25th May 2024 SATURDAY.

Solving the ICPC A problem.

We have 5 questions, giving us 5 equations of the form

$$\left.\begin{aligned} aj_1 + bk_1 + cl_1 &= r_1 \\ aj_2 + bk_2 + cl_2 &= r_2 \\ aj_3 + bk_3 + cl_3 &= r_3 \\ aj_4 + bk_4 + cl_4 &= r_4 \\ aj_5 + bk_5 + cl_5 &= r_5 \end{aligned}\right\} \text{System of equations.} \quad \ldots 1$$

Fig 1.

In the problem, one, or four of the equations is false.
With my current solution, that wouldn't even matter, we'll always be able to get the solution.
We just need a set of 4 of these equations to have same values.
Using a matrix to simplify my expression.

$$\left|\begin{matrix} j_1 & k_1 & l_1 \\ j_2 & k_2 & l_2 \\ j_3 & k_3 & l_3 \\ j_4 & k_4 & l_4 \\ j_5 & k_5 & l_5 \end{matrix}\right| = \left|\begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{matrix}\right| \begin{matrix} ----- q_1 \\ ----- q_2 \\ ----- q_3 \\ ----- q_4 \\ ----- q_5 \end{matrix}$$

Fig 2.

The number of possible 4 set combination $= {}^5C_4$
$= 5$

set of 4 set combo $(A) = \{e_1, e_2, e_3, e_4, e_5\}$

Where the members of A represent a 4 equation combo.

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$$\left.\begin{aligned} E_1 &= \{q_1, q_2, q_3, q_4\} \\ E_2 &= \{q_1, q_2, q_3, q_5\} \\ E_3 &= \{q_1, q_2, q_4, q_5\} \\ E_4 &= \{q_1, q_3, q_4, q_5\} \\ E_5 &= \{q_2, q_3, q_4, q_5\} \end{aligned}\right\} A$$

Fig 3.

Only one of $E_1, E_2 \cdots E_5$ will be true.

Proof: Only one q_n $\{n \in N : 1 \leq n \leq 5\}$ is false. Every q_n appears in every E_n $\{n \in N : 1 \leq n \leq 5\}$ except one E_n. hence the 4 values of E_n, q_n appears in will be false, and only one E_n out of the 5 will be true.

How to verify if E_n is true.

E_n is given by 4 equations of the form q_n. In order to verify E_n, these 4 equations must be solved simultaneously to provide integer solutions for (a, b, c).

Algorithm for Solution: Using E_1 as a case study.

We have E_1 given by:

$$\left.\begin{vmatrix} j_1 & k_1 & l_1 \\ j_2 & k_2 & l_2 \\ j_3 & k_3 & l_3 \\ j_4 & k_4 & l_4 \end{vmatrix} = \begin{vmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{vmatrix} \begin{matrix} \cdots q_1 \\ \cdots q_2 \\ \cdots q_3 \\ \cdots q_4 \end{matrix}\right\} E_1$$

Fig 4

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$$\left.\begin{vmatrix} (j_1+j_2) & (k_1+k_2) & (l_1+l_2) \\ j_3 & k_3 & l_3 \\ j_4 & k_4 & l_4 \end{vmatrix} = \begin{vmatrix} r_1+r_2 \\ r_3 \\ r_4 \end{vmatrix} \begin{matrix} \cdots & (q_1+q_2) \\ \cdots & q_3 \\ \cdots & q_4 \end{matrix}\right\} E_1$$

This can further be resolved using Crammer's Rule.
Now to generalize this for E_i we have the following:
N.B: $i \in N : 1 \leq i \leq 5$

Algorithm for Verifying E_i is true.

Let $n = 5$ represent the number of equations given, I know this will always be 5, but I just like place holders. Please do not let this n confuse you with the n used earlier in the solution. Or better yet in order to avoid this confusion:

Let $k = 5$ represent the number of equations given.
We have E_i given by:

$$\left.\begin{vmatrix} m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \\ m_3 & n_3 & p_3 \\ m_4 & n_4 & p_4 \end{vmatrix} = \begin{vmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{vmatrix}\right\} \cdot E_i$$

$$\left.\begin{vmatrix} (m_1+m_2) & (n_1+n_2) & (p_1+p_2) \\ m_3 & n_3 & p_3 \\ m_4 & n_4 & p_4 \end{vmatrix} = \begin{vmatrix} (h_1+h_2) \\ h_3 \\ h_4 \end{vmatrix} \cdot \begin{vmatrix} a \\ b \\ c \end{vmatrix}\right\} E_i$$

$$Y \qquad \cdot \qquad X$$

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Solving using Crammer's Rule we have Y as:

$$Y = (m_1+m_2)\begin{vmatrix} n_3 & p_3 \\ n_4 & p_4 \end{vmatrix} - (n_1+n_2)\begin{vmatrix} m_3 & p_3 \\ m_4 & p_4 \end{vmatrix} + (p_1+p_2)\begin{vmatrix} m_3 & n_3 \\ m_4 & n_4 \end{vmatrix}$$

$$Y = (m_1+m_2)[(n_3 p_4) - (n_4 p_3)] - (n_1+n_2)[(m_3 p_4) - (m_4 p_3)]$$

$$+ (p_1+p_2)[(m_3 n_4) - (m_4 n_3)]$$

$V_{00} \quad V_{01} \quad V_{02}$

$V_{10} \quad V_{11} \quad V_{12}$

$V_{20} \quad V_{21} \quad V_{22}$

$V_{00} * [V_{11} * V_{22} - V_{12} * V_{21}]$

$-V_{01} * [V_{10} * V_{22} - V_{12} * V_{20}]$

$V_{02} * [V_{10} * V_{21} - V_{11} * V_{20}]$

$$a = \frac{\begin{vmatrix} (h_1+h_2) & (n_1+n_2) & (p_1+p_2) \\ h_3 & n_3 & p_3 \\ h_4 & n_4 & p_4 \end{vmatrix}}{Y}$$

$$b = \frac{\begin{vmatrix} (m_1+m_2) & (h_1+h_2) & (p_1+p_2) \\ m_3 & h_3 & p_3 \\ m_4 & h_4 & p_4 \end{vmatrix}}{Y}$$

$$c = \frac{\begin{vmatrix} (m_1+m_2) & (n_1+n_2) & (h_1+h_2) \\ m_3 & n_3 & h_3 \\ m_4 & n_4 & h_4 \end{vmatrix}}{Y}$$

If $(a, b, c) \in N$ then it is our solution.

Python Implementation of Solution

Step 1: Choose values for $j_1 \ldots j_5$, $k_1 \ldots k_5$, $l_1 \ldots l_5$

These values should be chosen such that no three equation subset of the first system of equations has $Y \Rightarrow$ undefined.

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I don't reall have energy to think of values for these guys so I'm just gonna use the values povided in test case 1.

1	1	1		r_1	...	q_1	9	0	9
1	1	1		r_2	...	q_2	0	9	0
5	0	1	=	r_3	...	q_3	0	0	9
1	0	0		r_4	...	q_4	1	1	1
1	1	0		r_5	...	q_5	0	0	1

I just realized this two are the same, so I'm gonna use test case 2 instead [illegible]

$r_1 \ldots r_5$ will be collected from the user so, I'm just gonna leave that blank

I would've used a loop for the next part, but I think not using a loop makes it easier to understand what's going on.

Please refer back to Fig 3, and you'll notice that, what we're basically doing there is this:

$$\left.\begin{array}{l} E_1 = \{\{\ldots h_1\}, \{\ldots h_2\}, \{\ldots h_3\}, \{\ldots h_4\}\} \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ E_5 = \{\{\ldots h_2\}, \{\ldots h_3\}, \{\ldots h_4\}, \{\ldots h_5\}\} \end{array}\right\}$$

This is simply because the value of $j_1 \ldots j_5$, $k_1 \ldots k_5$, $l_1 \ldots l_5$ are constant.

Sorry, my bad I forgot I'm not hardcoding it.

Continuing In Code