MTH 218

Exercises on Complex Variables

(1) Let $z_1 = \sqrt{2} + i\sqrt{2}$ and $z_2 = 2\sqrt{2} + 2i\sqrt{2}$
find (i) $z_1 + z_2$ (ii) $z_1 - z_2$ (iii) $z_1 \times z_2$
(iv) z_1/z_2 (v) $|z_1 + z_2|$ (vi) $|z_1/z_2|$

(vii) Express z_1 and z_2 in Euler' form.

(2) Using the principle of mathematical induction, Prove ~~that~~ the De Moivre's theorem ie Prove that
$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

(3) Using the ~~____~~ De Moivre's theorem Evaluate (i) $\dfrac{1}{(\cos\theta + i\sin\theta)^3}$ (ii) $\dfrac{(\cos\theta - i\sin\theta)^4}{(\cos\theta + i\sin\theta)^5}$

4 Express $\cos 5\theta$ in terms of powers of $\cos\theta$.

(5) If $z = 2+3i$, Express the following in the powers of exponential e.

(6) Express $z = (4+3i)e^{i\pi/3}$ in the form $u+iv$ where u and v are real numbers.

(7) Express the following in Polar form
(i) $e^{i\theta}$ (ii) $e^{i\pi/2}$ (iii) $e^{i\pi}$ (iv) $e^{i3\pi/2}$
(v) $e^{2\pi i}$ (vi) $e^{n\pi i}$

(8) Let $z_1 = re^{\pi i/6}$ and
$z_2 = re^{\pi i/6 + \pi i/2}$
show that $z_2 = z_1 e^{i\pi/2}$

(9) If $z = 2+3i$, Express the followings in the powers of exponential e

(10) Find $\lim_{z \to i} \frac{(z^2+1)}{z-i}$

(11) If given a complex number $\omega^2 = z$, find ω.

(12) find the primitive ~~root~~ 12th roots of 1.

(13) Find z if $az^2+bz+c=0$. Hence ~~find~~ solve the: $z^2+(2i-3)z+(5-i)=0$

evaluate $\frac{\cos 4\theta}{\sin 3\theta}$

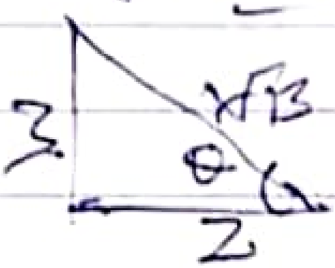
Sin

$3\sin\theta - 4\sin^3\theta$

Express the following in Polar form
(i) $e^{i\theta}$ (ii) $e^{i\pi/2}$ (iii) $e^{i\pi}$ (iv) $e^{i3\pi/2}$
(v) $e^{2\pi i}$ (vi) $e^{n\pi i}$

$z = x + iy$

$r^2 = 2^2 + 3^2 \Rightarrow r = \sqrt{4+9} = \sqrt{13} =$



$\sin\theta = \frac{3}{r}$. $\cos\theta = \frac{2}{r}$

$3 = r\sin\theta$

$2 = r\cos\theta$

$\tan\theta = \frac{3}{2}$

~~$\theta = \tan^{-1} 3/2 = 1.5$~~