# Reaching high laser intensity by a radiating electron

M. Jirka, et al, Phys. Rev. A 103, 053114

Notebook: Óscar Amaro, June 2021/Jan 2022 @ GoLP-EPP

## Figure 1

Problem: what is the definition of T($\tau$)? By inspection it's possible to determine an approximate relation, though it does not seem to be explicit in the text.

```
In[1]:= Clear[χe, ℰe, γe, Wγ, pc, tc, ω0, E0, ℰc, me, c, e, α, ℏ, T, ES, λμm, τ]

χe = 2 γe E0 / ES;
γe = ℰe / (me c ^ 2);
Wγ = 3 ^ (2 / 3) × 28 Gamma[2 / 3] α me ^ 2 c ^ 4 χe ^ (2 / 3)  / (54 ℏ ℰe) ;
pc  =  Wγ tc;
tc = τ / (2 Sqrt[2 Log[2]]);
ω0 = 2 π c / (λμm 10 ^ - 6);
E0 = 0.855 λμm Sqrt[I0 10 ^ - 18] me ω0 c / e;


ℰc = (1 - 16 / 63) ^ pc  ℰe
(*Refine[//Simplify,{c>0,me>0,E0>0,ES>0,ℏ>0,ℰe>0,τ>0}]//FullSimplify*)


me = 9.11 × 10 ^ - 31; (*[Kg]*)
c =  299 792 458; (*[m/s]*)
e = 1.602176634 × 10 ^ - 19; (*[C]*)
α = 1 / 137; (*[]*)
ℏ = 1.054571817 × 10 ^ - 34; (*[J s]*)
```

$$T = 0.67\,\frac{\lambda\mu m\ 10\,\hat{}\,{-6}}{c};$$

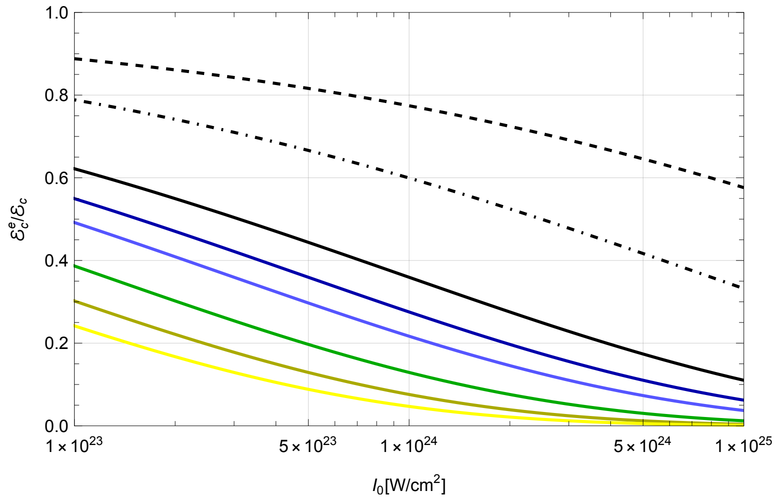
```
ES = me ^ 2 c ^ 3 / (e ℏ); (*[V/m]*)

LogLinearPlot[
```

$$\left\{\left(\left(\frac{47}{63}\right)^{\frac{0.030199136711634034\grave{}\ c^4\, me^2 \left(\frac{\sqrt{I0}\ \mathcal{E}e}{e\, ES}\right)^{2/3} \alpha\, \tau}{\mathcal{E}e\, \hbar}}\right) \text{ //. } \{\mathcal{E}e \to 100 \times 10\,\hat{}\,9\ e,\ \tau \to T,\ \lambda\mu m \to 0.25\},\right.$$

```
((47/63)^((0.030199136711634034` c^4 me^2 ((Sqrt[I0] ℰe)/(e ES))^(2/3) α τ)/(ℰe ℏ)) //. {ℰe → 100 × 10^9 e, τ → T, λμm → 0.5},
((47/63)^((0.030199136711634034` c^4 me^2 ((Sqrt[I0] ℰe)/(e ES))^(2/3) α τ)/(ℰe ℏ)) //. {ℰe → 100 × 10^9 e, τ → T, λμm → 1},
((47/63)^((0.030199136711634034` c^4 me^2 ((Sqrt[I0] ℰe)/(e ES))^(2/3) α τ)/(ℰe ℏ)) //. {ℰe → 50 × 10^9 e, τ → T, λμm → 1},
((47/63)^((0.030199136711634034` c^4 me^2 ((Sqrt[I0] ℰe)/(e ES))^(2/3) α τ)/(ℰe ℏ)) //. {ℰe → 30 × 10^9 e, τ → T, λμm → 1},
((47/63)^((0.030199136711634034` c^4 me^2 ((Sqrt[I0] ℰe)/(e ES))^(2/3) α τ)/(ℰe ℏ)) //. {ℰe → 100 × 10^9 e, τ → 2 T, λμm → 1},
((47/63)^((0.030199136711634034` c^4 me^2 ((Sqrt[I0] ℰe)/(e ES))^(2/3) α τ)/(ℰe ℏ)) //. {ℰe → 50 × 10^9 e, τ → 2 T, λμm → 1},
((47/63)^((0.030199136711634034` c^4 me^2 ((Sqrt[I0] ℰe)/(e ES))^(2/3) α τ)/(ℰe ℏ)) //. {ℰe → 30 × 10^9 e, τ → 2 T, λμm → 1}},
{I0, 10^23, 10^25}, PlotRange → {{10^23, 10^25}, {0, 1}}, Frame → True,
GridLines → Automatic, ImageSize → 400, FrameLabel → {"I₀[W/cm²]", "ℰ_c^e/ℰ_c"},
PlotStyle → {Directive[Black, Dashed], Directive[Black, DotDashed],
  Directive[Black], Directive[Darker[Blue]], Directive[Lighter[Blue]],
  Directive[Darker[Green]], Directive[Darker[Yellow]], Directive[Yellow]}]
```

Out[9]= $\left(\frac{47}{63}\right)^{\frac{0.0301991\, c^4\, me^2 \left(\frac{\sqrt{I0}\, \mathcal{E}e}{e\, ES}\right)^{2/3} \alpha\, \tau}{\mathcal{E}e\, \hbar}}\ \mathcal{E}e$

Out[17]=




# Figure 2

Confirm with figure

In[18]:=
```
Clear[χe, ℰe, γe, Wγ, pc, tc, tf, ω0, E0, ℰc, me, c, e, α, ℏ, T, ES, λμm, τ]

χe = 2 γe E0 / ES;
γe = ℰe / (me c^2);
Wγ = 3^(2/3) × 28 Gamma[2/3] α me^2 c^4 χe^(2/3) / (54 ℏ ℰe);
tc = τ / (2 Sqrt[2 Log[2]]);
tf = τ / (Sqrt[2 Log[2]]);
pc = Wγ tc;
pf = Wγ tf;
ω0 = 2 π c / (λμm 10^-6);
E0 = 0.855 λμm Sqrt[I0 10^-18] me ω0 c / e;

me = 9.11 × 10^-31; (*[Kg]*)
c = 299 792 458; (*[m/s]*)
e = 1.602176634 × 10^-19; (*[C]*)
α = 1 / 137; (*[]*)
ℏ = 1.054571817 × 10^-34; (*[J s]*)
T = 0.67 (λμm 10^-6)/c;
ES = me^2 c^3 / (e ℏ); (*[V/m]*)

I0 = 10^24;
ℰe = 50 × 10^9 e;
τ = T;
λμm = 1;

ℰc = (1 - 16/63)^pc ℰe;
γec = ℰc / (me c^2);
ℰf = (1 - 16/63)^pf ℰe;

(* Figure 2 a) blue dot ℰ_e^c *)
ℰc / (10^9 e)
(* Figure 2 a) purple dot ℰ_e^f *)
ℰf / (10^9 e)
(* Figure 2 b) blue dot χ_e^c (χ_e^f=0 always) *)
χec = 2 γec E0 / ES
```

Out[42]= 13.7695

Out[43]= 3.79196

Out[44]= 111.784

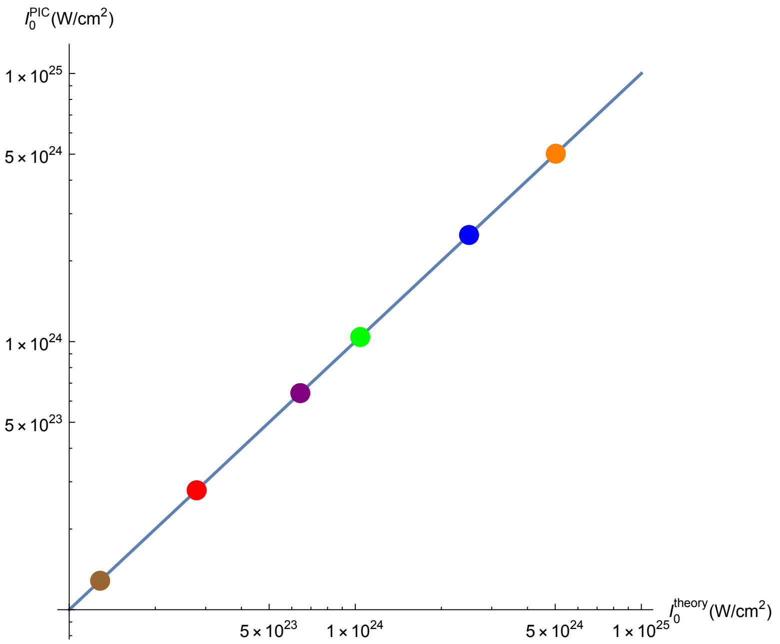
# Figure 3

Threshold intensity required for electron reflection by the laser pulse. Equation 2 gives the necessary a0 (intensity I0) for reflection. However, the energy of the electron at reflection will also be a function of a0. The intensity is found numerically.

In[45]:=
```
Clear[χe, ℰe, γe, Wγ, pc, tc, ω0, E0, ℰc, me, c, e, α, ℏ, T, ES, λμm, τ, I0, getI0]

me = 9.11 × 10^-31; (*[Kg]*)
c = 299792458; (*[m/s]*)
e = 1.602176634 × 10^-19; (*[C]*)
α = 1 / 137; (*[]*)
ℏ = 1.054571817 × 10^-34; (*[J s]*)
ES = me^2 c^3 / (e ℏ); (*[V/m]*)
λμm = 1;
ω0 = 2 π c / (λμm 10^-6);
T = 0.67 (λμm 10^-6)/c;

getI0[ℰe_, τ_] := Module[{I0, tc, tr, Wγ, E0, pr, ℰr, sol, γe, χe, eq},

  χe = 2 γe E0 / ES;
  γe = ℰe / (me c^2);
  Wγ = 3^(2/3) × 28 Gamma[2/3] α me^2 c^4 χe^(2/3) / (54 ℏ ℰe) ;
  pr = Wγ tr;
  tc = τ / (2 Sqrt[2 Log[2]]);
  E0 = 0.855 λμm Sqrt[I0 10^-18] me ω0 c / e;

  (*tr definition a bit ambiguous in the text*)
  tr = tc / Sqrt[2] (Sqrt[2 Log[2]])^(τ / T - 1);

  ℰr = (1 - 16 / 63)^pr ℰe // N // Simplify;

  (*ℰr will be a function of I0 *)
  eq = I0 10^-18 == 2 ((ℰr / (me c^2))^2 - 1) / 0.855^2;

  (* solve numerically *)
  sol = NSolve[eq, I0, Reals];
  Return[sol[[1, 1, 2]]]
 ]

Show[{LogLogPlot[I0, {I0, 10^23, 10^25}, AspectRatio → 1,
   AxesLabel → {"I0^theory (W/cm^2)", "I0^PIC (W/cm^2)"}, ImageSize → 400],
  ListLogLogPlot[getI0[6 × 10^9 e, T] × Transpose[{{1}, {1}}],
   PlotStyle → Directive[Blue, PointSize → 0.1 / 3]],
  ListLogLogPlot[getI0[10 × 10^9 e, T] × Transpose[{{1}, {1}}],
   PlotStyle → Directive[Orange, PointSize → 0.1 / 3]],
  ListLogLogPlot[getI0[30 × 10^9 e, 3 T] × Transpose[{{1}, {1}}],
   PlotStyle → Directive[Green, PointSize → 0.1 / 3]],
```

```
ListLogLogPlot[getI0[50 × 10^9 e, 5 T] × Transpose[{{1}, {1}}],
 PlotStyle → Directive[Red, PointSize → 0.1 / 3]],
ListLogLogPlot[getI0[100 × 10^9 e, 5 T] × Transpose[{{1}, {1}}],
 PlotStyle → Directive[Purple, PointSize → 0.1 / 3]],
ListLogLogPlot[getI0[100 × 10^9 e, 7 T] × Transpose[{{1}, {1}}],
 PlotStyle → Directive[Brown, PointSize → 0.1 / 3]]}]
```

Out[56]=




# Figure 2

γmin=1/Sqrt[2 Δn]

In the limit *χγ*<<1 Δn=8*α* E0^2/(45 *π* ES^2)

Limit *α χ*^2/3 = 1 ( it needs to be multiplied by a factor of 2 to reproduce Fig4 )

In[57]:=

```
Clear[χe, ℰe, γe, Wγ, pc, tc, ω0, E0, ℰc, me, c, e, α, ℏ, T, ES, λμm, τ, Δn]
χe = 2 γe E0 / ES;
γe = ℰe / (me c^2);
ω0 = 2 π c / (λμm 10^-6);
E0 = 0.855 λμm Sqrt[I0 10^-18] me ω0 c / e;
ES = me^2 c^3 / (e ℏ); (*[V/m]*)
Solve[χe == (1 / α)^1.5, ℰe][[1, 1, 2]]
```

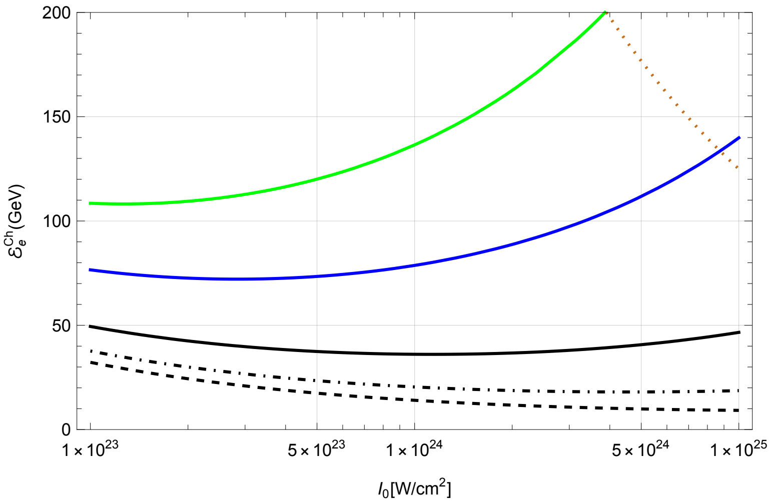
Out[63]=

$$\frac{93.0731\, c^3\, me^2 \left(\frac{1}{\alpha}\right)^{1.5}}{\sqrt{I0}\,\hbar}$$

We need to invert the relation $\mathcal{E}$^c_e ($\mathcal{E}$_e) and replace $\mathcal{E}$^c_e by the corresponding *γ*min given by the Cherenkov condition

```
In[64]:= Clear[χe, εe, γe, Wγ, pc, tc, ω0, E0, εc, me, c, e, α, ℏ, T, ES, λμm, τ, Δn, getεCh]

me = 9.11 × 10^-31;(*[Kg]*)
c = 299 792 458; (*[m/s]*)
e = 1.602176634 × 10^-19; (*[C]*)
α = 1 / 137; (*[]*)
ℏ = 1.054571817 × 10^-34;(*[J s]*)
ES = me^2 c^3 / (e ℏ);(*[V/m]*)

getεCh[I0_, τT_, λμm_] := Module[{εCh, Δn, χe, γe, εe, pc, Wγ, tc, ω0, E0, T, τ},
  χe = 2 γe E0 / ES;
  γe = εe / (me c^2);
  Wγ = 3^(2/3) × 28 Gamma[2/3] α me^2 c^4 χe^(2/3)  / (54 ℏ εe) ;
  pc  = Wγ tc;
  tc = τ / (2 Sqrt[2 Log[2]]);
  ω0 = 2 π c / (λμm 10^-6);
  E0 = 0.855 λμm Sqrt[I0 10^-18] me ω0 c / e;
  T = 0.67 (λμm 10^-6)/c;
  τ = τT T;
  Δn = 8 α E0^2 / (45 π ES^2);
  εCh = NSolve[
     εe (47/63)^((0.030199136711634034` c^4 me^2 ((Sqrt[I0] εe)/(e ES))^(2/3) α τ)/(εe ℏ)) == 1 / Sqrt[2 Δn] me c^2, εe, Reals][[1, 1, 2]];
  Return[εCh / (10^9 e)]
 ]

LogLinearPlot[
 {getεCh[I0, 3, 1], getεCh[I0, 2, 1], getεCh[I0, 1, 1], getεCh[I0, 1, 0.5],
  getεCh[I0, 1, 0.25], 2 (93.07306613561131` c^3 me^2 (1/α)^1.5`)/(Sqrt[I0] ℏ) / (10^9 e)},
 {I0, 10^23, 10^25}, PlotPoints → 2, PlotRange → {0, 200},
 PlotStyle → {Directive[Green], Directive[Blue], Directive[Black],
   Directive[Black, DotDashed], Directive[Black, Dashed], Directive[Dotted],
   Directive[Darker[Yellow]], Directive[Yellow]}, Frame → True,
 GridLines → Automatic, ImageSize → 400, FrameLabel → {"I_0[W/cm^2]", "ε_e^Ch(GeV)"}]
```

Out[72]=


200
150
100
50
0
$\mathcal{E}_e^{Ch}$(GeV)
1 × 10^23
5 × 10^23
1 × 10^24
5 × 10^24
1 × 10^25
$I_0$[W/cm^2]