# Model-independent inference of laser intensity

T. G. Blackburn, E. Gerstmayr, S. P. D. Mangles, and M. Marklund,

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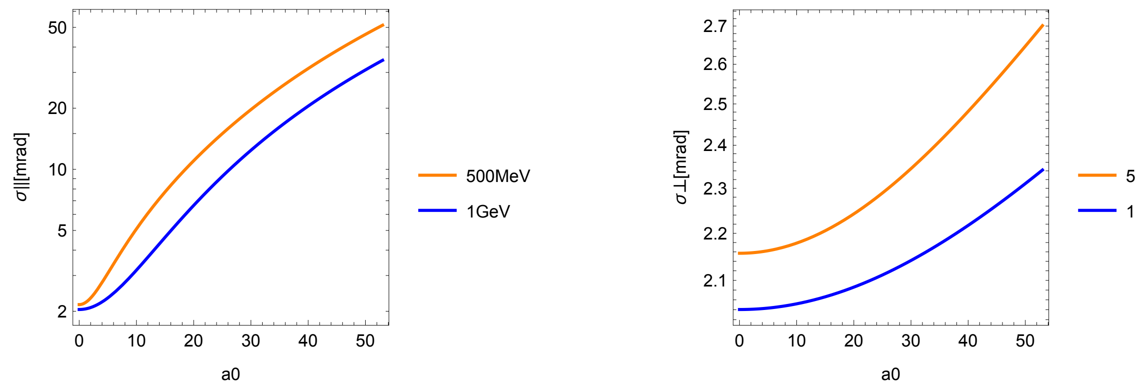
**Abstract**

"measurement of the variances of this profile in the planes parallel and perpendicular to the laser polarization, and the mean initial and final energies of the electron beam, allows the intensity of the laser pulse to be inferred in a way that is independent of the model of the electron dynamics"

# Figure 1

In[395]:=
```
Clear[σpll, σprp, γi, γf, a0, f2, f4, R, ω0, ω0eV, τ]

m = 0.511 × 10^6; (*[eV]*)
α = 1 / 137; (*[]*)
f2 = ω0 τ Sqrt[π / (4 Log[2])];
f4 = f2 / Sqrt[2];

(*equation 1*)
σprp = Sqrt[5/(8 γi γf) + δ^2];
(*equation 2*)
σpll = Sqrt[a0^2/(4 γi γf) f4/f2 + σprp^2];

λμm = 0.8; (*[μm]*)

ω0 = 2 π 3 × 10^8 / (λμm 10^-6); (*[s-1]*)
ω0eV = ω0 1.05 × 10^-34 / (9.11 × 10^-31 × (3 × 10^8)^2) m;  (*[eV]*)
τ = 40 × 10^-15; (*[s]*)

δ = 2 × 10^-3; (*[rad]*)

(*equation 3*)
γf = γi/(1 + R γi);
R = (2 α a0^2 ω0eV)/(3 m) f2;

(* plot *)
GraphicsRow[
 {LogPlot[{10^3 σpll /. {γi → 500 × 10^6 / m}, 10^3 σpll /. {γi → 1000 × 10^6 / m}},
   {a0, 0, 53}, AspectRatio → 1, ImageSize → 200,
   PlotStyle → {Orange, Blue}, Frame → True,
   FrameLabel → {"a0", "σ||[mrad]"}, PlotLegends → {"500MeV", "1GeV"}],
  LogPlot[{10^3 σprp /. {γi → 500 × 10^6 / m}, 10^3 σprp /. {γi → 1000 × 10^6 / m}},
   {a0, 0, 53}, AspectRatio → 1, ImageSize → 200, PlotStyle → {Orange, Blue},
   Frame → True, FrameLabel → {"a0", "σ⊥[mrad]"},
   PlotLegends → {"500MeV", "1GeV"}]}, ImageSize → 700]
```

Out[403]=



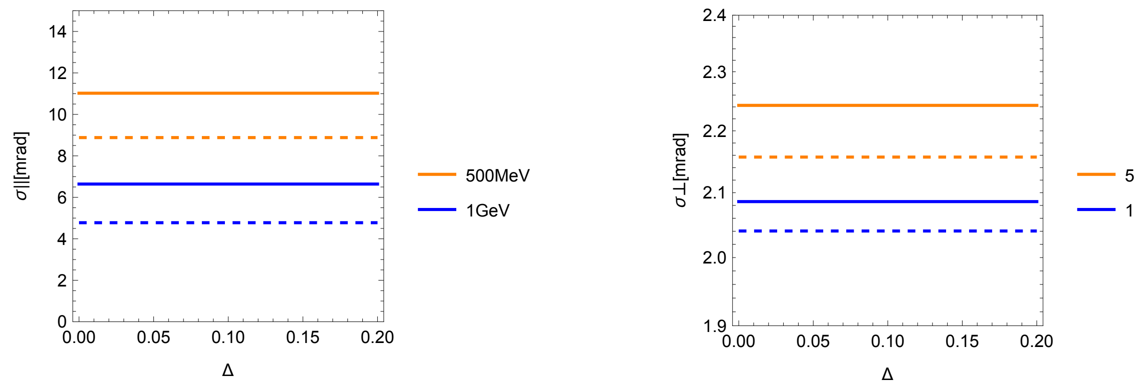

# Figure 2

In[383]:=

```
Clear[σpll, σprp, γi, γf, a0, f2, f4, R, ω0, ω0eV, τ]

m = 0.511 × 10^6; (*[eV]*)
α = 1 / 137; (*[]*)
f2 = ω0 τ Sqrt[π / (4 Log[2])];
f4 = f2 / Sqrt[2];

(*equation 1*)
σprp = Sqrt[5/(8 γi γf) + δ^2];
(*equation 2*)
σpll = Sqrt[a0^2/(4 γi γf) f4/f2 + σprp^2];

(*equation 1 noRR*)
σprpnoRR = Sqrt[5/(8 γi γi) + δ^2];
(*equation 2 noRR*)
σpllnoRR = Sqrt[a0^2/(4 γi γi) f4/f2 + σprp^2];

λμm = 0.8; (*[μm]*)

ω0 = 2 π 3 × 10^8 / (λμm 10^-6); (*[s-1]*)
ω0eV = ω0 1.05 × 10^-34 / (9.11 × 10^-31 × (3 × 10^8)^2) m; (*[eV]*)
τ = 40 × 10^-15; (*[s]*)

δ = 2 × 10^-3; (*[rad]*)

(*equation 3*)
```

```
γf = γi / (1 + R γi);
R = (2 α a0^2 ω0eV / (3 m)) f2;
a0 = 20;

(* plot *)
GraphicsRow[
 {Plot[{10^3 σpll /. {γi → 500 × 10^6 / m}, 10^3 σpll /. {γi → 1000 × 10^6 / m},
    10^3 σpllnoRR /. {γi → 500 × 10^6 / m}, 10^3 σpllnoRR /. {γi → 1000 × 10^6 / m}},
   {Δ, 0, 0.2}, AspectRatio → 1, ImageSize → 200,
   PlotStyle → {Directive[Orange], Directive[Blue], Directive[Orange, Dashed],
     Directive[Blue, Dashed]}, Frame → True, FrameLabel → {"Δ", "σ||[mrad]"},
   PlotLegends → {"500MeV", "1GeV"}, PlotRange → {0, 15}],
  LogPlot[{10^3 σprp /. {γi → 500 × 10^6 / m}, 10^3 σprp /. {γi → 1000 × 10^6 / m},
    10^3 σprpnoRR /. {γi → 500 × 10^6 / m}, 10^3 σprpnoRR /. {γi → 1000 × 10^6 / m}},
   {Δ, 0, 0.2}, AspectRatio → 1, ImageSize → 200, PlotStyle → {Directive[Orange],
     Directive[Blue], Directive[Orange, Dashed], Directive[Blue, Dashed]},
   Frame → True, FrameLabel → {"Δ", "σ⊥[mrad]"}, PlotLegends → {"500MeV", "1GeV"},
   PlotRange → {1.9, 2.4}]}, ImageSize → 700]
```

Out[394]=



# Equation 6: infered a0

In[46]:=
```
Clear[a, a0, n, a2, a4, x, y, W0, xb, ξ, P, Q, nnorm]

(* laser profile at z=0 *)
a = a0 Exp[- (x ^ 2 + y ^ 2) / W0 ^ 2];

(* density profile at z=0 *)
n = Exp[-0.5 ((x - xb) ^ 2 + y ^ 2) / rb ^ 2];

nnorm = 2 π rb^2;

(* <a^2> *)
a2 = (Integrate[ a ^ 2 n, {x, -∞, ∞}, {y, -∞, ∞}] / nnorm) // Normal

(* <a^4> *)
a4 = (Integrate[ a ^ 4 n, {x, -∞, ∞}, {y, -∞, ∞}] / nnorm) // Normal

(* <a^4>/<a^2> *)
a42 = Refine[Refine[a4 / a2, {W0 > 0, rb > 0}] // Simplify, {W0 > 0, rb > 0}] // Factor

Clear[P, Q, ξ, ρ]
P = 1 + 4 ρ ^ 2;
Q = 1 + 8 ρ ^ 2;
ξ = xb / W0;
ρ = rb / W0;
(* equation 6 *)
a0inf2 = a0 ^ 2 (P / Q) Exp[-2 ξ ^ 2 / (P Q)]

(* the two expressions are equivalent, confirming equation 6 *)
(a0inf2 - a42 ) // N // Simplify
```

Out[50]= $$\frac{0.282095\, a0^2\, e^{-\frac{0.5\, xb^2}{1.\, rb^2+0.25\, W0^2}}}{rb^2 \sqrt{\frac{0.159155}{rb^2}+\frac{0.63662}{W0^2}}\, \sqrt{\frac{0.5}{rb^2}+\frac{2.}{W0^2}}}$$

Out[51]= $$\frac{0.282095\, a0^4\, e^{-\frac{0.5\, xb^2}{1.\, rb^2+0.125\, W0^2}}}{rb^2 \sqrt{\frac{0.159155}{rb^2}+\frac{1.27324}{W0^2}}\, \sqrt{\frac{0.5}{rb^2}+\frac{4.}{W0^2}}}$$

Out[52]= $$\frac{1.\, e^{-\frac{2.\, W0^2\, xb^2}{32.\, rb^4+12.\, rb^2\, W0^2+1.\, W0^4}} \left(4.\, a0^2\, rb^2+1.\, a0^2\, W0^2\right)}{8.\, rb^2+1.\, W0^2}$$

Out[58]= $$\frac{a0^2\, e^{-\frac{2\, xb^2}{\left(1+\frac{4\, rb^2}{W0^2}\right)\times\left(1+\frac{8\, rb^2}{W0^2}\right) W0^2}} \left(1+\frac{4\, rb^2}{W0^2}\right)}{1+\frac{8\, rb^2}{W0^2}}$$

Out[59]= 0.

# Figure 5

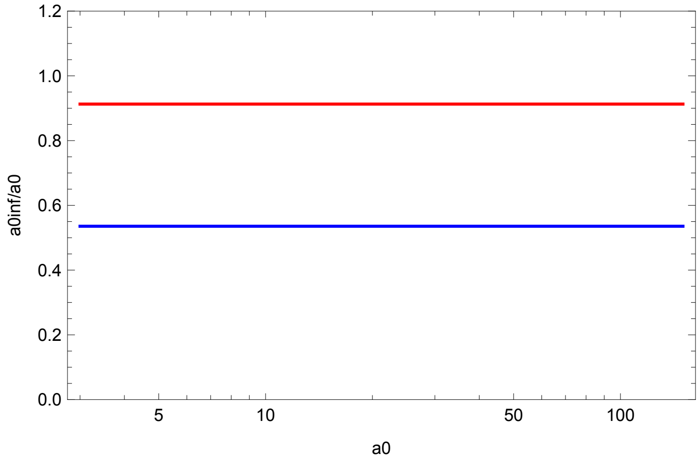
In[404]:= 

$$\text{LogLinearPlot}\left[\left\{e^{-\frac{xb^2}{\left(1+\frac{4\,rb^2}{W0^2}\right)\times\left(1+\frac{8\,rb^2}{W0^2}\right)W0^2}}\sqrt{\frac{1+\frac{4\,rb^2}{W0^2}}{1+\frac{8\,rb^2}{W0^2}}}\ \text{//. \{W0}\to 2,\ \text{rb}\to 0.5,\ \text{xb}\to \text{W0\}},\right.\right.$$

$$\left.e^{-\frac{xb^2}{\left(1+\frac{4\,rb^2}{W0^2}\right)\times\left(1+\frac{8\,rb^2}{W0^2}\right)W0^2}}\sqrt{\frac{1+\frac{4\,rb^2}{W0^2}}{1+\frac{8\,rb^2}{W0^2}}}\ \text{//. \{W0}\to 2,\ \text{rb}\to 0.5,\ \text{xb}\to 0\text{\}}\right\},$$

```
{a0, 3, 150}, PlotRange → {0, 1.2}, PlotStyle → {Blue, Red},
Frame → True, FrameLabel → {"a0", "a0inf/a0"}]
```

Out[404]=



# Figure 6

In[405]:=
$$\texttt{Plot}\Big[\Big\{ e^{-\frac{xb^2}{\left(1+\frac{4\,rb^2}{W0^2}\right)\times\left(1+\frac{8\,rb^2}{W0^2}\right)W0^2}} \sqrt{\frac{1+\frac{4\,rb^2}{W0^2}}{1+\frac{8\,rb^2}{W0^2}}} \;\texttt{//.}\; \{W0\to 2,\; xb\to W0,\; rb\to x\,W0\},$$

$$e^{-\frac{xb^2}{\left(1+\frac{4\,rb^2}{W0^2}\right)\times\left(1+\frac{8\,rb^2}{W0^2}\right)W0^2}} \sqrt{\frac{1+\frac{4\,rb^2}{W0^2}}{1+\frac{8\,rb^2}{W0^2}}} \;\texttt{//.}\; \{W0\to 2,\; xb\to 0,\; rb\to x\,W0\}\Big\},$$

```
{x, 0, 1}, PlotRange → {0, 1.2}, PlotStyle → {Blue, Red},
Frame → True, FrameLabel → {"a0", "a0inf/a0"}]
```

Out[405]=