DEPARTMENT OF MATHEMATICS

FACULTY OF PHYSICAL SCIENCES

UNIVERSITY OF BENIN, BENIN CITY, NIGERIA.

CONTINOUS ASSESSMENT 2023/2024 SESSION

COURSE TITLE: MTH 230 (LINEAR ALGEBRA)

MAT. NO:_____   DEPT._____

1. What is the quadratic form of the matrix $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$? (a) $x+xy+y^2$ (b) x^2+xy (c) x^2+y^2

(d) x^2+xy+y^2

2. The solution of the given matrix equation is _____

$A = \begin{pmatrix} 3 & 0 & 2 \\ 6 & 1 & 1 \\ 2 & 8 & 91 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ (a) $x_1 = 1, x_2 = 1, x_3 = 2$ (b) $x_1 = 0, x_2 = 0, x_3 = 0$

(c) $x_1 = 3, x_2 = -1, x_3 = -1$ (d) $x_1 = 0, x_2 = -2, x_3 = 4$

3. Which one of the following is not a criterion for linearity of an equation?
(a) The dependent variable y should be of second order
(b) The derivatives of the dependent variable should be of second order
(c) Each coefficient does not depend on the independent variable
(d) Each coefficient depends only on the independent variable

4. Which of the following is true for matrices? (a) $(AB)^{-1} = B^{-1}A^{-1}$ (b) $(A^T) = A$
(c) $AB = BA$ (d) $A*I = I$

5. Which of the following matrix is not orthogonal?

(a) $A = \begin{pmatrix} 0.33 & 0.67 & -0.67 \\ -0.67 & 0.67 & 0.33 \\ 0.67 & 0.33 & 0.67 \end{pmatrix}$ (b) $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$ c) $A = \begin{pmatrix} 0.33 & -0.67 & 0.67 \\ 0.67 & 0.67 & 0.33 \\ -0.67 & 0.33 & 0.67 \end{pmatrix}$

(d) $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & -\cos x \end{pmatrix}$

6. The determinant of the matrix whose eigen values are 7, 1, 9 is given by _____
(a) 7 (b) 63 (c) 9 (d) 17

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7. Find the values of x and y in the matrix below if the matrix is a symmetric matrix.

$$A = \begin{pmatrix} 0 & x+y & 6 \\ 3 & 0 & 9 \\ x & 9 & 0 \end{pmatrix}$$ (a) $x = -6$, $y = 3$ (b) $x = 3$, $y = 3$ (c) $x = 6$, $y = -3$

(d) $x = 0$, $y = 3$

8. 10. The sum of two symmetric matrices is also a symmetric matrix_____
(a) False (b) True (c) All of the above (d) None of the above

9. Which of the following is not a condition for a given real nonsingular quadratic form, $Q = X^T AX$, to be a negative definite quadratic form?
(a) The number of positive square terms in the quadratic form is equal to zero
(b) The rank of the matrix A is equal to the number of variables in the quadratic form (index)
(c) All the eigen values of A are negative
(d) The rank and index are equal

10. Signature of a quadratic form is the difference between the positive and negative terms in the canonical form. (a) True (b) False (c) All of the above (d) None of the above

11. Determine the nature of the given matrix.

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$ (a) Indefinite (b) Positive definite (c) Negative definite (d) Positive semi-definite

12. If A is a matrix, such that, $A^k = 0$, for positive integer k, then, A is known as Nilpotent matrix. (a) True (b) False (c) All of the above (d) None of the above

13. Write the expression for spur of a matrix for a 3×3 matrix whose entries are in the form of a_{ij}. (a) $a_{11}+ a_{12}+a_{13}$ (b) $a_{11}+ a_{21}+a_{31}$ (c) $a_{12}+ a_{22}+a_{32}$ (d) $a_{11}+ a_{22}+a_{33}$

14. The inverse of a symmetric matrix (if it exists) is? (a) A symmetric matrix (b) A skew symmetric matrix (c) A diagonal matrix (d) A triangular matrix

$$A = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & e^{\beta} \end{pmatrix}$$ (a) A $(-\alpha, \beta)$ (b) A $(-\alpha, -\beta)$ (c) A $(\alpha, -\beta)$ (d) A (α, β)

15. What is the rank of the following matrix is $A = \begin{pmatrix} 3 & 2 & -1 \\ 4 & 2 & 6 \\ 7 & 4 & 5 \end{pmatrix}$ is? (a) 1 (b) 3 (c) 2 (d) 0

16. If every minor of order 'r' of a matrix is zero then $\rho(A) = ?$ (a) $>r$ (b) $=r$ (c) $\le r$ (d) $<r$

17. Test for consistency and solve to find the value of x. Given that:

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$5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$. (a) Consistent, $x=1$
(b) Consistent, $x=-1$ (c) Inconsistent system, solution does not exist
(d) Consistent, infinite number of solutions possible.

18. Test for consistency and solve the system of equations if possible to get the value of z. $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$. (a) Consistent, $z = -1$ (b) Consistent, $z = 0$ (c) Inconsistent (d) Consistent, $z = 5$

19. Which of the following matrix are singular? (a). $\begin{pmatrix} 31 & 12 \\ 26 & 8 \end{pmatrix}$ (b). $\begin{pmatrix} 1 & 11 \\ 2 & 8 \end{pmatrix}$ (c). $\begin{pmatrix} 13 & 12 \\ 2 & 8 \end{pmatrix}$ (d). $\begin{pmatrix} 3 & 12 \\ 2 & 8 \end{pmatrix}$

20. Find the Eigen values for the following 2×2 matrix. $A = \begin{pmatrix} 1 & 8 \\ 2 & 1 \end{pmatrix}$. (a) -3 (b) 2 (c) 6 (d) 4

21. Find the Eigenvalue for the given matrix, $A = \begin{pmatrix} 4 & 1 & 3 \\ 1 & 3 & 1 \\ 2 & 0 & 5 \end{pmatrix}$ (a) 13 (b) -3 (c) 7.1 (d) 8.3

22. Find the Eigen vector for value of $\lambda = -2$ for the given matrix. $A = \begin{pmatrix} 3 & 5 \\ 2 & 1 \end{pmatrix}$ (a) $(0, -1)^T$
(b) $(1, -1)^T$ (c) $(-1, -1)^T$ (d) $(1, 0)^T$

23. Find the Eigen vector for value of $\lambda = 3$ for the given matrix, $A = \begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$. (a) (-1, -1, 2)^T (b) (-1, 1, 2)^T (c) (-1, -1, -2)^T (d) (-1, -2, 2)^T.

24. If A = {1, 2, 3} and B = {3, 4}, find the Cartesian product of A and B. (a). {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)} (b). {(1, 3), (1, 4), (4, 3), (2, 4), (3, 3), (3, 4)} (c). {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (4, 4)} (d). {(1, 4), (2, 4), (2, 3), (2, 4), (3, 3), (4, 4)}

25. If A = {3, 4, 5}, B = {5, 6} and C = {6, 7, 8}, then find A × (B ∩ C). (a). {(3, 4), (4, 6), (5, 6)} (b). {(3, 6), (4, 6), (5, 5)} (c). {(3, 6), (4, 6), (5, 6)} (d). {(3, 2), (4, 6), (5, 6)}

26. If A = {3, 4, 5}, B = {5, 6} and C = {6, 7, 8}, then find (A × B) ∩ (A × C). (a) {(3, 7), (4, 6)} (b) {(3, 6), (4, 6)} (c) {(3, 6), (4, 8)} (d) {(3, 5), (4, 6)}.

27. If A = {3, 4, 5}, B = {5, 6} and C = {6, 7, 8}, then find A × (B ∪ C)
(a) {(3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8), (5, 3), (5, 6), (5, 7), (5, 8)}
(b) {(3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8), (5, 3), (5, 6), (5, 7), (5, 8)}
(c) {(3, 8), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8), (5, 5), (5, 6), (5, 7), (5, 8)}
(d) {(3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8), (5, 5), (5, 6), (5, 7), (5, 8)}

28. If A = {3, 4, 5}, B = {5, 6} and C = {6, 7, 8}, then find (A × B) U (A × C)
(a). {(3, 5), (3, 6), (4, 5), (4, 6), (4, 4), (4, 8), (5, 5), (5, 6), (5, 7), (5, 8)}
(b). {(3, 5), (3, 6), (4, 5), (4, 6), (4, 4), (4, 8), (5, 5), (5, 6), (5, 7), (5, 8)}.
(c). {(3, 5), (3, 6), (4, 5), (4, 6), (4, 7), (4, 8), (5, 4), (5, 6), (5, 7), (5, 8)}
(d). {(3, 5), (3, 6), (4, 5), (4, 6), (4, 7), (4, 8), (5, 5), (5, 6), (5, 7), (5, 8)}

29. If A× B is an empty set then which of the following is a null set? (a) only A (b) only B (c) either A or B (d) both A and B.

30. If set A has 4 elements and set B has 5 elements then find the number of elements in A× B.
(a) 9 (b) 4^5 (c) 20 (d) 5^4

31. If (x+2, y-3) = (5, 7), then find values of x and y.
(a) x=3 and y=10 (b) x=3 and y=4 (c) x=7 and y=4 (d) x=7 and y=10

32. If A × B has 10 elements then which is not possible? (a) n(A)=1 and n(B)=10
(b) n(A)=10 and n(B)=1 (c) n(A)=2 and n(B)=5 (d) n(A)=5 and n(B)=4

33. If A = B then A × B = B × A is true or not? (a) True (b) False (c) All the solution (d) None of the above

If set A has 2 elements and set B has 3 elements then how many subsets does A × B have?
(a) 6 (b) 8 (c) 32 (d) 64

35. Which of the following set is finite? (a) {1,2,3,4,...} (b) {4,7,9} (c) {1,4,9,16,...}
(d) {1,8,27,...}.

36. Which of the following is infinite set? (a) Set of days of week (b) Set of prime numbers less than 99 (c) Set of months in a year (b) Set of points on a line.

37. Which of the set is singleton set? (a) Set of odd prime numbers (b) Set of even prime numbers (c) Set of odd numbers (d) Set of prime numbers.

Which of the following is not the element of power set of {2, 3}? (a) Φ (b) {2} (c) {{2, 3}}
(d) {2, 3}

39. If set A = {1, 2, 3} then which of the following is incorrect? (a) ΦЄA (b) ΦЄP(A) (c) Φ⊂A
(d) Φ⊂P(A),

40. Cardinality of the power set of {0, 1, 2, 3, 4, 5, 6} is __ (a) 1024 (b) 4096 (c) 512 (d) 2048

41. If set A={Φ} then P(A) is ___ (a) {Φ} (b) {{Φ}} (c) {Φ, {Φ}} (d) Φ

42. A relation is a subset of Cartesian products. (a) True (b) False (c) None of the above (d) All of the above

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43. Let $A = \{1,2,3,4,5\}$ and R be a relation from A to A, $R = \{(x, y): y = x + 1\}$. Find the domain.
(a) $\{1,2,3,4,5\}$ (b) $\{2,3,4,5\}$ (c) $\{1,2,3,4\}$ (d) $\{1,2,3,4,5,6\}$.

44. Let $A = \{1,2,3,4,5\}$ and R be a relation from A to A, $R = \{(x, y): y = x + 1\}$. Find the codomain. (a) $\{1,2,3,4,5,6\}$ (b) $\{2,3,4,5\}$ (c) $\{1,2,3,4\}$ (d) $\{1,2,3,4,5\}$.

45 If set A has 2 elements and set B has 4 elements then how many relations are possible?
(a) 32 (b) 128 (c) 256 (d) 64

46. Which of these is not a type of relation? (a) Reflexive (b) Surjective (c) Symmetric (d) Transitive

47. An Equivalence relation is always symmetric. (a) True (b) False (c) None of the above (d) All of the above

48. Which of the following relations is symmetric but neither reflexive nor transitive for a set $A = \{1, 2, 3\}$. (a) $R = \{(1, 2), (1, 3), (1, 4)\}$ (b) $R = \{(1, 2), (2, 1)\}$ (c) $R = \{(1, 1), (2, 2), (3, 3)\}$ (d) $R = \{(1, 1), (1, 2), (2, 3)\}$.

49. Which of the following relations is transitive but not reflexive for the set $S = \{3, 4, 6\}$?
(a) $R = \{(3, 3), (4, 4), (6, 6)\}$ (b) $R = \{(1, 2), (1, 3), (1, 4)\}$ (c) $R = \{(3, 4), (4, 6), (3, 6)\}$ (d) $R = \{(3, 4), (4, 3)\}$

50. Let R be a relation in the set N given by $R = \{(a, b): a+b=5, b>1\}$. Which of the following will satisfy the given relation? (a) $(4, 2) \in R$ (b) $(2,3) \in R$ (c) $(2,1) \in R$ (d) $(5,0) \in R$

51. Which of the following relations is reflexive but not transitive for the set $T = \{7, 8, 9\}$?
(a) $R = \{(7, 7), (8, 8), (9, 9)\}$ (b) $R = \{(7, 8), (8, 7), (8, 9)\}$ (c) $R = \{0\}$ (d) $R = \{(7, 8), (8, 8), (8, 9)\}$

52. Let I be a set of all lines in a XY plane and R be a relation in I defined as $R = \{(l_1, l_2): l_1 \text{ is parallel to } l_2\}$. What is the type of given relation? (a) Reflexive relation (b) Transitive relation (c) Symmetric relation (d) Equivalence relation

53. Which of the following relations is symmetric and transitive but not reflexive for the set $I = \{4, 5\}$? (a) $R = \{(4, 4), (5, 4), (5, 5)\}$ (b) $R = \{(4, 4), (5, 5)\}$ (c) $R = \{(4, 5), (5, 4)\}$ (d) $R = \{(4, 5), (5, 4), (4, 4)\}$.

54. $(a, a) \in R$, for every $a \in A$. This condition is for which of the following relations?
(a) Reflexive relation (b) Symmetric relation (c) Equivalence relation (d) Transitive relation

55. $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$. This condition is for which of the following relations? (a) Equivalence relation (b) Reflexive relation (c) Symmetric relation (d) Universal relation.

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56. If f:R→R, g(x)=3x²+7 and f(x)=√x, then gof(x) is equal to ____ (a) 3x-7 (b) 3x-9 (c) 3x+7
(d) 3x-8

57. If f:R→R is given by f(x)=(5+x⁴)^{1/4}, then fof(x) is ____ (a) x (b) 10+x⁴ (c) 5+x⁴ (d) (10+x⁴)^{1/4}

58. A function is invertible if it is ____ (a) surjective (b) bijective (c) injective (d) neither surjective nor injective.

59. Let M={7,8,9}. Determine which of the following functions is invertible for f:M→M.
(a) f = {(7,7),(8,8),(9,9)} (b) f = {(7,8),(7,9),(8,9)} (c) f = {(8,8),(8,7),(9,8)}
(d) f = {(9,7),(9,8),(9,9)}

60. Let f: R→[9,∞) given by f(x)=x²+9. Find the inverse of f. (a) √(x-9) (b) √(9-x) (c) √(x²-9)
(d) x²+9.