Nonlinear Breit–Wheeler pair creation with bremsstrahlung γ rays

T G Blackburn and M Marklund 2018 Plasma Phys. Control. Fusion **60** 054009

Notebook: Óscar Amaro, Feb/Apr 2022 @ GoLP-EPP

Physical constants

```
In[45]:= (*α fine structure constant*)
α = 1 / 137; (*[♯]*)
(*c speed of light in vacuum*)
c = 299 792 458; (*[m/s]*)
(*m electron mass*)
m = 0.5109989461 / 1000; (*[GeV]*)
(*ℏ Planck's constant*)
ℏ = 6.626070040 * 10 ^ - 34  / (2 π);
(*e electron abs charge*)
e = 1.6021766208 * 10 ^ - 19;
```

Figure 2

P± probability of pair creation

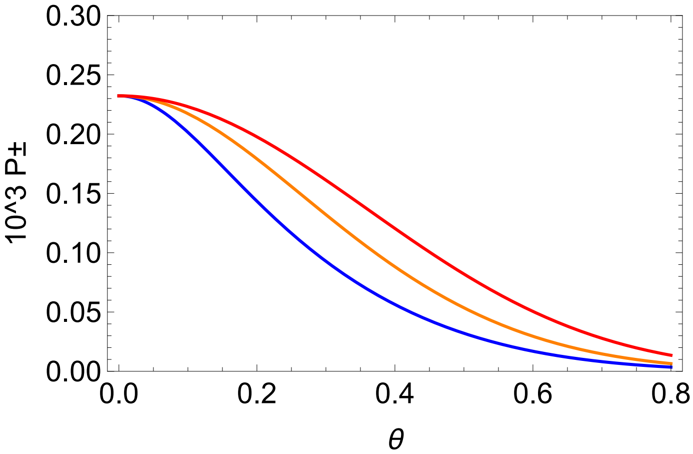
...

Need to multiply probability by ~2500 to reproduce Figure 2 ...

```
In[788]:= Clear[Ppm, ℛ, neff]
Clear[a0, λ, ω0, ω]

(*fixed parameters*)
a0 = 30 (*[#]*);
λ = 0.8 * 10^-6 (*[m]*);
ω0 = 2 π c / λ (*[rad s-1]*);
ω0m = 2 π c / λ ℏ / e  10^-9(*[GeV]*);
ω = 1000 m (*[GeV]*);
ℛ[x_] := 0.453 BesselK[1 / 3, 4/(3 x)]^2 / (1 + 0.145 x^0.25  Log[1 + 2.26 x] + 0.330 x)
neff[θ_, r0_, τ_] := (ω0 τ)/(2 π) (1 + ((c τ)/r0)^2 (Tan[θ / 2]^2)/(Log[4]))^-0.5
Ppm[θ_, r0_, τ_] := 2.5 × 10^3 α a0 neff[θ, r0, τ] × ℛ[(a0 ω0m ω (1 + Cos[θ]))/(m^2)]

(* (c 30 10^-15)/λ = (ω0 30 10^-15)/(2π) ~ 11.24*)
(* (a0 ω0m ω)/(m^2) ~0.091 *)
(*(ω0 30 10^-15)/(2π) ~2.35  10^15*)


Plot[{1000 Ppm[θ, 1 λ, 30 × 10^-15],
  1000 Ppm[θ, 2 λ, 30 * 10^-15], 1000 Ppm[θ, 10 λ, 30 * 10^-15]},
 {θ, 0, 0.8}, Axes → False, Frame → True, FrameStyle → Directive[Black, 15],
 PlotStyle → {Blue, Orange, Red}, FrameLabel → {"θ", "10^3 P±"},
 PlotLabel → "τ=30fs, r0/λ=(1,2,10)", PlotRange → {0, 0.3}]

Plot[{1000 Ppm[θ, 1 λ, 100 * 10^-15],
  1000 Ppm[θ, 2 λ, 100 * 10^-15], 1000 Ppm[θ, 10 λ, 100 * 10^-15]},
 {θ, 0, 0.8}, Axes → False, Frame → True, FrameStyle → Directive[Black, 15],
 PlotStyle → {Blue, Orange, Red}, FrameLabel → {"θ", "10^3 P±"},
 PlotLabel → "τ=100fs, r0/λ=(1,2,10)", PlotRange → {0, 1}]

LogPlot[{1000 Ppm[0, r0 λ, 100 * 10^-15],
  1000 Ppm[π / 8, r0 λ, 100 * 10^-15], 1000 Ppm[π / 4, r0 λ, 100 * 10^-15]},
 {r0, 0, 8}, Axes → False, Frame → True, FrameStyle → Directive[Black, 15],
 PlotStyle → {Blue, Orange, Red}, FrameLabel → {"r0/λ", "10^3 P±"},
 PlotLabel → "θ=(0,π/8,π/4)", PlotRange → {10^-2, 2}]

LogLogPlot[{1000 Ppm[π / 8, 1 λ, τ 10^-15],
  1000 Ppm[π / 8, 2 λ, τ 10^-15], 1000 Ppm[π / 8, 10 λ, τ 10^-15]},
 {τ, 10, 200}, Axes → False, Frame → True, FrameStyle → Directive[Black, 15],
 PlotStyle → {Blue, Orange, Red}, FrameLabel → {"τ[fs]", "10^3 P±"},
 PlotLabel → "θ=π/8, r0/λ=(1,2,10)", PlotRange → {0.05, 1}]
```

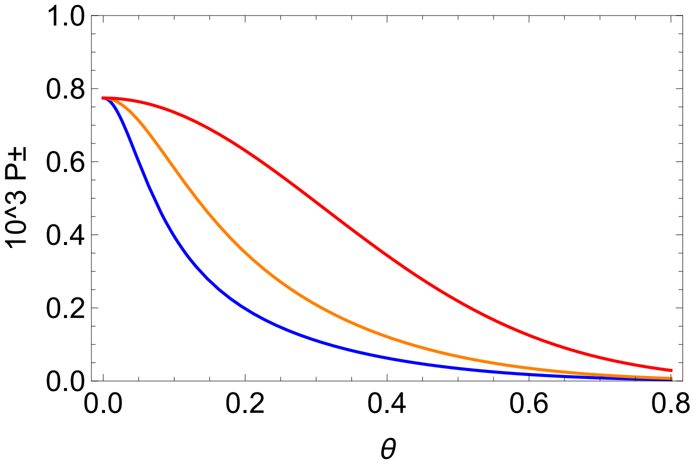
Out[798]=

τ=30fs, r0/λ=(1,2,10)



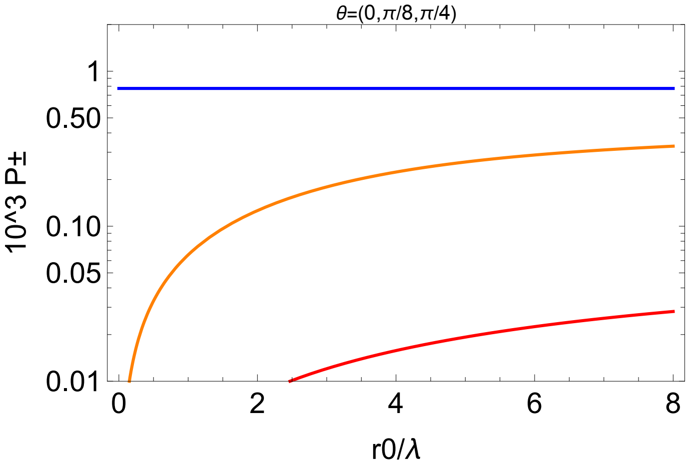
Out[799]=

τ=100fs, r0/λ=(1,2,10)



Out[800]=

θ=(0,π/8,π/4)



Out[801]=

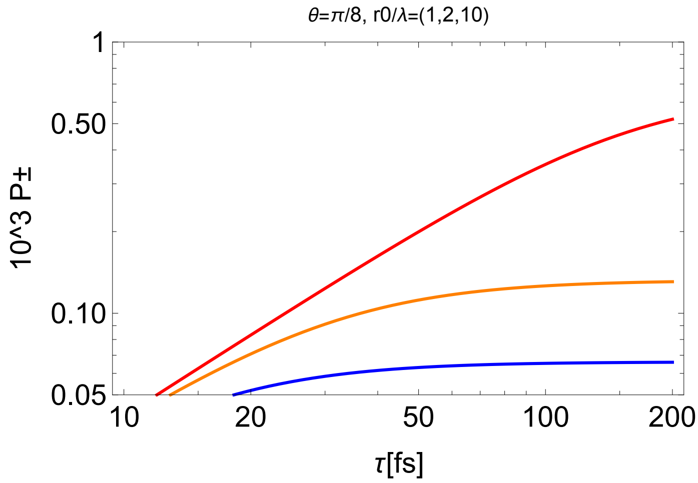
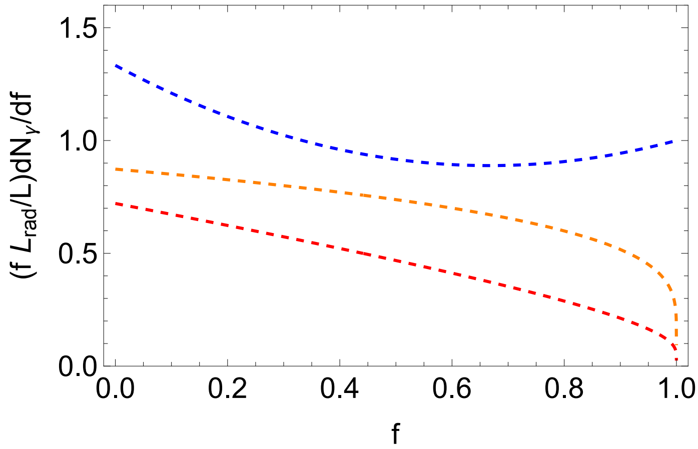


Figure 3

Bremsstrahlung photon generation

```
In[219]:= Clear[Lrad, dNγ8, dNγ9, E0, f, l, χe, L]
Lrad = 5.6 (*[mm]*);
dNγ8[f_, L_] := (Lrad f / L) (L / (Lrad f)) (4/3 - 4/3 f + f^2) (*(8)*)
dNγ9[f_, L_] := (Lrad f / L) ((1 - f) ^ (4 (L / Lrad) / 3) - Exp[-7 (L / Lrad) / 9]) / (f (7/9 + 4/3 Log[1 - f])) (*(9)*)

Plot[{dNγ8[f, 0.2], dNγ9[f, 2], dNγ9[f, 5]}, {f, 0, 1}, PlotRange → {0, 1.6},
 Axes → False, Frame → True, FrameStyle → Directive[Black, 15],
 FrameLabel → {"f", "(f L_rad/L) dN_γ/df"}, PlotStyle →
  {Directive[Blue, Dashed], Directive[Orange, Dashed], Directive[Red, Dashed]}]

(* Figure 3 c *)
E0 = 2000;
LogLogPlot[1000 Sqrt[l] / (E0 / 19.2), {l, 0.001, 1}, PlotRange → {{0.001, 1}, {1, 10}},
 PlotStyle → Directive[Dashed, Black], AxesLabel → {"L/Lrad", "RMS θ (mrad)"}]
```

Out[223]=



Out[225]=

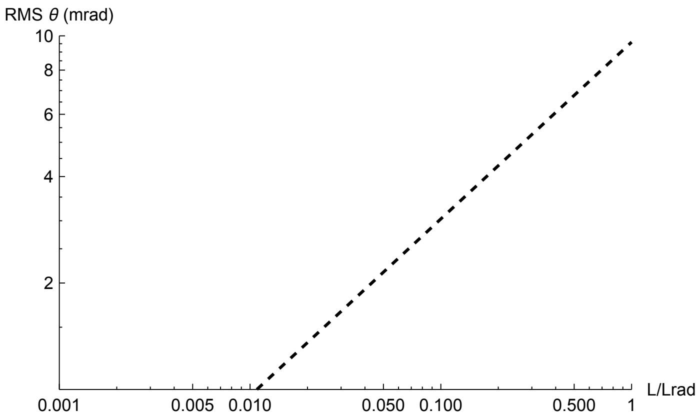
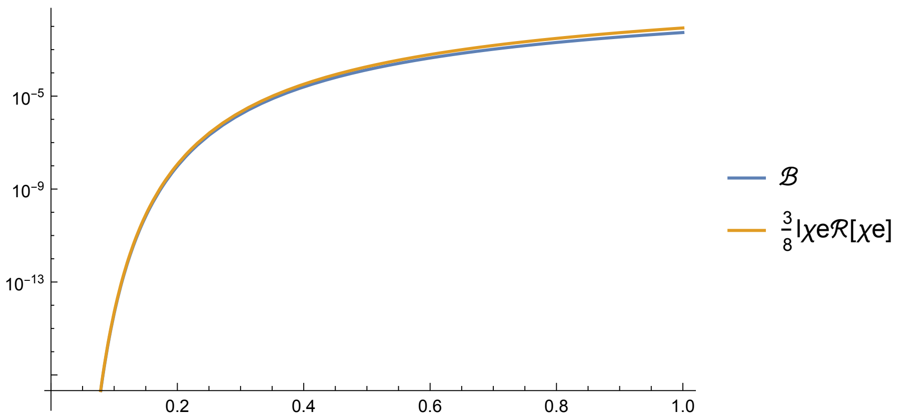


Figure 5

Number of positrons

In[237]:=
```
(*prove ℬ~3/8 lχeℛ[χe] for χe<<1*)
Clear[ℛ, ℬ, ℬ2, l, χe]
ℛ[x_] := (0.453 BesselK[1 / 3, 4/(3 x)] ^ 2)/(1 + 0.145 x ^ 0.25 Log[1 + 2.26 x] + 0.330 x)
ℬ[l_, χe_] := (0.375 l χe ℛ[χe])/(1 + 0.574 χe ^ (2 / 3))
ℬ2[l_, χe_] := 3/8 l χe ℛ[χe]
l = 1;
LogPlot[{ℬ[l, χe] / l, ℬ2[l, χe] / l},
 {χe, 10 ^ -2, 1}, PlotLegends → {"ℬ", "3/8 lχeℛ[χe]"}]


Clear[l, a0, τ, λ, E0, ω0, neff, ℬ]
ℬ[l_, χe_] := (0.375 l χe ℛ[χe])/(1 + 0.574 χe ^ (2 / 3))
a0 = 30;
τ = 40 (*[fs]*);
λ = 0.8 (*[μm]*);
E0 = 2 (*[GeV]*);
ω0 = 2 π c / (λ 10 ^ -6);
neff = (ω0 τ)/(2 π) 10 ^ -15 (*[] should be ~15*)
ω0m = ω0 ℏ / e 10 ^ -9;
χe = E0 a0 ω0m / m ^ 2;

LogLogPlot[α a0 neff ℬ[l, 1] (10 ^ 6), {l, 10 ^ -3, 2}, Axes → False, Frame → True,
 PlotStyle → Directive[Black, Dashed], FrameStyle → Directive[Black, 15],
 FrameLabel → {"L/Lrad", "positrons per pC"}, PlotRange → {8, 1 × 10 ^ 4}]
```

Out[242]=



Out[250]= 14.9896

Out[253]=

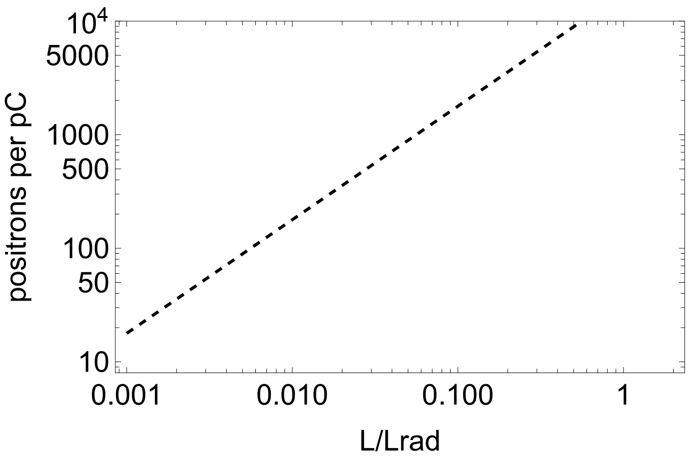


Figure 6

Target thickness which maximizes Breit-Wheeler pair creation

...

The equation 15 and expression in-text don’t match Figure 6 but are consistent with each other ...

In[399]:=
```
Clear[χe, d2Nγ9, Lrad, l, L, dNγdf, getRootl, lopt2]

(* in-text fitted expression *)
lopt2[χe_] := 0.693 χe ^ (1 / 4) + 0.0447 χe ^ (-1 / 5)

(* double differential *)
```

$$\text{dN}\gamma\text{df} = \frac{(1 - f) \wedge (4\, l / 3) - \text{Exp}[-7\, l / 9]}{f\, (7 / 9 + 4 / 3\, \text{Log}[1 - f])};$$

```
D[dNγdf, l]
Lrad = 5.6 (*[mm]*);
```

$$\text{d2N}\gamma 9[\text{f\_}, \text{l\_}] := \frac{\frac{7}{9}\, e^{-7\, l/9} + \frac{4}{3}\, (1 - f)^{4\, l/3}\, \text{Log}[1 - f]}{f \left(\frac{7}{9} + \frac{4}{3}\, \text{Log}[1 - f]\right)}$$

```
(*use (15)*)
(*Get root of equation:
  do we have to do it numerically or is there an analytic solution?*)
getRootl[χe_] := Module[{tab, indx},
  tab = Table[{l, NIntegrate[ℛ[f χe] × d2Nγ9[f , l ], {f, 0, 1}] // Quiet},
    {l, 0.3, 1.3, 1.3 / 10}];
  indx = Position[Abs[tab⟦All, 2⟧ ], Min[ Abs[tab⟦All, 2⟧ ] ]]⟦1, 1⟧;
  Return[tab⟦All, 1⟧⟦indx⟧];
 ]

tab = ParallelTable[{χe, getRoot[χe]}, {χe, 0.1, 5, 0.2}];

plt1 = ListPlot[tab, PlotStyle → Gray,
   AxesLabel → {"χe", "L/Lrad"}, PlotRange → {0, 1.5}];
plt2 = Plot[{0.693 χe ^ 0.25 + 0.0447 χe ^ -0.2}, {χe, 0.1, 5}];
Show[{plt1, plt2}]
```

Out[402]=

$$\frac{\frac{7}{9}\, e^{-7\, l/9} + \frac{4}{3}\, (1 - f)^{4\, l/3}\, \text{Log}[1 - f]}{f \left(\frac{7}{9} + \frac{4}{3}\, \text{Log}[1 - f]\right)}$$

Out[409]=