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DEPARTMENT OF MATHEMATICS
FACULTY OF PHYSICAL SCIENCES
UNIVERSITY OF BENIN, BENIN CITY

CA 2022/2023 SESSION

COURSE TITLE: Linear Algebra (MTH 230)

1. In a certain gathering of 200 students 50% of them like Commerce while 87% of them like Social Studies. How many students like both Social Studies and Commerce (a) 70 (b) 74 (c) 50 (d) 60 (e) None of the above
2. Consider the set $A=\{1,2,3,6,8\}$ $B=\{2,5,6,7,9\}$ and $C=\{3,7,6,9\}$. Determine $(A-B)\cup(B-A)$ (a) $\{1,3,5,7,8\}$ (b) $\{1,3,7\}$ (c) $\{1,3,5,7,8,9\}$ (d) $\{5,7,9\}$ (e)None of the above
3. Given vectors $u=(1,2,-1)$, $v=(6,4,2)$ and $w=(9,2,7)$ in IR^3. Which of the following is correct? (a) w is linearly independent on the vectors u and v (b) w is a linear combination of the vectors u and v (c) (a) and (b) (d) w is a linear transformation of the vectors u and v (e) None of the above
4. A mapping θ is said to be Onto if and only if (a) every element in the co-domain is an image from the domain (b) some elements in the co-domain are image from the domain (c) no element in the co-domain are image from the domain (d) (a) and (c) (e) None of the above
5. Let $f:V\to W$ be a function from the vector space V to the vector space W. f is a linear transformation if and only if (a) $f(x+y)=f(x)+f(y)$ and $k\,f(x)=f(kx)$ $\forall$ vectors x and $y\in V$ and k, a scalar (b) $f(x+y)=f(x)+f(y)$ $\forall$ vectors x and $y\in V$ (c) $f(x+y)=k\,f(x)$ $\forall$ vectors x and $y\in V$ (d) $k\,f(x)=f(kx)$ where k is a scalar (e) None of the above
6. Let $U=\{x:x \text{ is a real number}\}$ be the universal set and let $A=\{a:a \text{ is a Natural numbers}\}$ $B=\{b:b \text{ is an even numbers}\}$. Find $(A-B)\cup(B-A)$ (a) $(A-B)\cup(B-A)=\{$Set of natural numbers$\}$ (b) $(A-B)\cup(B-A)=\{$set of positive rational numbers$\}$ (c) $(A-B)\cup(B-A)=\{$set of Negative integers$\}$ (d) (b) and (c) (e) None of the above
7. Consider the sets $X=\{2,3,4,7\}$ $Y=\{3,5,4,8\}$ $Z=\{5,6,9\}$. Find $(Z/Y)\cup(X/Y)$. (a) $(Z/Y)\cup(X/Y)=\{2,3,6,7,5\}$ (b) $(Z/Y)\cup(X/Y)=\{3,6,7\}$ (c) $(Z/Y)\cup(X/Y)=\{2,3,6,7,9\}$ (d) $(Z/Y)\cup(X/Y)=\{5,6,9\}$ (e) None of the above
8. Which of the following is correct about vectors $v_1=(1,1,2)$, $v_2=(1,0.1)$ and $v_3=(2,1,3)$ in IR^3. (a) v_1,v_2 and v_3 spans IR^3 (b) v_1 and v_3 spans IR^3 (c) v_1 and v_2 spans IR^3 (d) v_1,v_2 and v_3 does not span IR^3 (e) None of the above
9. Given the vectors $w=(4,-1,8)$, $u=(2,4,-1)$ and $v=(6,4,-5)$. Which of the following statements is correct? (a) w is not a linear combination of the vectors u and v (b) w is a linear combination of the vectors u and v (c) w is linearly dependent on the vectors u and v (d) (b) and (c)
10. The function $T:R^2\to R^2$ subject to $T(x,y)=(x+y,x-y+3)$ is not a linear transformation because? (a) $T(x+y)=T(x)+T(y)$ (b) $T(x+y)\neq T(x)+T(y)$ and $k\,T(x)\neq T(kx)$ for $k\in IR$ (c) $k\,T(x)=T(kx)$ for $k\in IR$ (d) All of the above (e) None of the above
11. Which of the following is true about vectors $v_1=(1,2,1)$, $v_2=(2,9,0)$ and $v_3=(3,3,4)$ in IR^3 (a) v_1,v_2 and v_3 does not span IR^3 (b) v_1 and v_3 spans IR^3 (c) v_1,v_2 and v_3 spans IR^3 (d) v_1,v_2 and v_3 spans IR^2 (e) None of the above

12. Find the scalars x, y, z in $v = xu_1 + yu_2 + zu_3$ such that $v = (3,7,-4)$, $u_1 = (1,2,3)$, $u_2 = (2,3,7)$ and $u_3 = (3,5,6)$ in $\mathbb{R}^3$ (a) $x=4 \quad y=2 \quad z=3$ (b) $x=-2 \quad y=4 \quad z=-3$ (c) $x=4 \quad y=-5 \quad z=3$ (d) $x=2 \quad y=-4 \quad z=3$ (e) None of the above

13. Let V be any vector space and $S=\{v_1, v_2, \ldots, v_n\}$ a finite set of vectors in V. S is called a basis for V if and only if? (a) S is linearly independent and S spans V (b) S is linearly independent (c) S is linearly dependent (d) S does not span V (e) None of the above

14. Which of the following is true about $M=\begin{bmatrix}5 & 7\\ 7 & 8\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 1\\ 3 & 5\end{bmatrix}$ and $C=\begin{bmatrix}1 & 2\\ 4 & 4\end{bmatrix}$?
(a) M is a linear combination of A, B and C (b) M is not a linear combination of A, B and C (c) A, B and C are linearly independent (d) A, B and C are linearly dependent (e) None of the above

15. Determine whether the vectors $v_1=(1,-2,3)$, $v_2=(5,6,-1)$ and $v_3=(3,2,1)$ are linearly dependent or linearly independent (a) v_1 and v_2 are linearly independent (b) v_1, v_2 and v_3 are linearly dependent (c) v_1, v_2 and v_3 are linearly independent (d) All of the above (e) None of the above

16. What values of scalars a, b and c in $au+bv+cw=0$ would make the vectors $u=(1,2,3)$, $v=(2,5,7)$, $w=(1,3,5)$ linearly independent (a) $a=2 \quad b=5 \quad c=0$ (b) $a=0 \quad b=2 \quad c=10$ (c) $a=-1 \quad b=3 \quad c=2$ (d) $a=0 \quad b=0, \quad c=0$ (e) None of the above

17. Let $\theta: X \to Y$ from a set X to a set Y be an Onto map, then Y is called ? (a) Range (b) Surjective (c) Bijective (d) One-to-one (e) None of the above

18. An infinite set is called countable if and only if (a) there exist a surjective correspondence between it and the set of Natural numbers (b) there exist a One-to-one correspondence between it and the set of Natural numbers (c) there exist a One-to-one correspondence between it and the set of Integers (d) there exist a bijective correspondence between it and the set of rational numbers (e) None of the above

19. Two sets A and B can have the same number of Element if they are (a) Complex (b) Real (c) Equal (d) Rational (e) None of the above

20. A mapping that is both injective and surjective is called (a) Surjective (b) Into (c) Onto (d) Bijective (e) None of the above

21. Let $v_1=(3,2,1)$, $v_2=(3,8,0)$, $v_3=(2,2,6)$, and $S=\{v_1, v_2, v_3\}$ then (a) S is a Basis for $\mathbb{R}^3$ (b) S is a Basis for $\mathbb{R}^2$ (c) S is not a Basis for $\mathbb{R}^3$ (d) S is not a Basis for $\mathbb{R}^2$ (e) None of the above

22. An Injective mapping is also called: (a) Onto (b) One-to-one (c) Into (d) Bijective (e) None of the above

23. A Surjective mapping is (a) Bijective (b) Into (c) Onto (d) One-to-one (e) None of the above

24. If $A=\begin{bmatrix}2 & 1 & 3\\ 0 & 4 & 5\\ 2 & 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 3\\ 0 & 1 & 2\\ 1 & 4 & y\end{bmatrix}$. Find the value of y if $|AB|=48$ (a)-12 (b) 8 (c) 12 (d) 6 (e) none of the above

25. Find the eigenvalues of A if $A=\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}$ (a) 7 and 3 (b) 2 and 3 (c) 1 and 3 (d) 0 and 3 (e) none of the above

26. Determine A^{-1} if $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \\ 2 & -2 & 5 \end{bmatrix}$

(a) $\frac{1}{9}\begin{bmatrix} 13 & 1 & -8 \\ -7 & 4 & -5 \\ -8 & -2 & 7 \end{bmatrix}$

(b) $\frac{1}{20}\begin{bmatrix} 13 & -1 & -8 \\ -7 & 4 & 5 \\ -8 & 2 & 7 \end{bmatrix}$ (c) $\frac{1}{6}\begin{bmatrix} 3 & -1 & -8 \\ 6 & -4 & 6 \\ -8 & 2 & 7 \end{bmatrix}$ (d) $\frac{1}{7}\begin{bmatrix} 13 & -1 & -8 \\ -7 & 4 & 8 \\ -8 & 2 & 7 \end{bmatrix}$ (e) None of the above.

27. A square matrix A is Symmetric if (a)$A = -A^T$ (b) $A = A^{-1}$ (c) $A = A^T$ (d) $A = -A^{-1}$ (e) none of the above .

28. Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$ (a)-1, 1, and 2 (b) 3, 2, and 6 (c) -1, 2, and -2 (d) -1, 5, and -2 (e)none of the above

29. A square matrix A is Skew Symmetric if (a)$A = A^T$ (b) $A = -A^T$ (c) $A = A^{-1}$ (d) $A = -A^{-1}$ (e) none of the above.

30. If equation $x_1^2 + 4x_1x_2 - 3x_1x_3 + x_2^2 + 4x_2x_3 - x_3^2$ is expressed in the form $x^T Ax$ Find A (a) $\begin{bmatrix} 2 & 0 & 1 \\ 7 & 3 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & -1.5 \\ 2 & 1 & 2 \\ -1.5 & 2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -3 \\ 1 & 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 & 0 \\ 5 & 1 & -9 \\ 1 & 0 & -1 \end{bmatrix}$ (e)none of the above.

31. Determine the soiution to these set of equations using Gaussian elimination method $\begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & -1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \\ -5 \end{bmatrix}$ (a) 2 ,-4, and -3 (b) 2, -3, and 5 (c)-3, 4, and 0 (d) 6, 1 and 12 (e) none of the above

32. A system of linear equations is inconsistent if (a) it has one solution (b) Infinite number of solutions (c) No solution (d) It cannot represent a real life system (e) None of the above

33. Find $|A|$ if $A = \begin{bmatrix} 5 & 2 & 1 \\ 0 & 6 & 3 \\ 8 & 4 & 7 \end{bmatrix}$ (a)150 (b) 20(c) 100(d) 120(e) none of the above.

34. Determine the inverse of A if $A = \begin{bmatrix} 2 & 7 & 4 \\ 3 & 1 & 6 \\ 5 & 0 & 8 \end{bmatrix}$ (a) $\frac{1}{28}\begin{bmatrix} 8 & 56 & 38 \\ -6 & -4 & 0 \\ -5 & -35 & -19 \end{bmatrix}$

(b) $\frac{1}{38}\begin{bmatrix} 8 & -56 & 38 \\ 6 & -4 & 0 \\ -5 & 35 & -19 \end{bmatrix}$ (c) $\frac{1}{28}\begin{bmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{bmatrix}$ (d) $\frac{1}{40}\begin{bmatrix} 3 & -5 & 7 \\ 23 & 15 & -8 \\ 10 & -3 & 2 \end{bmatrix}$ (e)none of the above

35. If $A=\begin{bmatrix}2&3&5&3\\1&-2&-3&2\\6&5&4&1\end{bmatrix}$ and $B=\begin{bmatrix}1&-1\\1&0\\0&0\\1&1\end{bmatrix}$ Find AB (a)$\begin{bmatrix}2&1&3\\1&0&1\\7&2&1\end{bmatrix}$ (b)$\begin{bmatrix}7&5\\-2&-3\\10&4\end{bmatrix}$

(c)$\begin{bmatrix}8&1\\1&1\\12&-5\end{bmatrix}$ (d)$\begin{bmatrix}1&8\\2&9\\3&1\end{bmatrix}$ (e)none of the above

36. Find the determinant of $A=\begin{pmatrix}3&2&5\\4&7&9\\2&8&6\end{pmatrix}$ (a)10 (b) -12 (c) 40 (d) 30 (e) none of the above

37. A matrix is singular if (a)$|A|=0$ (b)$|A|<0$ (c)$|A|=2$ (d)$|A|>0$ (e) none of the above.

38. If $A=\begin{bmatrix}1&x&1\\3&-4&-2\\5&3&5\end{bmatrix}$ find the value of x if $|A|=-35$ (a) 0(b)2(c)3(d) 5 (e) None of the above.

39. Find the Adjoint of A if $A=\begin{bmatrix}2&3&5\\4&1&6\\1&4&0\end{bmatrix}$ (a)$\begin{bmatrix}20&-24&11\\7&2&4\\15&-5&-10\end{bmatrix}$(b) $\begin{bmatrix}-24&20&13\\6&-5&8\\15&-5&-10\end{bmatrix}$

(c)$\begin{bmatrix}-24&11&15\\0&-3&4\\14&-6&0\end{bmatrix}$ (d)$\begin{bmatrix}30&10&7\\-1&2&3\\15&-5&10\end{bmatrix}$ (e) None of the above.

40. Solve the system of equations $\begin{aligned}2x+4y+z&=20\\-2y+z&=11\\z&=1\end{aligned}$. Find $z^2+(y)^2-x$ (a)$\frac{13}{2}$ (b)$\frac{1}{4}$ (c)$\frac{3}{2}$ (d)$\frac{5}{7}$ (e) 9

41. Which of the following is/are not true? (a) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ (b) $A-B=A\cap B'$ (c) $(A\cap B)'=A'\cup B'$ (d) If $\cup$ is the universal set then $B'=\{t:t\in B,t\notin\cup\}$ (e) None of the above

42. Consider the set $A=\{1,2,3,5,7\}$, $B=\{0,3,6,7,9\}$ and $C=\{4,5,6,8\}$. Determine $(A-B)\cup(B-A)$ (a) (1, 2, 7, 1) (b) (0, 6, 9, 1) (c) (φ) (d) {0, 1, 2, 5, 6, 9} (e) None of the above

43. A relation from a set E to a set F is a subset of (a) $E\cap F$ (b) $E\cup F$ (c) E x F(d) E F(e) None of the above

44. Let A = {a, c} and B = { a, b, e, f }. What is n(A x B) (a) 16 (b) 6 (c) 8 (d) 4 (e) None of the above

45. Which of the following is not a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$? (a) T (x, y, z) = (x, 2y, 3x – y) (b) T (x, y, z) = (0, 0, 0) (c) T (x, y, z) = (x, 2y, 5z) (d) T (x, y, z) = (1, x, z) (e) None of the above

46. Which of the following is not true? (a) If $T:U\to V$ is any linear transformation from U to V, then T (x,y) = T (x) T (y) for all vectors. (b) The set A of all linear transformations of a vector space

into itself is also a vector space. (c) The set T of all linear transformations of a vector space into itself. (d) The set A of all linear transformations of a vector space into itself is a ring with respect to addition and multiplication.(e) None of the above

47. $T: R^3 \to R^3$ such that $T(x, y, z) = (-x, y-z, x-1)$ is not a linear map because it is: (a) Not additive (b) Not well defined (c) Neither homogeneous nor additive (d) Not closed with respect to x, y and z (e) None of the above

48. Given a 2 x 3 column vector $A = \begin{bmatrix} 0 & 1 \\ -2 & 2 \\ 1 & 0 \end{bmatrix}$, then the linear transformation $T: IR^2 \to IR^3$ is defined by (a) $T(x, y) = (-2x + z, x + 2y)$ (b) $T(x, y) = (x + 2y, 2x + y, 0)$ (c) $T(x, y) = (y, -2x + 2y, x)$ (d) $T(x, y) = (-x + 3y)$ (e) None of the above

49. Suppose $f: IR^n \to IR^m$ is linear, then (a) The kernel of f is a subspace of IR^n (b) The range of f is a subspace of IR^m (c) f (u + v) = f (u) + f (v) for all u, v, $\in IR^n$ (d) f (ku) = k f (u) for all u $\in IR^n$ and k, a scale (e) None of the above

50. 10. Which of the following function is not a linear transformation (a) $f: IR^3 \to IR^2$ such that f (x, y, z) = (x, -y, -z) (b) $f: IR^3 \to IR^3$ such that f (x, y, z) = (x, y, z)+(0, -1, 0) (c) $h: IR^2 \to IR^2$ such that h (x, y) = (2x, y-x) (d) $t: IR^3 \to IR^3$ such that t(x, y, z) = (x+ y, y- z, x) (e) None of the above

51. Which of the following statements is incorrect? (a) The empty set is a subspace of every vector space (b) Every subspace of a vector space is also a vector space (c) Every vector space is an additive abelian group (d) Every vector space is also a subspace (e) None of the above

52. Which of the following is a vector space? (a) The set V of all m x n matrices with real entries together with the operation of matrix addition and scalar multiplication. (b) The points on a plane V through the origin in IR^3 with addition and scalar multiplication (c) The points on a line passing through the origin in IR^2 with addition and scalar multiplication (d) All of the above (e) None of the above

53. Which of these vector space is finite dimensional even though it does not have a linearly independent set and therefore no basis? (a) The n-dimensional vector space (b) The zero vector space (c) The countable dimensional vector space (d) The infinite dimensional space (e) None of the above

54. Let V be a vector space, then (i) The set IR of real numbers is an element of V (ii) The set S of all linear combinations of the subspaces of V is also a subspace of V (iii) V is an additive group that is also commutative: (a) (i) and (iii) only (b) (ii) only (c) (ii) and (iii) only (d) (i), (ii) and (iii) (e) None of the above

55. Which of these is not a vector space? (a) The set of all vectors b for which a given system Ax = b has a solution (b) The set of vectors with positive entries (c) The vector V consisting of the single object zero (d) The set $V = IR^n$ with standard operations of addition and scalar multiplication. (e) None of the above