OPTION B

DEPARTMENT OF MATHEMATICS
FACULTY OF PHYSICAL SCIENCES
UNIVERSITY OF BENIN, BENIN CITY

FIRST SEMESTER EXAMINATIONS 2015/2016 SESSION

COURSE TITLE: Mathematical Methods I (MTH 218) **TIME ALLOWED: 2 Hours.**

INSTRUCTIONS: (i) Write and circle your attendance list serial number on the objective answer paper. (ii) Attempt all questions by SHADING (using HB pencil) the letter box that corresponds to the correct option. Information about your Mat. No., Name, Course code, Faculty code and Department code must be clearly written and **CORRECTLY SHADED. YOU MUST SUBMIT YOUR QUESTION PAPER ALONG WITH YOUR ANSWER SHEET.**

NAME_____**MAT. NO.**_____

1. Evaluate $\int \frac{4x}{3-x^2}dx$. (a) $2Log_e\left|3-x^2\right|+C$ (b) $4Log_e(3-x^2)$ (c) $-4Log_e\left|3-x^2\right|+C$ (d) $-2Log_e\left|3-x^2\right|+C$ (e) none of the above
2. Solve $\int (cos^3x+2)sinx\,dx$ (a) $(\frac{cos^4x}{4}-2cosx)+C$ (b) $-(\frac{cos^4x}{4}+2cosx)+C$ (c) $-(\frac{2sinx}{4}+2cos^4x)+C$ (d) $(\frac{sin^4x}{4}-2sinx)+C$ (e) none of the above
3. Solve $\int x\sqrt{x+1}\,dx$ (a) $\frac{2(x+1)^{2/5}}{5}-\frac{2(x+1)^{2/3}}{3}+C$ (b) $\frac{(x+1)^{2/5}}{10}+\frac{(x+1)^{2/3}}{6}+C$ (c) $\frac{x(x+1)^{2/5}}{5}+\frac{(x+1)^{2/3}}{3}+C$ (d) $\frac{(x+1)^{2/5}}{5}-\frac{(x+1)^{2/3}}{3}+C$ (e) none of the above
4. Express $cos3\theta$ in terms of multiple of angles (a) $4cos^2\theta+3cos\theta$ (b) $4cos^2\theta-3cos\theta$ (c) $3cos^2\theta-4cos\theta$ (d) $4cos^2\theta-sin^2\theta$ (e) none of the above
5. Determine the first root of the complex number given as $z=\log(1+i)$ for k=0 (a) $log\sqrt{2}+i\frac{\pi}{4}$ (b) $log\sqrt{2}+i(\frac{\pi}{4}+2\pi k)$ (c) $log\sqrt{3}+i\frac{\pi}{4}$ (d) $log\sqrt{3}-i\frac{\pi}{3}$ (e) none of the above
6. Let $z=z+iy$ be a complex number with the polar form given as $z=r(cos\theta+isin\theta)$, then θ is given as (a) $tan^{-1}\left(\frac{x}{y}\right)$ (b) $tan^{-1}\left(\frac{1}{y}\right)$ (c) $tan^{-1}\left(\frac{y}{x}\right)$ (d) $tan^{-1}\left(\frac{1}{x}\right)$ (e) none of the above
7. Find the stationary points to the surface $z=x^3+xy+y^2$ (a) (0,0), $(\frac{1}{6},-\frac{1}{12})$ (b) (1,1), $(\frac{1}{6},-\frac{1}{12})$ (c) (0,0), $(\frac{1}{6},\frac{1}{12})$ (d) all of the above (e) none of the above
8. A function $f(x,y)$ is said to have a relative maximum at point (x_o, y_o) if which of the following condition holds.

   (a) $f_{xx}<0$ and $f_{xx}.f_{yy}>(f_{xy})^2$ (b) $f_{xx}>0$ and $f_{xx}.f_{yy}>(f_{xy})^2$ (c) $f_{xx}.f_{yy}<(f_{xy})^2$

   (d)) $f_{xx}.f_{yy}>(f_{xy})^2$ (e) none of the above

9. Solve $\int tan^3 x sec^2 x\, dx$ (a) $\frac{tan^4 x}{4} + \frac{sec^3 x}{3} + C$ (b) $\frac{tan^4 x}{4} - \frac{tan^2 x}{2} + C$
(c) $\frac{sec^4 x}{4} - \frac{sec^2 x}{2} + C$ (d) $\frac{tan^4 x}{4} - \frac{sec^3 x}{3} + C$ (e) none of the above

10. Solve $\int sin^3 x cos^2 x\, dx$ (a) $\frac{sin^4 x}{4} + \frac{sin^6 x}{6} + C$ (b) $-\frac{cos^4 x}{4} - \frac{cos^6 x}{6} + C$
(c) $\frac{sin^4 x}{4} - \frac{sin^6 x}{6} + C$ (d) $\frac{cos^4 x}{4} - \frac{cos^6 x}{6} + C$ (e) none of the above

11. Obtain the reduction formula for $I_n = \int x^n lnx\, dx.$ (a) $I_n = \frac{1}{n+1} I_{n+1} - x$
(b) $I_n = \frac{1}{n+1} I_{n+1} - \frac{x^{n+1}}{(n+1)^2}$ (c) $I_n = I_{n+1} - \frac{x^{n+1}}{n+1}$ (d) $I_n = (n+1) I_{n+1} - \frac{x^n}{(n+1)^2}$ (e) none of the above

12. Evaluate $\frac{cos4\theta + isin4\theta}{cos\theta - isin\theta}$ (a) $cos4\theta + isin5\theta$ (b) $cos4\theta - isin5\theta$ (c) $cos5\theta + isin5\theta$
(d) $cos5\theta - isin5\theta$ (e) none of the above

13. Evaluate $Arg\left(\frac{\sqrt{3}+i}{1+i}\right)$ (a) $\tan^{-1}\left(\frac{\sqrt{3}+1}{1-\sqrt{3}}\right)$ (b) $\tan^{-1}\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right)$
(c) $\tan^{-1}\left(\frac{1+\sqrt{3}}{1+\sqrt{3}}\right)$ (d) $\tan^{-1}\left(\frac{\sqrt{3}-1}{1-\sqrt{3}}\right)$ (e) none of the above

14. Express $cos^3\theta$ in terms of multiple angles (a) $\frac{1}{4}(cos3\theta + 3cos\theta)$ (b) $4cos^3\theta - 3cos\theta$
(c) $3cos\theta - 4cos^3\theta$ (d) $\frac{1}{4}(cos3\theta - 3cos\theta)$ (e) none of the above

15. Express $-1 + i$ in terms of $re^{i\theta}$ (a) $\sqrt{2}e^{i\frac{4}{3}\pi}$ (b) $\sqrt{2}e^{-i\frac{4}{3}\pi}$ (c) $\sqrt{2}e^{i\frac{3}{4}\pi}$ (d) $\sqrt{2}e^{i\frac{1}{4}\pi}$
(e) none of the above

16. A function $f(x, y)$ is said to have a saddle point at point (x_o, y_o) if which of the following condition holds.

(a) $f_x \cdot f_{yy} < (f_{xy})^2$ (b) $f_{xx} \cdot f_{yy} < (f_{xy})^2$ (c) $f_{xx} < f_{xy}$ (d)) $f_x \cdot f_{yy} < f_{xy}$

(e) none of the above

17. Determine the reduction formula for $I_n = \int_0^{\pi} x^n cosx\, dx$ (a) $I_n = -n\pi^{n-1} - n(n-1)I_{n-1}$

(b) $I_n = n\pi^{n-1} - n(n-1)I_{n-2}$ (c) $I_n = -n\pi^{n-1} - n(n-1)I_{n-2}$

(d) $I_n = -n\pi^{n-1} + n(n-1)I_{n-1}$ (e) none of the above

18. Find the minimum and maximum values of $f(x, y) = xy$ subject to the constraint
$x^2 + y^2 - 8$ (a) (8, 4) (b) (8, -1) (c) (4, -4) (d) (2, 1) (e) none of the above

Use the information in* L1 *to solve the following questions.

L1: Given $Q = \frac{x}{(x+2)(x^2-2x-3)}$ to be integrated with respect to x.

19. Obtain the values of the numerators of the partial fractions of Q in **L1.** (a) $\frac{1}{4}, -\frac{2}{5}, \frac{3}{20}$

(b) $-\frac{1}{4}, \frac{2}{5}, \frac{3}{20}$ (c) $\frac{1}{4}, \frac{2}{5}, \frac{3}{20}$ (d) $-\frac{1}{4}, \frac{2}{5}, -\frac{3}{20}$ (e) none of the above

20. Solve $\int Q dx$ using **L1** (a) $-\frac{1}{4}\log_e|x-1| - \frac{2}{5}\log_e|x+2| + \frac{3}{20}\log_e|x+3|$
(b) $\frac{1}{4}\log_e|x+1| + \frac{2}{5}\log_e|x+2| + \frac{3}{20}\log_e|x-3| + C$
(c) $\frac{1}{4}\log_e|x+1| - \frac{2}{5}\log_e|x+2| + \frac{3}{20}\log_e|x-3| + C$
(d) $-\frac{1}{4}\log_e|x-1| + \frac{2}{5}\log_e|x+2| - \frac{3}{20}\log_e|x+3| + C$ (e) none of the above

21. Find the value of the langrage multiplier for $f(x,y,z) = x + y + 2z$ subject to the constraint $x^2 + y^2 + z^2 = 3$. (a) $\pm\frac{1}{2}$ (b) $\pm\frac{1}{\sqrt{2}}$ (c) $\pm\frac{1}{\sqrt{4}}$ (d) $\pm\frac{1}{4}$ (e) none of the above

22. Find the smallest value of the function $f(x,y) = x^2 + y^2$ subject to the constraint $y + 3x = 3$
(a) $(0.9, 0.3)$ (b) $(0.9, 0.3)$ (c) $(0.9, 0.3)$ (d) $(0.9, 0.3)$ (e) none of the above

23. Find the values of the langrage multiplier that gives the smallest value to the function $f(x,y) = x^2 + y^2$ Subject to the constraint $y + 3x = 3$ (a) 1 (b) 2 (c) 0.2 (d) 0.6
(e) none of the above

24. Find the nth differential coefficient of $y = \log_e x$ (a) $(-1)^n n!\, x^{-n}$ (b) $(-1)^{n-1}(n-1)!\, x^{-n}$
(c) $n!\, x^{-n}$ (d) $(1)^n(n-1)!\, x^{-n}$ (e) none of the above

25. Determine the second root of the complex number given by $z^4 = -1$. For $k = 1$
(a) $\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$ (b) $\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$ (c) $\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}$ (d) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$
(e) none of the above

26. Express the complex number $z = 3.5e^{1.12i}$ in the form $a + ib$ (a) $\log z = 3.5 + 1.12i$
(b) $\log z = 3.5 - 1.12i$ (c) $\log z = 3.5$ (d) $\log z = 1.12i$ (e) none of the above

27. If $f(x) = e^{2x}$. Find the nth derivative of $f(x)$. (a) e^{2x} (b) $2e^{2x}$ (c) $2^n e^{2x}$
(d) none of the above

28. Expand $f(x) = \ln x$ about a point $x = 1$.
(a) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \cdots .. + \frac{1}{n}(x-1)^n + E_n(x)$
(b) $(x+1) - \frac{1}{2}(x+1)^2 + \frac{1}{3}(x+1)^3 - \frac{1}{4}(x+1)^4 + \cdots .. + \frac{1}{n}(x+1)^n + E_n(x)$
(c) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \cdots .. + \frac{1}{n}(x-1)^n$
(d) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^2 + \cdots + E_n(x)$ (e) none of the above

29. Let $z = cos\theta + isin\theta$ and $z^{-1} = cos\theta - isin\theta$. What is the expression for $z - \frac{1}{z}$
(a) $2isin\theta$ (c) $2cos\theta$ (b) $2\sin n\theta$ (d) $2icos\theta$ (e) none of the above

30. Express $1 - i\sqrt{3}$ in polar form. (a) $\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$ (b) $2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$
(c) $\left(\cos\frac{7\pi}{3} + i\sin\frac{7\pi}{3}\right)$ (d) $4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$ (e) none of the above

31. Find the value of $\frac{z_1}{z_2}$ in polar form if $z_1 = 1 + i\sqrt{3}$ and $z_2 = 1 + i$
(a) $\frac{\sqrt{2}}{2}\left(\frac{\cos\pi}{12} + i\frac{\sin\pi}{12}\right)$ (b) $\sqrt{2}\left(\frac{\cos\pi}{12} + i\frac{\sin\pi}{12}\right)$ (c) $\sqrt{5}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
(d) $\sqrt{2}\left(\frac{\cos\pi}{12} - i\frac{\sin\pi}{12}\right)$ (e) none of the above

32. Evaluate $\int x^2 \sin x \ dx$ (a) $\sin x\,(2-x^2)+x\cos x+C$ (b) $\cos x\,(2-x^2)+x\sin x+C$
(c) $\sin x\,(2+x^2)-x\cos x+C$ (d) $\sin x\,(2-x^2)+x\sin x+C$ (e) none of the above

33. Evaluate $\int xe^x\,dx$ (a) $(x-1)e^x+C$ (b) $(x+1)e^x+C$ (c) $\frac{x^2}{2}e^x+x+C$
(d) x^2e^x+C (e) none of the above

34. Evaluate $\int x^2 \ln x\,dx$. (a) $\frac{x^3}{3}(\ln x+1)+C$ (b) $\frac{x^3}{27}(3\ln x+1)+C$ (c) $\frac{x^3}{9}(3\ln x-1)+C$
(d) $x^3(3\ln x-1)+C$ (e) none of the above

35. Solve $\int_0^{1/2} 4x\,e^{2x}dx$. (a) $2-e$ (b) $2e$ (c) 2 (d) $2+e$ (e) none of the above

36. Solve $\int x(x^2+2)^3dx$. (a) $\frac{(x^2+2)^4}{4}+C$ (b) $\frac{x(x^2+2)^4}{8}+C$ (c) $\frac{x(x^2+2)^4}{16}+C$
(d) $\frac{(x^2+2)^4}{8}+C$ (e) none of the above

37. Evaluate $\int \frac{x}{\sqrt{1+2x^2}}dx$. (a) $\frac{(1+2x^2)^{-1/2}}{2}+C$ (b) $\frac{(1+2x^2)^{1/2}}{2}+C$ (c) $\frac{x(1+2x^2)^{1/2}}{2}+C$
(d) $x(1+2x^2)^{1/2}+C$ (e) none of the above

38. Solve $\int \sin x\cos x\,dx$ (a) $\frac{sin^2x}{2}+C$ (b) $\frac{sin^2x\cos x}{2}+C$ (c) $\frac{cos^2x\sin x}{2}+C$
(d) $\cot x+C$ (e) none of the above

39. Evaluate $(i-\sqrt{3})(1+\sqrt{3})$ in the form $a+ib$ (a) $\sqrt{3}-2i$ (b) $\sqrt{3}+2i$ (c) $2\sqrt{3}-2i$
(d) $-2(i-\sqrt{3})$ (e) none of the above

40. Express the following complex number $\frac{(i-\sqrt{3})^2}{1-i}$ in the form $x+iy$
(a) $\left(1-\sqrt{3}\right)+i(1+\sqrt{3})$ (b) $\left(1+\sqrt{3}\right)+i(1-\sqrt{3})$ (c) $\left(1+\sqrt{3}\right)-i(1-\sqrt{3})$
(d) $\left(1-\sqrt{3}\right)-i(1+\sqrt{3})$ (e) none of the above