Simple Linear Regression Examples

Question 1: Predicting House Prices

A real estate company collected data on houses sold in a suburban area. The dataset includes the size of the house in square feet and the selling price in thousands of dollars.

| House ID | Size (sqft) | Price (\$1000s) |
|----------|-------------|-----------------|
| 1 | 1200 | 200 |
| 2 | 1500 | 240 |
| 3 | 1600 | 250 |
| 4 | 1700 | 265 |
| 5 | 1850 | 275 |
| 6 | 1900 | 295 |

- a) Fit a simple linear regression model to predict Price from Size.
- b) Interpret the slope and intercept.
- c) Predict the price of a 1800 sqft house.
- d) Calculate the residual for a house of 1500 sqft sold for \$240,000.
- e) Report the R² value and explain the model's fit.
- f) Perform an F-test to determine the model's significance.

Question 2: Analyzing Study Time vs Exam Scores

A group of students recorded the number of hours they studied and their resulting exam scores.

| Student | Study Hours | Exam Score |
|---------|-------------|------------|
| А | 2 | 55 |
| В | 4 | 65 |
| С | 6 | 80 |
| D | 8 | 88 |
| E | 10 | 94 |
| F | 12 | 95 |

- a) Fit a linear regression model with Study Hours as the predictor.
- b) Interpret model coefficients.
- c) Predict the exam score for a student who studied 9 hours.
- d) Compute the residual for a student who studied 4 hours and scored 65.
- e) What does R² tell you in this case?
- f) Is the relationship statistically significant at the 0.05 level?

Question 3: Marketing Spend vs Customer Acquisition

A marketing team measured how different levels of digital ad spend impacted the number of new customers acquired.

| Campaign | Ad Spend (\$100s) | New Customers |
|----------|-------------------|---------------|
| 1 | 2 | 10 |
| 2 | 3 | 14 |
| 3 | 5 | 22 |
| 4 | 6 | 25 |
| 5 | 7 | 29 |
| 6 | 8 | 35 |

- a) Fit a simple linear regression model.
- b) Interpret the intercept and slope.
- c) Predict the number of customers if \$600 is spent.
- d) Calculate the residual for a campaign that spent \$500 and got 22 customers.
- e) Evaluate model strength using R².
- f) Construct an ANOVA table and perform the F-test.

Question 4: Website Load Time and Bounce Rate

A UX team wants to explore whether page load time affects the website's bounce rate (percentage of users who leave immediately).

| Website | Load Time (sec) | Bounce Rate (%) |
|---------|-----------------|-----------------|
| А | 1.2 | 20 |
| В | 2.1 | 35 |
| С | 3.0 | 50 |
| D | 4.2 | 60 |
| Е | 5.3 | 72 |
| F | 6.1 | 81 |

- a) Create a linear regression model predicting Bounce Rate from Load Time.
- b) Interpret coefficients.
- c) Estimate the bounce rate for a load time of 3.5 seconds.
- d) Calculate the residual for a 2.1-second load time with a 35% bounce rate.
- e) Report R² and comment on predictive strength.
- f) Use the F-test to assess model significance.