DEPARTMENT OF MATHEMATICS

UNIVERSITY OF BENIN, BENIN CITY.

MTH230 (LINEAR ALGEBRA MOCK) 2019/2020 SESSION **Time allowed: 1Hr**

1. Express $v = (1,-2,5)$ in R^3 as a linear combination of the vectors $u_1 = (1,1,1)$, $u_2 = (1,2,3)$, $u_3 = (2,-1,1)$
   (a) $v = -6u_1 + 3u_2 + 2u_3$ (b) $v = 6u_1 - 3u_2 - 2u_3$ (c) $v = -8u_1 + 7u_2 - 6u_3$ (d) $v = -4u_1 + 7u_2 + 8u_3$ (e) None
2. Solve by crammers rule the systems of equations
   $$x_1 + 2x_2 + x_3 = 4$$
   $$3x_1 - 4x_2 - 2x_3 = 2$$
   $$5x_1 + 3x_2 + 5x_3 = -1$$
   (a) 4, 3 and 4 (b) 2, 3 and -4 (c) 9, 0 and -3 (d) 2, 3 and 4 (e) none of the above
3. Find the cofactor matrix A_{ij} of the matrix $A = \begin{bmatrix} 1 & 7 & 5 \\ -4 & 4 & 8 \\ 2 & 6 & 9 \end{bmatrix}$ (a) $\begin{bmatrix} 2 & 0 & 36 \\ -33 & 2 & k \\ 30 & 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -12 & 52 & -32 \\ -33 & -1 & 8 \\ 36 & -k & 32 \end{bmatrix}$ (c) $\begin{bmatrix} 15 & 30 & 32 \\ 4 & 1 & 54 \\ 0 & k & -11 \end{bmatrix}$
   (d) $\begin{bmatrix} -12 & 1 & 12 \\ 10 & 35 & 16 \\ k & 0 & 34 \end{bmatrix}$ (e) none of the above
4. Which of the following is a standard basis of R^3? (a) $\{(0,0,1),(1,2,5),(0,1,2)\}$ (b) $\{(1,2,3),(2,3,4),(3,4,5)\}$ (c) Both (a) and (b) (d) $\{(1,0,0),(0,1,0),(0,0,1)\}$ (e) none of the above
5. Obtain scalars a, b, c in $V = ax_1 + bx_2 + cx_3$ such that the vector $V = (5,2,4)$ in R^3 is written as a linear combination of the vectors $x_1 = (1,2,3)$, $x_2 = (2,3,7)$, and $x_3 = (3,5,6)$ (a) $a = -2, b = -4, c = 3$ (b) $a = 2, b = -4, c = -3$
   (c) $a = 2, b = 4, c = 3$ (d) $a = 0, b = 0, c = 0$ (e) none of the above
6. Consider the set A={1,2,3,5,7} and B={0,3,6,7,9} Determine $(A-B)^c \cup (B-A)$ (a) {1,2,7,1} (b) {0,6,9,1} (c) $\emptyset$
   (d) {0,1,2,5,6,9} (e) none of the above
7. Given the matrix $A = (A_{ij}) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$ find the determinant of A (a) $a_{11}.a_{23}.a_{24}.a_{33}$ (b) $a_{11}, a_{22}, a_{33}, a_{44}$
   (c) $a_{11}.a_{22}.a_{33}.a_{44}$ (d) $a_{11}.a_{12}.a_{22}.a_{44}$ (e) none of the above
8. Given the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 7 & 4 \\ 8 & 0 & 6 \end{bmatrix}$ find the product $\boldsymbol{A^{-1}A}$, (a) $\begin{bmatrix} 2 & 3 & 5 \\ 1 & 7 & 4 \\ 8 & 0 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & 8 \\ 3 & 7 & 0 \\ 5 & 4 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 5 & 8 \\ 4 & 3 & 2 \end{bmatrix}$ (e) none
9. Given a matrix $A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 4 & -2 \\ 2 & 10 & -5 \end{bmatrix}$ find the cofactor A_{13} (a) 5 (b) 2 (c) -2 (d) -8 (e) None of the above
10. Express $U = (6,3,4)$ as a linear combination of $u_1 = (1,1,2)$, $u_2 = (1,-1,-3)$, $u_3 = (-1,1,2)$ $in\ IR^3$
    (a) $U = -\frac{9}{2}u_1 + 8u_2 + \frac{13}{2}u_3$ (b) $U = \frac{9}{2}u_1 + 2u_2 + \frac{1}{2}u_3$ (c) $U = 6u_1 - 8u_2 + u_3$ (d) $U = \frac{9}{2}u_1 - 8u_2 + \frac{1}{2}u_3$
11. Find a matrix A such that $\left(2A^T + \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}\right)^T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (a) $(2,-1)$ (b) $\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ (c) $(-\frac{1}{2}, \frac{1}{2})$ (d) $\begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$
    (e) None of the above
12. Suppose $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & 2 \end{bmatrix}$ find $\boldsymbol{A^{-I}I}$ (a) $\frac{1}{28}\begin{bmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{bmatrix}$ (b) $\frac{1}{16}\begin{bmatrix} 2 & 6 & -2 \\ 22 & -16 & 7 \\ -6 & 12 & 8 \end{bmatrix}$ (c) $\frac{1}{28}\begin{bmatrix} 2 & -2 & 7 \\ 22 & -4 & 8 \\ 4 & 12 & -7 \end{bmatrix}$ (d) $\frac{1}{8}\begin{bmatrix} 4 & 27 & 7 \\ 22 & 16 & 8 \\ -6 & 12 & -7 \end{bmatrix}$ (e) none
13. Evaluate $A = \begin{bmatrix} 2 & 4 & 1 & 3 \\ -1 & -2 & 1 & 0 \\ 0 & 0 & 2 & 2 \\ 3 & 6 & 2 & 5 \end{bmatrix}$ (a) 4 (b) 8 (c) 2 (d) 1 (e) None of the above
14. Find the scalars of $\boldsymbol{x}$ *and* $\boldsymbol{y}$ such that the vector is a linear combination of $u = (1,2,-1)$, $v = (6,4,3)$,
    (a) $x = 3, y = 2$ (b) $x = -2, y = -3$ (c) $x = -3, y = -2$ (d) $x = 1, y = 5$ (e) None of the above
15. Let $A = \{a, c\}$ *and* $B = \{a, b, e, f\}$ what is $n(B)$ (a) 16 (b) 6 (c) 8 (d) 4 (e) None of the above
16. Find the possible eigenvalues of the matrix $\begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & -2 \end{bmatrix}$ (a) 0,1,1 (b) $0,-1,1$ (c) 0,2,3 (d) $0,-2,3$ (e) None of the above
17. Let $P(A)$ denote the power set of A. If $P(A) \subseteq B$ then (a) $2^{|A|} \geq |B|$ (b) $2^{|A|} = |B|$ (a) $2^{|A|} \leq |B|$ (a) $2^{|A|} < |B|$
    (e) None of the above

18. Given a 2×3 column vector $A = \begin{bmatrix} -2 & 0 \\ 3 & 2 \\ 1 & 0 \end{bmatrix}$, then the linear transformation $T: IR^2 \to IR^3$ is defined by
(a) $T(x,y) = (-2x + z, x + 2y, x)$ (b) $T(x,y) = (-2x + 2y, 3x + y, 0)$ (c) $T(x,y) = (-2x, 3x + 2y, x)$
(d) $T(x,y) = (-x + 3y, 2x + 3y, 2x)$ (e) None of the above

19. $T: R^2 \to R^3$, which of these is not a linear transformation? (a) $T(x,y) = (-2x + z, x + 2y)$ (b) $T(x,y) = (x + 2y, 2x, 0)$
(c) $T(x,y) = (y, -2x + 2y, x)$ (d) $T(x,y) = (-x + 3y, 4, x + y)$ (e) None of the above

20. Which of the following is not a condition for a Subset W of a vector space V to be a subspace of V (a) $0 \in W$ (b) $u + v \in W \; \forall \, u, v \in W$ (c) $\alpha u \in W \; \forall \; \alpha \in F, u \in W$ (d) $uv \in W \; \forall \; , u, v \in W$ (e) None of the above

21. Find the rank of $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 4 & 2 \\ 0 & 2 & 2 & 1 \end{bmatrix}$ (a) 4 (b) 3 (c) 2 (d) 1 (e) None of the above

22. Let $A = \begin{bmatrix} \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{15} \\ \mathbf{8} & \mathbf{9} & \mathbf{1} & \mathbf{7} \end{bmatrix}$ find $\det(A)$ (a) -20 (b) -20 (c) -27 (d) -25 (e) None of the above

23. If A is an orthogonal matrix, which of the following is wrong (a) $A \cdot A^T = I$ (b) $A^{-1} = A^T$ (c) $A \cdot A^T \cdot A = A$
(d) $A^T \cdot A = I$ (e) None of the above

24. Suppose $AB = \begin{pmatrix} 5 & 4 \\ -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$. Find A (a) $\begin{pmatrix} 1 & -3 \\ -2 & 7 \end{pmatrix}$ (b) $\begin{pmatrix} 11 & -17 \\ -27 & 41 \end{pmatrix}$ (c) $\begin{pmatrix} 13 & 43 \\ 4 & 18 \end{pmatrix}$ (d) $\begin{pmatrix} -2 & 13 \\ -8 & 27 \end{pmatrix}$
(e) None of the above

25. Express the polynomial $v = t^2 + 4t - 3$ in $P(t)$ as a linear combination of the polynomials $p_1 = t^2 - 2t + 5$, $p_2 = 2t^2 - 3t$, $p_3 = t + 1$ (a) $v = 8p_1 - 4p_2 - 7p_3$ (b) $v = -3p_1 + 2p_2 + 4p_3$ (d) $v = -5p_1 + 8p_2 + 7p_3$
(d) $v = -9p_1 + 4p_2 + 3p_3$ (e) None of the above

26. Which of the following pairs of vectors, u and v, are linearly dependent? (a) $u = (1,2), v = (3,-5)$
(b) $u = (1,-3), v = (-2,6)$ (c) $u = (1,2,-3), v = (4,5,-6)$ (d) All of the above (e) None of the above

27. Let $C = \{a, c, 2,4,6\}$ *and* $B = \{a, b, e, f\}$ what is the number of elements in $P(C)$ (a) 16 (b) 64 (c) 48 (d) 32 (e) None of the above

28. Which of the following vectors forms a basis of R^3? (a) $(1,1,1), (1,0,1)$ (b) $(1,2,3), (1,3,5), (1,0,1), (2,3,0)$ (c) $(1,1,1), (1,2,3), (2,-1,1)$ (d) None of the above

29. If W is a subspace of the vector space $V = R^3$ where $W = \{(a,b,c): a + b + c = 0\}$ then (a) $\dim W = 3$ (b) $\dim W = 4$
(c) $\dim W = 2$ (d) $\dim W = 5$ (e) None of the above

30. Given that $B = \begin{pmatrix} 4 & 2 & 6 \\ 1 & 8 & 7 \end{pmatrix}$ determine $B \cdot B^{-1} \cdot B \cdot B^T$ (a) $\begin{pmatrix} -56 & 62 \\ 60 & 114 \end{pmatrix}$ (b) $\begin{pmatrix} 56 & 114 \\ 62 & 62 \end{pmatrix}$ (c) $\begin{pmatrix} 56 & 62 \\ 62 & 144 \end{pmatrix}$ (d) None of the above

31. Which of the following is wrong? (a) if A is non-singular then $det(A) \neq 0$ (b) if A^{-I} is the inverse of an $n \times n$ matrix A, then A^{-I} is $n \times n$ (c) if A is singular matrix, then A^{-I} does not exist (d) A singular matrix is invertible (e) if A is an $n \times n$ matrix then $A^{-I} = \frac{1}{det\, A} adjA$, provided $\det A \neq 0$

32. If A and B are symmetric matrices of the same order, then $AB - BA$ is a (a) Null matrix (b) symmetric matrix (c) skew-symmetric matrix (d) All of the above (e) None of the above

33. A square matrix $A = [a_{ij}]$ such that $a_{ij} = 0$ whenever $i > j$ and $a_{ij} = 0$ whenever $i < j$ is called (a) Upper triangular matrix
(b) Lower triangular matric (c) Diagonal matrix (d) Canonical matrix (e) None of the above

34. The main diagonal of an $n \times n$ skew-symmetric matrix consists of only the element (a) 0 (b) 1 (c) $^1/_2$ (d) 2 (e) None of the above

35. Let $A = \begin{bmatrix} \mathbf{1} & \mathbf{-2} & \mathbf{3} & \mathbf{5} \\ \mathbf{2} & \mathbf{3} & \mathbf{1} & \mathbf{-1} \\ \mathbf{-7} & \mathbf{6} & \mathbf{7} & \mathbf{-15} \\ \mathbf{4} & \mathbf{6} & \mathbf{2} & \mathbf{-2} \end{bmatrix}$ find $\det(A)$ (a) 5 (b) 3 (c) -4 (d) 12 (e) None of the above