26th August 2024, Monday, 09:02 PM.

Encryption Method.

1. Choose (e, m)

2. Convert letter string to number string.

3. Group number string into blocks of same length, use "dummy digits" to make up incomplete blocks.

4. Encrypt plaintext block by:

$$C = B^e \bmod m$$

28th August 2024 C1–C4
New Lecturer first shift
MTH 213 C3–C4
Dr. Mrs. Akhigbe

Vector Analysis.

C3: Vector Valued Functions: Limits, Continuity and Differentiation.

C4: Gradient, Divergence and Curl.

Vector Valued Function: maps real numbers to vectors.

vector markers → i, j, k

Definition:

If $\bar{A} = (A_1(u), A_2(u), A_3(u))$ is a vector valued function. We define limit of $\bar{A}$ at a point p as

$$\lim_{u \to p} \bar{A}(u) = \lim_{u \to p} A_1(u)i + \lim_{u \to p} A_2(u)j + \lim_{u \to p} A_3(u)k$$

Continuity

$\bar{A}(u)$ is continuous at a point p if each of the components are continuous at the point i.e $\bar{A}$

$|A_1(u) - A_1(p)| <$

is continuous at p if given any positive number c, we can find some positive number l such that

$|A_1(u) - A_1(p)| < c$, whenever $|u - p| < l$

$|A_2(u) - A_2(p)| < c$, whenever $|u - p| < l$

$|A_3(u) - A_3(p)| < c$, whenever $|u - p| < l$

Derivative

The derivative of $\bar{A}(u)$ with respect to u exist if the derivative of each components exists and is defined thus:

$$\frac{\partial \bar{A}}{\partial u} = \frac{\partial A_1}{\partial u}\hat{i} + \frac{\partial A_2}{\partial u}\hat{j} + \frac{\partial A_3}{\partial u}\hat{k}$$

where $\frac{\partial A_s}{\partial u} = \lim_{\Delta u \to 0} \frac{A_s(u+\Delta u) - A_s(u)}{\Delta u}$

$s = 1, 2, 3$

Application:

Ex 1, Given

~~$r = xi + yj + zk$~~

~~find~~ $r(t) = 2e^{t}i + 3e^{t}j + 6e^{-t}k$

find

$\frac{dr}{dt}$ at $t = 0$

$$\frac{dr}{dt} = 2e^{t}i + 3e^{t}j - 6e^{-t}k$$

at $t = 0$

$= 2i + 3j - 6k$.

Frenet - Serret formulas

(a) $\frac{d\bar{T}}{ds} = k\bar{N}$

(b) $\frac{d\bar{B}}{ds} = -\tau\bar{N}$

(c) $\frac{d\bar{N}}{ds} = \tau\bar{B} - k\bar{T}$

where $\bar{T} = \frac{r'(t)}{|r'(t)|}$ is the unit tangent.

$\bar{N} = \frac{T'(t)}{|T'(t)|}$ is the principal normal vector

$\bar{B} = \bar{T} \times \bar{N}$ is the binomial

$k = \frac{|dT|}{|ds|}$ is the curvature of the curve

$e = 1/k$ is the radius of curvature of the curve

Example

For the space curve

$x = 3\cos t$, $y = 3\sin t$, $z = 4t$

find $\bar{T}, \bar{N}, \bar{B}, k, \tau, e$

$r(t) = xi + yj + zk$

$r(t) = 3\cos t\, i + 3\sin t\, j + 4t\, k$

$r'(t) = -3\sin t\, i + 3\cos t\, j + 4k$

$T = \frac{r'(t)}{|r'(t)|}$

$$T = \frac{-3\sin t\, i + 3\cos t\, j + 4k}{\sqrt{9\sin^2 t + 9\cos^2 t + 16}}$$

$$= \frac{-3\sin t\, i + 3\cos t\, j + 4k}{\sqrt{9(\sin^2 t + \cos^2 t) + 16}}$$

$$T = \frac{-3\sin t}{5} i + \frac{3\cos t}{5} j + \frac{4}{5} k$$

Find $\bar{N}$

Assignment:

* ~~Equation~~ of a straight line
* Vector equation of a plane:

Gradient, Divergence, Curl.

Gradient:

If ϕ is a scalar function of the coordinates x, y, z, the values of the partial derivatives $\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}$ at a point represent the rate of change of ϕ with respect to distance in the x, y, z directions respectively.

div $= \nabla \cdot c$

Curl $= \nabla \times c$

$$\text{grad } \phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$$

Let $v = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$

The component of v in any direction is called the directional derivative in that direction.

$\text{grad } \phi = \nabla\phi$

Where $\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$

Divergence and Curl.

If A is a vector function of x, y, z the dot product of the operator ∇ and $\bar{A}$ is called the divergence of A written as:

$$\nabla \cdot A = \text{div } A = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \cdot (A_1 i + A_2 j + A_3 k)$$

$$= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

The cross product of the operator ∇ and A is called the curl of $\bar{A}$ written as:

71.

Grad $\rightarrow \nabla$

$\nabla \times A = $ Curl A

$$= \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \times (A_1 i + A_2 j + A_3 k)$$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= i\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) - j\left(\frac{\partial A_3}{\partial x} - \frac{\partial A_1}{\partial z}\right) + k\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

Examples

If $\Phi(x, y, z) = 3x^2y - y^3z^2$ find the gradient of ϕ at $(1, -2, -1)$

(2) If $A = x^2yi - 2xzj + 2yzk$ find the curl of A.

Solution.

(1) Given

$$\phi = 3x^2y - y^3z^2$$

$$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$$

$$= 6xyi + (3x^2 - 3y^2z^2)j - 2y^3zk$$

$\nabla\phi(1, -2, -1)$

$$= -12i - 9j - 16k$$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & -2xz & 2yz \end{vmatrix}$$

$$= \left(\frac{\partial 2yz}{\partial y} + \frac{\partial 2xz}{\partial z}\right)i - \left(\frac{\partial 2yz}{\partial x} - \frac{\partial x^2y}{\partial z}\right) + \left(-\frac{\partial 2xz}{\partial x} - \frac{\partial x^2y}{\partial y}\right)k$$

$$\nabla \times A = (2z + 2x)i - (0 - 0)j + (-2z - x^2)k$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2z+2x & 0 & -2z-x^2 \end{vmatrix}$$

$$\nabla \times (\nabla \times A) = (0)i - (-2x - 2)j + (0)k$$
$$= (2x + 2)j$$

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