1. This is a problem to get you familiar with Matlab, calling **M-functions**, **rand**, **for** loops, and calling the **plot** function.

   (a) Suppose that a genie gave you a number equally likely to be anywhere on the $[0,1]$ line (Matlab does this by calling **rand**). Suppose that you wanted to generate a number that is 1 with probability 0.75 and 0 with probability 0.25. Generate Matlab code of the form

   ```
   u = rand;
   if % PUT SOME COMPARISON HERE BETWEEN U and 0.75
    coinFlip=1;
   else
    coinFlip=0;
   end
   ```

   (b) Now create an M-function **generateRandomCoinFlip.m** that generates a random coin flip that is 1 with probability p. This is a function where p is an input and "outcome" is the output. Start your function with

   ```
   function outcome = generateRandomCoinFlip(p)
   ```

   It should use code similar to what you wrote in part (a). After bulding this function, you should do a sanity check by going to the Matlab prompt, making sure Matlab is in the same working directory as your M-file, and typing **result=generateRandomCoinFlip(0.75)** or **result=generateRandomCoinFlip(0.21)** over and over again and watching what happens.

   (c) Now, in the same directory as **generateRandomCoinFlip.m**, create a new M-function called **generateRandomBinaryVector(p,n)**. Here, create a vector of length n where each entry is the outcome of a coin flip where the probability of heads was p. Start your function with

   ```
   function outcomes = generateRandomBinaryVector(p,n)

   % begin by setting up the vector, with n rows and 1 column
   vector = zeros(n,1);
   for index=1:n,
    % assign the index entry of vector to an outcome of a coin flip
   end
   ```

   Specifically, I want you to call the function **generateRandomCoinFlip** here, otherwise you do not get full credit.

   (d) I want you to let $n = 20$ and $p = 0.3$. I want you to use the **stem** function and plot the outcomes of the 20 coin flips. Then do likewise when $p = 0.8$. Also **xlabel**, **ylabel**, and **title** to describe that the x axis pertains to which of the 20 coin flips, the y-axis corresponds to the outcome of the coin flip, and title denotes that there are 20 coin flips and it and what p is. You should have two plots: one for when $n = 20$ and $p = 0.3$, and another for $n = 20$ and $p = 0.8$. You should first call **generateRandomBinaryVector** and then do your plotting.

2. In this problem, we discuss equivalence of events.

   (a) Suppose that the sample space Ω is the set of all non-negative numbers: $\Omega = [0, \infty)$. Consider two events A and B as:

   $$\begin{aligned} A &= \{\omega \in \Omega : \omega \geq 2\} \\ B &= \{\omega \in \Omega : \log(\omega) \geq \log(2)\} \end{aligned}$$

   Show that if $\omega \in A$, then $\omega \in B$. This means that $A \subset B$. Next, show that if $\omega \in B$, then $\omega \in A$. This means that $B \subset A$. Thus you have shown that $A = B$.

   (b) Suppose that the sample space Ω is the real line: $\Omega = \mathbb{R}$. Consider some two events C and D as:

   $$\begin{aligned} C &= \{\omega \in \Omega : \omega \leq -5\} \\ D &= \left\{\omega \in \Omega : e^{-\omega} \geq e^{5}\right\} \end{aligned}$$

   Show that $C = D$.

   (c) Suppose that Ω is the set of all non-negative numbers: $\Omega = [0, \infty)$. Consider some two events E and F as:

   $$\begin{aligned} E &= \{\omega \in \Omega : \omega \leq 9\} \\ F &= \left\{\omega \in \Omega : \sqrt{\omega} \leq 3\right\} \end{aligned}$$

   Show that $E = F$. What happens if $\Omega = \mathbb{R}$. Does $E = F$ or no? Explain. Specifcally, if you think no, either argue that there is an $\omega \in E$ for which $\omega \notin F$ or vice versa.

   (d) Consider now the general case that $\Omega = \mathbb{R}$, and two events A, B, C described as

   $$\begin{aligned} A &= \{\omega \in \Omega : a \leq \omega \leq b\} \\ B &= \{\omega \in \Omega : g(a) \leq g(\omega) \leq g(b)\} \\ C &= \{\omega \in \Omega : g(a) \geq g(\omega) \geq g(b)\} \end{aligned}$$

   where g is some function. Use your reasoning in previous parts of the problem to argue that: (i) if g is invertible and monotonically increasing over Ω, then $A = B$; (ii) if g is invertible and monotonically decreasing over Ω, then $A = C$. What happens if g is not invertible on Ω? Can we in general say that $A = B$ or $A = C$?

3. You will use these approximations later in this class and over and over again in your engineering career. Consider some fixed number $\lambda > 0$.

   (a) Show using a Taylor series expansion around the point $f(x) = \log(1-x)$ that the following holds for small Δ:

   $$\log(1 - \lambda\Delta) \simeq -\lambda\Delta.$$

   (b) Show that for a fixed $t > 0$,

   $$\lim_{\Delta \to 0} (1 - \lambda\Delta)^{\frac{t}{\Delta}} = e^{-\lambda t}$$

   (*hint: remember from calculus that if* $\lim_{\Delta \to 0} f(\Delta) = a$, *and if the function* g *is continuous, then* $\lim_{\Delta \to 0} g(f(\Delta)) = g(a)$)

4. Show that

   $$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

   by using two different procedures to count the set of all subsets, 2^Ω, of a set Ω with n elements. *Hint: First start with* $\Omega = \{a, b, c\}$. *In one strategy, try to count the total number of subsets of size* 0*, plus the total number of subsets of size* 1*, up to the total number of subsets of size* n*. Then generalize for any* Ω *with* $|\Omega| = n$*. In your other approach, consider describing any subset as an ordered tuple, for example* $\underbrace{(0, 1, ..., 0)}_{\text{length } n}$*, and argue that any subset can be described this way. From this, argue from counting that there must be* 2^n *such subsets.*

5. Consider a situation where there an election where there are 8 total candidates, 4 democrats and 4 republicans. First, in parallel, there is a democratic runoff and republican runoff. For the democratic primary, the top 3 democrats are selected. Then, in the second runoff, the top two democrats go to another runoff. Finally, the top vote-getter of that runoff wins the primary. On the republican side, first the top two republican vote-getters are selected. Afterwards, the second runoff takes place to select the republican candidate. Then, the democrat and republicans standoff.

   (a) what is the total number of different ways that the republican and democrat primaries can occur?

   (b) A car dealer in La Jolla says that you will win a Tesla if you correctly guess the sequence of the first, and second democrat primaries correctly. Another car dealder in Del Mar says that you will win a Tesla if you correctly guess the sequence of first and second republican primaries correctly? Calculate the probability of winning a Tesla for either dealership. Which dealership should you go to if you want to get a free Tesla?