OPTION B

DEPARTMENT OF MATHEMATICS
FACULTY OF PHYSICAL SCIENCES
UNIVERSITY OF BENIN, BENIN CITY

FIRST SEMESTER EXAMINATIONS 2016/2017 SESSION

COURSE CODE: MATHEMATICAL MEYTHOD I (MTH 218) **TIME ALLOWED: 2 Hours.**

INSTRUCTIONS: (i) Write and circle your attendance list serial number on the objective answer paper. (ii) Attempt all questions by SHADING (using HB pencil) the letter box that corresponds to the correct option. Information about your Mat. No., Name, Course code, Faculty code and Department code must be clearly written and **CORRECTLY SHADED. YOU MUST SUBMIT YOUR QUESTION PAPER ALONG WITH YOUR ANSWER SHEET.**

NAME_____**MAT. NO.**_____

1. Solve $\int tan^3 x sec^2 x\, dx$ (a) $\frac{tan^4 x}{4} + \frac{sec^3 x}{3} + C$ (b) $\frac{tan^4 x}{4} - \frac{tan^2 x}{2} + C$
   (c) $\frac{sec^4 x}{4} - \frac{sec^2 x}{2} + C$ (d) $\frac{tan^4 x}{4} - \frac{sec^3 x}{3} + C$ (e) none of the above
2. Solve $\int sin^3 x cos^2 x\, dx$ (a) $\frac{sin^4 x}{4} + \frac{sin^6 x}{6} + C$ (b) $-\frac{cos^4 x}{4} - \frac{cos^6 x}{6} + C$
   (c) $\frac{sin^4 x}{4} - \frac{sin^6 x}{6} + C$ (d) $\frac{cos^4 x}{4} - \frac{cos^6 x}{6} + C$ (e) none of the above
3. If $I_n = \int -x^n e^x dx$, obtain the reduction formula for I_n . (a) $(n-1)I_{n-1} - x^{n-1}e^x$
   (b) $(n+1)I_{n+1} - x^{n+1}e^x$ (c) $nI_{n-1} - x^n e^x$ (d) $(n+1)I_{n-1} - x^n e^x$ (e) none of the above
4. If $I_n = \int_0^{\pi/2} x^n \sin x dx$ (a) $n(\pi/2)^{n-1} + n(n-1)I_{n-1}$ (b) $n(\pi/2)^{n-1} - n(n-1)I_{n-2}$
   (c) $n(\pi/2)^{n} - (n+1)I_{n-1}$ (d) $(n-1)(\pi/2)^{n-1} - nI_{n-2}$ (e) none of the above
5. Determine the reduction formula for $I_n = \int_0^{\pi} x^n cosx\, dx$ (a) $I_n = -n\pi^{n-1} - n(n-1)I_{n-1}$
   (b) $I_n = n\pi^{n-1} - n(n-1)I_{n-2}$ (c) $I_n = -n\pi^{n-1} - n(n-1)I_{n-2}$
   (d) $I_n = -n\pi^{n-1} + n(n-1)I_{n-1}$ (e) none of the above
6. Solve $\int \frac{dx}{9-x^2}$ (a) $\frac{1}{6}\ln\left(\frac{3-x}{3+1}\right) + C$ (b) $\frac{1}{6}\ln\left(\frac{3+1}{3-1}\right) + C$ (c) $\frac{1}{3}\ln\left(\frac{3-x}{3+1}\right) + C$
   (d) $\frac{1}{3}\ln\left(\frac{3+x}{3-1}\right) + C$ (e) none of the above
7. Differentiating $f(x,y,z) = e^{1-x\cos y} + ze^{\frac{-1}{(1+y^2)}}$ once with respect to x at $(1,0,\pi)$ gives
   (a) 1 (b) -1 (c) -2 (d) 0 (e) none of the above

OPTION B

8. Find the first partial derivative of $f(x,y) = x^3y^2 + 3xe^{-y}$ with respect to y.
(a) $3x^2y^2 + 3e^{-y}$ (b) $2x^3y - 3xe^{-y}$ (c) $2x^3y + 3xe^{-y}$ (d) $3x^2 - 2xe^{-y}$
(e) none of the above

9. Solve $\int x^2 \sin(\pi - x^3)\ dx$ (a) $\frac{\cos(\pi - x^3)}{3} + B$ (b) $\frac{\sin(\pi - x^3)}{3} + B$ (c) $-\frac{\cos(\pi - x^3)}{3} + B$
(d) $-\frac{\sin(\pi - x^3)}{3} + B$ (e) none of the above

10. Solve $\int (cos^3x + 2)sinx\ dx$ (a) $(\frac{cos^4x}{4} - 2cosx) + C$ (b) $-(\frac{cos^4x}{4} + 2cosx) + C$
(c) $-(\frac{2sinx}{4} + 2cos^4x) + C$ (d) $(\frac{sin^4x}{4} - 2sinx) + C$ (e) none of the above

11. Find the nth differential coefficient of $y = \log_e x$ (a) $(-1)^n n!\, x^{-n}$ (b) $(-1)^{n-1}(n-1)!\, x^{-n}$
(c) $n!\, x^{-n}$ (d) $(1)^n(n-1)!\, x^{-n}$ (e) none of the above

12. Find the modulus of $-5 - 5i$. (a) $\sqrt{25}$ (b) $-5\sqrt{2}$ (c) $3\sqrt{2}$ (d) $5\sqrt{2}$ (e) none of the above

13. If $z = -1 + i$ and $w = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ then $z * w$ is (a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) $i\sqrt{2}$
(d) $-i\sqrt{2}$ (e) none of the above

14. Express the complex number $z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ in polar form (a) $\cos\frac{\pi}{4} - \text{isin}\frac{\pi}{4}$
(b) $\sqrt{2}\left(\cos\frac{\pi}{4} + \text{isin}\frac{\pi}{4}\right)$ (c) $\cos\frac{\pi}{2} + \text{isin}\frac{\pi}{2}$ (d) $\cos\frac{\pi}{4} + \text{isin}\frac{\pi}{4}$ (e) none of the above

15. Express the complex number $z = -1 + i$ in polar form (a) $\sqrt{2}\left(\cos\frac{3\pi}{4} + \text{isin}\frac{3\pi}{4}\right)$
(b) $\sqrt{2}\left(\cos\frac{3\pi}{4} - \text{isin}\frac{3\pi}{4}\right)$ (c) $\left(\cos\frac{3\pi}{4} + \text{isin}\frac{3\pi}{4}\right)$ (d) $\sqrt{2}\left(\cos\frac{\pi}{4} + \text{isin}\frac{\pi}{4}\right)$
(e) none of the above

16. If $z_1 = -1 + i$ and $z_2 = -1 + 5i$ find the real part of z_1-z_2 (a) -4 (b) 0 (c) 2
(d) 1 (e) none of the above

17. If $z_1 = x_1 + \mathbf{i}y_1$ and $z_2 = x_2 + \mathbf{i}y_2$ then (a) $|z_1 + z_2| \geq |z_1| + |z_2|$ (b) $|z_1 + z_2| \leq |z_1| + |z_2|$
(c) $|z_1 - z_2| \geq |z_2|$ (d) $|z_1| = |z_2|$ (e) none of the above

18. Express the complex number $z = 3.5e^{1.12i}$ in the form $a + ib$ (a) $\log z = 3.5 + 1.12i$
(b) $\log z = 3.5 - 1.12i$ (c) $\log z = 3.5$ (d) $\log z = 1.12i$ (e) none of the above

19. Express $1 - i\sqrt{3}$ in Exponential form. (a) $2e^{i\frac{3}{5}\pi}$ (b) $2e^{i\frac{5}{3}\pi}$ (c) $2e^{-i\frac{5}{3}\pi}$ (d) $e^{i\frac{5}{3}\pi}$
(e) none of the above

20. If $z = x + iy$ then $z\bar{z}$ = (a) $x^2 - y^2$ (b) $x^2 + y^2$ (c) $x^2 + iy^2$ (d) $x^2 - iy^2$
(e) none of the above

21. A function $f(x,y)$ is said to have a saddle point at point (x_o, y_o) if which of the following condition holds.

(a) $f_x \cdot f_{yy} < (f_{xy})^2$ (b) $f_{xx} \cdot f_{yy} < (f_{xy})^2$ (c) $f_{xx} < f_{xy}$ (d)) $f_x \cdot f_{yy} < f_{xy}$ (e) none of the above

22. Express $cos^3\theta$ in terms of multiple angles (a) $\frac{1}{4}(cos3\theta + 3cos\theta)$ (b) $4cos^3\theta - 3cos\theta$
(c) $3cos\theta - 4cos^3\theta$ (d) $\frac{1}{4}(cos3\theta - 3cos\theta)$ (e) none of the above

23. Expand $f(x) = \ln x$ about a point $x = 1$.
(a) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \cdots .. + \frac{1}{n}(x-1)^n + E_n(x)$
(b) $(x+1) - \frac{1}{2}(x+1)^2 + \frac{1}{3}(x+1)^3 - \frac{1}{4}(x+1)^4 + \cdots .. + \frac{1}{n}(x+1)^n + E_n(x)$
(c) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \cdots .. + \frac{1}{n}(x-1)^n$
(d) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^2 + \cdots + E_n(x)$ (e) none of the above

24. Find the stationary points to the surface $z = x^3 + xy + y^2$ (a) (0,0), $(\frac{1}{6}, -\frac{1}{12})$
(b) (1,1), $(\frac{1}{6}, -\frac{1}{12})$ (c) (0,0), $(\frac{1}{6}, \frac{1}{12})$ (d) all of the above (e) none of the above

25. If $f(x) = e^{ax}$. Find the nth derivative of $f(x)$. (a) e^{ax} (b) $2e^{ax}$ (c) $2^n e^{ax}$
(d) none of the above

26. Let $z = cos\theta + isin\theta$ and $z^{-1} = cos\theta - isin\theta$. What is the expression for $z - \frac{1}{z}$
(a) $2isin\theta$ (c) $2cos\theta$ (b) $2\sin n\theta$ (d) $2icos\theta$ (e) none of the above

27. A function $f(x, y)$ is said to have a relative minimum at point (x_o, y_o) if which of the following condition holds.
(a) $f_{xx} < 0$ and $f_{xx} \cdot f_{yy} > (f_{xy})^2$ (b) $f_{xx} > 0$ and $f_{xx} \cdot f_{yy} > (f_{xy})^2$ (c) $f_{xx} \cdot f_{yy} < (f_{xy})^2$
(d)) $f_{xx} \cdot f_{yy} > (f_{xy})^2$ (e) none of the above

28. If $f(x) = (x^2 + 1)e^{2x}$, find $f^n(x)$, where $f^n(x)$ is the nth derivative of $f(x)$
(a) $2^{n-2}e^{2x}[4x^2 + 4nx + n^2 - n + 4]$ (b) $2^{n+2}e^{2x}[4x^2 + 4nx + n^2 - n + 4]$
(c) $2^{n-2}e^{2x}[4x^2 - 4nx + n^2 - n + 4]$ (d) $2^{n-2}e^{2x}[4x^2 - 4nx + n^2 - n - 4]$
(e) none of the above

29. Expand $f(x) = e^{ax}$ about the point $x = 0$
(a) $1 + (ax) + \frac{(ax)^2}{2!} + \frac{(ax)^3}{3!} + \cdots ... + \frac{(ax)^n}{n!} + E_n(x)$
(b) $1 - (ax) + \frac{(ax)^2}{2!} - \frac{(ax)^3}{3!} + \cdots ... + \frac{(ax)^n}{n!} + E_n(x)$
(c) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots ... + \frac{x^n}{n!} + E_n(x)$
(d) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots ... + \frac{x^n}{n!} + E_n(x)$ (e) none of the above

30. Determine the second root of the complex number given by $z^4 = -1$.
(a) $\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$ (b) $\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$ (c) $\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}$ (d) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$
(e) none of the above

31. Evaluate $\int x^2 \sin x \, dx$ (a) $\sin x\,(2 - x^2) + x\cos x + C$ (b) $\cos x\,(2 - x^2) + x\sin x + C$
(c) $\sin x\,(2 + x^2) - x\cos x + C$ (d) $\sin x\,(2 - x^2) + x\sin x + C$ (e) none of the above

32. Evaluate $\int xe^x\,dx$ (a) $(x-1)e^x+C$ (b) $(x+1)e^x+C$ (c) $\frac{x^2}{2}e^x+x+C$
(d) x^2e^x+C (e) none of the above

33. Evaluate $\int x^2\ln x\,dx$. (a) $\frac{x^3}{3}(\ln x+1)+C$ (b) $\frac{x^3}{27}(3\ln x+1)+C$ (c) $\frac{x^3}{9}(3\ln x-1)+C$
(d) $x^3(3\ln x-1)+C$ (e) none of the above

34. Solve $\int_0^{1/2} 4x\,e^{2x}dx$. (a) $2-e$ (b) $2e$ (c) 2 (c) $2+e$ (d)4e (e) none of the above

35. Solve $\int x(x^2+2)^3dx$. (a) $\frac{(x^2+2)^4}{4}+C$ (b) $\frac{x(x^2+2)^4}{8}+C$ (c) $\frac{x(x^2+2)^4}{16}+C$
(d) $\frac{(x^2+2)^4}{8}+C$ (e) none of the above

36. Evaluate $\int \frac{x}{\sqrt{1+2x^2}}dx$. (a) $\frac{(1+2x^2)^{-1/2}}{2}+C$ (b) $\frac{(1+2x^2)^{1/2}}{2}+C$ (c) $\frac{x(1+2x^2)^{1/2}}{2}+C$
(d) $x(1+2x^2)^{1/2}+C$ (e) none of the above

37. Solve $\int \sin x\cos x\,dx$ (a) $\frac{sin^2x}{2}+C$ (c) $\frac{sin^2x\cos x}{2}+C$ (c) $\frac{cos^2x\sin x}{2}+C$
(d) $\cot x+C$ (e) none of the above

38. Evaluate $\int \frac{4x}{3-x^2}dx$. (a) $2Log_e\left|3-x^2\right|+C$ (b) $4Log_e(3-x^2)$ (c) $-4Log_e\left|3-x^2\right|+C$
(d) $-2Log_e\left|3-x^2\right|+C$ (e) none of the above

39. Solve $\int (cos^3x+2)sinx\,dx$ (a) $(\frac{cos^4x}{4}-2cosx)+C$ (b) $-(\frac{cos^4x}{4}+2cosx)+C$
(c) $-(\frac{2sinx}{4}+2cos^4x)+C$ (d) $(\frac{sin^4x}{4}-2sinx)+C$ (e) none of the above

40. Solve $\int x\sqrt{x+1}\,dx$ (a) $\frac{2(x+1)^{2/5}}{5}-\frac{2(x+1)^{2/3}}{3}+C$ (b) $\frac{(x+1)^{2/5}}{10}+\frac{(x+1)^{2/3}}{6}+C$
(c) $\frac{x(x+1)^{2/5}}{5}+\frac{(x+1)^{2/3}}{3}+C$ (d) $\frac{(x+1)^{2/5}}{5}-\frac{(x+1)^{2/3}}{3}+C$ (e) none of the above