Problem 2: Algorithms Osama Al-Wardi

a) $T(n) = 36T(n/6) + 2n$

Master Method

$a = 36$, $b = 6$, $f(n) = 2n$

$n^{\log_b a} = n^{\log_6 36} = n^2$

$n < n^2$

$\therefore$ Case 1:

$F(n) = (n^{\log_b a - \varepsilon})$ wherein this case $\varepsilon = 1$

$\therefore \quad T(n) = \Theta(n^2)$

b) $T(n) = 5T(n/3) + 17n^{1.2}$

$a = 5$, $b = 3$, $f(n) = 17n^{1.2}$

$n^{\log_b a} = n^{\log_3 5} = n^{1.4}$

$n^{1.2} < n^{1.4}$

Case 1:

$T(n) = (n^{\log_b a - \varepsilon})$ where in this case $\varepsilon = 0.2$

$T(n) = \Theta(n^{1.4})$

c) $T(n) = 12T(n/2) + n^2 \log n$

$a = 12$, $b = 2$, $f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_2 12} = n^{3.5}$

$n^2 \log n < n^{3.5}$

Case 1:

$T(n) = (n^{\log_b a - \varepsilon})$ where in this case $\varepsilon =$

$T(n) = \Theta(n^{3.5})$

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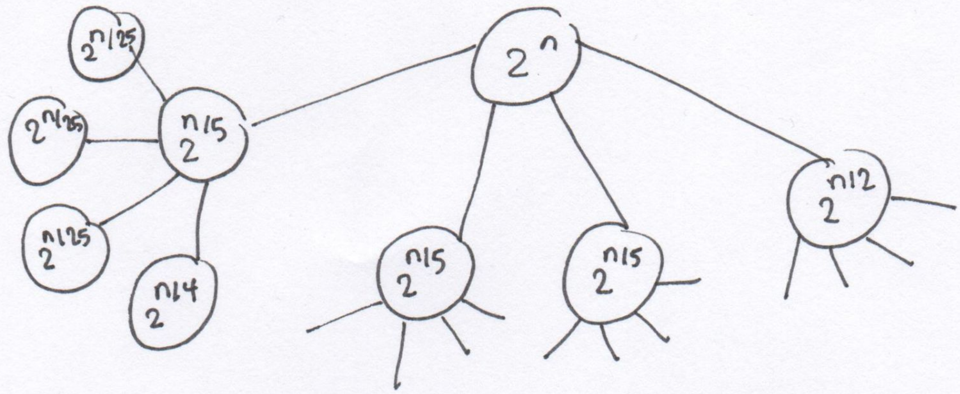
d) $T(n) = 3T(n/5) + T(n/2) + 2^n$



$3(2^{n/5}) + 2^{n/2}$ ... 1st Level

$12(2^{n/25}) + 4(2^{n/4})$ ... 2nd Level

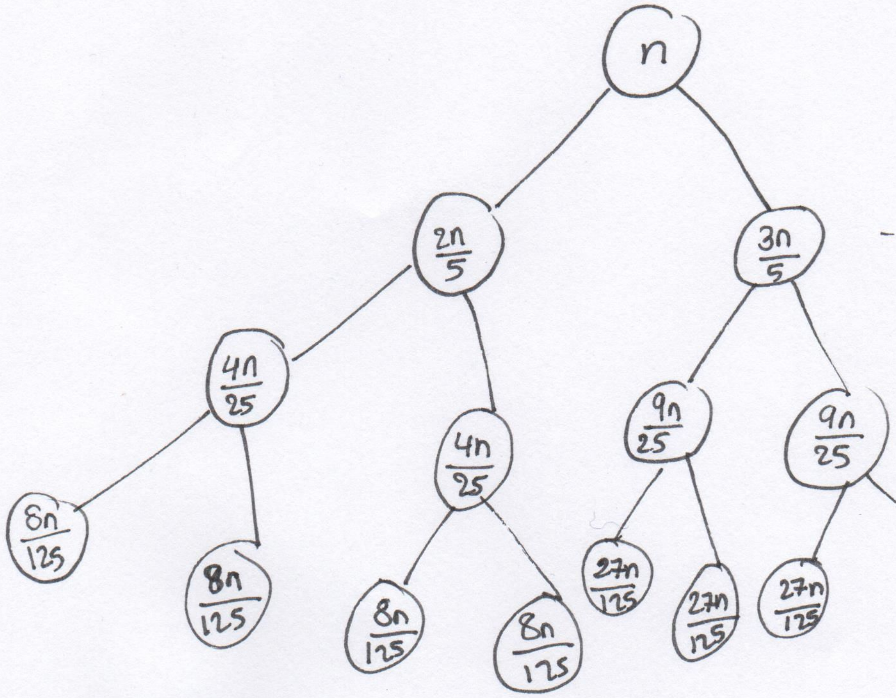
$24(2^{n/125}) + 16(2^{n/8})$ ... 3rd Level

Sum of Level ?

$\text{Total} = (3(2^{n/5}) + 2^{n/4} + 12(2^{n/25}) + 4(2^{n/4}) + 24(2^{n/125}) + 16(2^{n/8}) \ldots)$

$(2^{n/5}) + (2^{n/4}) \; (2(2^{n/5}) + 2^{n/4} + 12(2^{n}$

$T(n) = T(2n/5) + T(3n/5) + \Theta(n)$

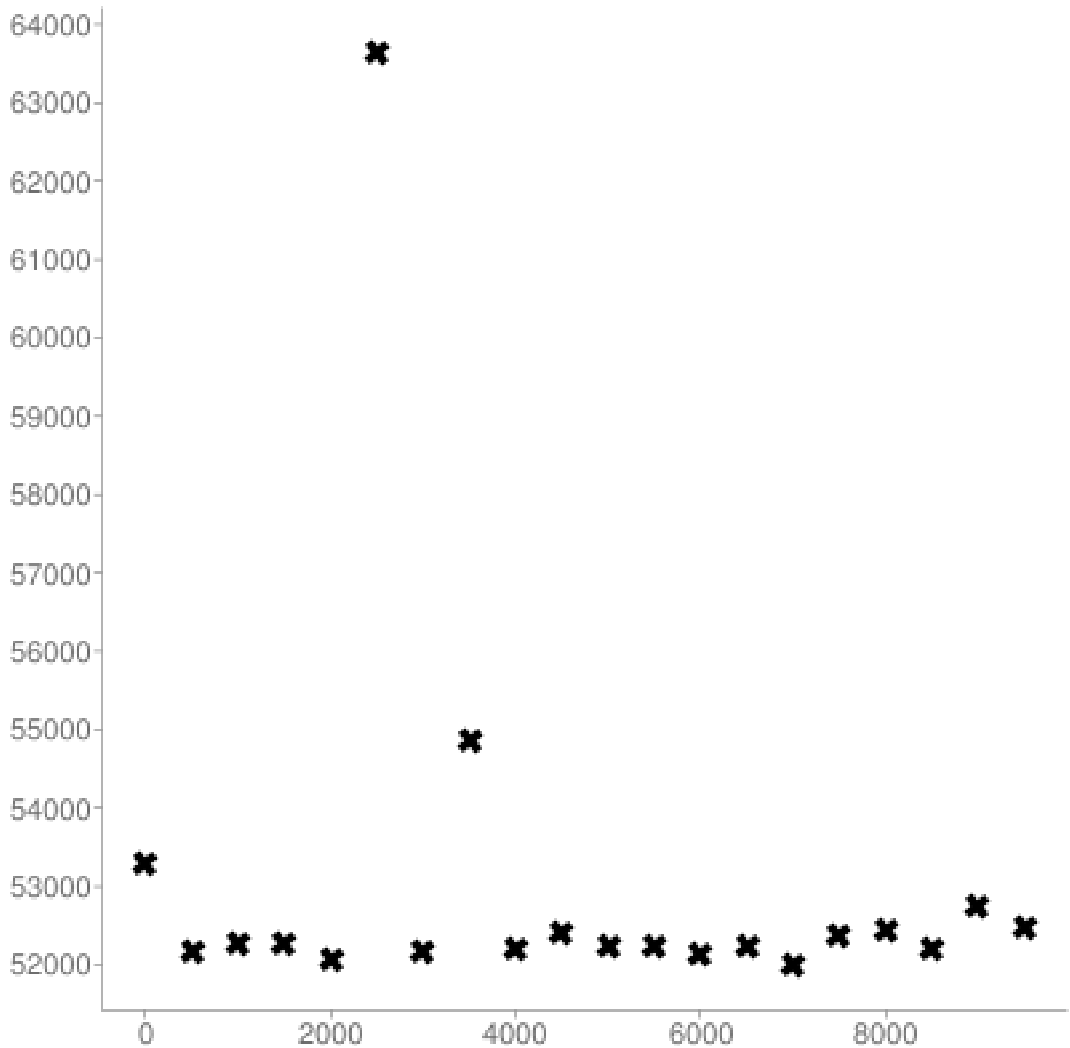


$\ldots \frac{2n}{5} + \frac{3n}{5}$

$\ldots 2\left(\frac{4n}{25}\right) + 2($

$4\left(\frac{8n}{125}\right) + 4$

Best Case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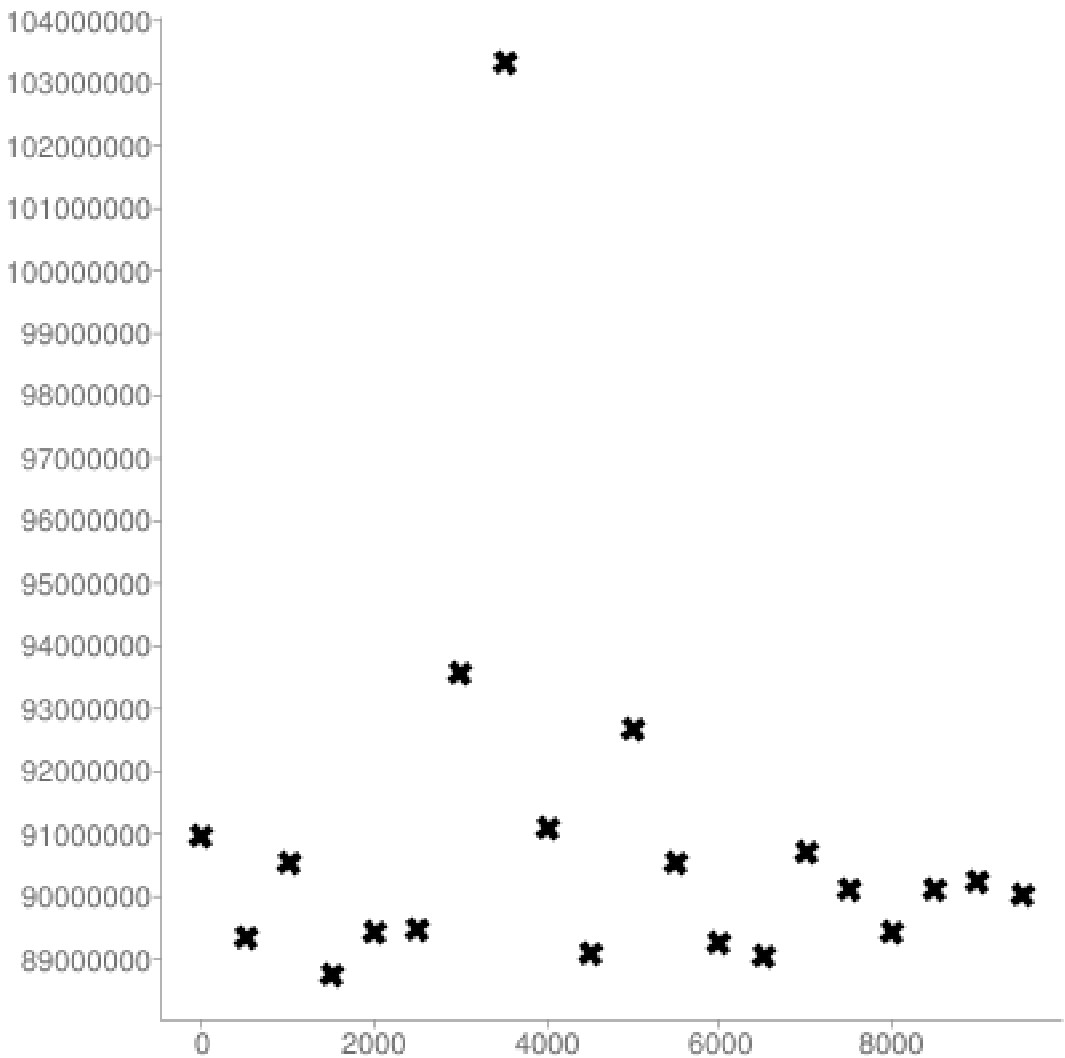
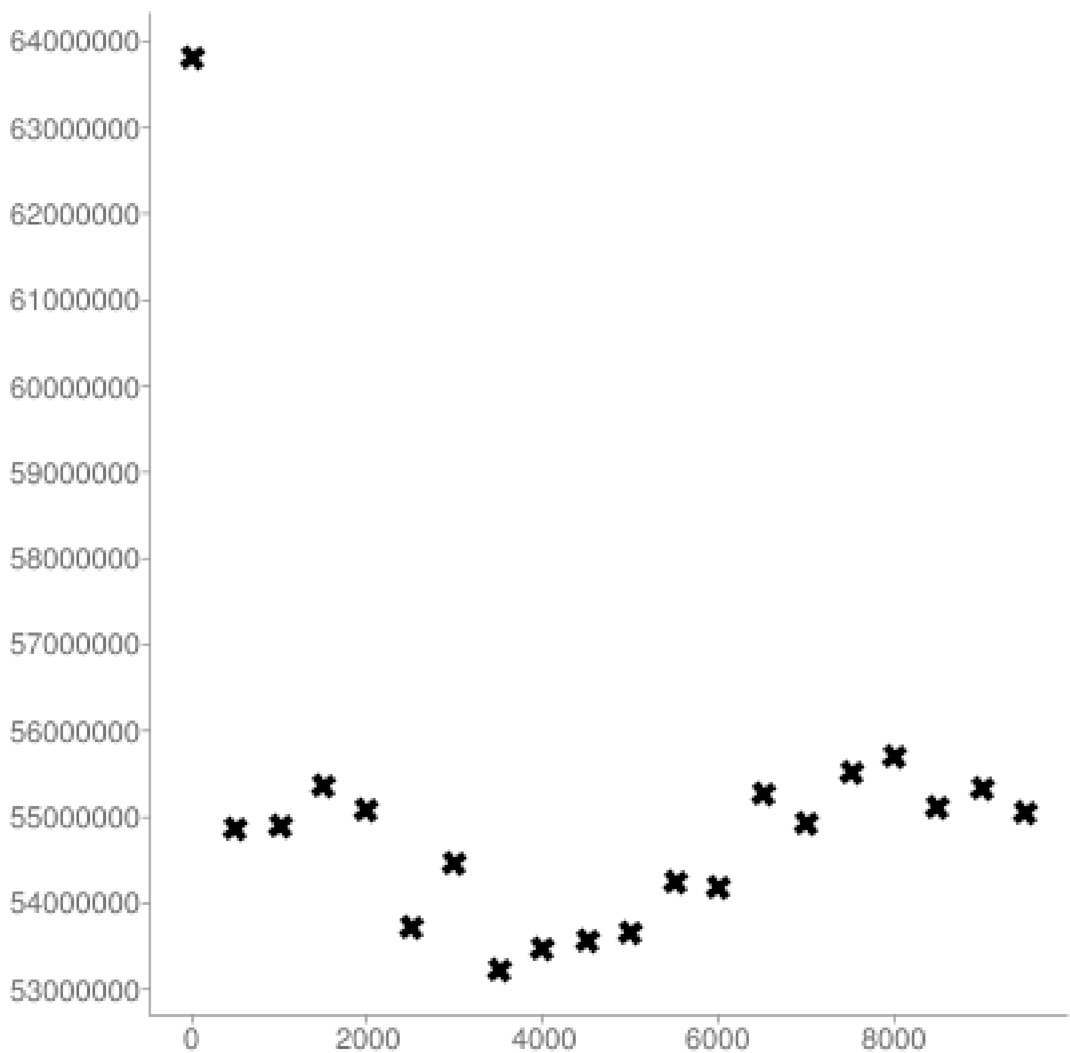
Average Case



Worst Case



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