STA211

BASIC PROBABILITY

Definition of Terms;

A random Experiment is an experiment whose outcome cannot be predicted with certainty

Examples include; Rolling a die <1,2,3,4,5,6> Tossing a coin once <H,T> or Twice <HH, HT, TH, TT>

Sample space: The set S consisting of all possible outcome of a random experiment is a Sample Space and each outcome is called a Sample point.

For rolling a die S = <1, 2, 3, 4, 5, 6> and flipping of coin once S = <H, T>

- If S has a finite number of points, it is called a finite Sample Space.
- If S has many points as there are natural number that is <1, 2, . . . 3> it is called a countably infinite S
- If S has a many points as there are for some interval on the x-axis such that $0 \leq x \leq 1$ it is called a non

Countably infinite S.

- A sample space that is finite or Countably infinite is often Called a discrete S.
- A sample space that is non-Countably infinite is Called a non-discrete

Events: An event is a Subset, A of the Sample Space S. It is a set of possible or likely outcome

Example: If the toss of a Coin twice, the event that only the head shows up is {HT, TH} from the sample space
$S = \{HH, HT, TH, TT\}$

- $A \cup B$ is the event either A or B or Both
- $A \cap B$ is the event both A and B

A' or A^T is the event not A
$A - B = A \cap B'$ is the event A but not B
in particular
$A' = S - A$

Mutually Exclusive Events

If the set Corresponding to A and B are disjoint that is $A \cap B = \phi$ we then say

that A and B are mutually exclusive
This simply means that A and B events both occur together

Probability
This is the measure of the occurance of an event
Given an event (E) the probability that E will occur is

$$Prob(E) = \frac{\text{Number of outcomes possible to Event (E)}}{\text{Total number of outcome}}$$

$$= \frac{(E)}{(S)}$$

Axioms of Probability

1. For every event A in the class C $P(A) \geq 0$

2. For the same or Certain event ~~So~~ So in the class C
$F(S) = 1$

3. For any number of mutually exclusive events A_1, A_2 - - - -
$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$
$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2)$

4. If A^c is the Complement of A, then
$P(A^c) = 1 - P(A)$

5. If $A = A_1 \cup A_2 \cup \cdots \cup A_n$ where $A_1, A_2 \ldots A_n$ are mutually exclusive then
$P(A) = P(A_1) + P(A_2) + \cdots + P(A_n)$

In particular if $A = S$, then
$P(A_1) + P(A_2) + \cdots P(A_n)$

6. If $A \subset A_2$, then $P(A_1) = P(A_2)$ and
$P(A_2 - A_1) = P(A_2) - P(A_1)$

7. If every event A $0 \leq P(A) \leq 1$

8. $P(\phi) = 0$

9 If A and B are any two events then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

More generally, if A_1, A_2 and A_3 are any 3 events, then
$P(A_1 \cup A_2 \cup A_3) - P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

10. For any events A and B
$P(A) = P(A \cap B) + P(A \cap B')$

11. If an events A must result in the occurance of one of the mutually exclusive

events
$A_1 A_2 \ldots\ldots$, And then
$P(A)^2 = P(A \cap A_1) + P(A \cap A_2) + \cdots\cdots + P(A \cap A_n)$

Conditional Probability & Marginal Probability

Suppose 150 students sat for STA 110 examinations and their performances were recorded and tabulated as shown

Gender	Pass	Fail
F	30	55
M	29	36

Conditional Probability

$$P(P/F) = \frac{P(P \cap F)}{P(F)}$$

$P(F)$
$P(M)$ $\Big\}$ marginal probability

$$P(F) = \frac{85}{150}$$

CamScanner

$P(P|M) = \cancel{P(P|M}\ \dfrac{P(P \cap M)}{P(M)} = \dfrac{29}{65}$

Let A denote the event that a Student is a male and B denote the event that the student is Benin. In a class of 100 students Suppose 60 are benin and 10 of the benin students are males, find the probability that a randomly chosen benin student is a male

Solution

A ⟶ Male

B ⟶ Benin

* $P(B) = 0.6$

* $P(A \cap B) = \dfrac{10}{100} = 0.1$

* $P(B/A) = \dfrac{P(B \cap A)}{P(B)} = \dfrac{0.1}{0.6} = 0.16$

The probability of an applicant to be admitted into a university is 0.8, the probability for a student to have a hostel accomodation is 0.6. What is the probability that an applicant will admitted to the university and given a hostel space?

Solution

$P(\text{Admitted}) = 0.8$

$P(\text{hostel}) = 0.6$

$$P(A \cap B) = \frac{P(A \cap B)}{P(}$$

$$P(B \cap A) = \frac{P(B \cap A)}{P(A)} = \frac{0.48}{0.8} = 0.48$$

$$P(A \cap B) = P(A)\,P(B/A)$$

$$= 0.8 \times 0.6$$

$$= 0.48$$

To show wether the ~~ve~~ events are independent

$$P(P|M) = \frac{P(P \cap M)}{P(M)} = \frac{\frac{29}{150}}{\frac{65}{180}} = \frac{29}{65} = 0.44$$

~~Adult~~	with Sugar	without sugar
Male	156	84
Female	169	90

Solu

M = Males

$$P(wts) = \frac{175}{500}$$

$$\frac{P(Wts \cap M)}{P(M)} = \frac{84}{\frac{\cancel{540}}{\cancel{240}}} \quad \frac{\frac{84}{500}}{\frac{240}{500}} = \frac{84}{240}$$

Let A and B be two independent event such that probability of B given $A \cup B$ $P(B/A \cup B)$ is equal to $\frac{2}{3}$ and Probability ~~of B~~ $P(A/B) = \frac{1}{2}$. Find $P(B)$?

Find the probability of getting four 6 and then another number in 5 random roles of a balanced die

Solution

1. $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{7776} = 0.00064$

In a Clinic the probability that a blood Sample shows Cancerous cells is 0.05. 4 blood samples are tested and the samples are independent.

a. What is the probability that none shows Cancerous cell.

b. What is the probability that exactly one Sample shows Cancerous Cell

c. What is the probability that at least one Sample shows Cancerous cell

Solution

1. $(1-0.05)^4 = 0.95^4 \quad P(C_1^c \cap C_2^c \cap C_3^c \cap C_4^c)$

$= 0.814$

2. $(0.05)(0.95)^3 = (0.0429)^{4}$

Bayes' Theorem pg (75)

A large manufacturer uses three different trucking Companies (A, B, C) to deliver products. The probability that a randomly selected shipment is delivered by each Company is $P(A) = 0.60$, $P(B) = 0.25$ and $P(C) = 0.15$.

Occasionally, a shipment is damaged (D) in transit with the probabilities $P(D/A) = 0.01$, $P(D/B) = 0.005$ and $P(D/C) = 0.015$.

Let us assume a shipment is selected at random

Soln

1. $P(D/B) = 0.005$

$P(B \cap D) = P(D/B)\, P(B)$

$= 0.005 \times 0.25$

$=$

2. $P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$

$= P(D/A)\, P(A) + P(D/B)\, P(B) + P(D/C)\, P(C)$

$=$

3. Probability

The Completion of a highway Construction maybe delayed because of a projected storm. The probabilities are 0.6 that there will be a storm, 0.85 that the Construction job will be Completed on time if there is no storm, and 0.35 that the Construction will be Completed on time if there is storm. What is the probability that the Construction job will be Completed on time.

Solution

$P(\text{storm}) = \cancel{0.85}\ 0.6$

$P(C/NS) = 0.85$

$P(C/S) = 0.35$