

On the new and old physics in the interaction of a radiating electron with the extreme electromagnetic field

M. Jirka, et al, Phys. Rev. A 103, 053114

Notebook: Óscar Amaro, April/May 2022 @ [GoLP-EPP](#)

Figure 1

Field components in tight-focusing.

```
In[ ]:= Clear[λ, x, y, z, Bx, E0]
Clear[xR, W, W0, r, ϕ, ϕG, R, ρ, k, ω0, ϕ0, ε]
Clear[S2, S3, S4, S5, S6]
Clear[C2, C3, C4, C5, C6, C7, C8]
Clear[E0max, Bxmax]

λ = 0.8; (*[μm] wavelength*)
W0 = 0.424 λ; (*[μm] spotsize, not explicit in the paper*)
E0 = 1; (*[E] maximum field *)
k = 2 π / λ; (*[μm-1] wavenumber*)
ε = λ / (π W0); (*[] focusing parameter*)
xR = k W0^2 / 2; (*[μm] Rayleigh range*)
r[y_, z_] := Sqrt[y^2 + z^2]; (*[μm] radius*)
ρ[y_, z_] := r[y, z] / W0; (*[] normalized radius*)
W[x_] := W0 Sqrt[1 + (x / xR)^2]; (*[μm] spatially-dependent spotsize*)
R[x_] := x + xR^2 / x; (*[μm] radius of curvature*)
ϕG[x_] = ArcTan[x / xR];
t = 0; (*[ω-1] time *)
ϕ0 = 0; (*[] initial phase*)
(*x = 0; (*[μm] focal plane*)*)
ϕ[x_, y_, z_] := ϕ0 + ω0 t - k x - k r[y, z]^2 / (2 (x + xR^2 / x)); (*[] phase *)

(* auxiliary tight focusing expansion terms*)
C2[x_, y_, z_] := (W0 / W[x])^2 Cos[ϕ[x, y, z] + 2 ϕG[x]];
C3[x_, y_, z_] := (W0 / W[x])^3 Cos[ϕ[x, y, z] + 3 ϕG[x]];
C4[x_, y_, z_] := (W0 / W[x])^4 Cos[ϕ[x, y, z] + 4 ϕG[x]];
C5[x_, y_, z_] := (W0 / W[x])^5 Cos[ϕ[x, y, z] + 5 ϕG[x]];
C6[x_, y_, z_] := (W0 / W[x])^6 Cos[ϕ[x, y, z] + 6 ϕG[x]];
```

```

C7[x_, y_, z_] := (W0 / W[x]) ^ 7 Cos[φ[x, y, z] + 7 φG[x]];
C8[x_, y_, z_] := (W0 / W[x]) ^ 8 Cos[φ[x, y, z] + 8 φG[x]];
(**)
S2[x_, y_, z_] := (W0 / W[x]) ^ 2 Sin[φ[x, y, z] + 2 φG[x]];
S3[x_, y_, z_] := (W0 / W[x]) ^ 3 Sin[φ[x, y, z] + 3 φG[x]];
S4[x_, y_, z_] := (W0 / W[x]) ^ 4 Sin[φ[x, y, z] + 4 φG[x]];
S5[x_, y_, z_] := (W0 / W[x]) ^ 5 Sin[φ[x, y, z] + 5 φG[x]];
S6[x_, y_, z_] := (W0 / W[x]) ^ 6 Sin[φ[x, y, z] + 6 φG[x]];

(* fields *)
Bx[x_, y_, z_] :=
  E0 Exp[-r[y, z] ^ 2 / W[x] ^ 2] (ε ^ 2 (S2[x, y, z] - ρ[y, z] ^ 2 × S3[x, y, z]) +
    ε ^ 4 (S3[x, y, z] / 2 + ρ[y, z] ^ 2 × S4[x, y, z] / 2 - 5 ρ[y, z] ^ 4 × S5[x, y, z] / 4 +
      ρ[y, z] ^ 6 × S6[x, y, z] / 4) ) // Quiet(*Bx equation 2*)

Eθ[x_, y_, z_] :=
  E0 Exp[-r[y, z] ^ 2 / W[x] ^ 2] (ε ρ[y, z] × C2[x, y, z] + ε ^ 3 (ρ[y, z] × C3[x, y, z] / 2 +
    ρ[y, z] ^ 3 × C4[x, y, z] / 2 × ρ[y, z] ^ 5 × C5[x, y, z] / 4) +
    ε ^ 5 (3 ρ[y, z] × C4[x, y, z] / 8 + 3 ρ[y, z] ^ 3 × C5[x, y, z] / 8 +
      3 ρ[y, z] ^ 5 × C6[x, y, z] / 16 - ρ[y, z] ^ 7 × C7[x, y, z] / 4 +
      ρ[y, z] ^ 9 × C8[x, y, z] / 32) ) // Quiet(*Eθ equation 3*)

(* find maximum fields for plotting *)
Eθmax = FindMaximum[Abs[Eθ[0, 0, z]], {z, 0, 3 λ}] [[1]];
Bxmax = FindMaximum[Abs[Bx[x, 0, 0]], {x, 0, 3 × 0.5 λ}] [[1]];

In[ ]:= Clear[pltBx, pltEθ, pnts]
pnts = 25;
pltBx = ContourPlot3D[Abs[Bx[x λ, y λ, z λ]], {x, -0.5, 0.5}, {y, 0, 1},
  {z, -1, 1}, Contours → Table[i, {i, 0.5, 1, 0.1 / 2}] Bxmax, Mesh → None,
  MaxRecursion → 0, PlotPoints → pnts, AxesLabel → {"x/λ", "y/λ", "z/λ"},
  ColorFunction → "BlueGreenYellow", PlotRange → All];
pltEθ = ContourPlot3D[Abs[Eθ[x λ, y λ, z λ]], {x, -0.5, 0.5}, {y, 0, 1}, {z, -1, 1},
  Contours → Table[i, {i, 0.5, 1, 0.1 / 2}] Eθmax, Mesh → None, MaxRecursion → 0,
  PlotPoints → pnts, ColorFunction → "ThermometerColors", PlotRange → All];
Show[{pltBx, pltEθ}]

```

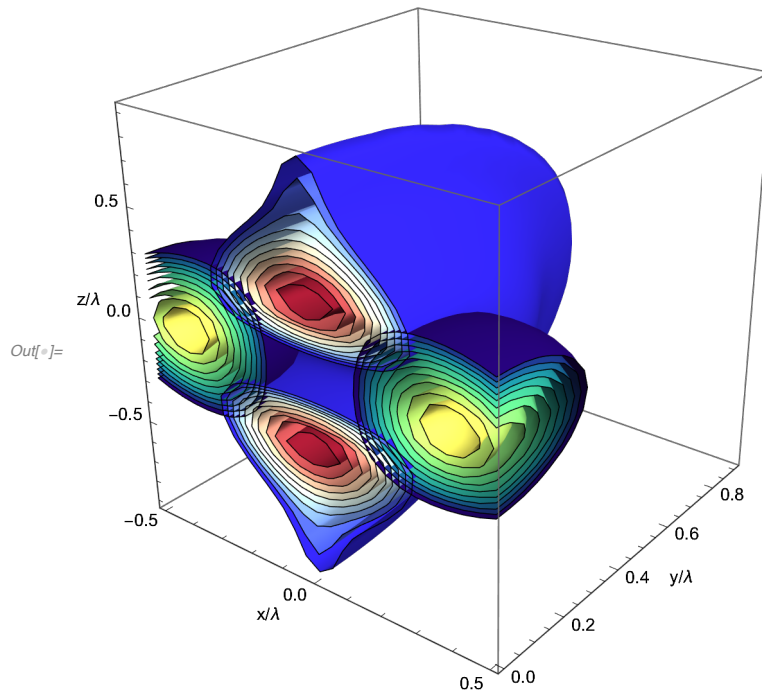


Figure 2

Ratio of electrons reaching $\alpha \chi e^{2/3} \sim 1$

For fixed χe and varying electron energy, the laser intensity varies accordingly. If the implementation of the equations is correct, then actually we are fixing $\alpha \chi e^{2/3} \sim 2.4$ (and not 1)...

```
In[159]:= Clear[χe, δe, γe, Wγ, pc, tc, ω0, E0, δc, me, c, e, α, ħ, T, ES, λμm, τ, W0, aS]
```

```
W0 = 0.424 λμm; (*[μm] spotsizes, not explicit in the paper*)
```

```
χe = 2 γe E0 / ES;
```

```
γe = δe / (me c ^ 2);
```

```
Wγ = 3 ^ (2 / 3) × 28 Gamma[2 / 3] α me ^ 2 c ^ 4 χe ^ (2 / 3) / (54 ħ δe);
```

```
pc = Wγ tc;
```

```
tc = τ / (2 Sqrt[2 Log[2]]);
```

```
ω0 = 2 π c / (λμm 10 ^ -6);
```

```
E0 = 0.855 λμm Sqrt[I0 10 ^ -18] me ω0 c / e;
```

```
δc = (1 - 16 / 63) ^ pc δe;
```

```
T = 0.67  $\frac{\lambda\mu m 10^{-6}}{c}$ ;
```

```
(*[s] the 0.67 factor is needed to reproduce the results *)
```

```
(* physical constants *)
```

```
me = 9.11 × 10 ^ -31; (*[Kg]*)
```

```
c = 299 792 458; (*[m/s]*)
```

```
e = 1.602176634 × 10 ^ -19; (*[C]*)
```

```
α = 1 / 137; (*[ ]*)
```

```
ħ = 1.054571817 × 10 ^ -34; (*[J s]*)
```

```
ES = me ^ 2 c ^ 3 / (e ħ); (*[V/m]*)
```

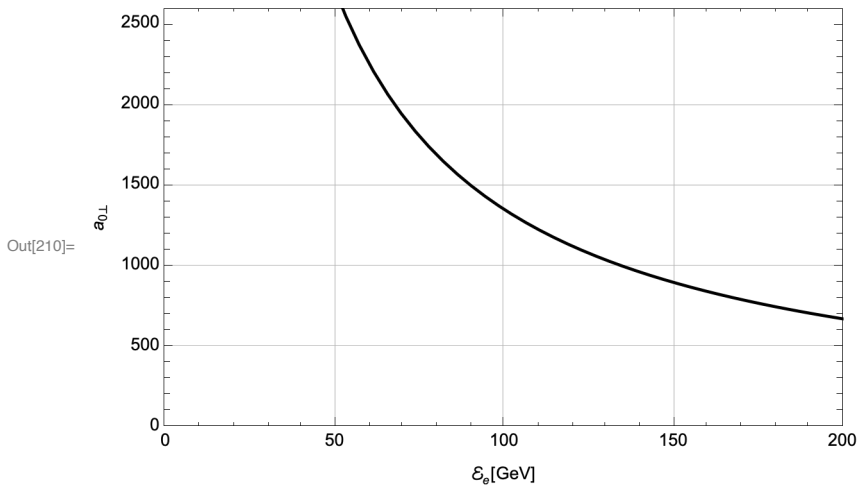
```
aS = 329 719; (*[ ] for λ=0.8*)
```

In[210]:= (* required a0⊥ for $\alpha \chi^2/3=1$ *)

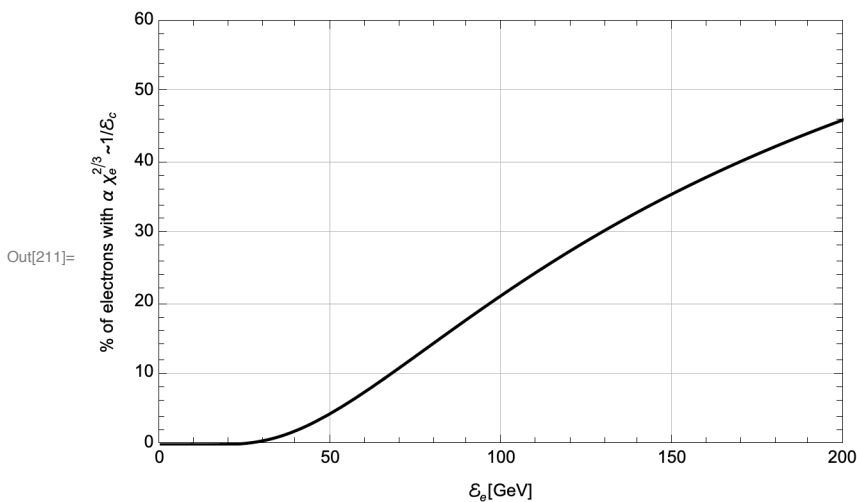
```
Plot[ $\left\{\frac{aS}{2 \delta_{ee} 10^9 e / (me c^2)} (1/\alpha)^{1.5}\right\}, \{\delta_{ee}, 0, 200\},$ 
  PlotRange → {{0, 200}, {0, 2600}}, Frame → True, ImageSize → 400,
  FrameLabel → {" $\delta_e$ [GeV]", " $a_{0\perp}$ "}, PlotStyle → Black, GridLines → Automatic]
```



```
Plot[ $\left(100 \left(\frac{47}{63}\right)^{\frac{0.030199136711634034 \cdot c^4 me^2 \left(\frac{\sqrt{10} \delta_e}{e ES}\right)^{2/3} \alpha \tau}{\delta_e \hbar}}\right) // . \{\delta_e \rightarrow \delta_{ee} 10^9 e, \tau \rightarrow T/2, \lambda \mu m \rightarrow 0.8,$ 
  I0 →  $\left(\frac{aS}{2 \delta_{ee} 10^9 e / (me c^2)} (2.4/\alpha)^{1.5} / (0.855 \lambda \mu m)\right)^2 10^{18},$ 
  { $\delta_{ee}, 0.1, 200\}$ , PlotRange → {{0, 200}, {0., 60}}, Frame → True,
  ImageSize → 400, FrameLabel → {" $\delta_e$ [GeV]", "% of electrons with  $\alpha \chi_e^{2/3} \sim 1/\delta_c$ "},
  PlotStyle → Black, GridLines → Automatic]
```



General: $\frac{1}{63^{5072.25}}$ is too small to represent as a normalized machine number; precision may be lost.



(* page 3/6 text says $\alpha_0=960$, $\varepsilon=140$ GeV \rightarrow $\%=1/3$ *)

$$\frac{aS}{2 \varepsilon e / (m e c^2)} (1 / \alpha)^{1.5} /. \{\varepsilon e \rightarrow 140 \times 10^9 e\}$$

(* indeed if the χe is corrected for, this is the percentage obtained *)

$$\left(100 \left(\frac{47}{63} \right) \frac{0.030199136711634034 \cdot c^4 m e^2 \left(\frac{\sqrt{10} \varepsilon e}{e E S} \right)^{2/3} \alpha \tau}{\varepsilon e h} \right) /. \{\varepsilon e \rightarrow 140 \times 10^9 e, \tau \rightarrow T / 2,$$

$$\lambda \mu m \rightarrow 0.8, I_0 \rightarrow \left(\frac{aS}{2 \varepsilon e / (m e c^2)} (2.4 / \alpha)^{1.5} / (0.855 \lambda \mu m) \right)^2 10^{18} \}$$

Out[208]= 964.975

Out[209]= 33.1261