

## MAE 8 - Winter 2015

### Homework 5

**Instructions:** Follow the homework solution template. Put all answers in a MATLAB script named **hw5.m**. For this homework, you will need to submit multiple files. Create a zip archive named **hw5.zip**. The zip archive should include the following files: **hw5.m**, and **testpi.m**. Submit **hw5.zip** through TED before 9 PM on 02/12/2015. Use double precision unless otherwise stated.

**Problem 1:** Perform the following exercises using function  $f(x)$ :

$$f(x) = \cos(x) \exp \left[ - \left( \frac{x}{15} \right)^2 \right].$$

(a,b) Compute  $f(x)$  for  $x = [-20:0.1:20]$ . Put  $x$  into **p1a** and  $f(x)$  into **p1b**.

(c,d,e) How many local maxima does  $f(x)$  have (excluding the end values of  $f$ )? Put the answer in **p1c**. Find the  $x$  values and  $y$  values of the maxima and put the answers in **p1d** and **p1e**, respectively. List the maxima in the order of increasing  $x$ .

(f,g,h) How many local minima does  $f(x)$  have (excluding the end values of  $f$ )? Put the answer in **p1f**. Find the  $x$  values and  $y$  values of the minima and put the answers in **p1g** and **p1h**, respectively. List the minima in the order of increasing  $x$ .

(i,j,k) How many times does  $f(x)$  cross zero value? Put the answer in **p1i**. Find the  $x$  values and  $y$  values of  $f(x)$  right before  $f(x)$  crosses zero value and put the answers in **p1j** and **p1k**, respectively. List the zero crossing in the order of increasing  $x$ .

(l) Make figure 1 to include a solid line for  $f(x)$  and different symbols for the maxima, the minima and the zero crossing. Label the axes and give title and legend. Set **p1l='See figure 1'**.

**Problem 2:** A ball is released from a 10 m high roof and bounces three quarters as high on each successive bounce.

(a,b) After traveling a total of 69.99 m (up and down motion), how many times did the ball bounce? Put the answer in **p2a**. What is the height of the most recent bounce? Put the answer in **p2b**.

(c,d) For the 32<sup>th</sup> bounce, how high did the ball go? Put the answer in **p2c**. How many meters did the ball travel (up and down motion) in total prior to the 32<sup>th</sup> bounce? Put the answer in **p2d**.

**Problem 3:** Leibniz found that  $\pi$  can be approximated by the following series:

$$\pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}.$$

Madhava later suggested an alternative series:

$$\pi = \sqrt{12} \sum_{n=0}^{\infty} \frac{(-3)^{(-n)}}{2n+1}.$$

In this exercise, you are asked to write a function **testpi.m** to compare how fast the two series can approximate the value of  $\pi$  for a given tolerance. The function should have the following declaration: **function [api nterm] = testpi(tol,method)** where **tol** is the input tolerance defined as the ratio of the approximated value of  $\pi$  to the default value of  $\pi$  in MATLAB and **method** is a string input being either '**Leibniz**' or '**Madhava**'. The function outputs are the approximated value of  $\pi$  **api** and the number of terms **nterm** in the series needed to compute the approximate value.

In the function, you may want to consider the relationship between **abs(api-pi)/pi** and **tol** as a condition to truncate  $n$  in the two series above. Give the function a description. In the following exercises, set the tolerance to 0.00001.

(a) Set **p3a=evalc('help testpi')**.

(b,c) For the Leibniz series, what is the approximated value of  $\pi$  and how many terms of the series are needed to compute that value? Put the answers in **p3b** and **p3c**, respectively.

(d,e) For the Madhava series, what is the approximated value of  $\pi$  and how many terms of the series are needed to compute that value? Put the answers in **p3d** and **p3e**, respectively.

(f) Which method converges faster? Give answer in **p3f = '... series converges faster'**.