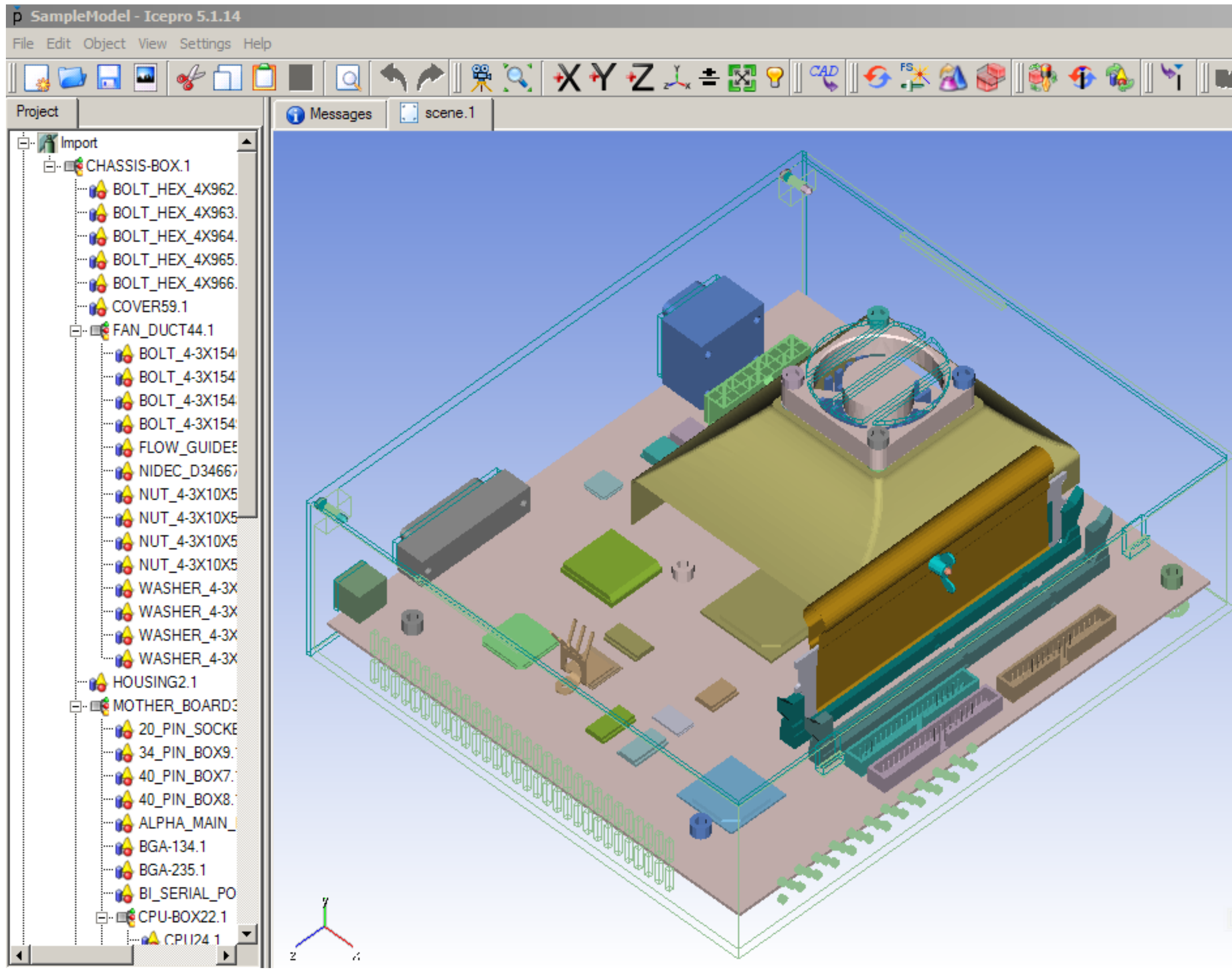
Geometry Simplification

- There are tools to simplify the model.

- Strategy:
  - Divide the model to functional components;
  - Simplify them and save into the Icepak library;
  - Load necessary components to the Icepak project from the library.



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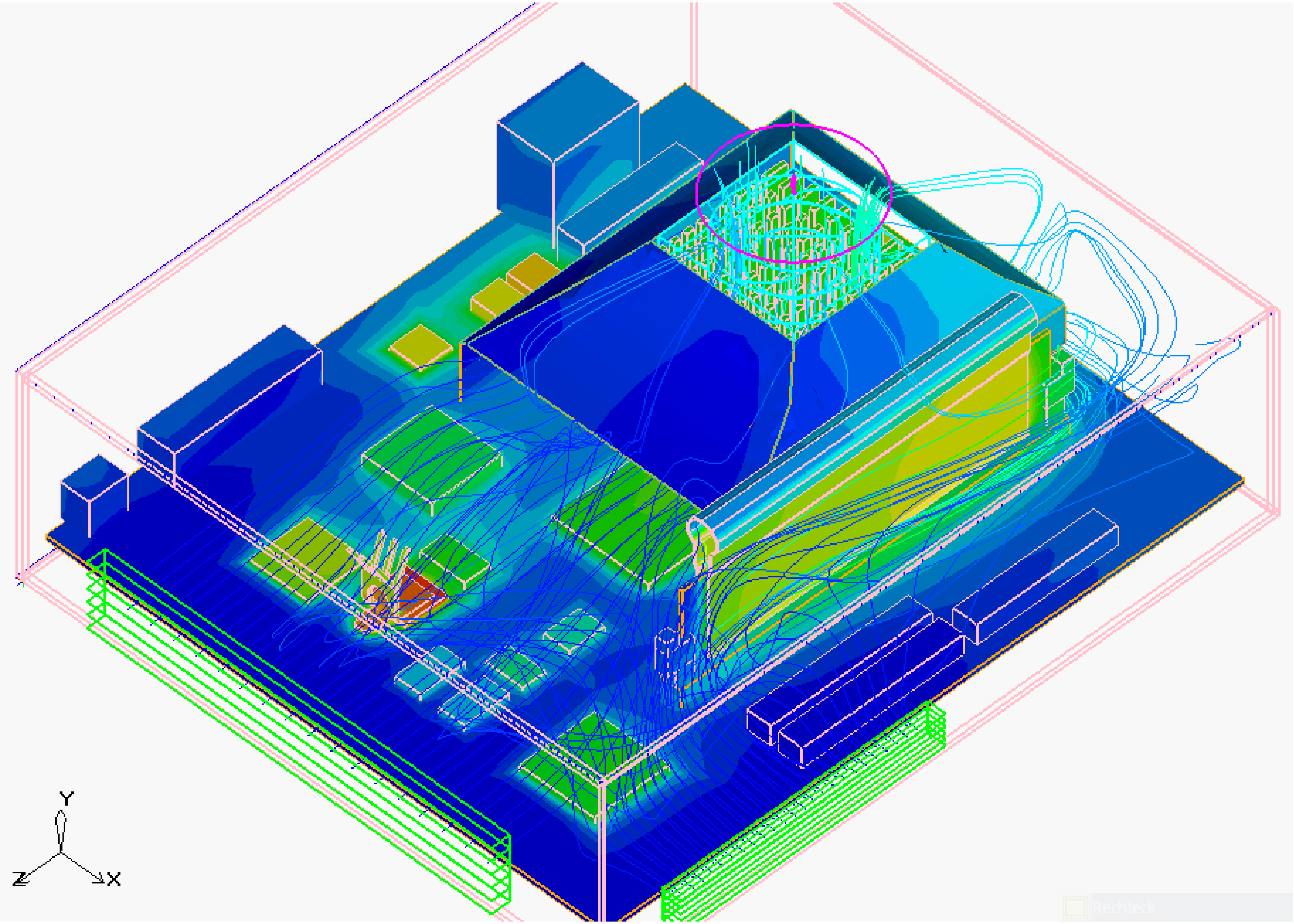
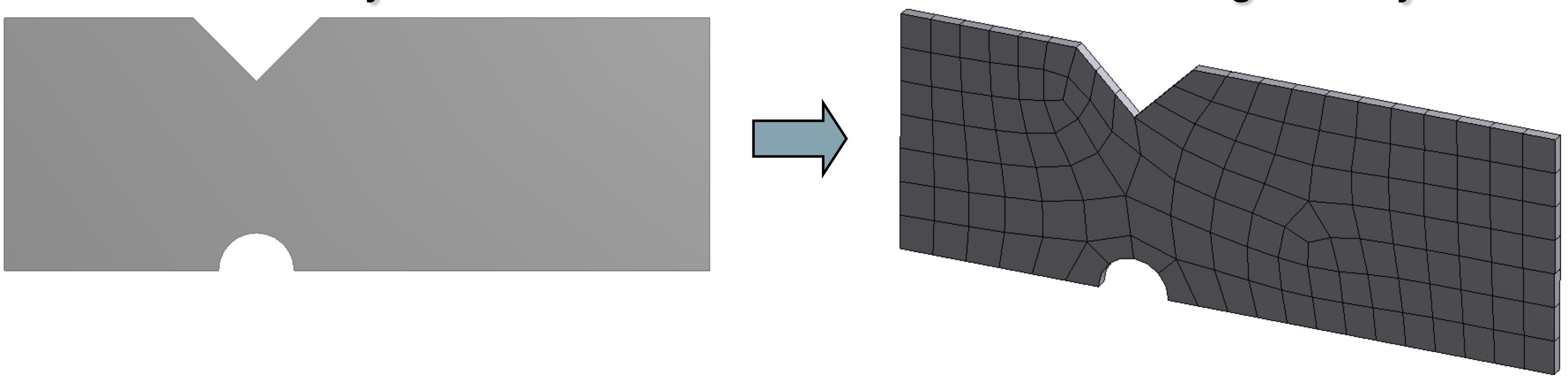
Simulation with simplified model



The Meshing Process

Geometry imported from CAD-System

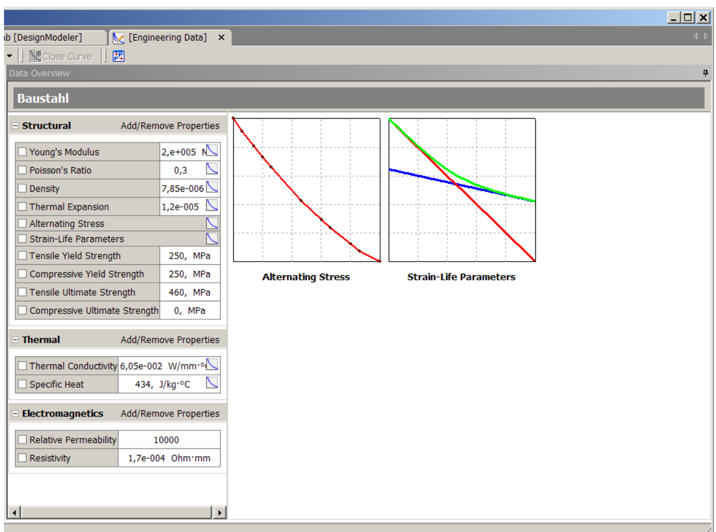
Mesh created on CAD-geometry



- The geometry is subdivided into and thus represented by elements and nodes
- The required mesh density is strongly governed by the
  - The size of geometric details
  - The desired accuracy
  - The physics to be solved

Simulation – Definition of Material Properties

For all elements physical properties must be defined.



Material Database

- **Structural Mechanics**
  - Elasticity Module, Poisson Number
  - Density, Material Damping
- **Heat Transfer**
  - Conductivity
  - Density, Heat Capacity
- **Fluid Dynamics**
  - Density, Viscosity
  - Conductivity, Heat Capacity
- **Electro-Magnetics**
  - Electric Conductivity
  - Magnetic Permeability

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Simulation – Definition of Boundary Conditions/Loads

The interaction with adjacent parts or the environment is modeled by boundary conditions or loads.

- **Structural Mechanics**
  - Supports, displacements
  - Forces, Pressures, Moments
- **Thermal Analysis**
  - Temperatures
  - Heat fluxes, convection, radiation
- **Fluid Analysis**
  - Mass flow, velocities
  - Pressures
- **Electro-Magnetics**
  - Electric Current
  - Electric Potential
  - Magnetic Flux Parallel

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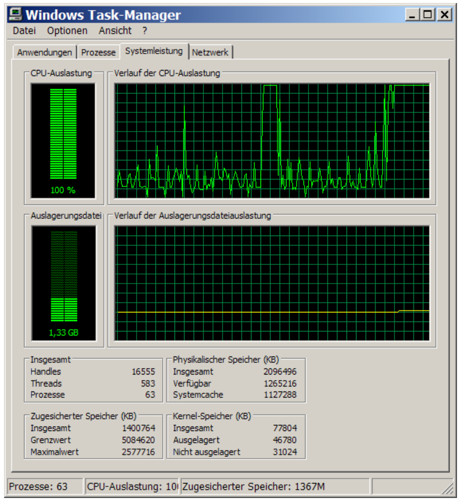
Simulation – Solving the Problem

The number of nodes defines the size of the algebraic system of equations to be solved numerically

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{12} & a_{22} & a_{23} & \cdots \\ \vdots & \vdots & a_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \end{bmatrix}$$

The numerical effort to solve this system of equations is limited by

- Amount of computer storage
- Acceptable CPU time



In practice one must often find a compromise between accuracy and numerical effort

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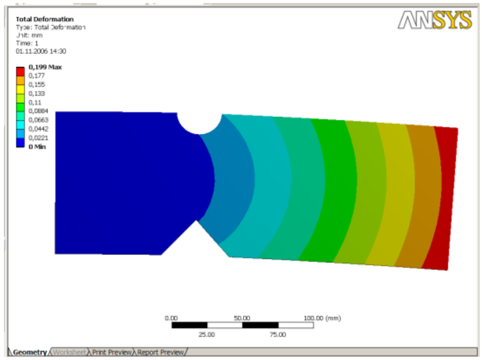
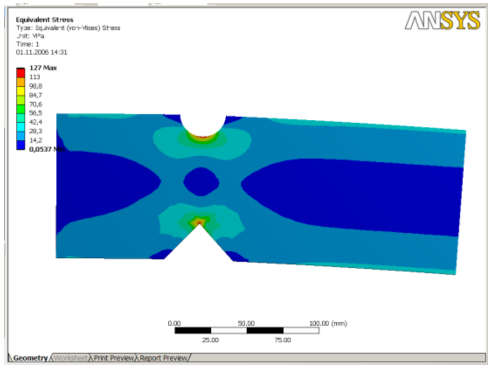
Post Processing – Structural Mechanics

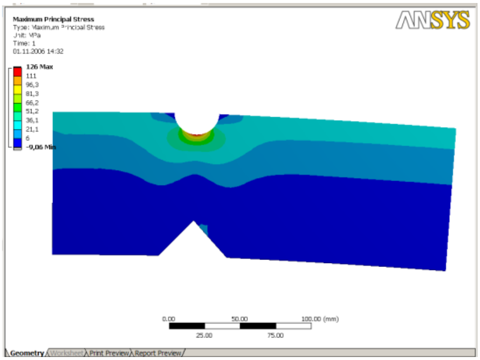
- Deformation – primary result
  - Vector sum
  - Directional Components
- Stresses – derived results
  - Equivalent stresses
  - Principle stresses
  - Stress components
  - Safety factors
  - ..



Total Displacement



Van Mises Equivalent Stress



Maximum Principle Stress

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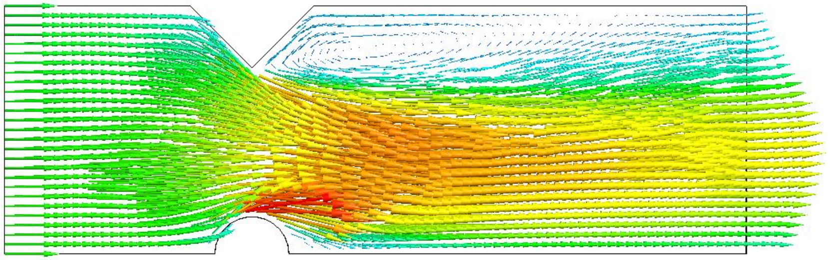
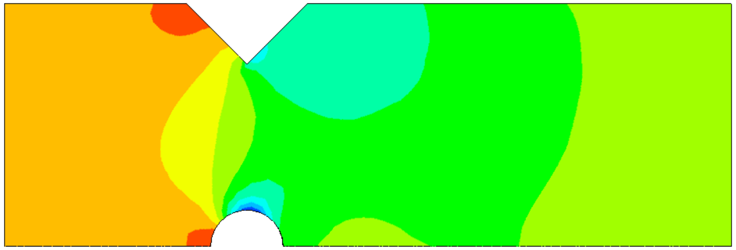
Post Processing – Fluid Mechanics

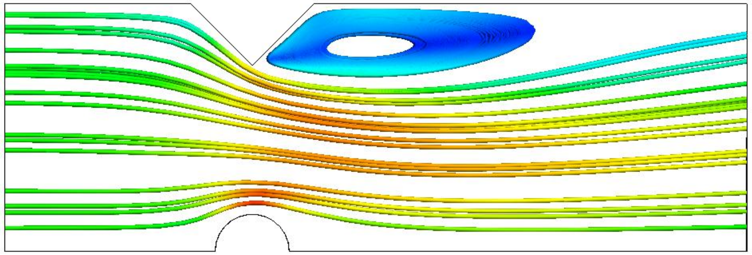
- Velocities
  - Vector Sum, Massflow
  - Directional Components
- Pressure
  - Absolute Pressure
  - Total Pressure
- Temperatures
  - Static and total temperature
  - Heat Film Coefficients
- Flow Forces
  - Pressure Forces
  - Friction Forces



Velocity Vectors



Pressure Contours



Streamlines

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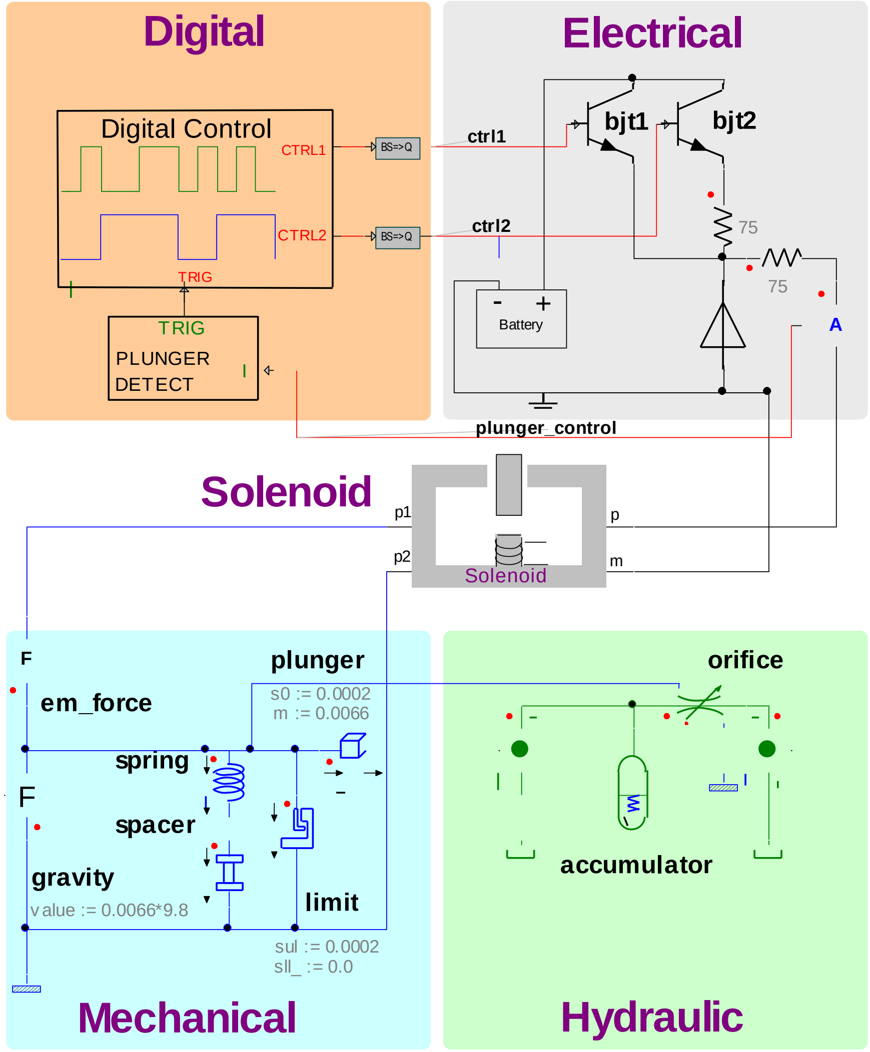
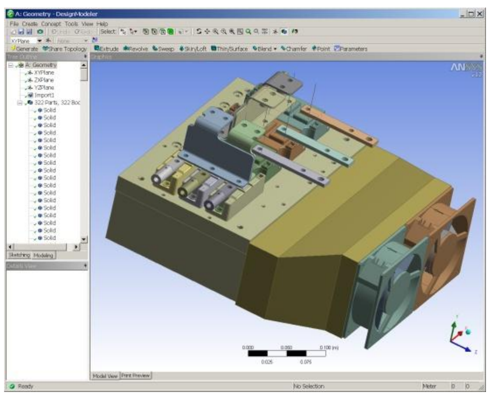
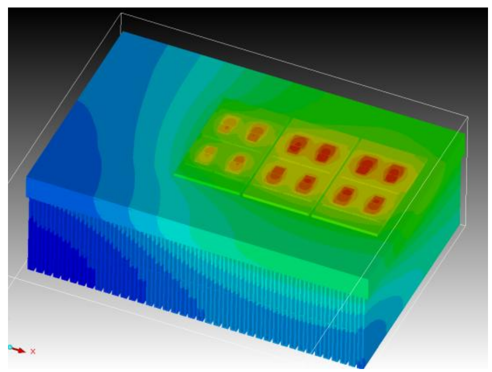
Example of system level simulation

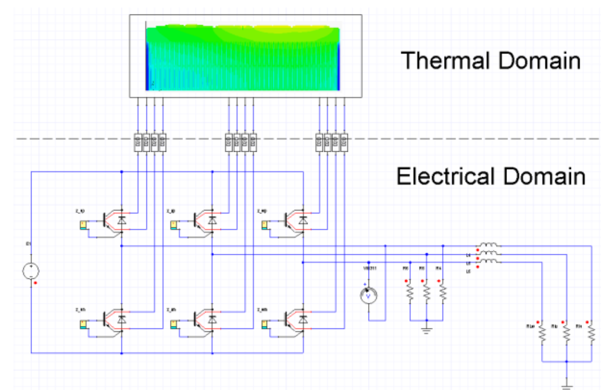


From Finite Elements to System Simulation

Physics & Geometry → **FEM** → System of n ODEs → **MOR** → Reduced System of $r << n$ ODEs







- http://ModelReduction.com

Summary

- It is important to understand, that the accuracy of a simulation strongly depends on
  - Correct boundary conditions and loads
  - Correct material properties
  - Sufficient mesh resolution

- All this is the ENGINEERS responsibility

However:

- The **Simulation Environment** provides all the tools that are needed for an efficient and accurate simulation