Reaching supercriticalfield strengths with intense lasers

T G Blackburn et al 2019 New J. Phys. **21** 053040

Notebook: Óscar Amaro, Feb 2022 @ GoLP-EPP

Attention: There is either a flawed implementation of neff or χmax (compare with all figures from the paper), and with this implementation the 50 GeV in Figure 4 are not consistent with the paper.

Physical constants

In[41]:=
```
(*α fine structure constant*)
α = 1 / 137; (*[#]*)
(*c speed of light in vacuum*)
c = 299 792 458; (*[m/s]*)
(*m electron mass*)
m = 0.5109989461 / 1000; (*[GeV]*)
(*ℏ Planck's constant*)
ℏ = 6.626070040 * 10^-34 / (2 π);
(*e electron abs charge*)
e = 1.6021766208 * 10^-19;
```

```
In[•]:= Clear[g, χc, χmax, δ, eq5a, Rcneff, W]

(*Gaunt factor*)
g[χ_] := (1 + 4.8 × (1 + χ) Log[1 + 1.7 χ] + 2.44 χ^2)^(-2 / 3)
(*(8) critical χc*)
χc[γ0_, a0_, ω0_, n_] :=
 FindRoot[χ^4 g[χ]^2 - (72 N[Log[2]])/(π^2 α^2) ((γ0 ω0)/(n m))^2 Log[(2 γ0 a0 ω0)/(m χ)], {χ, 10^-10}][[1, 2]]
(*(4) χmax, here χ=χ/χ0*)
χmax[Rcneff_, θ_, mm_] := FindRoot[
   χ^4 g[χ mm]^2 - (9 Log[2] (1 + Cos[θ])^2)/((π Rcneff)^2) Log[(1 + Cos[θ])/(2 χ)], {χ, 10^-10}][[1, 2]]
(*(5b)*)
δ[θ_, Rcneff_] := (π Rcneff (1 + Cos[θ]))/(3 Sqrt[2 Log[2]])
(*(5a)*)
eq5a[θ_, Rcneff_] := (1 + Cos[θ])/2 Exp[-W[δ[θ, Rcneff]^2] / 5]
(* Lambert function W*)
W[x_] := ProductLog[x]
```

Figure 1

Analytical prediction of χmax of electron

In[46]:=
```
(*FIG.1.*)
(*Gaunt factor*)
g[χ_] := (1 + 4.8 × (1 + χ) Log[1 + 1.7 χ] + 2.44 χ^2) ^ (-2 / 3)
(*(4)  χmax, here χ=χ/χ0*)
χmax[Rcneff_, θ_, mm_] := FindRoot[
   χ^4 g[χ mm] ^2 - (9 Log[2] (1 + Cos[θ]) ^2)/((π Rcneff) ^2) Log[(1 + Cos[θ])/(2 χ)], {χ, 10^-10}] [[1, 2]]
Rcneff = ParallelTable[10^i, {i, -0.7, 4.5, 0.1}];
χ0 = {1, 10, 100};
χmaxx = ParallelTable[{Rcneff[[i]], χmax[Rcneff[[i]], 0, χ0[[j]]]},
   {i, 1, Length[Rcneff]}, {j, 1, Length[χ0]}];
fig1 = ListLogLinearPlot[{χmaxx[[All, 1]], χmaxx[[All, 2]], χmaxx[[All, 3]]},
   Joined → True, Axes → False, PlotStyle → Directive[Dashed, Black, 15],
   PlotRange → {{10^-0.5, 10^4}, {0, 1.05}}, Frame → True,
   FrameStyle → Directive[Black, 15], FrameLabel → {"Rcneff", "χmax/χ0"}];
fig1
```

Out[52]=

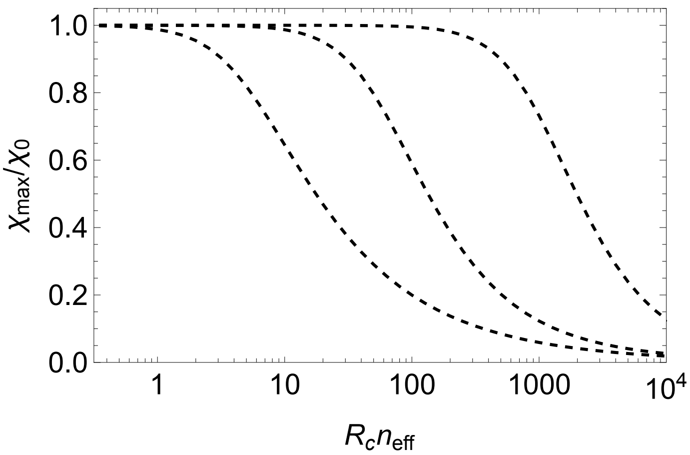


Figure 2

Angle at which χmax is largest

In[675]:=
```
Clear[a0, γ0, λ, ω0, ω0m, dθ, χ0, Rc, χmax, χ0, da0]

λ = 0.8; (*μm*)
r0 = 2 ; (*μm*)
ω0 = 2 π c / (λ / 10 ^ 6);
ω0m = 2 π c / (λ / 10 ^ 6) ℏ / e / 10 ^ 9;

(*Gaunt factor*)
g[χ_] := (1 + 4.8 × (1 + χ) Log[1 + 1.7 χ] + 2.44 χ ^ 2) ^ (-2 / 3)
(*(3) neff *)
neff[θ_, τ_] := (ω0 τ 10 ^ -6 / c)/(2 π) (1 + τ ^ 2/r0 ^ 2 Tan[θ / 2] ^ 2/Log[4]) ^ (-1 / 2)
(*(4) find χmax*)
χmax[a0_, θ_, γ0_, τ_] :=
 FindRoot[χ ^ 4 g[χ] ^ 2 / (2 a0 γ0 ω0m / m) ^ 4 - (9 Log[2] (1 + Cos[θ]) ^ 2)/(π α a0 2 a0 γ0 ω0m / m  neff[θ, τ]) ^ 2
     Log[((1 + Cos[θ]) × (2 a0 γ0 ω0m / m))/(2 χ)], {χ, 10 ^ -7}]〚1, 2〛 // Quiet
(*χ0 is χe at θ=0, χ0=2 a0 γ0 ω0/m;*)
(*Rc=α a0 χ0*)

optθ[a0_, γ0_, τ_] := Module[{tab, dθ},
  dθ = 0.1 / 4 / 2;
  tab = Table[{θ, χmax[a0, θ, γ0, τ]}, {θ, 0, π / 2, dθ}];
  Return[tab〚Position[tab〚All, 2〛, Max[tab〚All, 2〛]]〚1, 1〛, 1〛];
 ]

(* create tables*)
da0 = 0.1 / 2;
tab50full = Table[{10 ^ a0, optθ[10 ^ a0, 2 × 10 ^ 4, 50 λ] 180 / π},
   {a0, Log10[10], Log10[1000], da0}];
tab50dash = Table[{10 ^ a0, optθ[10 ^ a0, 5 × 10 ^ 3, 50 λ] 180 / π},
   {a0, Log10[10], Log10[1000], da0}];
tab10full = Table[{10 ^ a0, optθ[10 ^ a0, 2 × 10 ^ 4, 10 λ] 180 / π},
   {a0, Log10[10], Log10[1000], da0}];
tab10dash = Table[{10 ^ a0, optθ[10 ^ a0, 5 × 10 ^ 3, 10 λ] 180 / π},
   {a0, Log10[10], Log10[1000], da0}];

(*Plot*)
ListLogLinearPlot[{tab50full, tab50dash, tab10full, tab10dash},
 Joined → True, PlotStyle → {Directive[Blue],
   Directive[Blue, Dashed], Directive[Orange], Directive[Orange, Dashed]},
 Axes → False, Frame → True, FrameStyle → Directive[Black, 15],
 FrameLabel → {"a₀", "optimal θ[°]"}, PlotRange → {0, 85}]
```

Out[687]=

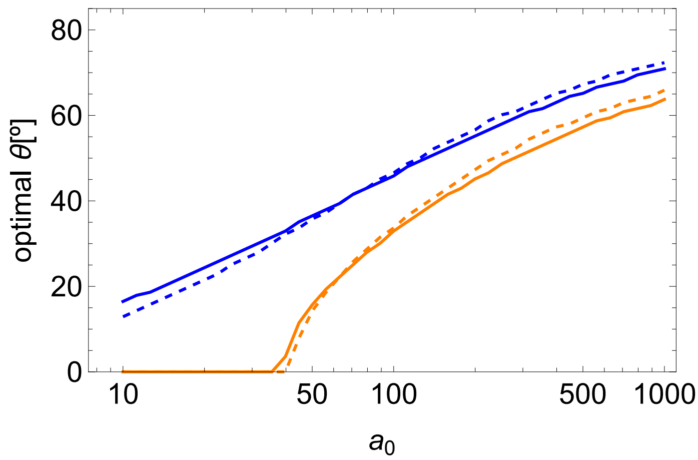


Figure 3

Enhanced quantum effects at oblique incidence

In[664]:=

```
Clear[a0, γ0, λ, ω0, ω0m, dθ, χ0, Rc, χmax, χ0, da0]

λ = 0.8; (*[μm]*)
r0 = 2 ; (*[μm]*)
a0 = 82.4; (*[]*)
ω0 = 2 π c / (λ / 10^6);
ω0m = 2 π c / (λ / 10^6) ℏ / e / 10^9;

(*Gaunt factor*)
g[χ_] := (1 + 4.8 × (1 + χ) Log[1 + 1.7 χ] + 2.44 χ^2) ^ (-2 / 3)
(*(3) neff *)
neff[θ_, τ_] := (ω0 τ 10^-6 / c)/(2 π) (1 + (τ^2)/(r0^2) (Tan[θ / 2]^2)/(Log[4])) ^ (-1 / 2)
(*(4) find χmax*)
χmax[a0_, θ_, γ0_, τ_] :=
 FindRoot[χ^4 g[χ]^2 / (2 a0 γ0 ω0m / m) ^4 - (9 Log[2] (1 + Cos[θ]) ^2)/((π α a0 2 a0 γ0 ω0m / m  neff[θ, τ]) ^2)
     Log[((1 + Cos[θ]) × (2 a0 γ0 ω0m / m))/(2 χ)], {χ, 10^-7}][[1, 2]] // Quiet
(*tables*)
tab1 = Table[{180 / π θ, χmax[a0, θ, 2 × 10^4, 10 λ]}, {θ, 0, π / 2, 0.1}];
tab2 = Table[{180 / π θ, χmax[a0, θ, 2 × 10^4, 50 λ]}, {θ, 0, π / 2, 0.1}];
(*plot*)
ListPlot[tab1, Joined → True, PlotStyle → Directive[Dashed, Black],
 PlotRange → {0, 11}, Axes → False, Frame → True, FrameStyle → Directive[Black, 15],
 FrameLabel → {"θ[°]", "χmax"}, PlotLabel → "(a i)"]
ListPlot[tab2, Joined → True, PlotStyle → Directive[Dashed, Black],
 PlotRange → {0, 11}, Axes → False, Frame → True, FrameStyle → Directive[Black, 15],
 FrameLabel → {"θ[°]", "χmax"}, PlotLabel → "(a ii)"]
```

Out[673]=

Out[674]=

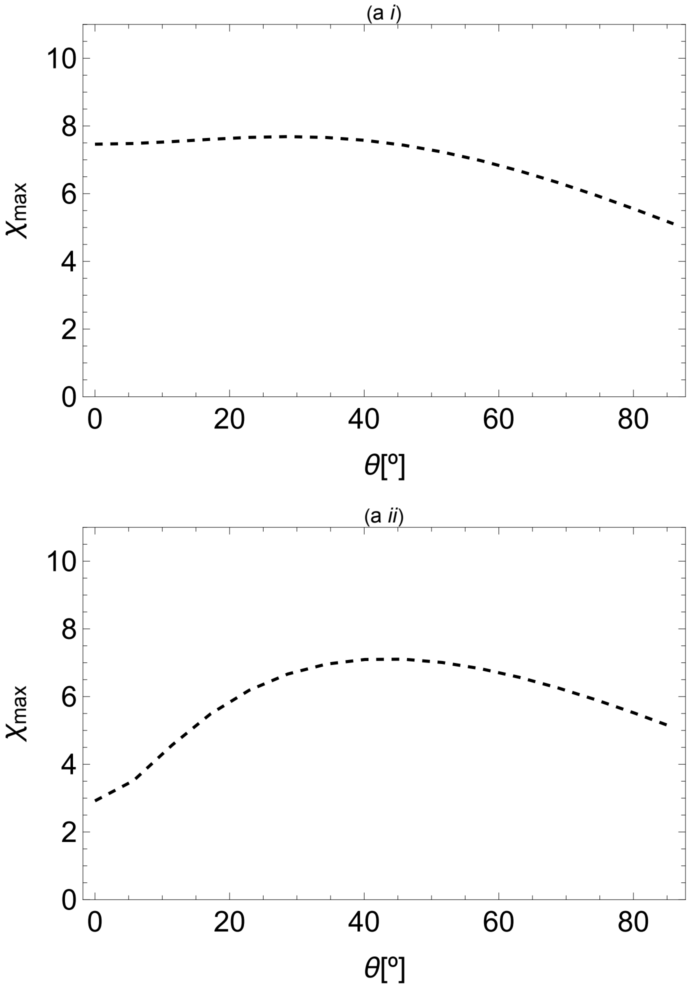
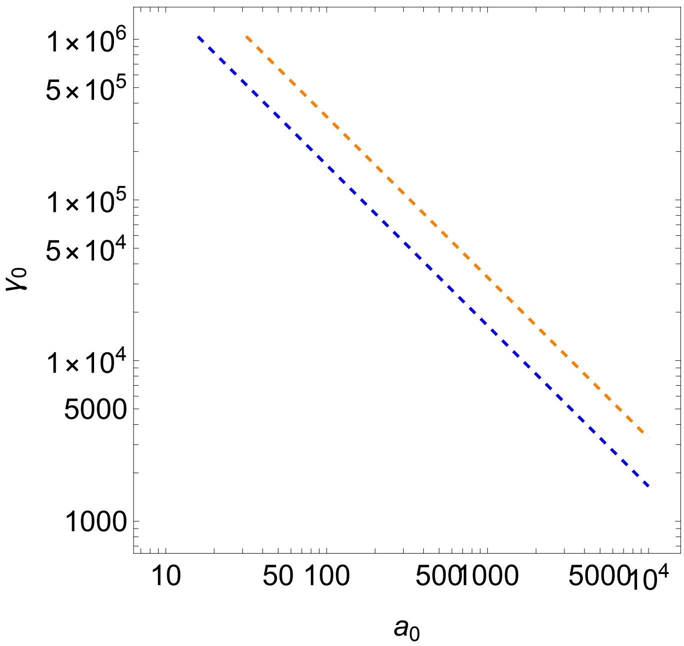


Figure 4

Minimum γ0 and a0 required for χe>=100

In[61]:=
```
(*FIG.4.*)
Clear[λ, ω0, γ0min, p1, p2]
λ = 0.8;
ω0 = 2 π c / (λ / 10^6) ℏ / e / 10^9;
γ0min[a0_, χe_, θ_] := m χe / (a0 ω0 (1 + Cos[θ]))
p1 = LogLogPlot[γ0min[a0, 100, 0], {a0, 10^1.2, 10^4},
   Axes → False, PlotStyle → Directive[Dashed, Blue, 15],
   PlotRange → {{10^0.8, 10^4.2}, {10^2.8, 10^6.2}}, Frame → True,
   FrameStyle → Directive[Black, 15], FrameLabel → {"a0", "γ0"}, AspectRatio → 1];
p2 = LogLogPlot[γ0min[a0, 100, π / 2], {a0, 10^1.5, 10^4},
   Axes → False, PlotStyle → Directive[Dashed, Orange, 15],
   PlotRange → {{10^0.8, 10^4.2}, {10^2.8, 10^6.2}}, Frame → True,
   FrameStyle → Directive[Black, 15], FrameLabel → {"a0", "γ0"}, AspectRatio → 1];
Show[
 p1,
 p2]
```

Out[67]=



In[688]:=
```
Clear[a0, γ0, λ, ω0, ω0m, dθ, χ0, Rc, χmax, χ0, da0]

λ = 0.8; (*[μm]*)
r0 = 2 ; (*[μm]*)
a0 = 1000; (*[]*)
ω0 = 2 π c / (λ / 10^6);
ω0m = 2 π c / (λ / 10^6) ℏ / e / 10^9;

(*Gaunt factor*)
g[χ_] := (1 + 4.8 × (1 + χ) Log[1 + 1.7 χ] + 2.44 χ^2) ^ (-2 / 3)
(*(3) neff *)
neff[θ_, τ_] := (ω0 τ 10^-6 / c)/(2 π) (1 + τ^2/r0^2 Tan[θ / 2]^2/Log[4]) ^ (-1 / 2)
(*(4) find χmax*)
χmax[a0_, θ_, γ0_, τ_] :=
 FindRoot[χ^4 g[χ]^2 / (2 a0 γ0 ω0m / m)^4 - (9 Log[2] (1 + Cos[θ])^2)/((π α a0 2 a0 γ0 ω0m / m neff[θ, τ])^2)
   Log[((1 + Cos[θ]) × (2 a0 γ0 ω0m / m))/(2 χ)], {χ, 10^-7}][[1, 2]] // Quiet
(*inset plot*)
plt1 = ListPlot[Table[{180 / π θ, χmax[a0, θ, 40 / m, 10 λ]}, {θ, 0, π / 2, 0.1}],
   Joined → True, PlotStyle → Directive[Dashed, Purple],
   PlotRange → {0, 170}, Axes → False, Frame → True,
   FrameStyle → Directive[Black, 15], FrameLabel → {"θ[°]", "χmax"}];
plt2 = ListPlot[Table[{180 / π θ, χmax[a0, θ, 40 / m, 50 λ]}, {θ, 0, π / 2, 0.1}],
   Joined → True, PlotStyle → Directive[Dashed, Red],
   PlotRange → {0, 170}, Axes → False, Frame → True,
   FrameStyle → Directive[Black, 15], FrameLabel → {"θ[°]", "χmax"}];
Show[{plt1, plt2}]
```

Out[698]=