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## Sheet 1 Answer

$$X_1 = 0,05$$

$$X_2 = 0,10$$

$$b_1 = 0,35$$

$$b_2 = 0,60$$

Initial weights

$$w_1 = 0,15$$

$$w_2 = 0,20$$

$$w_3 = 0,25$$

$$w_4 = 0,30$$

$$w_5 = 0,40$$

$$w_6 = 0,45$$

$$w_7 = 0,50$$

$$w_8 = 0,55$$

Output value

$$O_1 = 0,01$$

$$O_2 = 0,99$$

Forward Pass

$$\text{In } H_1 = X_1 * w_1 + X_2 * w_2 + b_1$$

$$= 0,05 * 0,15 + 0,10 * 0,20 + 0,35$$

$$= 0,3775$$

activation function:

$$\text{out } H_1 = \frac{1}{1 + e^{-H_1}} = \frac{1}{1 + e^{-0,3775}} = 0,593259992$$

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$$\text{In } H_2 = x_1 * w_3 + x_2 * w_4 + b_1 = 0,05 * 0,25 + 0,10 * 30 + 0,35 \\ = 0,3925$$

activation function:

$$\text{out } H_2 = \frac{1}{1 + e^{-0,3925}} = 0,596884378$$

$$\text{In } O_1 = \text{out } H_1 * w_5 + \text{out } H_2 * w_6 + b_2 \\ = 0,593269992 * 0,40 + 0,596884378 * 0,45 + 0,60 \\ = 1,105905967$$

$$\text{out } O_1 = \frac{1}{1 + e^{-1,105905967}} = 0,75136507$$

$$\text{In } O_2 = 0,50 * 0,593269992 + 0,596884378 * 0,55 + 0,60 \\ = 1,224921409$$

$$\text{out } O_2 = \frac{1}{1 + e^{-1,224921409}} = 0,772928465$$

Calculate Total Error

$$E_{\text{total}} = \sum \frac{1}{2} (\text{predict} - \text{actual})^2 \\ = \frac{1}{2} (P_1 - O_1)^2 + \frac{1}{2} (P_2 - O_2)^2$$

$$= \frac{1}{2} (0,01 - 0,75136507)^2 + \frac{1}{2} (0,99 - 0,772928465)^2$$

$$= 0,274811083 + 0,02350026 = 0,298311343$$



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## Back Propagation

$$w_5 = \frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial o_1} * \frac{\partial o_1}{\partial \ln o_1} * \frac{\partial \ln o_1}{\partial w_5}$$

$$\frac{\partial E_{total}}{\partial \text{out } o_1} = 2 * \frac{1}{2} (p_1 - \text{out } o_1)^{2-1} * -1 + 0$$

$$= -(p_1 - \text{out } o_1) = -(0,01 - 0,75136507)$$

$$\frac{\partial E_{total}}{\partial \text{out } o_1} = \boxed{0,74136607}$$

$$\text{out } o_1 = \frac{1}{1 + e^{-o_1}}$$

$$\frac{\partial \text{out } o_1}{\partial o_1} = \text{out } o_1 (1 - \text{out } o_1) = 0,75136507 (1 - 0,75136507)$$

$$\frac{\partial \text{out } o_1}{\partial o_1} = \boxed{0,186815602}$$

$$\frac{\partial o_1}{\partial w_5} = 1 * \text{out } H_1 * w_5^{(1-1)} + 0 + 0$$

$$= \text{out } H_1$$

$$\frac{\partial y_1}{\partial w_5} = 0,593269992 \Rightarrow \frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial \text{out } o_1} * \frac{\partial \text{out } o_1}{\partial o_1} * \frac{\partial o_1}{\partial w_5}$$

$$\frac{\partial E_{total}}{\partial w_5} = 0,74136607 * 0,186815602 * 0,593269992$$

$$= \boxed{0,082167041} \rightarrow \text{Change in } w_5$$

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updating  $w_5$

Learning rate = 0.5

$$w_5 = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5}$$

$$= 0.4 - 0.5 * 0.082167041$$

$$w_5 = 0.35891848$$

In the same way we will get new updated weights as

$$w_6 = 0.408665186$$

$$w_7 = 0.511301270$$

$$w_8 = 0.061370121$$

For hidden layer update

$w_1, w_2, w_3, w_4$

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out H_1} * \frac{\partial out H_1}{\partial H_1} * \frac{\partial H_1}{\partial w}$$

$$\frac{\partial E_{total}}{\partial out H_1} = \frac{\partial E_1}{\partial out H_1} + \frac{\partial E_2}{\partial out H_2}$$

$$\frac{\partial E_1}{\partial out H_1} = \frac{\partial E_1}{\partial o_1} * \frac{\partial o_1}{\partial out H_1}$$

$$\frac{\partial E_1}{\partial o_1} = \frac{\partial E_1}{\partial out o_1} * \frac{\partial out o_1}{\partial y_1}$$

$$= 0.74136607 * 0.188815602$$

$$= 0.138498562$$

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$$\frac{\partial H_1}{\partial \text{out} H_1} = w_5 = 0,40$$

$$\frac{\partial E_1}{\partial \text{out} H_1} = 0,138998562 * 0,40 = \boxed{0,055399425}$$

$$\frac{\partial E_2}{\partial \text{out} H_1} = -0,019049119$$

$$\frac{\partial E_{\text{total}}}{\partial \text{out} H_1} = 0,055399425 + (-0,019049119)$$

$$\frac{\partial H_1}{\partial w_1} = x_1 = 0,05 \quad \frac{E_{\text{total}}}{\partial w_1} = \frac{\partial E_{\text{total}}}{\partial \text{out} H_1} * \frac{\partial \text{out} H_1}{\partial H_1} * \frac{\partial H_1}{\partial w_1}$$

$$\frac{\partial E_{\text{total}}}{\partial w_1} = 0,03635 * 0,241300 * 0,05$$

$$= 0,000438568$$

updating  $w_1$

$$w_1 = w_1 - \eta * \frac{\partial E_{\text{total}}}{\partial w_1} = ,16 - 0,5 * 0,000438$$

$$= ,149780716$$

in the same way

$$w_2 = 0,19956143$$

$$w_3 = 0,24975114$$

$$w_4 = 0,29950229$$