Option C

DEPARTMENT OF MATHEMATICS
FACULTY OF PHYSICAL SCIENCES
UNIVERSITY OF BENIN, BENIN CITY
FIRST SEMESTER B.Sc. (FULL-TIME) EXAMINATIONS 2022/2023 SESSION

COURSE TITLE: MTH 213 (Vector Analysis) **TIME ALLOWED:** 1½ Hours.

INSTRUCTIONS: (i) Write and circle your attendance list serial number on the objective answer paper.(ii) Attempt all questions by SHADING (**using HB pencil**) the letter box that corresponds to the correct option. Information about your **Mat. No., Name, Course code, Faculty code** and **Departmental code** must be clearly written and **CORRECTLY SHADED. YOU MUST SUBMIT YOUR QUESTION PAPER ALONG WITH YOUR ANSWER SHEET**

NAME________________________________ MAT. NO.________________

1. Given that $U = Xz^2i + 2yj - 3xzk$ and $V = 3xzi + 2yzj - z^2k$, evaluate $(U \times \nabla) \times V$ at points (1,-1,2)
   (a) $-7i$ (b) $(12xz + 3z)i$ (c) $0i + 0j + 0k$ (d) $-12i + 3j$ (e) None of the above
2. If $A = x^2yi + (xy + yz)j + xz^2k, B = yzi - 3xzj + 2xyk$ and $\phi = 3x^2y + xyz - 4y^2z^2 - 3$, determine curl curl A at the point (1,2,1). (a) $14i - 12j - 30k$ (b) $5i - 2j - 4k$ (c) $7i + 2j + 3k$ (d) $3i + 2j + k$
   (e) None of the above
3. Suppose A = 3i +5j - 2kandB = 4i – 8j +7k find |3A-2B|

   (a) 36.910 (b) 23.685 (c) 23.706) (d) 6.164 (e) None of the above.

4. The divergence of the gradient of $\phi = 2x^3y^2z^4$ is ______
   (a) $2xz - 6y^2z^2 + xy^2$ (b) $-2xz + 6y^2z^2$ (c) $12xy^2z^4 - 4x^3z^4 - 24x^3y^2z^2$
   (d) $12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2$ (e) None of the above
5. The domain of the vector valued function r(t) $=\sqrt{t}i + (4 - t)j$ is...(a) t> 0(b) t ≥ 0(c) $t \in \mathcal{R}$
   (d) $t \in \mathbb{Z}$ (e) None of the above
6. Simplify:2A + B + 3C- {A- 2B - 2(2A-3B-C)} (a) 5A -7B + C (b) 5A – 7B - C (c) 5A – 3B +

   C (d) All of the above (e) None of the above.

7. The unit normal to the surface $x^2y + 2xz = 4$ at the points (2,-2,3) is ______

   (a) $-2i + 4j + 4k$ b) $2i - 4j - 4k$ (c) $\frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}k$ (d) $-\frac{1}{3}i + \frac{2}{3}j + \frac{2}{3}k$ (e) None of the above

8. The cross product of two vectors is ______ to the surface producing these vectors.
   (a) Perpendicular (b) parallel (c) solenoid (d) Irrotational (e) None of the above
9. If $\bar{\nabla}.\bar{v} = 0$ and $\bar{\nabla} \times \vec{A} = 0$ then vectors $\bar{v}$ and $\vec{A}$ are said to be ______and ______ respectively
   (a) Solenoid and irrotational (b) subnormal and solenoid (c) irrotational and parallel (d) solenoid and perpendicular (e) none of the above
10. Find constants a, b and c, so that $\bar{v} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + z)k$ is irrotational.
    (a) (4,2,-1) (b) (-1,4,2) (c) (2,-1,4) (d) (-1,2,4) (e) None of the above
11. Find the constants C_1 and C_2 so that the surface $c_1x^2 - c_2yz = (c_1 + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at points (1,-1-,2).

(a) (-1, 5/2) (b) (5/2, 1) (c) (2/5, 1) (d) (1, 2/5) (e) None of the above

12. One of the following is not a Frenet-Serret formula (a) $\frac{dT}{ds} = k N$ (b) $\frac{dB}{ds} = -\tau N$ (c) $\frac{dN}{ds} - \tau B \quad kT$

(d) $-\frac{1}{\tau}\frac{dN}{ds} = B$ (e) None of the above

13. Suppose ABCDEF are vertices of a regular hexagon. Find the resultant of the forces represented by vectors AB, AC, AD, AE and AF (a) 5A (b) 3AF (c) Zero (d) 3AD (e) None of the above.

14. Suppose A,B and C are vectors and m is a scalar. Then the following laws hold except. (a) (A.B)C = A.(B.C) (b) A.(B XC) - B.(CXA)(c) AX(B XC) ≠(AXB)XC (d) AX(B XC) = (A.C).C-(AB)C

(e) None of the above.

15. Suppose A = 3i – j - 2kand B = 2i + 3j + k. find (A + 2B) X (2A – B) (a) $\sqrt{195}$ (b) $2\sqrt{195}$

(c) -25i + 34j -55k (d) -2i - 7j -11k (e) None of the above.

16. Suppose A, B and C are vectors and m is scalar. Then one of the following is not correct. (a) AXB- BXA(b) AX(B –C)= (AXB) + (AXC)(c) m(AXB) = mAXB (d) iXi = 0 (e) None of the above.

17. Find the constant αso that 3i - 3j - k, -3i - 2j + 2k and6i – αj - 3kare coplanar.(a) -4 (b) -15 (c) -13

(d) 3 (e) None of the above.

18. If $r(t) = (t^3 + 2t)i - 3e^{-2t}j + 2\sin 5tk$ is a position vector, what is the speed of the vector at t – 0?
(a) √37 (b) √104 (c) √140 (d) √137 (e) None of the above

19. The ……. is perpendicular to the normal plane. (a) unit tangent T (b) unit normal N (c) binormalB
(d) All of the above (e) None of the above

20. Given that ϕ is a differentiable scalar and $\vec{v}$ is a vector field, then the following operations $\bar{\nabla}\phi$, $\bar{\nabla}\cdot\vec{v}$ and $\bar{\nabla}\times\vec{v}$ will respectively yield ______
(a) Scalar, vector and vector (b) vector, vector, scalar (c) vector, scalar and vector (d) vector, vector, vector (e) None of the above

21. Find the projection of A = i – 2j +3k onB = i + 2j + 2k (a) 1 (b) 5 (c) $-\frac{1}{3}i+\frac{2}{3}j\frac{2}{3}k$ (d) $\sqrt{9}$

(e) None of the above.

22. Given that $U = Xz^2i + 2yj - 3xzk$ and $V = 3xzi + 2yzj - z^2k$, evaluate $U \times (\nabla \times V)$ at point (1,-1,2)
(a) $-12yi + 3xj$ (b) $18i - 12j + 16k$ (c) $9x^2zi + 6xyzj + (3x^2z^2 + 4y^2)k$ (d) $+12i + 3j - 6k$ (e) None of the above

23. Which of the following is a vector quantity?

(a) Specific heat (b) Speed (c) magnetic field intensity (d) distance (e) None of the above.

24. Given the radius vectors $r_1 = 3i - 2j + 2k$, $r_2 = 3i + 4j + 9k$, $r_3 = -i + 2j + 2k$. Find the magnitude of

$r_1 - r_2 + 4r_3$ (a) 3 (b) 13 (c) $2\sqrt{2}$ (d)2.928 (e) None of the above.

25. Which of the following dot product axiom is not correct? (a) A.B = B.A (b) A.(B+C) = (A+B).C

(c) I.I=J.J=K.K=1 (d) m(A.B) = mA.B (e) None of the above.

26. If A.B=0 and A and B are not null vectors. Then A and B are (a) Parallel (b) vertical (c) horizontal

(d) perpendicular (e) None of the above.

27. Find the arc length of the position vector **r**(t) =4 sin t **i** + 3 t **j** + 4 cos t **k** defined on $0 \leq t \leq 2\pi$.
(a) 5 (b) (4, 3, 4) (c) 10 π (d) 4 (e) None of the above

28. With the position vector **r**(t) =cos t **i** + sin t **j** + t **k**, obtain the binormal at $t = \frac{\pi}{2}$ (a)$\frac{\sqrt{2}}{2}(\mathrm{i} - \mathrm{k})$
(b)$\frac{\sqrt{2}}{2}(\mathrm{i} + \mathrm{k})$ (c) $\frac{\sqrt{2}}{2}(-\mathrm{i} + \mathrm{k})$ (d) $\frac{\sqrt{2}}{2}(-\mathrm{i} - \mathrm{j} - \mathrm{k})$ (e) None of the above

29. Find the equation of the oscillating plane of the position vector **r**(t) =cos t **i** + sin t **j** + t **k** at the point $t = \frac{\pi}{2}$. (a) $x + z = \frac{\pi}{2}$ (b) $-x + z = \frac{\pi}{2}$ (c) $x - z = \frac{\pi}{2}$ (d) $x + z = -\frac{\pi}{2}$ (e) None of the above

30. If $\phi = 2xz^4 - x^2y$ then $\bar{\nabla}\phi$ and $|\bar{\nabla}\phi|$ are respectively given by__________ and ________

(a) $6i + 4j + 16k, \sqrt{308}$ (b) $-12i - 9j - 16k, \sqrt{481}$ (c) $-6i - 4j - 16k, \sqrt{308}$ (d) $12i + 9j + 16k, \sqrt{481}$

(e) None of the above

31. Evaluate the dot product of (2i – j) and (3i +k)(a) -3 (b) 6 (c) 6i (d) 0 (e) None of the above.

32. Find the angle between A = 2i + 2j - k and B = 7i +24k (a) 0.1333^0 (b) 98^0 (c) 90^0 (d) 836^0

(e) None of the above.

33. If $\mathbf{A} = x^2yi + (xy + yz)j + xz^2k$, $\mathbf{B} = yzi - 3xzj + 2xyk$ and $\phi = 3x^2y + xyz - 4y^2z^2 - 3$, evaluate |grad div A| at (1,2,1). (a) 5.08 (b) -8 (c) -5.08 (d) 7.28 (e) None of the above

34. Find the region of continuity of the vector function $\mathbf{r}(t) = \frac{\cos t - 1}{t}\mathrm{i} + \frac{\sqrt{t}}{1+2t}\mathrm{j} + te^{-\frac{1}{t}}\mathrm{k}$ (a) $t > 0$
(b) $t \geq 0$ (c) $t \in \mathcal{R}$ (d) $t \in \mathbb{Z}$ (e) None of the above

35. $\frac{d}{du}(A.B) = ?$ Choose the correct option. Here $A(u)$ and $B(u)$ are vector functions of the real variable u. (a)$B.\frac{dA}{du} + A.\frac{dB}{du}$ (b) $A.\frac{dB}{du} - B.\frac{dA}{du}$ (c) $A.\frac{dA}{du} + B.\frac{dB}{du}$ (d) $\left(A + \frac{dB}{du}\right).\left(B + \frac{dA}{du}\right)$ (e) None of the above