Class # 7

Date 6-4-25.

*) Continuous Variables:-

↳ length of time here.

e.g: waiting time.

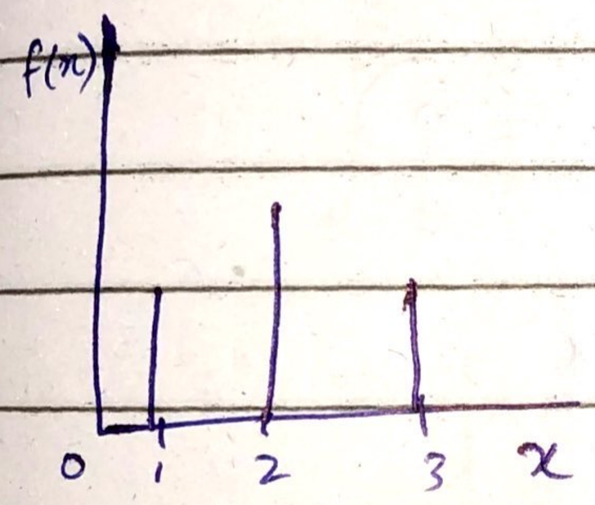
↳ range from min. value to max. value.

↳ all points included b/w max & min value

*) Discrete R.V & their distribution

1) $f(x) \geq 0$

$x = 0, 1, 2, 3.$



2) $\sum_x f(x) = 1$

Sum is equal to '1'

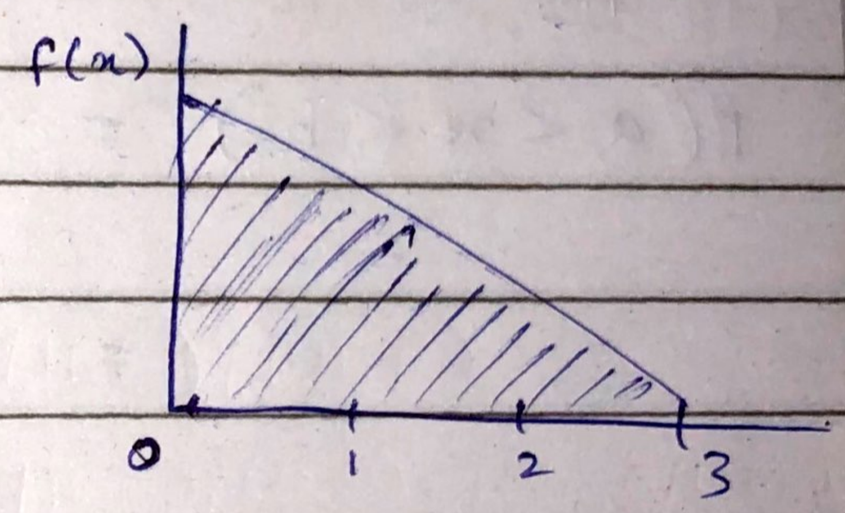
3) $E(x) = \sum x f(x)$

$E(x^2) = \sum x^2 f(x)$

Continuous R.V & their distribution

1) $f(x) \geq 0$

where, $0 \leq x \leq 3$



2) $\int_x f(x)\, dx.$

Integration replacing sum here.

3) $E(x) = \int_x x f(x)\, dx$

$E(x^2) = \int_x x^2 f(x)\, dx.$

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*) $f(x) \rightarrow$ pdf (probability density function)

*) $F(x) \rightarrow$ CDF (cummulative distribution function)

$$\boxed{F(x) = P(x \leq x) = \int_{x=L.L}^{x} f(x)\,dx.}$$

*) $$\boxed{f(x) = \frac{\partial F(x)}{\partial x}}$$

*) **Properties Of Continuous R.V:-**

Note:- for continuous; $<$ or $\leq$ are same
$>$ or $\geq$ " " "

$$P(a < x < b) = P(a \leq x \leq b)$$

$$= \int_{x=a}^{b} f(x)\,dx.$$

Or; $P(a < x < b) = F(x=b) - F(x=a)$

$\because$ where; a & $b \in X$

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4.1

$$f(x) = \begin{cases} \dfrac{K}{x^4} & \text{for } x \geq 1 \\ 0 & \text{for } x < 1 \end{cases}$$

..for above function; $\underset{L.L}{1} \leq x < \underset{U.L}{\infty}$

Overall; we can write;

$$f(x) = \frac{K}{x^4} \quad ; \quad x \geq 1$$

(∴ fn. only exist when $x = 1$ or >1)

A/c to 2nd property of Continuous R.V.:-

$$\int_{x=1}^{\infty} \frac{K}{x^4}\, dx = 1$$

$$\Rightarrow K \left[\frac{x^{-4+1}}{-4+1} \right]_1^{\infty} = 1$$

$$\Rightarrow \frac{K}{-3} \left[\frac{1}{x^3} \right]_1^{\infty} = 1$$

$$\Rightarrow \frac{-K}{3} \left[\frac{1}{\infty} - \frac{1}{1} \right] = 1$$

$$\Rightarrow \frac{-K}{3} (0 - 1) = 1$$

$$\Rightarrow K/3 = 1$$

$$\Rightarrow \boxed{K = 3}$$

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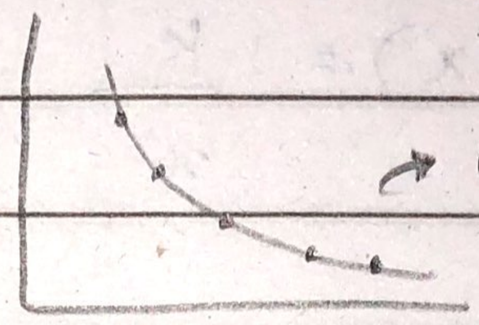
Now;

$$\boxed{f(x) = \frac{3}{x^4} \quad ; x \geq 1}$$

↳ with $k=3$; it's a valid fn. (for any value of $x \geq 1$; this fn. will integrate to 1)

i.e. $\int_{x=1}^{\infty} \frac{3}{x^4}\, dx = 1$

Note: It's curve will be:



→ life will decrease.

CDF:

$$F(x) = P(x \leq x) = \int_{x=1}^{x} \frac{3}{x^4}\, dx$$

$$= \frac{3}{-3}\left[\frac{1}{x^3}\right]_1^x$$

$$= -\left[\frac{1}{x^3} - 1\right]$$

$$\boxed{F(x) = 1 - \frac{1}{x^3}}$$

Verification of CDF:

$F(x = U.L) = 1$ → at $x = \infty$; $F(x=\infty) = 1$

$F(x = L.L) = 0$ → " " $= 1$; $F(x) = 0$.

↳ Verified

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✓ In continuous; calculate prob. via:-
i) Integration ii) Cdf.

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*) $P(x>2) = P(2 \le x \le \infty] = \int_{x=2}^{\infty} f(x)\,dx.$

OR:- $P(x>2) = P(2 \le x \le \infty]$

$= F(x=\infty) - F(x=2)$

$= \left[1 - \frac{1}{\infty}\right] - \left[1 - \frac{1}{2^3}\right]$

$= 1 - \left[1 - \frac{1}{8}\right]$

$$\boxed{P(x>2) = \frac{1}{8}}$$

4.2: Don't calculate CDF here. (Solve it)

-> Integrate from 1 to 2 here.

4.3 Solve it.

4.4

$$f(x) = \begin{cases} K - \frac{x}{50} & \text{for } 0 < x < 10 \text{ years.} \\ 0 & \text{for all other } x. \end{cases}$$

(i) Here; $0 \le x \le 10$

-> To find value of 'K':-

$\therefore \int_{x=0}^{10} K - \frac{x}{50}\,dx = 1$

$\Rightarrow \left[Kx - \frac{x^2}{100}\right]_0^{10} = 1$

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✓ Life time → always contin

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$\Rightarrow [10k - 1] = 1$

$\Rightarrow 10k = 1 + 1$

$\Rightarrow k = 2/10$

$\Rightarrow \boxed{k = 1/5}$

So;

$$\boxed{f(x) = \left(\frac{1}{5} - \frac{x}{50}\right) \quad ; \quad 0 < x < 10}$$

ii) $P(x \leq 5)$

$= P(0 \leq x \leq 5)$

$= \int_{x=0}^{5} \left(\frac{1}{5} - \frac{x}{50}\right) dx.$

$= \left[\frac{x}{5} - \frac{x^2}{100}\right]_0^5$

$= \left(1 - \frac{1}{4}\right)$

$= 3/4$

$= 0.75 = \boxed{75\%}$

OR can calculate it via CDF as done in 4.1

(iii) $E(x) = \int_{x=L.L}^{U.L} x f(x) dx.$

$= \int_0^{10} \left(\frac{x}{5} - \frac{x^2}{50}\right) dx.$

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$$= \left(\frac{x^2}{10} - \frac{x^3}{150}\right)_0^{10}$$

$$= \left(\frac{100}{10} - \frac{1000}{150}\right)$$

$$= 10 - 6.67$$

$$\boxed{E(x) = 3.33}$$

So; on avg. the life is 3years 4 months.

Extra:- $E(x^2) = \int_{x=L.L}^{U.L} x^2 f(x)\, dx$.

$$= \int_0^{10} \left(\frac{x^2}{5} - \frac{x^3}{50}\right) dx.$$

$$= \left(\frac{x^3}{15} - \frac{x^4}{200}\right)_0^{10}$$

$$= \left(\frac{1000}{15} - \frac{10000}{200}\right)$$

$$= \boxed{16.67}$$

$$Var(X) = E(x^2) - [E(x)]^2$$

$$= 16.67 - (3.33)^2$$

$$= 5.581$$

$$S.D(x) = \sqrt{5.581} = \boxed{2.362}$$

Deviation is 2 years, 4 months.

i $0 < x < 10$

OR can calculate it via CDF as done in w/.

Class # 8

Date 13-4-25.

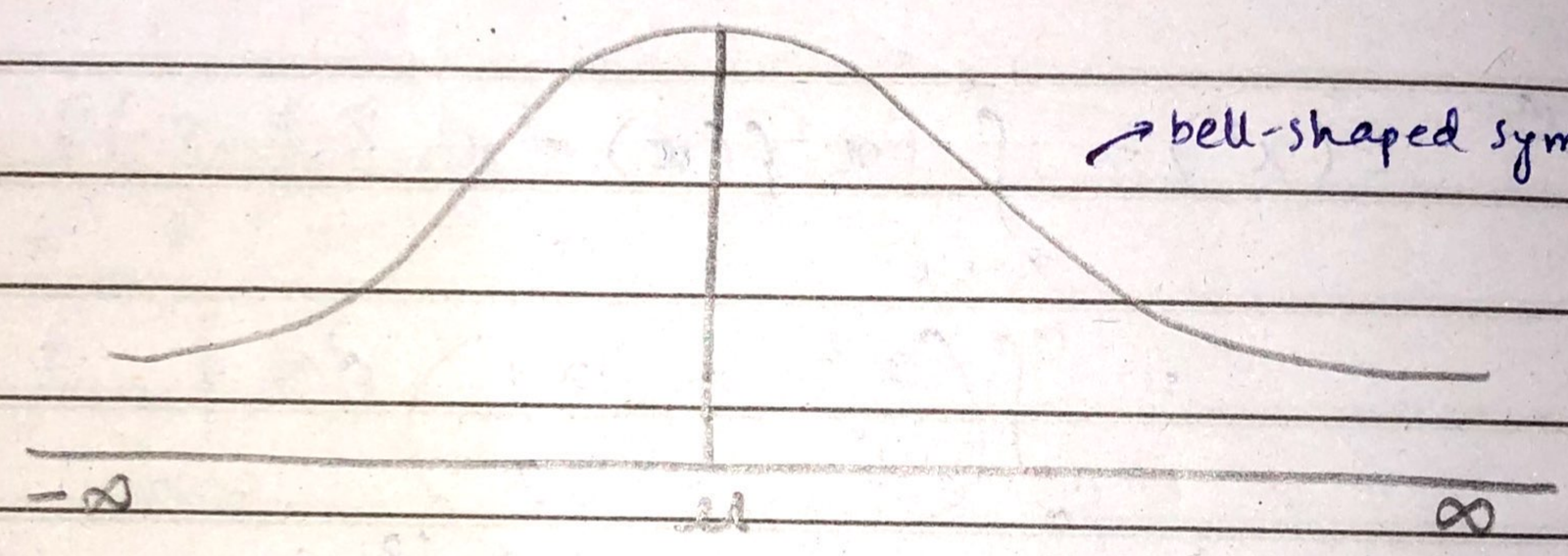
Normal Distribution :-

↳ Cont. prob. distri. with following pdf :-

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; -\infty < x < \infty$$

where; $\mu \rightarrow$ Mean ; $\sigma \rightarrow$ S.D.

Area of Normal Curve :-



→ bell-shaped symmetric.

*) Normal $\xrightarrow{\text{to}}$ Standard Normal form. (Transformation)

$$\boxed{Z = \frac{x-\mu}{\sigma}}$$

where;

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \quad ; -\infty < z < \infty$$

Mean $(z) = 0$ & S.D$(z) = 1$.

C.D.F :-

$$F(z) = P[z \leq z] = \int_{z=-\infty}^{z} f(z)\,dz.$$

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For Normal Distri..

(i) Convert into z form. (ii) Use CDF & Calculate Prob.
(iii) Use z - table.

$$\boxed{F(z) = \int_{z=-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \, dz}$$

Use C.D.F to calculate probabilities:-

(i) $P[z \le a] = P[z < a]$
$= F[z = a]$

(ii) $P[z \geq a] = P[z > a]$
$= 1 - F(z = a)$

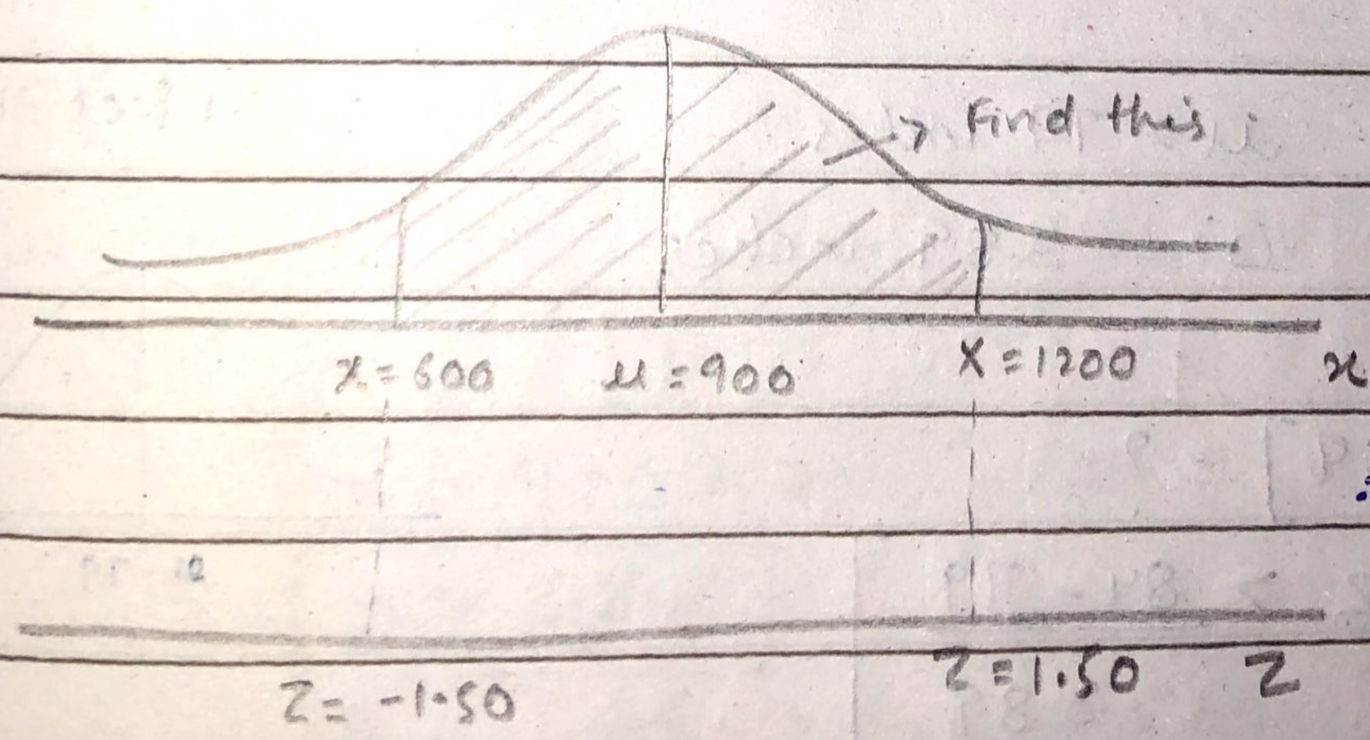
(iii) $P[a < z < b] = F[z = b] - F[z = a]$

Example # 4.11:-

Given; $\mu = 900$; $\sigma = 200$

X : income (continuous variable)

$P[600 < x < 1200] = P[-1.50 < z < 1.50]$



$\therefore z = \frac{X - 900}{200}$

$\therefore P[-1.50 < z < 1.50] = F[z = 1.50] - F[z = -1.50]$

read 1st decimal from rows, 2nd decimal place from col.

z	.00		0.04 → 2nd decimal place
-1.5	-1.50 ✓ → this value		-1.54

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from z - table;

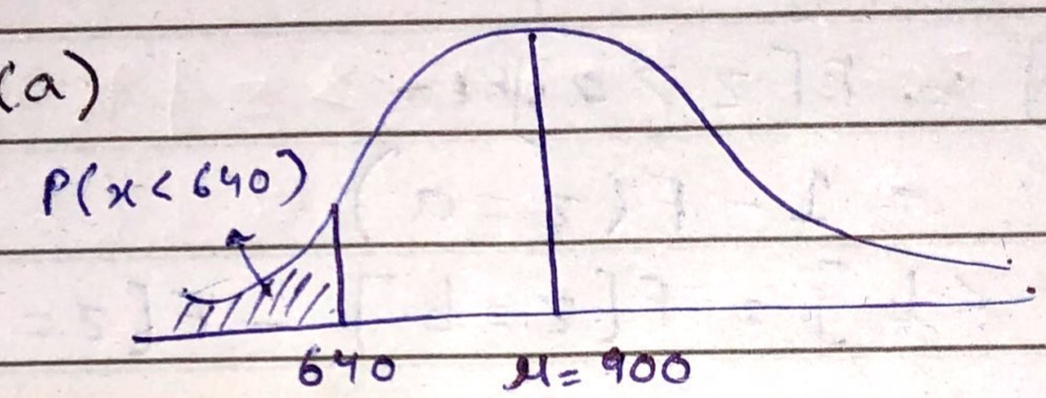
$F[z = -1.50] = 0.0668$

$F[z = 1.50] = 0.9332$

So; $P[-1.50 < z < 1.50] = 0.9332 - 0.0668$

$= \boxed{0.8664}$

4.22 (a)



$P(x < 640) = P\left(z < \frac{640-900}{200}\right)$

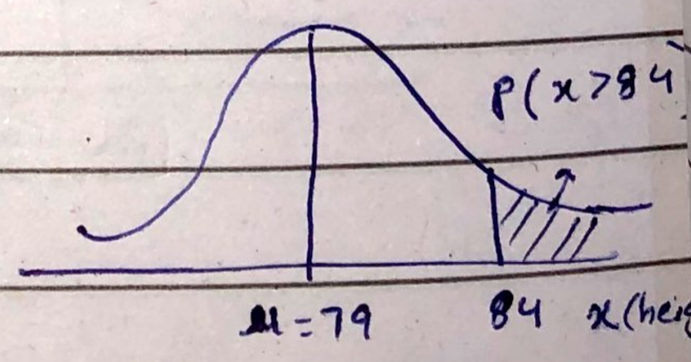
$= P(z < -260/200)$

$= P(z < -1.30)$

$= \boxed{0.0968}$ or $\boxed{9.68\%}$

4.21 (a) $\mu = 79$ inches. ∴ 1 feet = 12 inches

$\sigma = 3.89$ inches.



$P[x > 84] = ?$

$= P\left[z > \frac{84-79}{3.89}\right]$

$= P[z > 1.285]$

$= P[z > 1.29]$

$= 1 - P[z \leq 1.29]$

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$= 1 - F[z = 1.29]$

$= 1 - 0.9015$

$= 0.0985 = \boxed{9.85\%}$

Ex# 4.12

$\mu = 900$

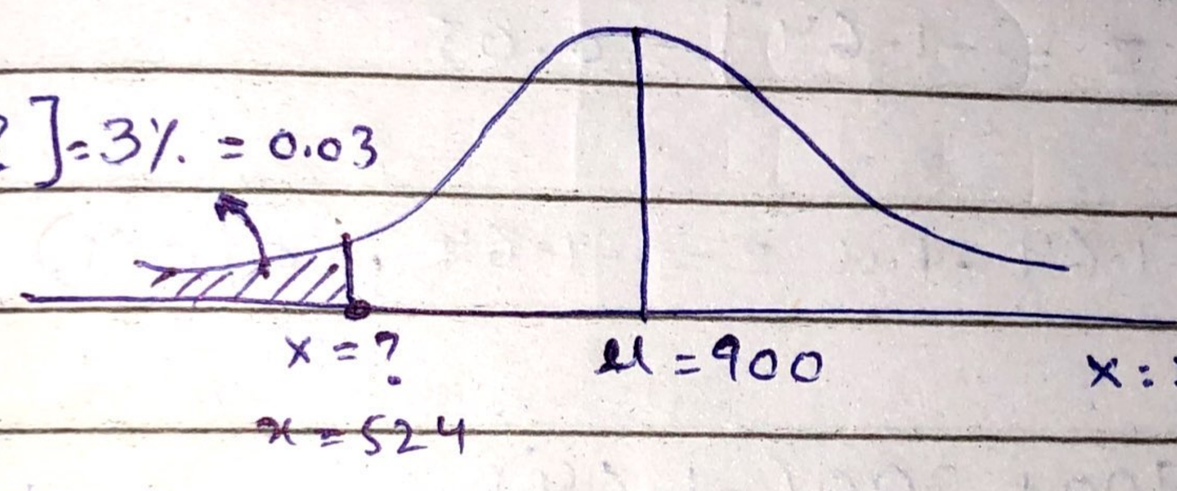
$\sigma = 200$

(Inverse prob. problem).

$x = \mu + \sigma z = 900 + 200z \rightarrow (1)$

Here; $z \rightarrow$ given ; $x \rightarrow$ asked

$P[x < ?] = 3\% = 0.03$



✓ poorest → at left side.

✓ richest → at right

x : Income

From Z-table:-

z	.08
-1.8	0.0301

$P[z < \boxed{-1.88}] = 0.03.$

$F[z = -1.88] = 0.03 \Rightarrow z = -1.88$

Put $z = -1.88$ in eqn (1) to find x :-

$x = 900 + 200(-1.88)$

$\boxed{x = 524}$

(Cross-verify by $x \rightarrow z \rightarrow$ CDF $\rightarrow$ table)

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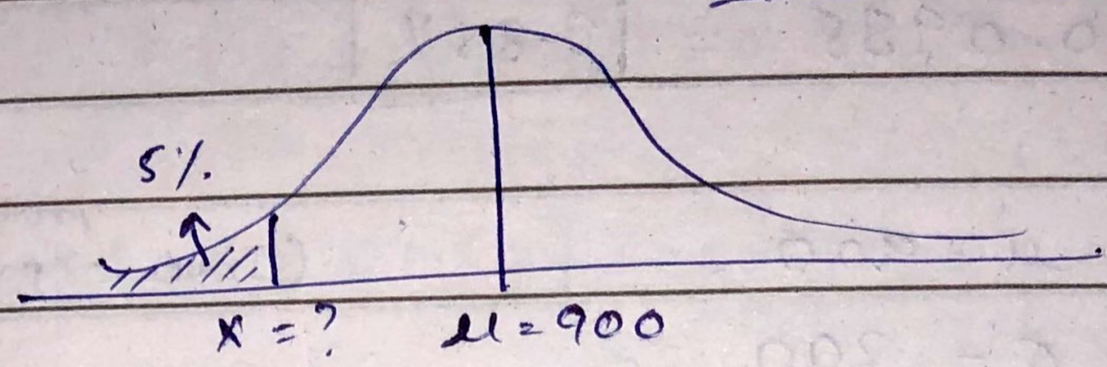
4.22(b):-

$\mu = 900$; $\sigma = 200$.

$x = \mu + \sigma z = \boxed{900 + 200z} \rightarrow (1)$



$P[x < ?] = 0.05$

From z - table,

$P[z < -1.64] = 0.05$

$F[z = -1.64] = 0.05$

So; $z = -1.64$. Put $z = -1.64$ in eqn (1) to find x:-

$x = 900 + 200(-1.64)$

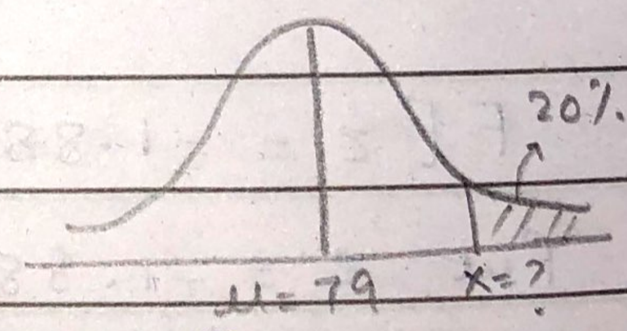
$\boxed{x = 572}$

4.21(b):-

$\mu = 79$ inches

$\sigma = 3.89$ inches.

$x = \mu + \sigma z = \boxed{79 + 3.89z} \rightarrow (1)$



$P[x > ?] = 0.20$ or $P[x \leq ?] = 0.80$

from z - table:-

$P[z < 0.84] = 0.7995$

$P[z = 0.84] \approx 0.80$

Put $z = 0.84$ in eqn (1)

$x = 79 + 3.89(0.84)$

$\boxed{x = 82.267}$

If;

$P(z < 0.85) = \boxed{0.8023}$

Diff = 0.0023

$P(z < 0.84) = \boxed{0.7995}$

Diff = 0.00

Small diff. (chose it)

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Central Limit

↳ If data

↳ Sample n

↳ (base

If $n \geq 30$;

↳ If data

& mean

x_1

*) $S_n = x_1$

$x \sim N$

$S_n \sim$

*) $\bar{x} = \frac{S_n}{n}$

Previously

Here;

*) Mean (

*) S.D (

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$\bar{X}$ = Sample mean.

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Central Limit Theorem:-

↳ If data whose distri. is not known → Central limit theorem.

↳ Sample must be larger.

↳ (based upon more than 1 sample)

(ii) If $n \geq 30$;

↳ If data follow any distribution, it's sum & mean will follow 'Normal Distribution)

$$\underbrace{x_1, x_2, \ldots\ldots, x_n}_{n \text{ obvs.}}$$

*) $S_n = X_1 + X_2 + \ldots\ldots + X_n = \Sigma X$.

$X \sim N[\mu, \sigma]$

$S_n \sim N[n\mu, \sqrt{n}\,\sigma]$

*) $\bar{X} = \dfrac{S_n}{n} = \dfrac{\Sigma X}{n}$.

Previously; $z = \dfrac{x - \mu}{\sigma}$

Here; $\boxed{z = \dfrac{S_n - n\mu}{\sqrt{n}\,\sigma}}$ or $\boxed{z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}}$

based upon $\bar{X}$ (mean given) or S_n (sum given) → use these transformations.

*) Mean $(S_n) = n\mu$

*) S.D $(S_n) = \sqrt{n}\,\sigma$

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Ex# 4.13

$\mu = 1$ — expected → mean.

avg. size '1' ← $\sigma = 0.5$ — S_n = sum of all imgs.

~~but some can~~ be greater — $n = 300$ — $S_n = x_1 + \cdots + x_{300}$

due to this deviation — $\sqrt{n}\,\sigma = \sqrt{300} \times 0.5 = 8.66.$

To find :-

$P[S_n \leq 330].$

Now; $z = \dfrac{S_n - n\mu}{\sqrt{n}\,\sigma} = \dfrac{S_n - 300}{8.66}.$

So; $P[S_n \leq 330] = P\left[z \leq \dfrac{330 - 300}{8.66}\right]$

$= P[z \leq 3.46]$

$= 0.9997$

$= \boxed{99.97\%}$

↳ space sufficient prob.

Ex# 4.14 :- $x_1, x_2, \ldots x_{10}$

$S_n = x_1 + x_2 + \cdots + x_{10}.$

$P[S_n + 150 \leq 2000] = P[S_n \leq 1850]$

Now; $E[S_n] = n\mu$

$= 10(165)$

$= \boxed{1650}$

$S.D(S_n) = \sqrt{n}\,\sigma$

$= \sqrt{10} \times 20$

$= 63.245$

$= \boxed{63.25}$

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$$Z = \frac{S_n - 1650}{63.25}$$

$\therefore Z_n = \frac{S_n - n\mu}{\sqrt{n}\,\sigma}$

Hence; $P[S_n \leq 1850] = P\left[Z \leq \frac{1850 - 1650}{63.25}\right]$

$= P[Z \leq 3.16]$

$= \boxed{0.9992}$

↳ 99.92% chances that we can go in lift.

4.24:- $n = 82$; $S_n = X_1 + X_2 + \cdots X_{82}$

$E[x_i] = 15$ seconds.

$Var = \sigma^2 = 16$; $S.D(x_i) = 4$.

To find:-

$P[S_n < 1200]$

As; $E[S_n] = n\mu = 82 \times 15 = 1230$

$S.D[S_n] = \sqrt{n}\,\sigma = \sqrt{82} \times 4 = 36.22$

Now; $P[S_n < 1200] = P\left[Z < \frac{1200 - 1230}{36.22}\right]$

$= P[Z < -0.828]$

$= 0.2033$

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4.23

$\mu = 5000$ hrs.

$\sigma = 100$ hrs

$n = 400$ (single individual $= x$)

($\bar{x}$ = avg.)

To find:-

$P[\bar{x} < 5012]$

As; $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{\bar{x} - 5000}{100/\sqrt{400}} = \dfrac{\bar{x} - 5000}{5}$

Now; $P[\bar{x} < 5012] = P\left[Z < \dfrac{5012 - 5000}{5}\right]$

$= P[Z < 12/5]$

$= P[Z < 2.4]$.

4.26 $E(x_i) = 0.6 = \mu$

$\sigma(x_i) = 0.4 = \sigma$

$n = 80$; $S_n = x_1 + x_2 + \cdots x_{80}$. (total)

To find:

$P[47 < S_n < 50]$

Now; $E(S_n) = n\mu$

$= 80 \times 0.6$

$= \boxed{48}$

$S.D(S_n) = \sqrt{n}\,\sigma$

$= \sqrt{80} \times 0.4$

$= \boxed{3.577}$

Class # 9

Date 20-4-25

Normal Approximation to Binomial Distribution:-

↳ use normal distribution for binomial problems when :-

↳ If np & nq both ≥ 10, then Normal approximation is good-

$$Z = \frac{x - \mu}{\sigma} = \frac{x - np}{\sqrt{npq}}$$

*) $np \rightarrow$ mean, (μ)

*) $\sqrt{npq} = \sigma =$ S.D.

Continuity Correction:-

To make discrete data into continuous, we subtract 0.5 in lower limit & add 0.5 in upper limit.

Ex # 4.15 $n = 200$, $p = 0.2$, $q = 0.8$

$P[x < 50] = P[x \leq 49] = ?$ 0.950647

($\because n = 200$ table ✗) Solve via Excel ← (Exact sol-) (from binomial distri)

↳ From calculator:

$$\sum_{x=0}^{49} \left(\left({}^{200}C_x \right) \left(0.2^{\wedge x} \right) \left(0.8^{\wedge (200-x)} \right) \right)$$

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↳ To solve via Normal Distribution:-

$np = 200 \times 0.2 = 40$

$nq = 160$

} verification ✓ (both ≥ 10)

Now; $\sigma = \sqrt{npq} = \sqrt{200 \times 0.2 \times 0.8} = \sqrt{32} = 5.657$

Upper limit (∵ less/equal to 49 means 49 is U.L)

$P(x \leq 49) = P(x \leq 49.5)$ → after applying continuity correction

$= P\left(z \leq \frac{49.5 - 40}{5.657}\right)$

$= \Phi(1.68)$ or $P(Z \leq 1.68)$

$= \boxed{0.9535}$

↳ approximated answer.

↳ If written in Exam that use approximation, then solve this way OR else we can also solve manually.

4.27 $n = 2400$

$p = 0.35$, $q = 0.65$

Condition check:- ✓ ($np \geq 10$ & $nq \geq 10$)

$P(800 \leq x \leq 850)$

Apply continuity correction:-

$= P(799.5 \leq x \leq 850.5)$

Now, we use z-transformation.

$\because z = \frac{x - np}{\sqrt{npq}} = \frac{x - 840}{23.67}$

$\because np = 2400(0.35) = 840$

$\because \sqrt{npq} = 23.67$

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$\therefore x \underset{\text{follow}}{\sim} \text{Normal}$

$$= P\left(\frac{799.5 - 840}{23.67} \le z \le \frac{850.5 - 840}{23.67}\right)$$

$$= P(-1.71 \le z \le 0.44)$$

$$= P(z \le 0.44) - P(z \le -1.71)$$

$$= F(z = 0.44) - F(z = -1.71)$$

$$= 0.67 - 0.0436$$

$$= 0.6264 = \boxed{62.64\%}$$

To do: 4.1 – 4.4, 4.16, 4.17, 4.21 – 4.27.

Continuous

Uniform Distribution :-

$a \to L.L$
$b \to U.L.$

*) p.d.f :- $f(x) = \dfrac{1}{b-a}$; $a < x < b$

integrate it to get

*) c.d.f :- $F(x) = \dfrac{x-a}{b-a}$; $a < x < b$

differentiate $F(x)$ to get $f(x)$

$$\text{Mean} = \frac{a+b}{2} \quad ; \quad \text{Var} = \frac{(b-a)^2}{12}$$

discussed earlier

$\therefore P(a < x < b) = F(x = b) - F(x = a)$

$\therefore P(x < a) = F(x = a)$

$\therefore P(x > a) = 1 - F(x = a)$

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$\therefore$ CDF $= F(x) = \int_{L.L}^{x} f(x)\,dx$

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Exponential Distribution:-

(In exp., Mean & S.D are same)

*) p.d.f.:- $f(x) = \lambda e^{-\lambda x}$; $x > 0$

↳ Integrate it to get $F(x)$

*) c.d.f.:-

$F(x) = 1 - e^{-\lambda x}$; $x > 0$

↳ Diff. it to get $f(x)$

Mean $= \frac{1}{\lambda}$; Var $= \frac{1}{\lambda^2}$

4.28

$n = 70$; $n\mu = 70 \times 0.2 = 14$

$\lambda = 5$; $\sqrt{n}\,\sigma = \sqrt{70} \times 0.2 = 1.67$

Mean $= \frac{1}{\lambda} = \frac{1}{5} = 0.2$ min $= \mu$

S.D $= \frac{1}{\lambda} = 1/5 = 0.2 = \sigma$

$P(S_n < 12)$

$\because Z = \frac{S_n - n\mu}{\sqrt{n}\,\sigma}$

$= P\left(Z < \frac{12 - 14}{1.67}\right)$

$\boxed{Z = \frac{S_n - 14}{1.67}}$

$= P(Z < -1.20) = 0.1151$

$= \boxed{11.51\%}$ Ans.

$= \int_{-\infty}^{x} f(x)\,dx$

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mean & SD are same)

λx ; $x > 0$

to get $f(x)$

λx ; $x > 0$

$f(x) = \frac{1}{\lambda^2}$

$n\mu = 70 \times 0.2 = 14$

$\sqrt{n}\,\sigma = \sqrt{70} \times 0.2 = 1.67$

$= \mu$

$\therefore Z = \frac{S_n - n\mu}{\sqrt{n}\,\sigma}$

$Z = \frac{S_n - 14}{1.67}$

No.

Q; $n \geq 30 \rightarrow$ large sample size.

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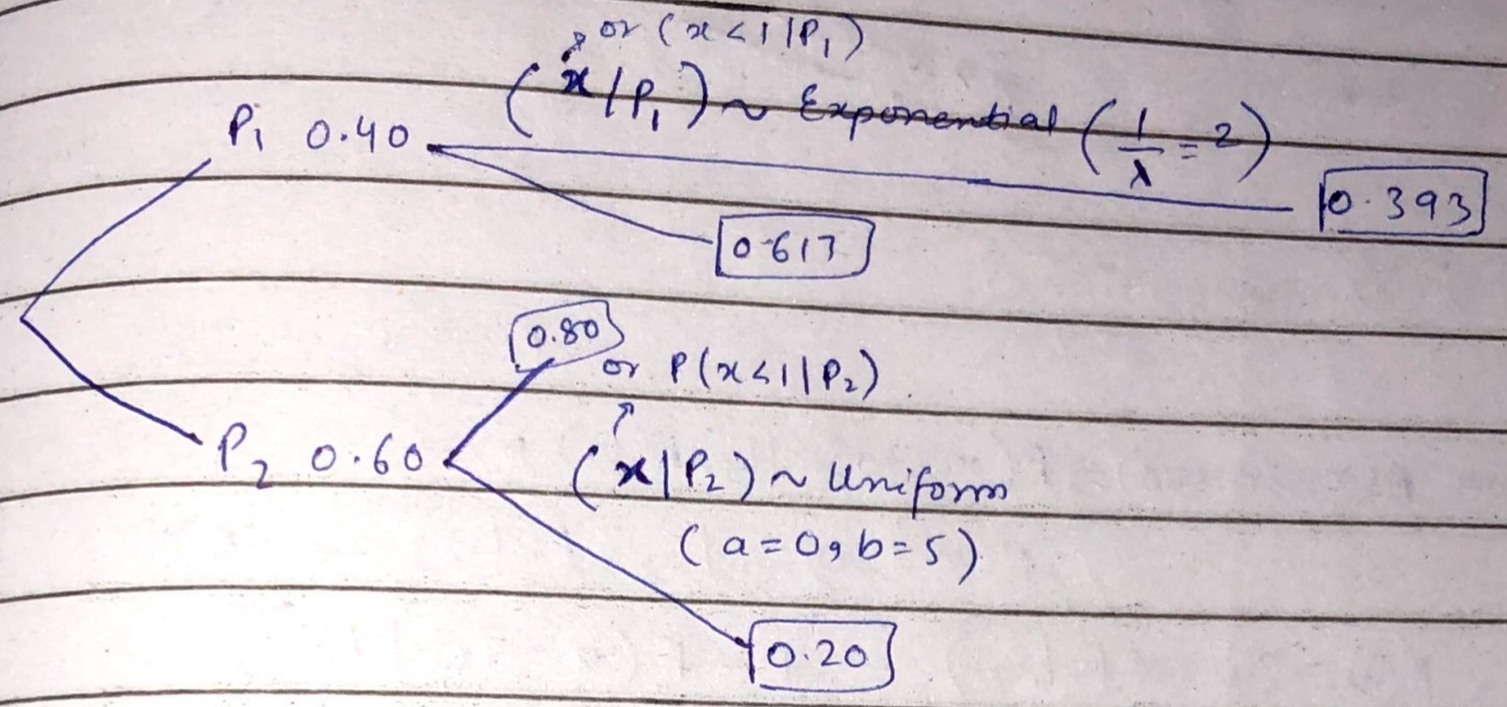
4.29:- X : printing time.

$\lambda = \frac{1}{2} = \frac{1}{\mu} = 0.5$.



Now; $P(P_1 | x<1) = \frac{P(x<1|P_1)\,P(P_1)}{P(x<1|P_1)P(P_1) + P(x<1|P_2)P(P_2)} \rightarrow (A)$

$\therefore P(x<1|P_2) = F(x=1)$ use uniform.

$F(x|P_2) = \frac{x-a}{b-a} = \frac{x}{5}$

$= 1/5 = \boxed{0.20}$.

$\therefore P(x<1|P_1) \left\{ = F(x|P_1) = 1 - e^{-0.5x} \; ; \; \lambda = 0.5 \right\}$

$= 1 - e^{-0.5}$

$= \boxed{0.393}$

Now; $(A) \Rightarrow \frac{(0.393)(0.40)}{(0.393)(0.40) + (0.20)(0.60)}$

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✓ If solve directly via calculator, take $\infty = 10000 / 100000$ etc → very big value) $\because e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

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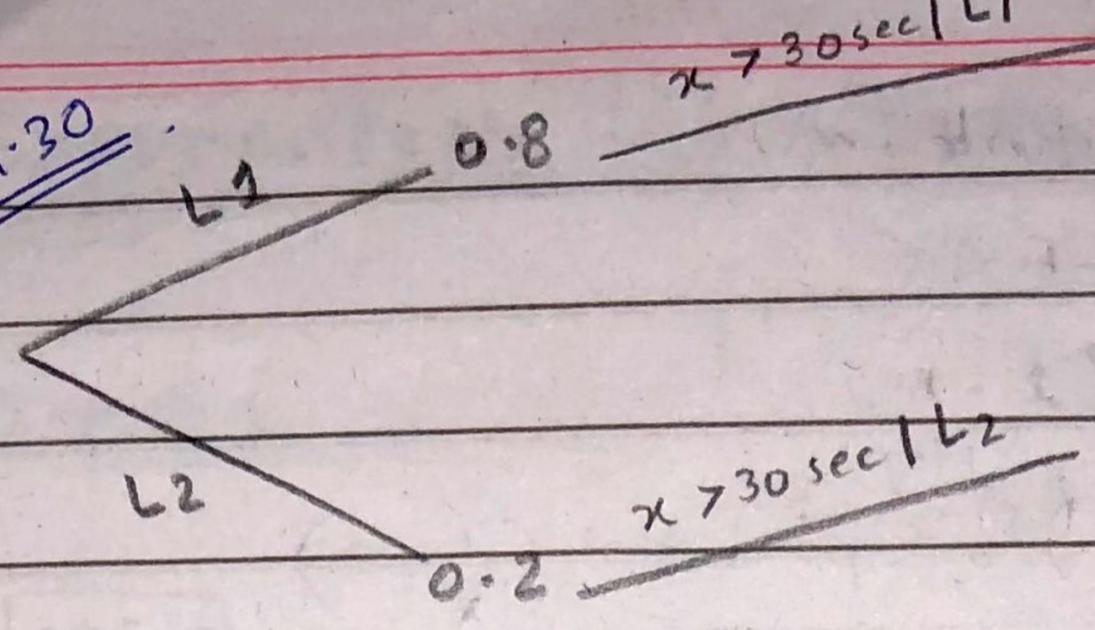
4.30



X : Connection Time.

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$$ → To be find here

Now, $P(x > 30 \text{ sec}) = P(x > 30 \text{ sec} | L_1) P(L_1) + P(x > 30 \text{ sec} | L_2) P(L_2) \rightarrow (A)$

(i) $\therefore P(x > 30 \text{ sec} | L_2) = 1 - \underbrace{F(x = 30 | L_2)}_{\text{uniform } (a = 20,\ b = 50)}$

$$= 1 - \left(\frac{30 - 20}{30}\right)$$

$$= 1 - \frac{10}{30} = 20/30 = 2/3 = \boxed{0.67}$$

At $\alpha = 3$ & $\lambda = 2$:- $f(x) = \frac{8}{2!} x^2 e^{-2x}$

↳ don't convert parameter into sec

$$\boxed{f(x) = 4x^2 e^{-2x}} \; ; \; x > 0$$

(ii) $P(x > 30 \text{ sec} | L_1) = P(x > 0.5 \text{ min} | L_1)$

$$= \int_{x=0.5}^{\infty} 4x^2 e^{-2x} \, dx = 1 - \int_{x=0}^{0.5} 4x^2 e^{-2x} \, dx.$$

$$\int (4x^2) e^{-2x} \, dx = \left[(4x^2)\left(\frac{e^{-2x}}{-2}\right) - (8x)\left(\frac{e^{-2x}}{4}\right) + (8)\left(\frac{e^{-2x}}{-8}\right) \right]_{0.5}^{\infty}$$

↓ take derivative ↳ integral

$$= -\left[(1)\left(\frac{e^{-1}}{-2}\right) - (4)\left(\frac{e^{-1}}{4}\right) + \left(\frac{e^{-1}}{-1}\right) \right]$$

(on putting U.L = ∞; ans = 0 ∴ $e^{-\infty} = 0$)

$$= \frac{e^{-1}}{2} + e^{-1} + e^{-1}$$

$$= 2.5 e^{-1} = \boxed{0.9196} \text{ Ans.}$$

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.. contd ..

If; $\alpha = n/2 \rightarrow$ Chi-sqr. distribution ← Contd.. x—
$\alpha = 1 \rightarrow$ Exp. distribution.

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Gamma Distribution:-

*) p.d.f:- $f(x) = \dfrac{\lambda^{\alpha}}{(\alpha-1)!} x^{\alpha-1} e^{-\lambda x}$; $x > 0$

$(\alpha-1)! \rightarrow \Gamma\alpha$

Mean $= \alpha/\lambda$; Var $= \alpha/\lambda^2$.

*) Relation b/w chi-sqr & Gamma Distribution:

$\nu = d.f$

$\alpha = \dfrac{\nu}{2}$

*) From Excel:- we're not using C.D.F (in gamma manually)

↳ (Excel has C.D.F)

$\alpha = 3$

use; $\beta = \dfrac{1}{\lambda} = \dfrac{1}{2} = 0.5$

$P(x < 0.5) = \text{gamma.dist}(x, \alpha, \beta, 1)$
$= \text{gamma.dist}(0.5, 3, 0.5, 1)$
$= 0.0803$

$P(x > 0.5) = 1 - 0.0803 = 0.9197$

*... Final ans. of 4.30 - - - *

$P(x > 30\,sec) = P(x > 30sec \mid L_1)\,P(L_1) + P(x > 30sec \mid L_2)\,P(L_2)$
$= (0.9197)(0.80) + (0.67)(0.20)$
$= \boxed{0.86}$ Ans. *

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Ch# 9.

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*) Descriptive Stats → findings from data (limited only to that data)

*) Inferential Stats → Theory of inference.

↳ use sample info. to predict about population.

Statistics

→ Descriptive Statistics

→ Inferential Statistics

→ Estimation

→ Hypothesis

*) Hypothesis → Assumption / statement about population.

↳ prove / disprove by experiment

*) Experiment require sample to test upon.

*) Consistent → less variation (more stable)

*) Null hypothesis → which we've to test. (claim).

9.10:

*) 24 defective items in sample of 200.

↳ Sample proportion $= p = \frac{24}{200} = 0.12 = \boxed{12\% \text{ prob.}}$

Claim: Atmost 10% items defective.

↳ Population proportion (P)

$H_0: P \leq 0.10$

$H_1: P > 0.10$

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*) Hypothesis → Test Statement

Class# 10

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*) Statistics
- Descriptive Statistics
- Inferential Statistics
  - Estimation
  - Hypothesis

*) Mean Test for One Sample :-

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \qquad \therefore \nu = n-1$$

*) Proportion Test for one Sample :-

$$z = \frac{\hat{p} - p}{\sqrt{\frac{PQ}{n}}}$$

Note: In hypothesis testing, we always test H_0 - If H_0 is false, it means H_1 is true - So, actually our prior assumption is that H_0 is true

*) Null hypothesis → H_0 ; Alternative hypothesis → H_1

*) Hypothesis Testing :-

1) Mean Test (μ)
   - ↳ for 1 sample
   - ↳ for 2 samples
2) Proportion Test (P)
   - ↳ for 1 sample
   - ↳ for 2 samples
3) Variance (σ^2)
   - ↳ for 1 sample
   - ↳ for 2 samples

4) Other Topics such as chi-square test for categorical data :
   - i) Goodness of fit Test
   - ii) Test of Association.

1) Mean Test (μ) :- (Average → Data follows normal distri.)

H_0 : Mean height is 68 ($\mu = 68$)
H_1 : $\mu < 68$

$n = 50$
$\bar{x} = 67$

sample

$(n=50)$ $\bar{x}_1 = 67$
$(n=40)$ $\bar{x}_2 = 60$ $\Rightarrow \bar{x}_1 - \bar{x}_2 = 7$

H_0 : $\mu_1 - \mu_2 = 6$
H_1 : $\mu_1 - \mu_2 > 6$

H_0 : $\mu_1 = \mu_2$ (No sig. diff.) → Varience exist if there is mean.
H_1 : $\mu_1 \,!= \mu_2$ (Sig. diff.)

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Hypothesis → Statement about population which is concluded based on some sample, presenting the data

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2) Proportion Test:- (Binomial Distri.).

↳ Result is based on True/False.

↳ $H_0: P \leq 0.05$ e.g. 10% defective → OK

$H_1: P > 0.05$ > 10% → Not OK

↳ Defective rates b/w 2 suppliers:-

$H_0: P_1 < P_2$ (company 1 has less defective than Comp. 2)

$H_1: P_1 \geq P_2$.

Nature Of Test :- It is always based on H_1

1) $H_1: \mu \neq \mu_0$ (2 tail-test)

2) $H_1: \mu > \mu_0$ (Right-tail test)

3) $H_1: \mu < \mu_0$ (left-tail test)

$\because \mu_0 \rightarrow$ any numeric value of 'μ'

Note: $H_0 \rightarrow$ always having $=, \geq, \leq$ signs.

$H_1 \rightarrow$ " " $\neq$ (two-tail), $<$ (left tail), $>$ (right tail) signs.

*) Mean Test.

σ known (Use Z)

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

σ unknown (Use t)

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}.$$

*) α = level of sig.

*) $1-\alpha \rightarrow$ Confidence level

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Ch#9

9.7 (b) $n = 100$

$\bar{x} = 37.7$ (Avg. concurrent users)

$\sigma = 9.2$; $\alpha = 0.10$ (inform either $\mu > 35$ or not?)

$1-\alpha = 0.90$

$\because H_0 : \mu \leq 35$; $H_1 : \mu > 35$ (Right-tail test)

$\therefore z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} \Rightarrow z = \dfrac{37.7 - 35}{9.2/\sqrt{100}} = \boxed{2.935}$

*) Decision Rule:-

Approach:- → Table-value method
→ P-value method

1) Based on table value of Critical values:-

Reject H_0 if $|z| \geq z_{tab}$; o/w don't reject H_0.

$\because z_{tab} = \begin{cases} z_\alpha \rightarrow \text{for One-tail test (right \& left)} \\ z_{\alpha/2} \rightarrow \text{for Two-tail test.} \end{cases}$

2) P-value Method:-

Reject H_0 if p-value $\leq \alpha$; o/w don't reject H_0.

$\because$ p-value $= P[z \geq |z|] \rightarrow$ for One-tail test

or p-value $= 2P[z \geq |z|] \rightarrow$ for Two-tail test

Sol: (b) $z_\alpha = z_{0.10} = 1.28$ {Reject $H_0 \Rightarrow \because 1.28 < 2.933$}

or p-value $= P[z > 2.93]$

$= 1 - P[z \leq 2.93]$

$= 1 - 0.9983$

$= \boxed{0.0017} \Rightarrow$ Reject H_0 $\because$ p-value $\leq \alpha = 0.10$

Hence; we conclude that $\mu > 35$ is true.

(a) $\bar{x} \pm z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$

Note; $\dfrac{\alpha}{2} = \dfrac{0.10}{2} = 0.05$

will be provided $\left\{ \begin{array}{l} Z_{0.01} = 2.33 \\ Z_{0.005} = 2.58 \end{array} \right.$ $\quad Z_{0.10} = 1.282$, $Z_{0.05} = 1.65$, $Z_{0.025} = 1.96$

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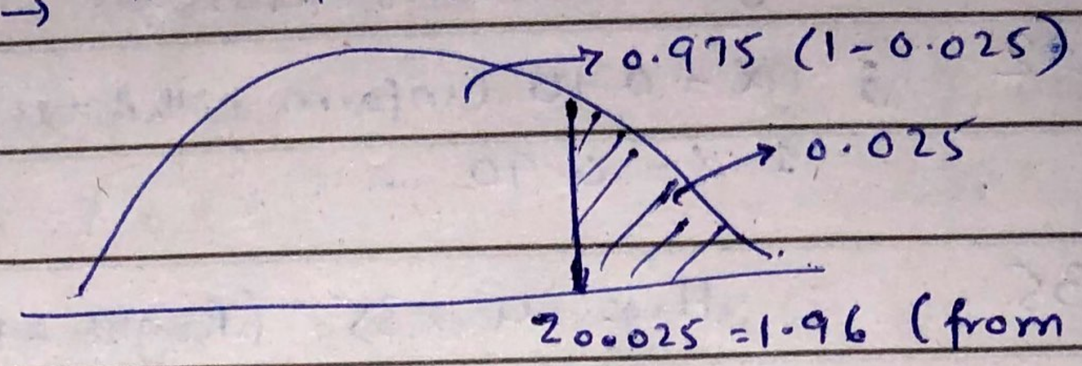
$\therefore Z_{\alpha/2} \rightarrow$ The value of 'z' against right area $= \alpha/2$

$\therefore Z_{\alpha} \rightarrow$ " " " " " " " $= \alpha$.

E.g:



$P[z < 1.96] = 0.9750$ (z-table)

$P[z > 1.96] = 0.0250$.

Now; (a) 90% Confidence Interval (C.I) of μ:-

$1 - \alpha = 0.9$; $\alpha = 0.1$; $\alpha/2 = 0.05$

$Z_{\alpha/2} = Z_{0.05} = 1.645 \cong 1.65$

From Table; $P[z > 1.65] = 0.05$ $\quad \therefore P[z < 1.65] = 0.95$

90% C.I of μ : $\bar{x} \mp Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$: 37.7 \mp (1.65)\left(\frac{9.2}{\sqrt{100}}\right)$

$: 37.7 \pm 1.518$

$: (36.18 - 39.22)$

9.8 (a) $\sigma = 5$; $n = 64$ (find 95% C.I of μ?)

$\bar{x} = 42$

*) $1 - \alpha = 0.95 \longrightarrow \alpha = 0.05$

$\alpha/2 = 0.025$

$\boxed{Z_{\alpha/2} = 1.96}$ $\quad \therefore \bar{x} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}$

*) 95% C.I of μ : $42 \pm (1.96)\left(\frac{5}{\sqrt{64}}\right)$

$= 42 \pm 1.218$

$= 40.78 - 43.218$

(b) $P(a < \bar{x} < b)$; $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ (do it yourself)

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9.9 *) Decision Rule:-

(1) Based on table-value of critical values:-

↳ Reject H_0 if $|t| \geq t_{tab}$; o/w don't reject

$$t_{tab} = \begin{cases} t_{\alpha} \rightarrow \text{One-tail (left/right)} \\ t_{\alpha/2} \rightarrow \text{Two-tail} \end{cases}$$

*) $\bar{X} = \frac{\Sigma x}{n}$; $s^2 = \frac{\Sigma (x-\bar{x})^2}{n-1}$.

(Sample variance)

*) In our question:- $\bar{X} = 50$.

x	$x - \bar{x}$	$(x-\bar{x})^2$
30	-20	400
50	0	0
70	20	400
		$\Sigma (x-\bar{x}) = 800$

Now, $s^2 = \frac{800}{3-1} = 400$

$s = \sqrt{400} = \boxed{20}$

-- With calculator find $\bar{X}$ & S:-

(1) Mode → Stats → 1 Var

(2) Input data

x
30
50
70

(3) Press $\boxed{AC}$

(4) Shift → $\boxed{1}$ → Var → $\bar{X}$ (Mean) / Optn → 1-Var stats (2)

(4) S_x or $X\sigma_{n-1} = \boxed{S}$ → Sample S.D

Ans:- $S = 20$, $\bar{X} = 50$

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$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$; $\bar{x} \pm t_{\alpha/2} \, s/\sqrt{n}$

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Now; $n = 3$; $\bar{x} = 50$; $s = 20$

(a) 90% C.I of μ:

$1 - \alpha = 0.90$; $\alpha = 0.10$; $\alpha/2 = 0.05$

$t_{0.05}$ (at df=2) from table. $= \boxed{2.920}$ $\therefore \nu = df = n-1 = 2$

(b) $\alpha = 0.10$

$H_0: \mu = 80$

$H_1: \mu \neq 80$ (Mean is diff^n from 80) $\rightarrow$ Two-tail.

$\therefore t_{tab} = t_{0.05} = 2.920 = t_{\alpha/2}$

And $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

$t = \frac{50 - 80}{20/\sqrt{3}}$

$t = -2.6$

$\boxed{|t| = 2.6}$ $\Rightarrow$ (Accept H_0)

$\therefore |t| = 2.6 < t_{tab}$ (table-value) i.e: 2.92, so we don't reject H_0 & conclude that mean is not different from 80,000.

$/2 \quad S/\sqrt{n}$

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; $\alpha/2 = 0.05$

$\nu = df = n-1 = 2$

o-tail.

2.92, so

hat mean

C.I of μ: $\bar{x} \mp Z_{\alpha/2}\, \sigma/\sqrt{n}$

Class# 11

Date 4-5-25.

sample proportion.

*) $Z = (\hat{p}) - (P)$ → pop. proportion of defective items. (for one proportion Test)

$\sqrt{\frac{PQ}{n}}$ → pop. proportion & pop. Q

*) $Z = \dfrac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\frac{PQ}{\blacksquare}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ (for two proportion test)

where; $P = \dfrac{x_1 + x_2}{n_1 + n_2}$; $Q = 1 - P$

9.10 $\hat{p} = \dfrac{x}{n} = \dfrac{24}{200} = 0.12$ = Sample of 200 has 12% def. items

$\hat{q} = 1 - \hat{p} = 0.88$

$n = 200$

*) C.I of P:- (Conf. Interval)

$$\hat{p} \mp Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

a) 96% C.I of P:-

$(1-\alpha) = 0.96$ → Confidence level

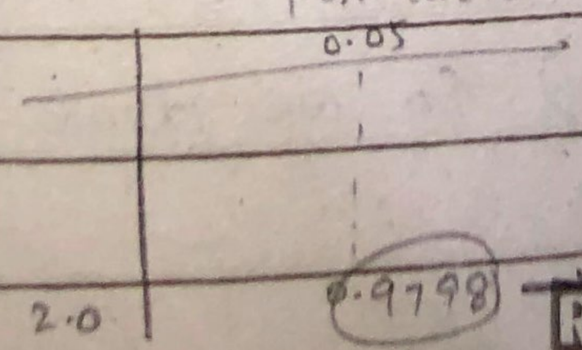
$\alpha = 0.04$ → level of significance.

$\alpha/2 = 0.02$

$Z_{\alpha/2} = Z_{0.02}$ → The value of 'z' against right area = 0.02.

$\boxed{Z_{0.02} = 2.05}$ ↓ from table

or: the value of 'z' against left area = 1-0.02 = 0.98



0.9798 → Inverse prob ⇒ when we've to find value of 'z' → see entries from table

No. _____ find

check for 0.98 in table.

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•) Three possible signs in H_0: $=, \leq, \geq$

•) " " " " H_1: $\neq, >, <$

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*) 96% C.I of P: $0.12 \mp (2.05)\sqrt{\frac{(0.12)(0.88)}{200}}$

$: 0.12 \mp 0.05$

$: 0.07 \text{ —— } 0.17$

$: \boxed{7\% \text{ to } 17\%}$

↳ The pop. having 7% to 17% def items.

b) Claim:- $P \rightarrow$ Population proportion of defective items

$P \leq 0.10$

Claim (Note: claim can also be H_1 in some Qs).

$H_0: P \leq 0.10$ vs $H_1: P > 0.10 \rightarrow$ One-tail-test

($\neq \Rightarrow$ two-tail)

•) $Z = \hat{P} - P / \sqrt{PQ/n}$

$$Z = \frac{0.12 - 0.10}{\sqrt{\frac{0.12 \times 0.90}{200}}} = 0.94.$$

•) p-value $= P[Z \geq |Z|]$ → probability

$= P[Z \geq 0.94]$

$= 1 - P[Z < 0.94]$ → Imp prob. now see:

$= 1 - 0.8264$

$\boxed{\text{p-value} = 0.1736}$

		0.04
0.9		$\boxed{0.8264}$

•) Decision Rule:-

Reject H_0 if p-value $\leq \alpha$.

otherwise (p-value $> \alpha$) so we don't reject H_0.

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$0.08667 \Rightarrow \underbrace{\overset{(2\text{-decimal})}{0.09} \text{ or } \overset{(3\text{-decimal})}{0.087}}_{\text{(write these)}}$ (4% or 15%)

•) Level of sig $= \alpha =$ prob. of error

•) Since p-value $> \alpha$; so we don't reject H_0 & we accept the claim. (i.e. giving $\leq 10\%$ defective items)

9.11

(1) $n_1 = 200$

$x_1 = 24$

$\hat{P}_1 = 0.12$

pop. prop. of def. items

(2) $n_2 = 150$

$x_2 = 13$

$\hat{P}_2 = \frac{x_2}{n_2} = \frac{13}{150} = 0.087$

Note: If $P_1 > P_2$; it means quality of items produced by Company 2 is higher than Company 1.

•) $H_0 : P_1 \leq P_2$; $H_1 : P_1 > P_2$

$P_1 = P_2$

$P_1 - P_2 = 0$

1-tail test.

•) $P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{24 + 13}{200 + 150} = \frac{37}{350} = 0.106$

•) $Q = 0.894$

•) $Z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$Z = \frac{(0.12 - 0.087) - (0)}{\sqrt{(0.106)(0.894)\left(\frac{1}{200} + \frac{1}{150}\right)}} = 0.99$

•) p-value $= P[Z \geq 0.99]$

$= 1 - P[Z < 0.99]$

$= 1 - 0.8389 = \boxed{0.1611}$

Note:- If α not given in Qs → take it as 5%.

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Conclusion:- Since p-value is more than $\alpha = 0.05$, so we don't reject H_0 & conclude P_1 is not significantly higher than P_2 which means quality of the products made by company 2 are same as company 1.

Ex# 9.14

- Data from Ex# 9.12:- $n_1 = 30$ $n_2 = 20$

$\bar{x}_1 = 6.7$ $\bar{x}_2 = 7.5$

$S_1 = 0.6$ $S_2 = 1.2$

If server A is faster than; $\mu_1 < \mu_2$

→ $H_0: \mu_1 \geq \mu_2$; $H_1 = \mu_1 < \mu_2$

or; $\mu_1 = \mu_2 \Rightarrow \mu_1 - \mu_2 = 0$

Note: In Exam, sir will give the value of 't'.

(Use; $t = -3.12$ & $d.f = 48$)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$S_p = \sqrt{\frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}}$$

$\therefore \boxed{S_p = 0.887}$

Now; $$t = \frac{(6.7 - 7.5) - (0)}{0.887 \sqrt{\frac{1}{30} + \frac{1}{20}}} = -3.12$$

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*) Decision Rule:-

Reject H_0 if $|t| \geq t_{tab}$

$$\therefore t_{tab} = \begin{cases} t_{\alpha}(v) & \rightarrow \text{for one-tail test} \\ t_{\alpha/2}(v) & \rightarrow \text{for 2-tail test.} \end{cases}$$

* Use; $\alpha = 0.05 \Rightarrow t_{0.05}(48) = 1.677$

(v) If this qs. was of 2-tail:-

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

$t_{\frac{0.05}{2}}(48) = t_{0.025}(48) = 2.011$

↳ For both one/two-tail test here; Reject H_0.

→ Note: (from Exam pov)

↳ for z-value ⇒ Better to calculate p-value.

↳ for t ⇒ Use table

→ Main thing is to set H_1:-

(1) Test $\mu \geq 5 \Rightarrow H_0: \mu = 5, H_1: \mu < 5$

(2) Test $\mu \leq 5 \Rightarrow H_0: \mu = 5, H_1: \mu > 5$

(3) Test $\mu = 5 \Rightarrow H_0: \mu = 5, H_1: \mu \neq 5$

(4) Test $\mu > 5 \Rightarrow H_0: \mu = 5, H_1: \mu > 5$

(5) Signature $\mu < 5 \Rightarrow H_0: \mu = 5, H_1: \mu < 5$

No.

✓ For single-variance = Chi-sqr. test
✓ For 2- " = F-test

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9.23 (a) For 2-mean test:- (Z or t distri.)

$H_0: \mu_1 = \mu_2$, $H_1: \mu_1 > \mu_2$

$t_{cal} = 0.93$ given; $d.f = 5 \Rightarrow t_\alpha(v) = t_{0.05,5} = 2.015 \Rightarrow$ Accept H_0

(b) For 2 variance:-

$H_0: \sigma_1^2 = \sigma_2^2$, $H_1: \sigma_1^2 > \sigma_2^2$

→ pop. variance

For 2-variance:

$\therefore F = \frac{S_1^2}{S_2^2} = \boxed{15.65}$ $\Rightarrow 12.8 = S_1$

$S_2 = 3.22$.

$\therefore \boxed{\text{p-value} = 0.004}$

$\therefore$ p-value $< 0.05^{(\alpha)}$ (standard-value)

↳ Reject H_0.

↳ α not given, take $\alpha = 0.05$.

PDF:-

Q#1 A company compares avg. CTR - - - -

$H_0: \mu_1 = \mu_2$ (two-tail)

$H_1: \mu_1 \neq \mu_2$.

For 2 mean:- $v = d.f = n_1 + n_2 - 2$

$t_{\alpha/2}(v) = t_{0.025}(56) = 2.003$.

$t_{given} = 1.96$

$\therefore t_{given} < t_{\alpha/2}(v) \Rightarrow$ Accept H_0.

Q#2. One-tail

$t_{0.05}$ (4

↳ Reject

Q#3 H_0:

H_1:

z =

p-value

$\therefore$ p-value

0.0322

Note: (ex

Q#5 :-

H

H

t

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Q#2. One-tail test.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

$t_{0.05(40)} = 1.684$

$\Rightarrow$ Reject H_0.

Q#3 $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

$z = 1.85$

$$\text{p-value} = P[z > 1.85]$$
$$= 1 - P[z \leq 1.85]$$
$$= 1 - 0.9678$$
$$= 0.0322.$$

$\therefore$ p-value $< \alpha$

$0.0322 < 0.05 \Rightarrow$ Reject H_0 & conclude $\mu_1 > \mu_2$.

Note: (extra) :- If z or t-value $> 2.05 \Rightarrow$ Reject

Q#5 :- $H_0: \mu_1 = \mu_2$.

$H_1: \mu_1 < \mu_2$.

t-statistic $= -2.10$ ($\mu_1 < \mu_2$ due to neg. sign).

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✓ For one-variance test:-

$$\chi^2 = \frac{(n-1)s^2}{\sigma}$$

Chi-sqr.

$$d.f = n-1$$

✓ For 2-variance:-

$$F = \frac{S_1^2}{S_2^2} \quad ; \nu_1 = n_1 - 1, \quad \nu_2 = n_2 - 1$$

Q#6. $H_0: \sigma^2 = 0.0025$

$H_1: \sigma^2 \neq 0.0025$ (~~Two-tail test~~ one-tail)

(•) $\chi^2 = \frac{(n-1)s^2}{\sigma} = 22.8$

Q#7. $H_0: \sigma^2 = 1$

$H_1: \sigma^2 \neq 1$

$\chi^2 = 33.6$

Q#8. $H_0: \sigma_1^2 = \sigma_2^2$; $n_1 = 20 \Rightarrow \nu_1 = 19$

$H_1: \sigma_1^2 \neq \sigma_2^2$; $n_2 = 25 \Rightarrow \nu_2 = 24$

$F = 1.39 = \frac{S_1^2}{S_2^2}$

$\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$; $1 - \frac{\alpha}{2} = 0.975$

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$\frac{\alpha}{2}$

$\therefore F_{0.025}$

$\therefore F_{0.975}(19, 19)$

Conclusion: Since

*) For 2-va

$\therefore F$

*) Decision

Q#9 F

F_0

F_0

$\therefore$

Signatur

$\frac{\alpha}{2}$

col

row

$\therefore F_{0.025}(\nu_1, \nu_2) = F_{0.025}(19, 24) = \boxed{2.33}$

$\therefore F_{0.975}(\nu_1, \nu_2) = \dfrac{1}{F_{0.025}(24, 19)}$

	15	20
24		$\boxed{} \rightarrow 2.33$

$= 1/(2.45)$ → this value will be given in exam.

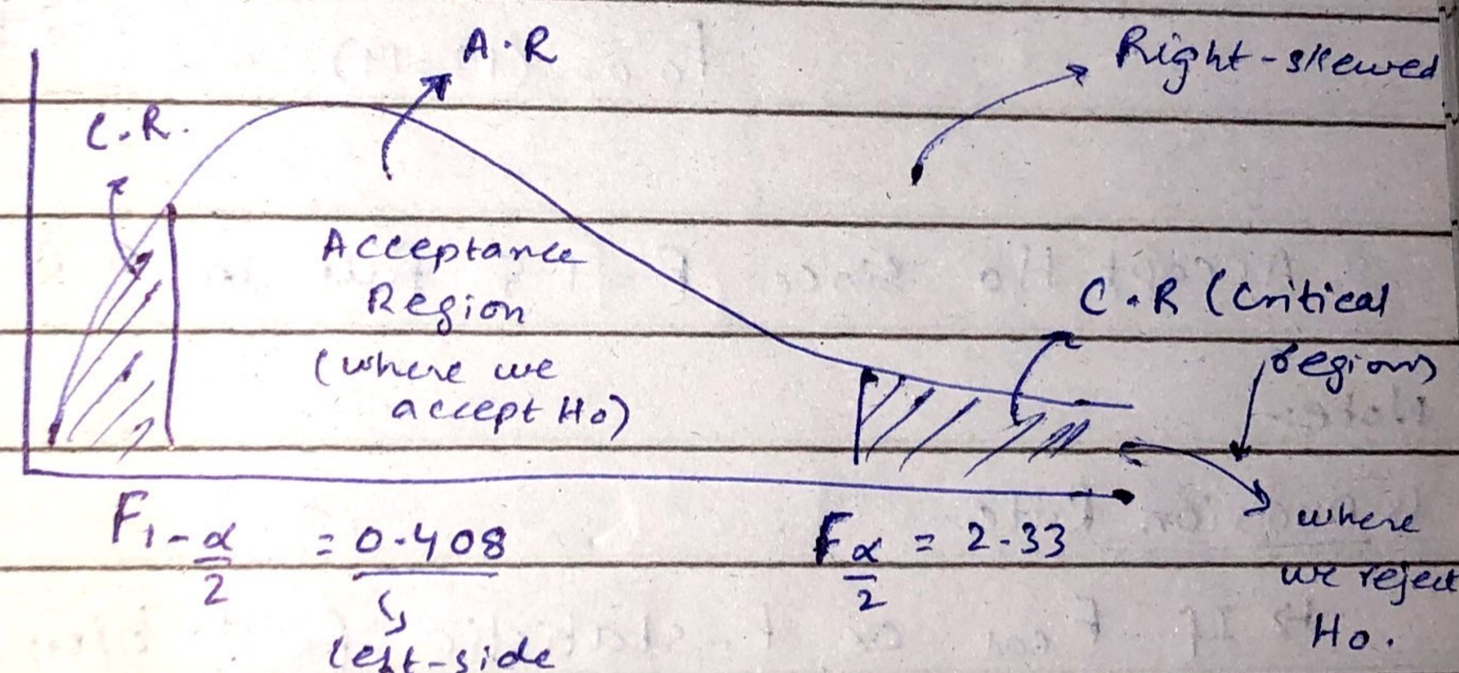
$= \boxed{0.408}$

Conclusion: Since $F = 1.39$ fall in A.R, so we accept H_0 & concluded $\sigma_1^2 = \sigma_2^2$

*) For 2-variance:-

$$\therefore F_{1-\frac{\alpha}{2}}(\nu_1, \nu_2) = \frac{1}{F_{\frac{\alpha}{2}}(\nu_2, \nu_1)}$$

*) Decision Rule:-



Q#9 $F = 2.5$; $n_1 = 10 \Rightarrow \nu_1 = 9$; $n_2 = 12 \Rightarrow \nu_2 = 11$

$F_{0.025}(9, 11) = \boxed{3.59}^{UL}$ (from F-table)

$F_{0.975}(9, 11) = \dfrac{1}{F_{0.025}(11, 9)} = \dfrac{1}{3.92} = \boxed{0.26}^{LL}$

from F-table

$\therefore H_0 : \sigma_1^2 = \sigma_2^2$ (variation is same)

$H_1 : \sigma_1^2 \neq \sigma_2^2$ (variation in two training time are diff).

Note Min. value of F can be zero.

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$\because F_{U.L} = F_{\alpha/2} = 3.59$ & $F_{L.L} = F_{1-\alpha/2} = 0.26$

$\therefore$ $F = 2.5$ fall in A.R, so we accept Ho & conclude that variation is same.

Q#10. $F = 1.5$

$n_1 = 15 \Rightarrow v_1 = 14$

$n_2 = 20 \Rightarrow v_2 = 19$

$\because$ $H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

$\therefore F_{0.025}(14, 19) = 2.62$

$\therefore F_{0.975}(14, 19) = \dfrac{1}{F_{0.025}(19, 14)} = \dfrac{1}{2.84} = 0.35$

$\because$ Accept Ho since $F = 1.5$ fall in A.R.

Note:-

↳ Decision Rule:-

↳ If F_{cal} or F-statistic fall b/w L.L & U.L of F-table value, we accept Ho & conclude $\sigma_1^2 = \sigma_2^2$. Otherwise; Reject Ho & conclude $\sigma_1^2 \neq \sigma_2^2$.

One-Way

↳ used

2 groups

*) Ho :

H1 :

*) F =

∴ N

*) Treatm

Medic

Example:-

*) Ho :

H1 :

Pa

(Analysis of Variance Approach)

Date

One-Way ANOVA :-

↳ used when we have >2 means (i.e. more than 2 groups ⇒ $K > 2$)

*) $H_0 : \mu_1 = \mu_2 ---- = \mu_K$ ⇒ There is no sig. diff among K-groups (all are same)

H_1 : Atleast one pair of mean is not equal (i.e: there is a sig. difference among K-groups)

*) $$\boxed{F = \frac{SS\ Treatment / K-1}{SS\ Error / N-K}}$$

$\therefore N = n_1 + n_2 + ---- n_K$.

*) Treatment ⇒ Group name (e.g: Type of students, Medicine if analyzing medicines)

Example:- Front, Middle, Back row.

$K = 3$; $N = n_1 + n_2 + n_3 = 7 + 9 + 8 = 24$.

*) $H_0 : \mu_1 = \mu_2 = \mu_3$

H_1: Atleast one pair of mean isn't equal.

Pairs : $\mu_1 = \mu_2$

$\mu_1 = \mu_3$

$\mu_2 = \mu_3$

✓ $\alpha = 0.05 \Rightarrow$ Use this if not given.

$\therefore y_{..} = y_{1.} + y_{2.} + y_{3.} \Rightarrow$ Grand-Total

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Given:

Sum:

Front (1)	$n_1 = 7$	$y_{1.} = 530$
Middle (2)	$n_2 = 9$	$y_{2.} = 604$
Back (3)	$n_3 = 8$	$y_{3.} = 428$

$\Sigma = 24$ $\quad$ $\Sigma = 1562$

Variance: $S_1^2 = 310.91$, $S_2^2 = 119.86$, $S_3^2 = 80.29$

$\bar{y}_1 = 75.71$, $\bar{y}_2 = 67.11$, $\bar{y}_3 = 53.5$ → Averages

Since; $v_1 = K - 1 = 2$

$v_2 = N - K = 21$.

$$\therefore F = \frac{\text{SS Treatment} / K-1}{\text{SS Error} / N-K}$$ → Not given this formula in Exams.

Note: Reject Ho if $F \geq F_\alpha(v_1, v_2) \Rightarrow$ One-tail test

$F_\alpha(v_1, v_2) = F_{0.05}(2, 21) = \boxed{3.47} \Rightarrow$ from table

Here; $y_{..} = y_{1.} + y_{2.} + y_{3.} = \boxed{1562}$.

Now; $$\text{SS Treatment} = \sum_{i=1}^{K} \frac{y_{i.}^2}{n_i} - \frac{(y_{..})^2}{N}$$ → Formula given in Exams.

$$= \left[\frac{y_{1.}^2}{n_1} + \frac{y_{2.}^2}{n_2} + \frac{y_{3.}^2}{n_3}\right] - \frac{(y_{..})^2}{N}$$

$$= \left[\frac{(530)^2}{7} + \frac{(604)^2}{9} + \frac{(428)^2}{8}\right] - \frac{(1562)^2}{24}$$

Mean of x^2

Signature

No.

SS Treatm

and SS Error

=

SS E

Thus; F =

Reject Ho

is not eq

∴ ANOVA Tab

Source of Variation
Treatment
Error
Total

Now;

Signature

Between Group → Treatment
Within " → Error.

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$$\boxed{SS_{Treatment} = 1901.516}$$

And; $SS\ Error = \sum_{i=1}^{K} (n_i - 1) S_i^2$ → Given in Exams.

$$= (n_1 - 1) S_1^2 + (n_2 - 1) S_2^2 + (n_3 - 1) S_3^2$$

$$= (6)(310.91) + (8)(119.86) + (7)(80.29)$$

$$\boxed{SS\ E = 3386.32}$$

Thus; $F = \dfrac{1901.516/2}{3386.32/21} = \boxed{5.896}$

$F = \dfrac{SS\ Treatment / K-1}{SS\ Error / N-K}$.

$\because F \geq F_{\alpha(\nu_1, \nu_2)}$

Reject Ho & conclude that atleast one-pair is not equal.

-. ANOVA Table :-

Source of Variation	SS	d.f	MS = SS/d.f	F
Treatment	SS Treatment 1901.52	K-1 = 2	$\frac{1901.52}{2} = 950.76 = \frac{SS\ Treat}{K-1}$ = MS Treatment	5.89
Error	SSE 3386.32	N-K = 21 = ν_2	$\frac{SSE}{N-K} = MSE = \frac{3386.32}{21} = 161.25$	
Total	SS Total = 5287.83	N-1 = 23	—	

Now; $F = \dfrac{MS\ Treatment}{MS\ Error} = \dfrac{950.76}{161.25} = \boxed{5.89}$

No.

Given:

i	n_i	$\bar{Y}_i$
1	7	75.71
2	9	67.11
3	8	53.5

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*) Post HOC Test → Applicable only if Ho is rejected to identify which pair isn't equal.

↓

Apply Multiple Comparison Test:-

*) LSD Test:-

Pair	Ho	$\lvert \bar{Y}_i - \bar{Y}_j \rvert$	LSD	Decision
1	$\mu_1 = \mu_2$	$\lvert \bar{Y}_1 - \bar{Y}_2 \rvert = 8.6$	13.5	Accept Ho $\mu_1 = \mu_2$
2	$\mu_1 = \mu_3$	$\lvert \bar{Y}_1 - \bar{Y}_3 \rvert = 22.2$	13.5	Reject Ho $\mu_1 \neq \mu_3$
3	$\mu_2 = \mu_3$	$\lvert \bar{Y}_2 - \bar{Y}_3 \rvert = 13.6$	13.4	Reject Ho $\mu_2 \neq \mu_3$

Rule:- Reject Ho if $\lvert \bar{Y}_i - \bar{Y}_j \rvert \geq LSD$

$$\therefore LSD = t_{\alpha/2;\, \nu_2} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

d.f of Error

$$= t_{0.025;\, 21} \sqrt{161.25\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

from table

$$= 2.08 \sqrt{161.25} \times \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

$$= (26.41)\sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

So; Pair 1: $(26.41)\sqrt{\frac{1}{7} + \frac{1}{9}} = 13.5$

Pair 2: $(26.41)\sqrt{\frac{1}{7} + \frac{1}{8}} = 13.5$

Pair 3: $(26.41)\sqrt{\frac{1}{9} + \frac{1}{8}} = 13.4$

13.16 $\therefore LSD = t_{\alpha/2;\, \nu_2}\sqrt{MSE\left(\frac{2}{n}\right)}$ when: $n_1 = n_2 = n_3 = \dots n_K$.

Here: $n_1 = 3$, $n_2 = n_3 = n_4 = 3 \Rightarrow \boxed{n = 3}$

$\nu_2 = N - K = 12 - 4 = 8$ (12 → Total Obvs.)

$K = 4$; $n = 3$ So: $N = Kn = 12$

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$\therefore LSD = t_{\frac{\alpha}{2}}$

$LSD = (2.$

$LSD = 4$

Pair	Ho
1	$\mu_1 = \mu_2$
2	$\mu_1 = \mu_3$
3	$\mu_1 = \mu_4$
4	$\mu_2 = \mu_3$
5	$\mu_2 = \mu_4$
6	$\mu_3 = \mu_4$

(\) Insights: R

*) If 5- groups:

Ch# 11

*) Final Mar

(Y)

↓

Dep. variabl

(Response)

↓

which we want

*) Simple linea re

intercep

*) Multiple L

Sig

Ho is rejected
pair isn't equal

LSD	Decision
13.5	Accept Ho $\mu_1 = \mu_2$
13.5	Reject Ho $\mu_1 \neq \mu_3$
13.4	Reject Ho $\mu_2 \neq \mu_3$

... nk.

$= 3$

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$\therefore LSD = t_{\frac{\alpha}{2}; \nu_2} \sqrt{MSE\left(\frac{2}{n}\right)}$

$LSD = (2.306)\sqrt{(5.615)\left(\frac{2}{3}\right)}$ → Given from table.

$\boxed{LSD = 4.46}$

Pair	Ho	$\lvert \bar{Y}_i - \bar{Y}_j \rvert$	Decision.
1	$\mu_1 = \mu_2$	0.26	Accept Ho
2	$\mu_1 = \mu_3$	2.7	Accept Ho
3	$\mu_1 = \mu_4$	6	Reject Ho
4	$\mu_2 = \mu_3$	2.9	Accept Ho
5	$\mu_2 = \mu_4$	5.7	Reject Ho
6	$\mu_3 = \mu_4$	8.6	Reject Ho

↳ Insights: Rejecting Ho when there is μ_4 => Remove 4th group. (Anamoly)

*) If 5- groups:- Total 10 pairs

Ch# 11 Regression

*) Final Marks vs. No. of hrs Studied.

(y) ↓ Dep. variable (Response) ↓ which we want to predict

(x_1) ↳ Indep. variable - (predictor)

*) Simple linear regression → when we've one indep. variable -

intercept ← $y = a + bx$ → slope. or $y = b_0 + b_1 x$.

*) Multiple linear Regression. $y = b_0 + b_1 x_1 + b_2 x_2 + \dots b_k x_k$.

No.

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year	code = Year − 1950	Pop. → base
1951	1	
1961	11	

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$\therefore \boxed{y = 3\times10^6 x + 1\times10^7}$

$b_0 \rightarrow$ base year (e.g: 1950)

$b_1 = 30,000000 \rightarrow$ increase in pop. per year.

✓ Exam:- In exams F-table won't be given (directly values will be provided)

↳ For 2-mean:- t-values given → conclude on the basis of values
or 2-prop or 2-variance (formula not provided) → create hypo. & check values from table & conclude

↳ For one mean → (formula provided) → Solve in exams (calculation)

Class #

Ch # 11

Date 18-5-2025

Indep. var

*) y vs x → Simple Regression ; y vs $x_1, x_2, \ldots x_k$ Multiple Regression

↳ Dep. variable (var. which we've to predict)

↳ Indep. variables

*) Simple Regression:-

$$y = a + bx \quad \text{or} \quad y = b_0 + b_1 x \quad \text{or} \quad y = mx + c$$

where; $b_1 = b$ = Slope (m) and; $b_0 = a$ = y-intercept (c) → constant term

↳ coeff. of 'x'

Ex # 11.6:-

Given; n = 7.

optional to add this col (we can directly calculate from calc.)

x	y	$(y-\bar{y})^2$
6	40	25
7	55	400
7	50	225
8	41	36
10	17	324
10	26	81
15	16	361

SS Total = 1452

Steps: (Calculator)

↳ Mode → Stats → A+BX
(3) (2)

Hypothesis:-

$H_0: \beta_1 = 0$ (There is no regression OR Reg. is insignificant)

↳ Press AC.

↳ Shift + 1 → Reg (5)

$H_1: \beta_1 \neq 0$ (Regression is sig.)

A (1)
B (2)
r (3)

From calculator:- $A = b_0 = 72.285 \simeq 72.3$

$B = b_1 = -4.14$

$r = -0.8136 \Rightarrow r^2 = 0.662$

And; $SS_{Total} = (n-1)\boxed{S_y^2}$ → Shift + 1 → Var → Sy (7) → $Ans^2 = S_y^2$

↳ $(\sigma_{n-1})^2$

$= (7-1)\,242$

$$\boxed{SS_{Total} = 1452}$$

Note: $S_y^2 = \dfrac{\sum (y-\bar{y})^2}{n-1}$ → SS Total

$\sum (y-\bar{y})^2 = (n-1)\,S_y^2$

So; our model is :- $\boxed{\hat{y} = 72.3 - 4.14X}$

Correlation:- tells us the association & strength b/w 2 variables.

↳ Two directions → +ve & -ve (lies b/w -1 to +1)

x↑, y↓ — both ↑s

↳ Pearson correlation coefficient (r)

*) Walmore → Ch#13 → ANOVA

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M T W T F S S

*) SS Total (SST) = SS Regression (SSReg) + SS Error (SSE) ⇒ SSE = SS Total − SS Regression

↳ $\Sigma(y-\bar{y})^2 = \Sigma(\hat{y}-\bar{y})^2 + \Sigma(y-\hat{y})^2$

*) $R^2 = 1 - \frac{SSE}{SS\ Total} = \frac{SSReg}{SS\ Total}$

Here; SS Total → Total variation in 'y'
SSReg → Total variation explained by Regression Model (Explained Variation)
SSE → Unexplained variation.

↳ Here;
R^2 → % of variation explained by Regression Model
↳ R^2 high → Choose that model

Note; $R^2_{Linear} = (r)^2$ → (coefficient of correlation)

Now; $SS_{Reg} = SS_{Total} - SSE$

$\therefore \frac{SSE}{SS\ Total} = 1 - R^2$

$SSE = (1-0.662)\times(1452)$

$\boxed{SSE = 491}$

Therefore; $SS\ Reg = 1452 - 491 = \boxed{961}$

..ANOVA Table:-

Source	SS	Degree of freedom (d.f)	MS = SS/d.f	$F = \frac{MSReg}{MSE} = \frac{SSReg/1}{SSE/n-2}$
Regression	961	1	MSReg = 961	9.79
Error	491	(n−2) = 5	MSE = 98.2	
Total	1452	(n−1) = 6		

✓ Regression is sig. when;
(i) $p < 0.05$
(ii) F-value > Table-value

From Table; $F_{0.05}(1,5) = 6.61$

Rule:- Reject Ho if $F \geq F_{\alpha}(v_1, v_2)$

$\therefore 9.79 > 6.61 \Rightarrow$ Reject Ho.

It means that Regression is Significant.

✓ Optional columns: $\hat{Y}$, Error1 = $Y - \hat{Y}$ (Residual), Error² = $(y-\hat{y})^2$
↳ SSE = Sum of this column

1 Qs → Multi-regression (conclusion-based) or (result-based)

Papers:
- ✓ 1 Qs from Ch #4
- ✓ Mids not included
- ✓ 1 Qs → Hypothesis (either calc. based or table-based)
- ✓ 1 Qs → ANOVA (calculation based)
- 1 Qs → Regression (calc2 based) → Linear Regression

1-mean, 2-means, proportion etc.. 1-var, 2-var.

M T W T F S S

*) Simple Linear Regression Examples:- (whatsapp PDF)

Q#2 (b) Through calc. (c) put $x = 9$ in model eqn.

(d) Residual $= 65 - \hat{y}$

(f) Do entire sol. (conclude from hypothesis)

*) Standard Residuals :- (just like we calculated z from x)

↳ $\dfrac{\text{Residual} - \text{Residual Mean}}{\text{Residual St.Dev of sample}}$

EXCEL RESULTS EXPLANATION (extra) (for 11.6).

Note: $R^2_{adjusted} = 1 - \dfrac{SSE / \text{d.f of Error}}{SS\ Total / \text{d.f of total}} = 1 - \dfrac{MSE}{MS_{Total}} = 1 - \dfrac{MSE}{S_y^2}$

For 11.6:- e.g: $R^2_{adjusted} = 1 - \dfrac{98.17}{242} = \boxed{0.594}$

$(\therefore MS_{Total} = S_y^2)$

*) Standard Error (S.E) $= \sqrt{MSE} = \sqrt{98.17} = 9.91$

↳ how much each predicted value is deviating from actual value, on avg.

Q# 11.10

x_1	x_2	y
6	4	40
7	20	55
7	20	50
8	10	41
10	10	17
10	2	26
15	1	16

From F-value:-
- ✓ overall regression is sig.
- ✓ x_2 → not significant
  - ↳ p-value of x_2 should be less than 10%.
- ✓ p-value of x_1 & x_2 etc. tells which factor is insig. & should be removed

Here; df Regression = 2 (since x_1, x_2)

Note:- For Quadratic Regression:-

x	x^2	y

Date ____ 20__
M T W T F S S

*) In ANOVA table; if $n = 7$:-

df Total = 6

↳ Regression d.f = (Max. 5)

↳ Error d.f = (Max 1)

i.e: no. of x-variables is maximum $n-2$..

*) $F = t^2$ (in Linear Regression only)