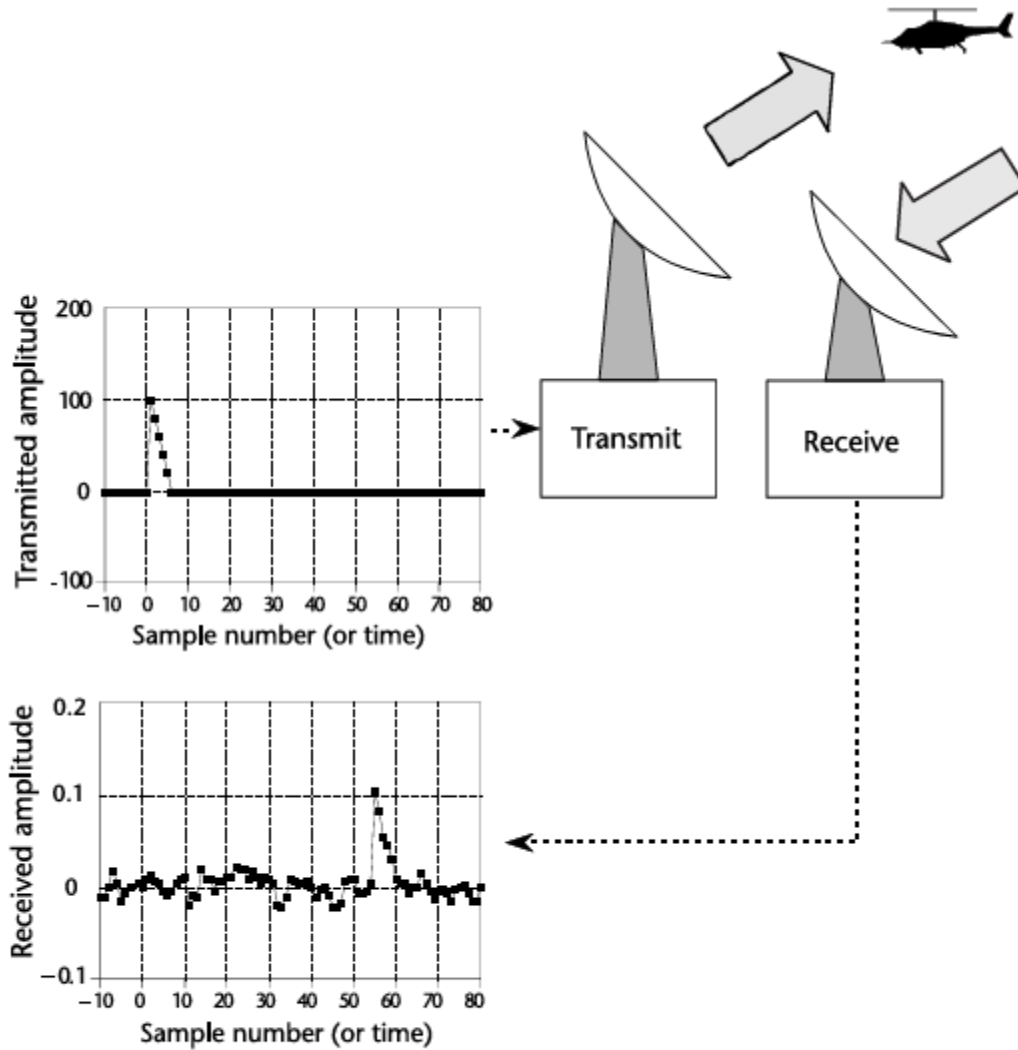


### Correlation



**Figure** Key elements of a radar system. Like other echo location systems, radar transmits a short pulse of energy that is reflected by objects being examined. This makes the received waveform a shifted version of the transmitted waveform, plus random noise. Detection of a known waveform in a noisy signal is the fundamental problem in echo location. The answer to this problem is *correlation*.

Cross-correlation and autocorrelation are two important concepts in DSP. Cross-correlation is a measure of similarity of two series as a function of the displacement of one relative to the other.

$$R_{x,y}(k) = \sum_{n=-\infty}^{\infty} x[n]y[n+k]$$

where k is the displacement.

## Task1: What is the relation between convolution and correlation? (Discuss the expression and the use)

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**Correlation** is a measure of how similar signals are:

For time-limited sequences

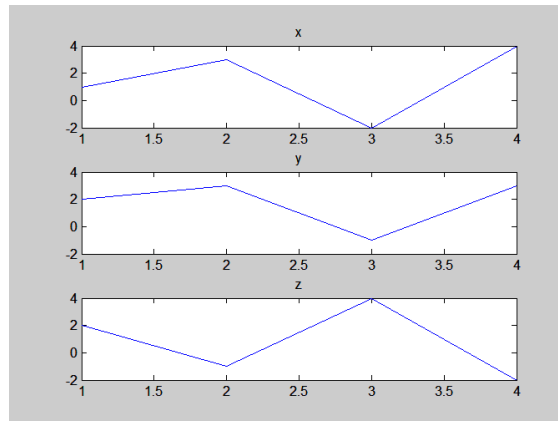
$$R_{x,y}(0) = \sum_{n=0}^{N-1} x[n]y[n]$$

### Example:

Let  $x = [1 \ 3 \ -2 \ 4]$   
 $y = [2 \ 3 \ -1 \ 3]$   
 $z = [2 \ -1 \ 4 \ -2]$

$$R_{x,y} = 25$$

$$R_{y,z} = -9$$



It is clear that **y** is very similar to **x** than **z** since  $R_{x,y} > R_{y,z}$ .

What about the following case:

If we modify **z** to **w** = [ 100 -1 4 -2] then  $R_{y,w} = 187$

In this case, normal correlation fails! We turn into **normalized correlation** (correlation coefficient) instead which is defined as:

$$\sigma_{x,y} = \frac{\sum_{n=0}^{N-1} x[n]y[n]}{\sqrt{\sum_{n=0}^{N-1} x^2[n]} \sqrt{\sum_{n=0}^{N-1} y^2[n]}}$$

In this case

$\sigma_{x,y} = 0.95 > \sigma_{y,w} = 0.38$ . It succeeds in identifying a similar sequence.

## Task2: check which one (standard or normalized) is more beneficial to identify how strongly one signal is present in two different signals. (Matlab)

Let  $S_i = \cos(2\pi f_i t)$

$s_1$ : 1 Hz

$s_2$ : 4 Hz

$s_3$ : 10 Hz

The following signals contain the three sinusoids above

$$X = 2S_1 + 4S_2 + S_3$$

$$Y = S_1 + S_2$$

- Plot the generated signals for 1 sec. let the sampling interval 0.01 sec
- Compare which is useful for identifying how strongly one signal (say  $S_1$ ) is present in another.

**More tasks will be given later!**