

Mobile Robot(Autonomous)

Locomotion



- **Locomotion** is the process of causing an autonomous robot to move
 - In order to produce motion, forces must be applied to the vehicle

Wheeled Mobile Robots (WMR)



Yamabico



MagellanPro



Sojourner



ATRV-2



Hilare 2-Bis



Koy

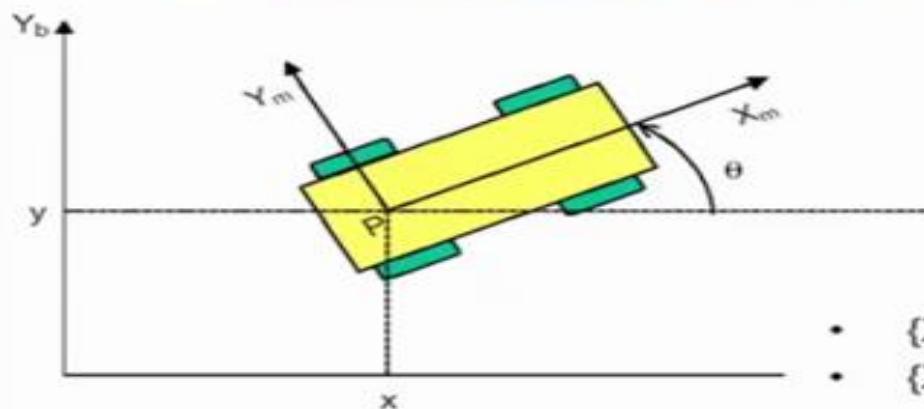
Wheeled Mobile Robots

- **Combination of various physical (hardware) and computational (software) components**
- **A collection of subsystems:**
 - **Sensing:** how the robot measures properties of itself and its environment
 - **Locomotion:** how the robot moves through its environment
 - **Control:** how the robot generate physical actions
 - **Reasoning:** how the robot maps measurements into actions
 - **Communication:** how the robots communicate with each other or with an outside operator

Wheeled Mobile Robots

- **Locomotion** — the process of causing a robot to move.
 - In order to produce motion, forces must be applied to the robot
 - Motor output, payload
- **Kinematics** – study of the mathematics of motion without considering the forces that affect the motion.
 - Deals with the geometric relationships that govern the system
 - Deals with the relationship between control parameters and the behavior of a system.
- **Dynamics** – study of motion in which these forces are modeled
 - Deals with the relationship between force and motion.

Notation



Posture: position(x, y) and orientation θ

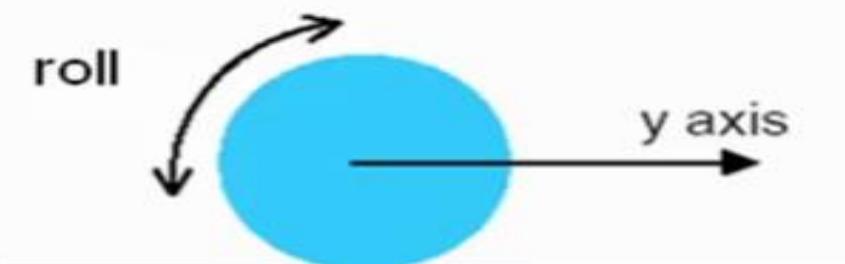
- $\{X_m, Y_m\}$ – moving frame
- $\{X_b, Y_b\}$ – base frame

$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad \text{robot posture in base frame} \rightarrow$$

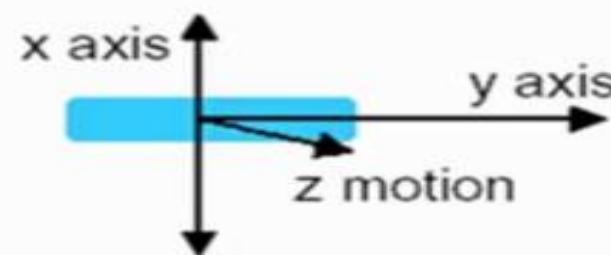
$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation matrix expressing the orientation of the base frame with respect to the moving frame

Wheels



Rolling motion

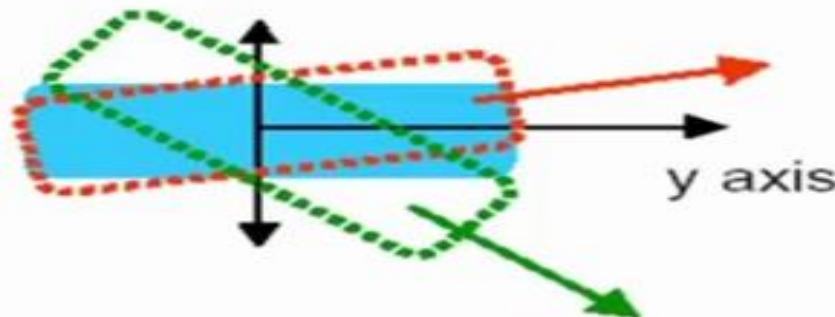
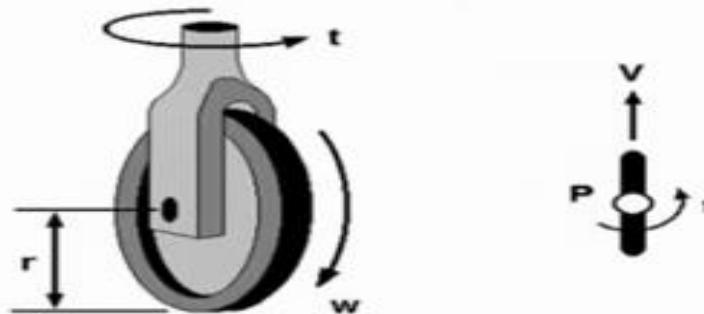


Lateral slip

Steered Wheel

- **Steered wheel**

- The orientation of the rotation axis can be controlled

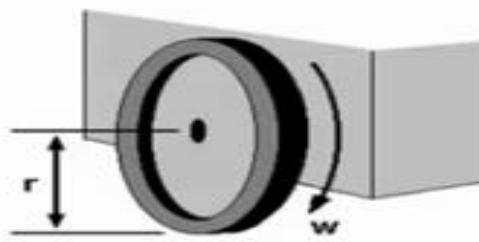


Robot wheel parameters

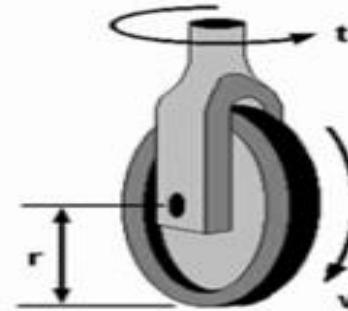
- For low velocities, rolling is a reasonable wheel model.
 - This is the model that will be considered in the kinematics models of WMR
- Wheel parameters:
 - r = wheel radius
 - v = wheel linear velocity
 - w = wheel angular velocity
 - t = steering velocity

Wheel Types

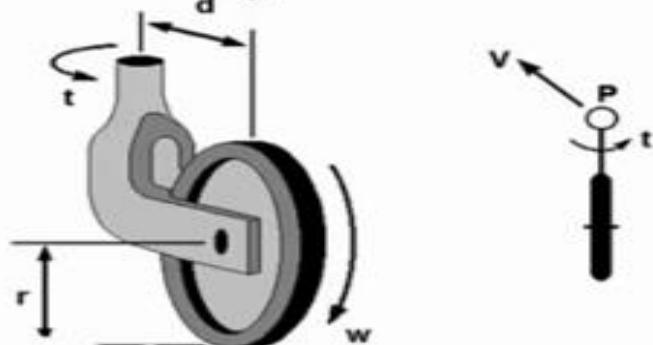
Fixed wheel



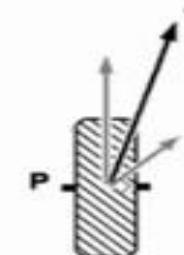
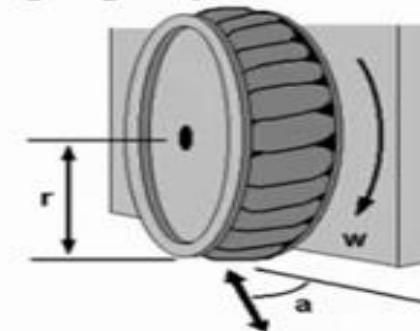
Centered orientable wheel



Off-centered orientable wheel
(Castor wheel)

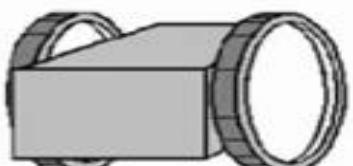


Swedish wheel: omnidirectional property



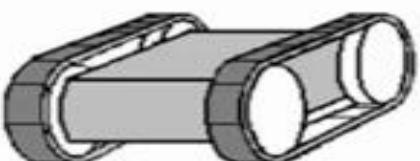
Examples of WMR

Example



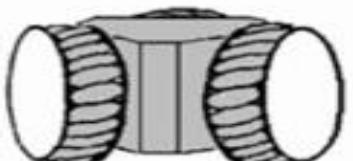
Bi-wheel type robot

- Smooth motion
- Risk of slipping
- Some times use roller-ball to make balance



Caterpillar type robot

- Exact straight motion
- Robust to slipping
- Inexact modeling of turning

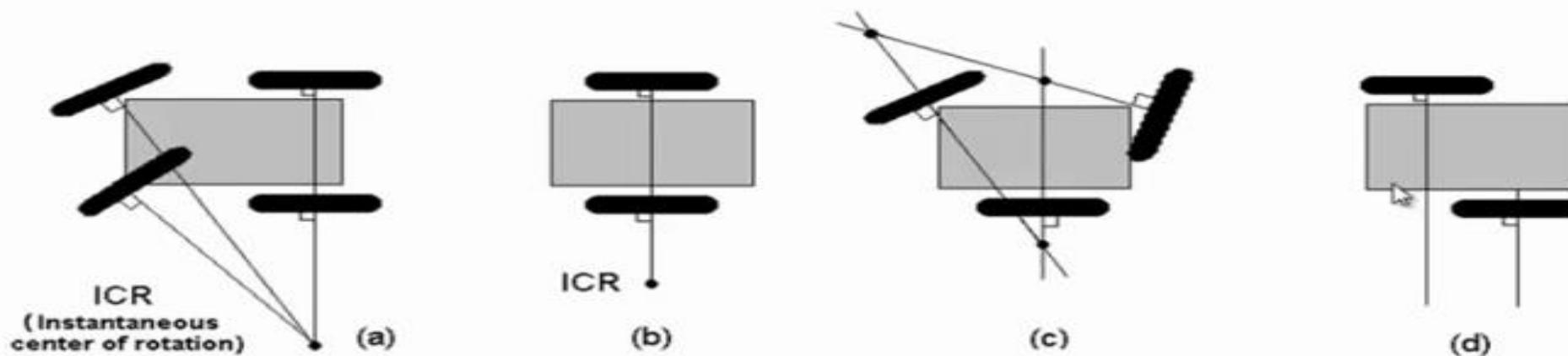


Omnidirectional robot

- Free motion
- Complex structure
- Weakness of the frame

Mobile Robot Locomotion

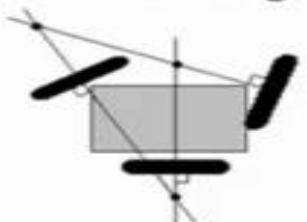
- Instantaneous center of rotation (ICR) or Instantaneous center of curvature (ICC)
 - A cross point of all axes of the wheels



Degree of Mobility

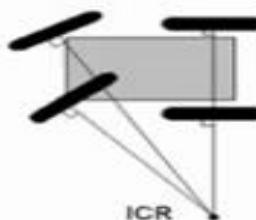
- **Degree of mobility**

The degree of freedom of the robot motion



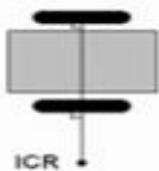
Cannot move anywhere (No ICR)

- Degree of mobility : 0



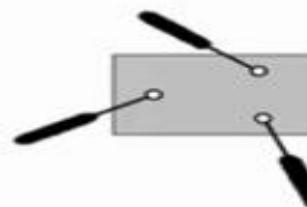
Fixed arc motion
(Only one ICR)

- Degree of mobility : 1



Variable arc motion
(line of ICRs)

- Degree of mobility : 2



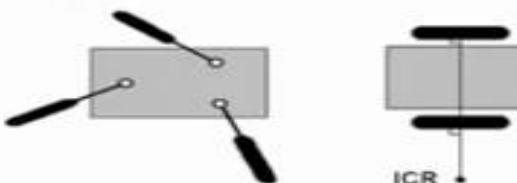
Fully free motion
(ICR can be located at any position)

- Degree of mobility : 3

Degree of Steerability

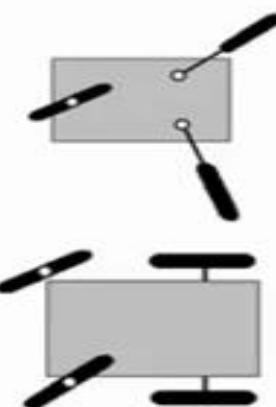
- **Degree of steerability**

The number of centered orientable wheels that can be steered independently in order to steer the robot



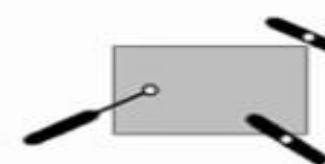
- Degree of steerability : 0

No centered orientable wheels



One centered orientable wheel

Two mutually dependent centered orientable wheels



- Degree of steerability : 1



Two mutually independent centered orientable wheels

- Degree of steerability : 2

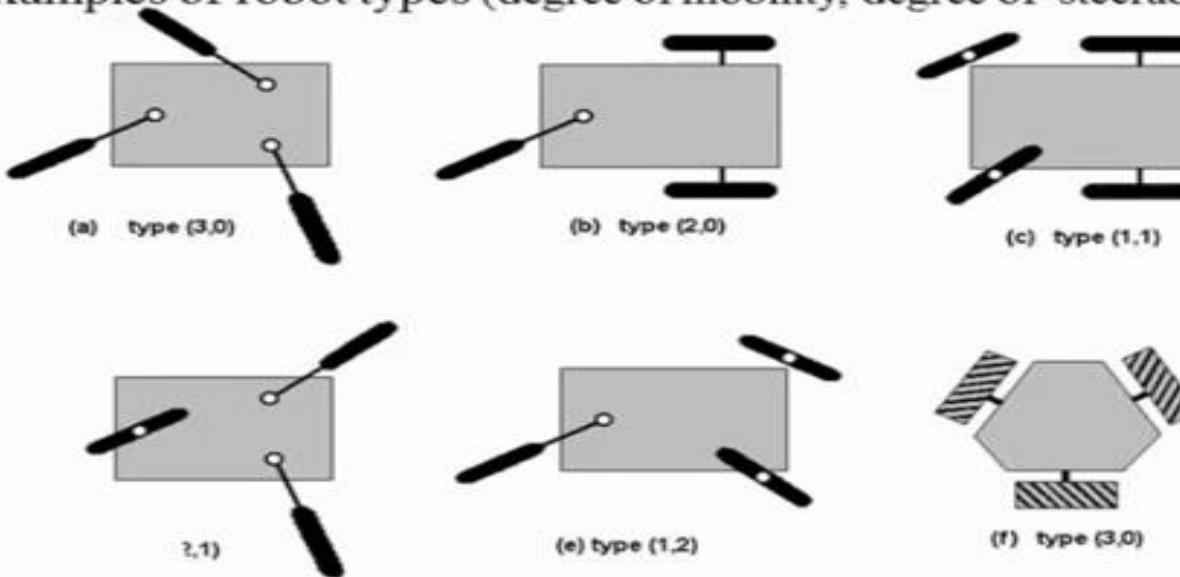
Degree of Maneuverability

- The overall degrees of freedom that a robot can manipulate:

$$\delta_M = \delta_m + \delta_s$$

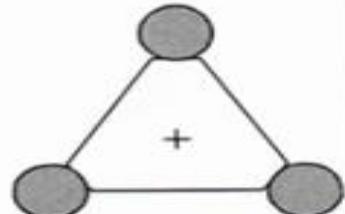
Degree of Mobility	3	2	2	1	1
Degree of Steerability	0	0	1	1	2

- Examples of robot types (degree of mobility, degree of steerability)

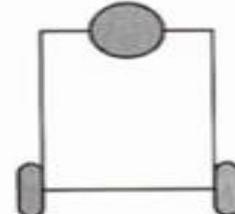


Degree of Maneuverability

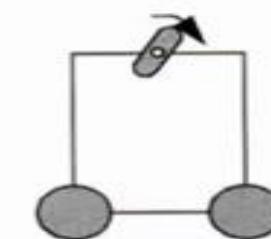
$$\delta_M = \delta_m + \delta_s$$



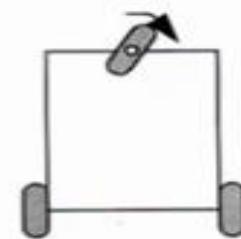
Omnidirectional
 $\delta_M = 3$
 $\delta_m = 3$
 $\delta_s = 0$



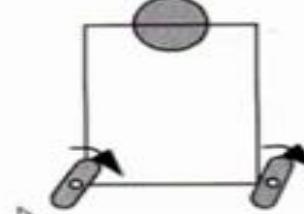
Differential
 $\delta_M = 2$
 $\delta_m = 2$
 $\delta_s = 0$



Omni-Steer
 $\delta_M = 3$
 $\delta_m = 2$
 $\delta_s = 1$



Tricycle
 $\delta_M = 2$
 $\delta_m = 1$
 $\delta_s = 1$



Two-Steer
 $\delta_M = 3$
 $\delta_m = 1$
 $\delta_s = 2$

Non-holonomic constraint

A non-holonomic constraint is a constraint on the feasible **velocities** of a body

So what does that mean?

Your robot can move in some directions (forward and backward), but not others (sideward).

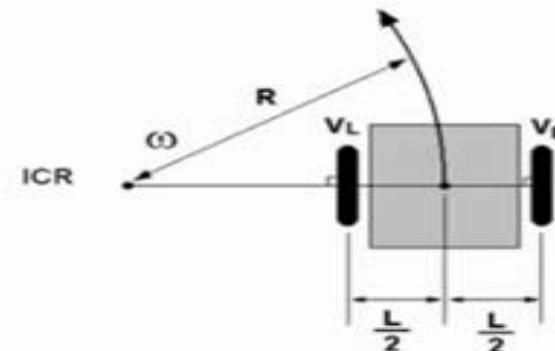
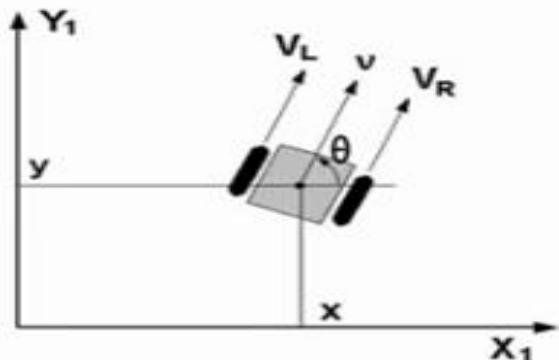
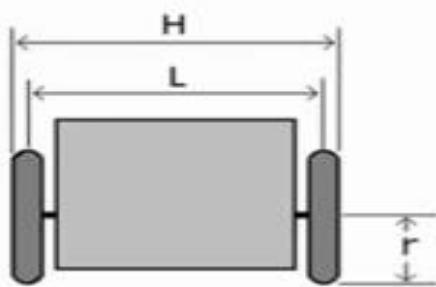
The robot can instantly move forward and backward, but can not move sideward



Types of driving (steering)

1. Differential Drive
 - two driving wheels (plus roller-ball for balance)
 - simplest drive mechanism
 - sensitive to the relative velocity of the two wheels (small error result in different trajectories, not just speed)
2. Steered wheels (tricycle, bicycles, wagon)
 - Steering wheel + rear wheels
 - cannot turn $\pm 90^\circ$
 - limited radius of curvature
3. Synchronous Drive
4. Omni-directional
5. Car Drive (**Ackerman Steering**)

1-Differential Drive



- Posture of the robot

$$P = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

(x,y) : Position of the robot
 θ : Orientation of the robot

- Control input

$$U = \begin{pmatrix} v \\ w \end{pmatrix}$$

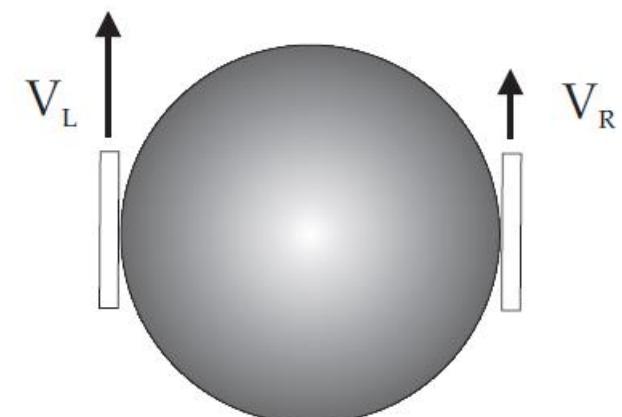
v : Linear velocity of the **robot**
w : Angular velocity of the **robot**
(notice: not for each wheel)

Kinematics

Kinematics is the process of determining the range of possible movements for a robot, without consideration of the forces acting on the robot, but taking into account the various constraints on the motion. The kinematic equations for a robot depend on the robot's structure, i.e. the number of wheels, the type of wheels used etc. Here, only the case of differentially steered two wheeled robots will be considered. For balance, a two-wheeled robot must also have one or several supporting wheels (or some other form of ground contact, such as a ball in a material with low friction). The influence of the supporting wheels on the kinematics and dynamics will not be considered.

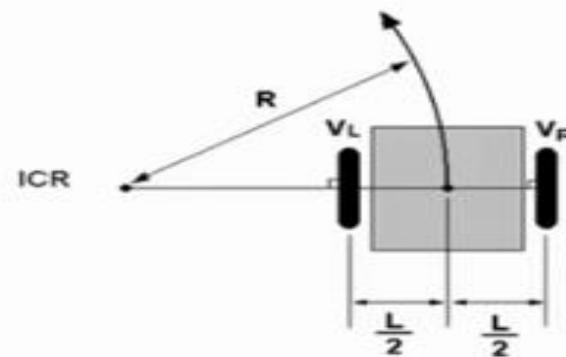
The differential drive

A schematic view of a differentially steered robot is shown in Fig. The Bobet that will be considered in the second half of the course (see the left panel



Motion Control

- Instantaneous center of rotation



$$(V_R - V_L) / L = V_R / (R + \frac{L}{2})$$

$$R = \frac{L}{2} \frac{V_R + V_L}{V_R - V_L}$$

R : Radius of rotation
 def

- Straight motion

$$R = \text{Infinity} \rightarrow V_R = V_L$$

- Rotational motion

$$R = 0 \rightarrow V_R = -V_L$$

mass of the robot is given by

$$V = \omega L.$$

Inserting Eq. (2.4) into Eqs. (2.2) and (2.3), L can be eliminated. V then be obtained in terms of v_L and v_R as

$$\begin{aligned} V &= \frac{v_L + v_R}{2}, \\ \omega &= -\frac{v_L - v_R}{2R}. \end{aligned}$$

Denoting the speed components of the robot V_x and V_y , and noting $V \cos \varphi$, $V_y = V \sin \varphi$, the position of the robot at time t_1 is given by

$$\begin{aligned} X(t_1) - X_0 &= \int_{t_0}^{t_1} V_x(t) dt = \int_{t_0}^{t_1} \frac{v_L(t) + v_R(t)}{2} \cos \varphi(t) dt, \\ Y(t_1) - Y_0 &= \int_{t_0}^{t_1} V_y(t) dt = \int_{t_0}^{t_1} \frac{v_L(t) + v_R(t)}{2} \sin \varphi(t) dt, \\ \varphi(t_1) - \varphi_0 &= \int_{t_0}^{t_1} \omega(t) dt = - \int_{t_0}^{t_1} \frac{v_L(t) - v_R(t)}{2R} dt, \end{aligned}$$

Dynamics

by contrast, considers the motion of the robot in response to the forces (and torques) acting on it. In the case of the two-wheeled, differentially steered robot, the two motors generate torques (as described above) that propel the wheels forward,. The frictional force at the contact point with the ground will try to move the ground backwards. By Newton's third law, a reaction force of the same magnitude will attempt to move the wheel forward. In addition to the torque from the motor (assumed to be known) and the reaction force \mathbf{F} from the ground, a reaction force $\mathbf{\rho}$ from the main body of the robot will act on the wheel, mediated by the wheel axis (the length of which is neglected in this derivation). Using Newton's second law, the equations of motion for the wheels take the form

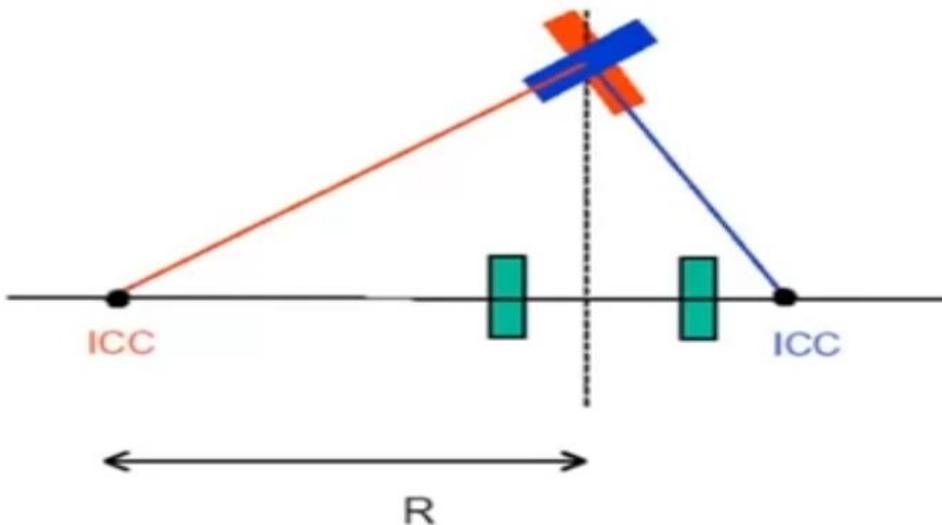
$$m\dot{v}_L = F_L - \rho_L,$$

$$m\dot{v}_R = F_R - \rho_R,$$

$$\bar{I}_w \ddot{\phi}_L = \tau_L - F_L r,$$

2- Tricycle

- Three wheels and odometers on the two rear wheels
- Steering and power are provided through the front wheel
- control variables:
 - steering direction $\alpha(t)$
 - angular velocity of steering wheel $w_s(t)$



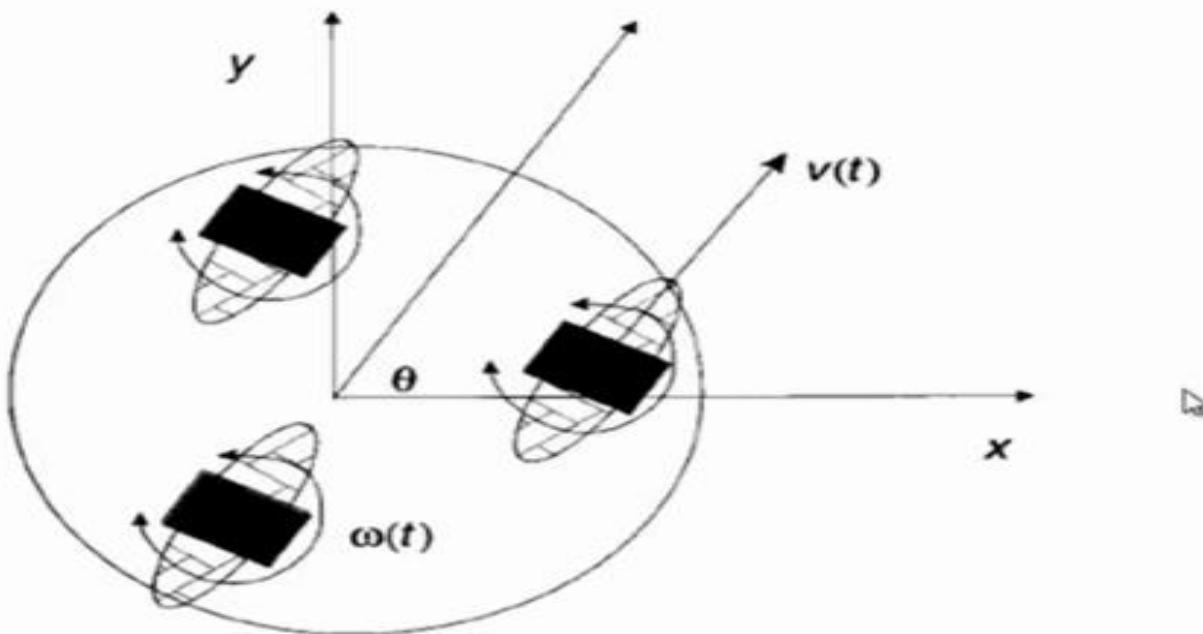
The ICC must lie on the line that passes through, and is perpendicular to, the fixed rear wheels

3-Synchronous Drive

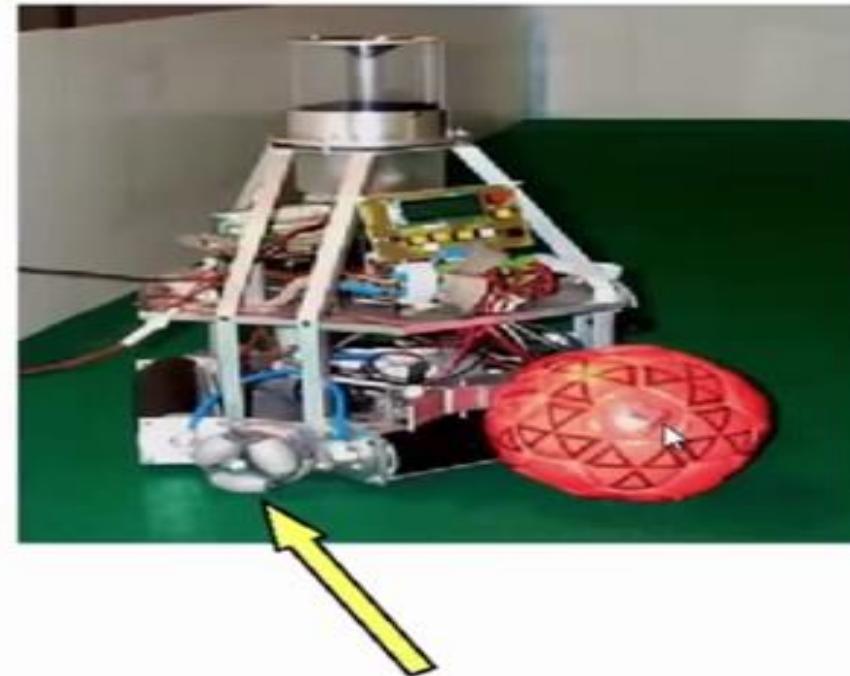
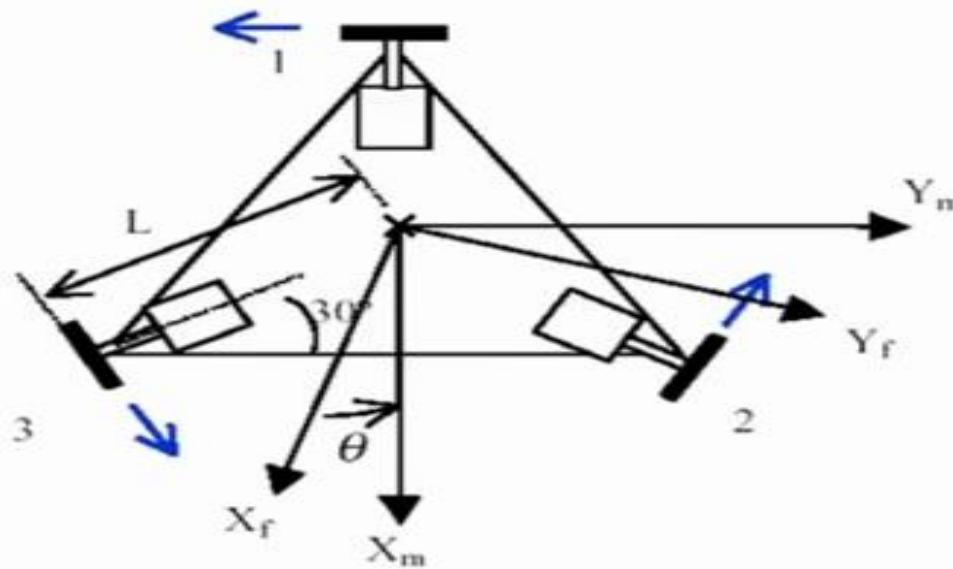
- In a synchronous drive robot (synchronous drive) each wheel is capable of being driven and steered.
- Typical configurations
 - Three steered wheels arranged as vertices of an equilateral triangle often surmounted by a cylindrical platform
 - All the wheels turn and drive in unison
- This leads to a holonomic behavior

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Synchronous Drive

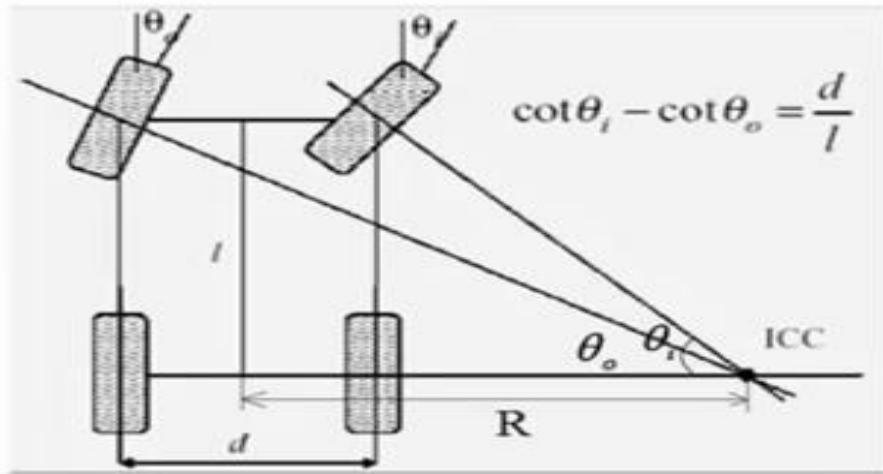


4-Omidirectional



Swedish Wheel

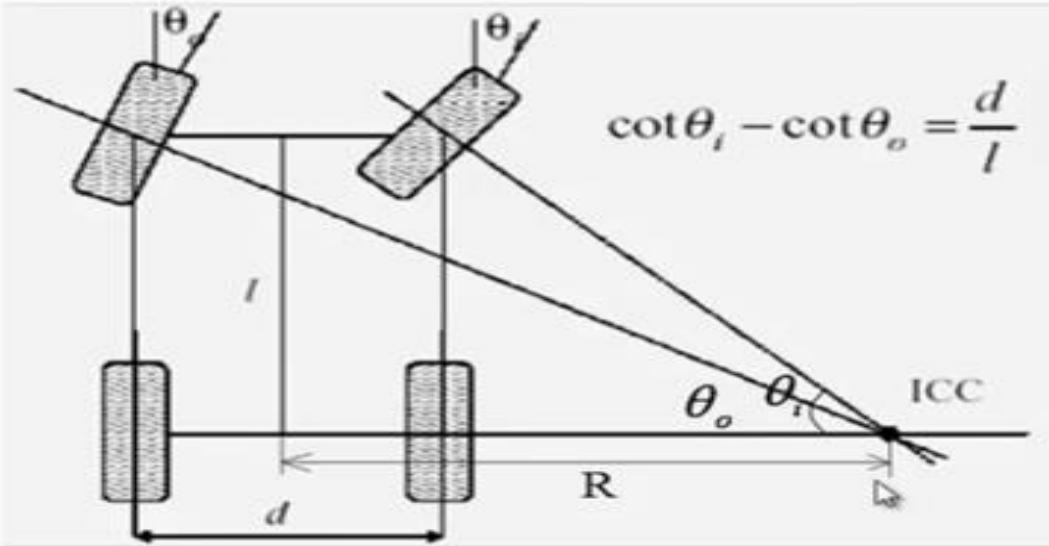
5-Car Drive (Ackerman Steering)



- Used in motor vehicles, the inside front wheel is rotated slightly sharper than the outside wheel (reduces tire slippage).
- Generally the method of choice for outdoor autonomous vehicles.



Ackerman Steering



where

d = lateral wheel separation

I = longitudinal wheel separation

θ_i = relative steering angle of inside wheel

θ_o = relative steering angle of outside wheel

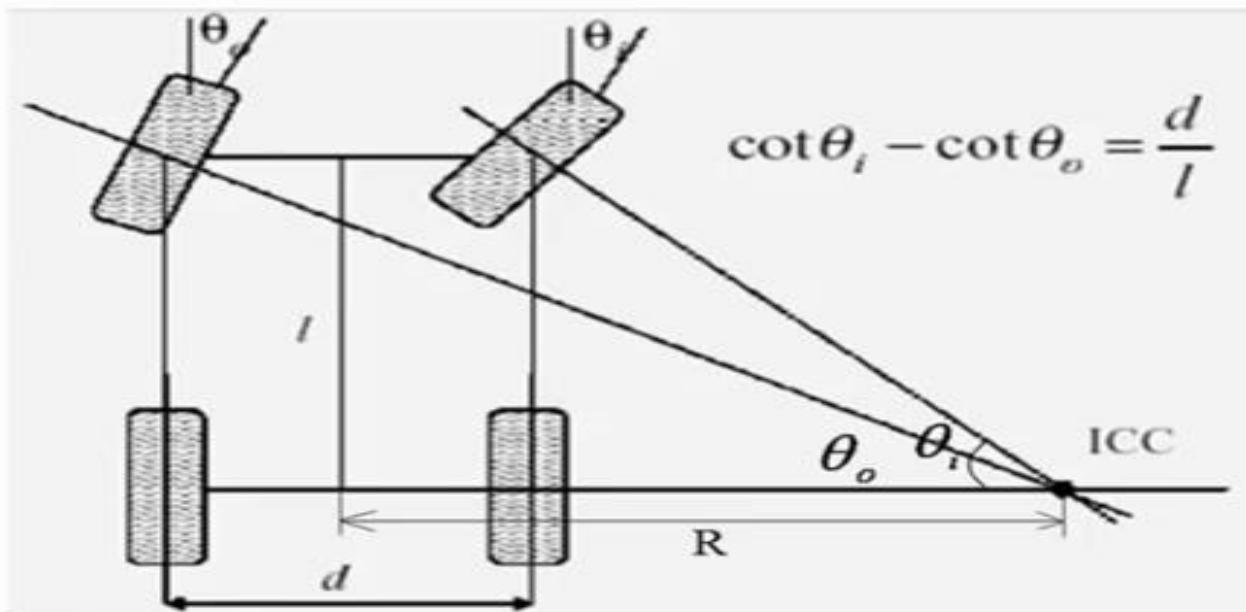
R =distance between ICC to centerline of the vehicle

Ackerman Steering

- The Ackerman Steering equation:

$$\cot \theta_i - \cot \theta_o = \frac{d}{l}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$



$$\cot \theta_i - \cot \theta_o = \frac{d}{l}$$

$$\begin{aligned}\cot \theta_i - \cot \theta_o &= \frac{R - d/2}{l} - \frac{R + d/2}{l} \\ &= \frac{d}{l}\end{aligned}$$

Summary

- Mobile Robot
- Classification of wheels
 - Fixed , Centered , Off-centered , Swedish wheel
- Degree of Maneuverability
 - Degrees of mobility+ Degree of steerability
- 5 types of driving (steering) methods
 - Differential Drive
 - Steered wheels (tricycle, bicycles, wagon)
 - Synchronous Drive
 - Omni-directional
 - Car Drive (Ackerman Steering)



The World consists of...

- Obstacles
 - Already occupied spaces of the world
 - In other words, robots can't go there
- Free Space
 - Unoccupied space within the world
 - Robots “might” be able to go here
 - To determine where a robot can go, we need to discuss what a *Configuration Space* is

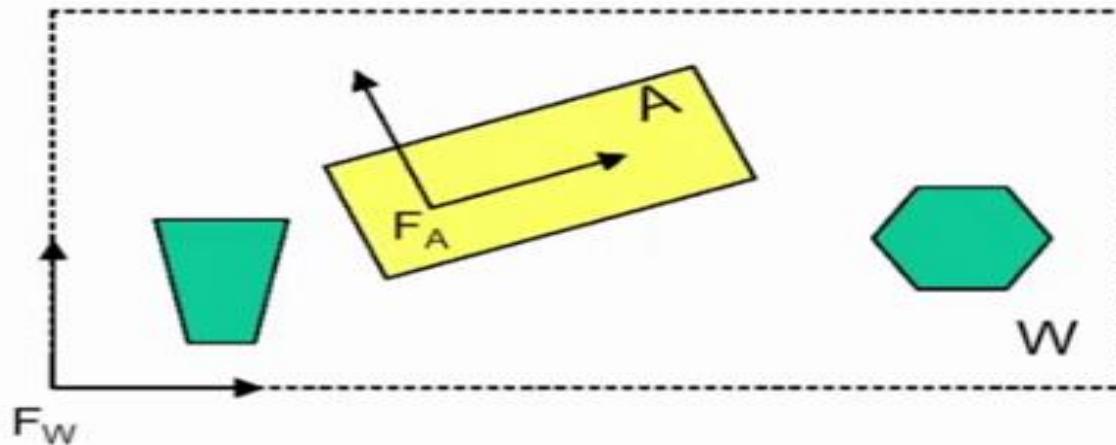
Configuration Space

Notation:

A: single rigid object –(the robot)

W: Euclidean space where **A** moves; $W = R^2 \quad or \quad R^3$

B₁,...,B_m: fixed rigid obstacles distributed in **W**



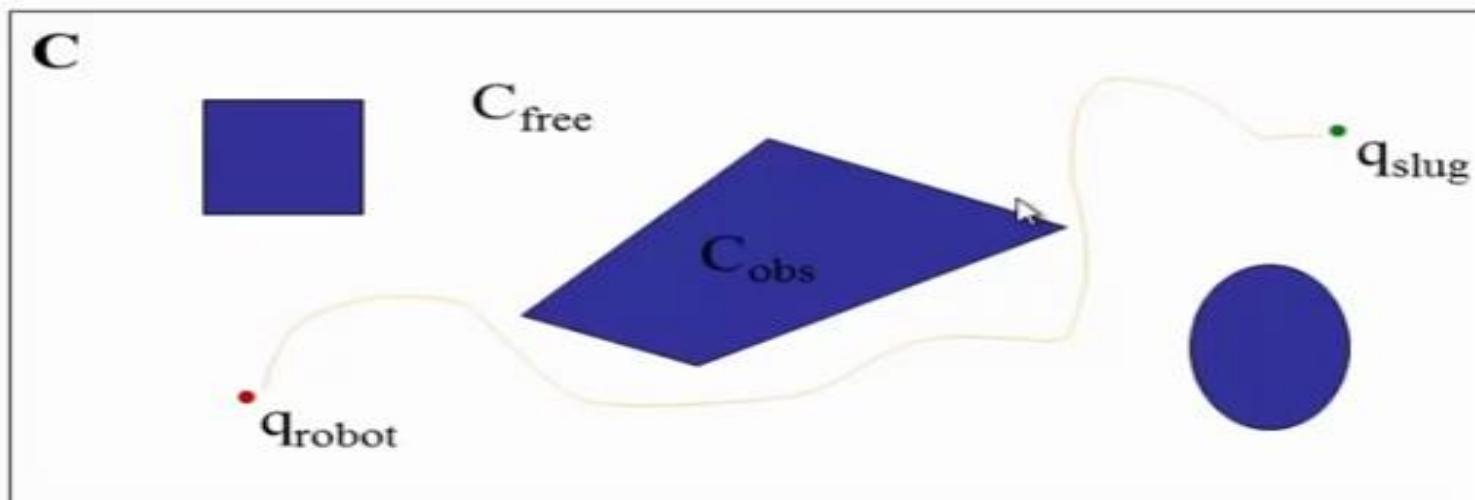
- F_w – world frame (fixed frame)
- F_A – robot frame (moving frame rigidly associated with the robot)

↳ Configuration q of **A** is a specification of the physical state (position and orientation) of **A** w.r.t. a fixed environmental frame F_w .

Configuration Space

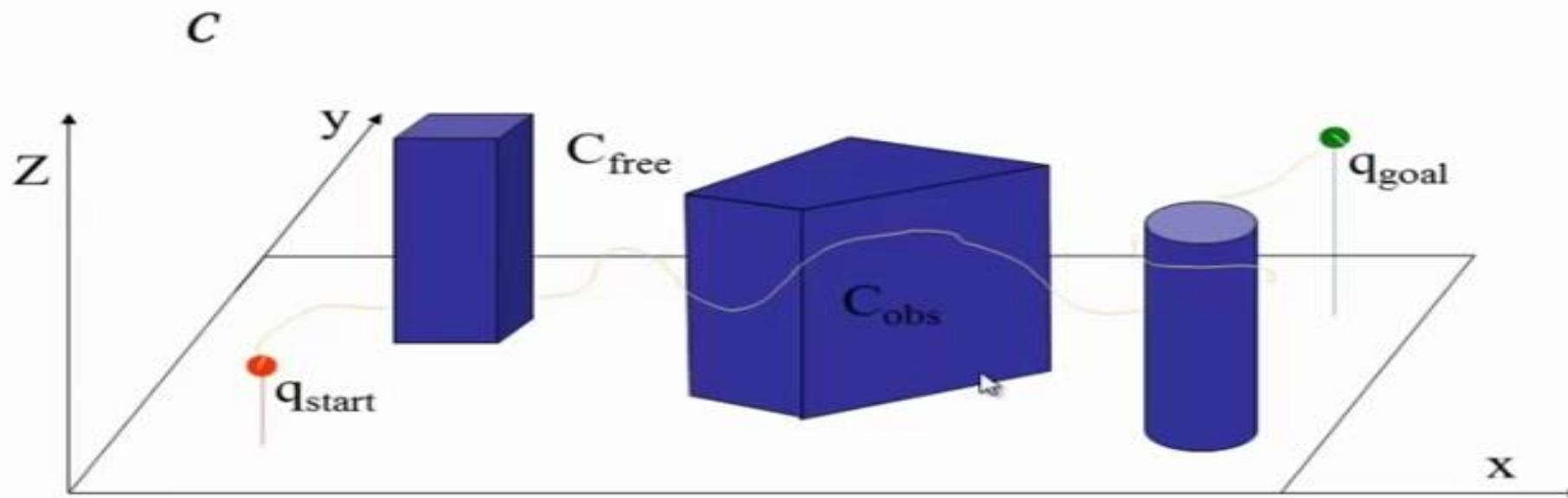
Configuration Space of A is the space (C) of all possible configurations of A .

Point robot (free-flying, no constraints)



For a point robot moving in 2-D plane, C -space is R^2

Configuration Space



For a point robot moving in 3-D, the C -space is R^3

Free Space

C-OBSTACLE REGION

B_1, B_2, \dots, B_m ————— obstacles

CB_i ————— C-obstacle

$\bigcup_{i=1}^m CB_i$ ————— C-obstacle region



FREE SPACE

$$C_{\text{free}} = C \setminus \bigcup_{i=1}^m CB_i = \{q \in C : A(q) \cap \bigcup_{i=1}^m CB_i = \emptyset\}$$

Free configuration q iff $q \in C_{\text{free}}$

Motion Planning Methods

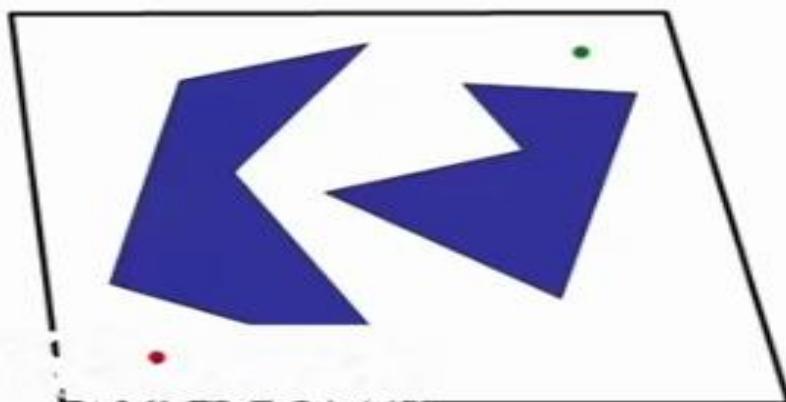
The motion planning problem consists of the following:

Input

- geometric descriptions of a robot and its environment (obstacles)
- initial and goal configurations

• q_{robot}

• q_{goal}



Output

- a path from start to finish (or the recognition that none exists)

Applications

- Robot-assisted surgery
- Automated assembly plans
- Drug-docking and analysis
- Moving pianos around...

What to do?

Motion Planning Methods

(1) Roadmap approaches

} **Goal** reduce the N-dimensional configuration space to a set of one-D paths to search.

(2) Cell decomposition

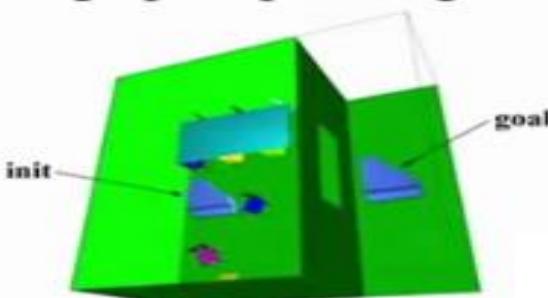
← **Goal** account for all of the free space

(3) Potential Fields

} **Goal** Create local control strategies that will be more flexible than those above
→

(4) Bug algorithms

Limited knowledge path planning

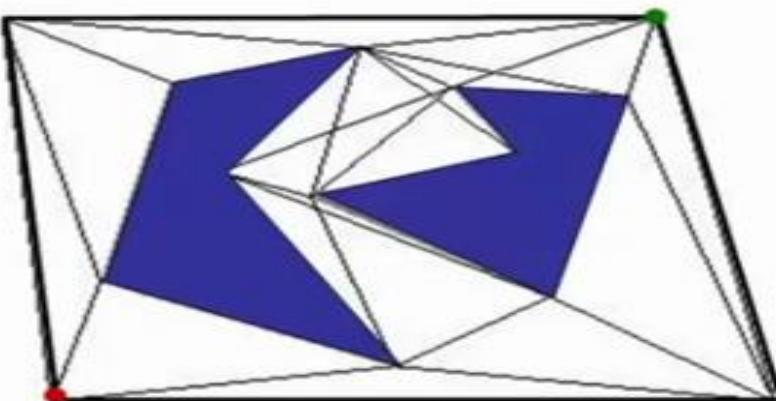


Roadmap: Visibility Graphs

Visibility graphs: In a polygonal (or polyhedral) configuration space, construct all of the line segments that connect vertices to one another (and that do not intersect the obstacles themselves).

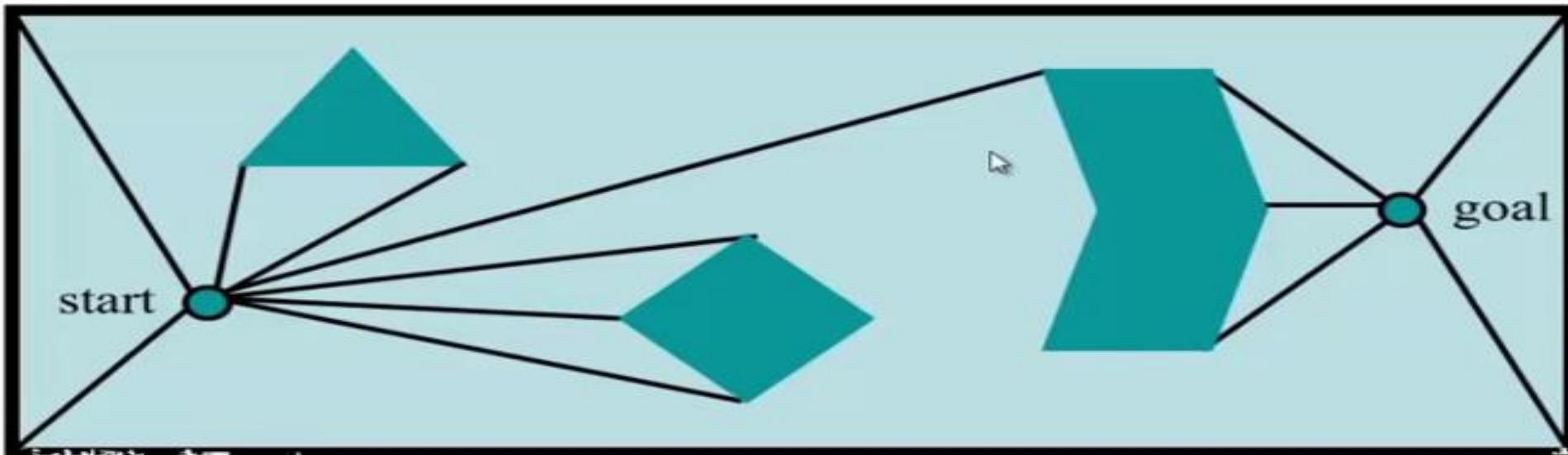
Formed by connecting all “visible” vertices, the start point and the end point, to each other.

For two points to be “visible” no obstacle can exist between them
Paths exist on the perimeter of obstacles



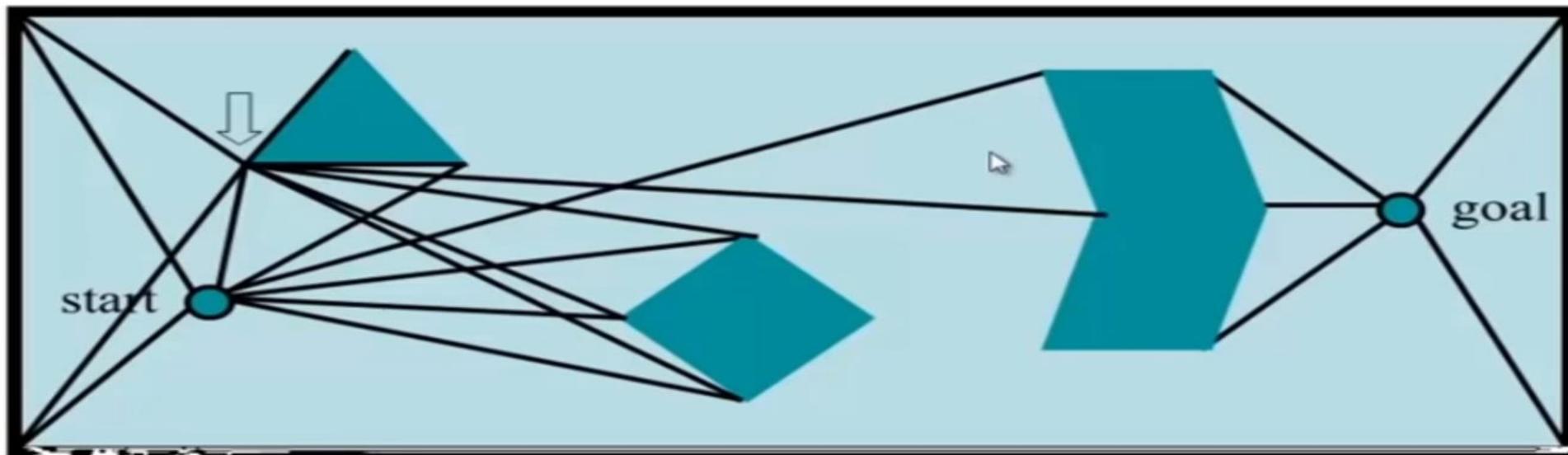
The Visibility Graph in Action (Part 1)

- First, draw lines of sight from the start and goal to all “visible” vertices and corners of the world.



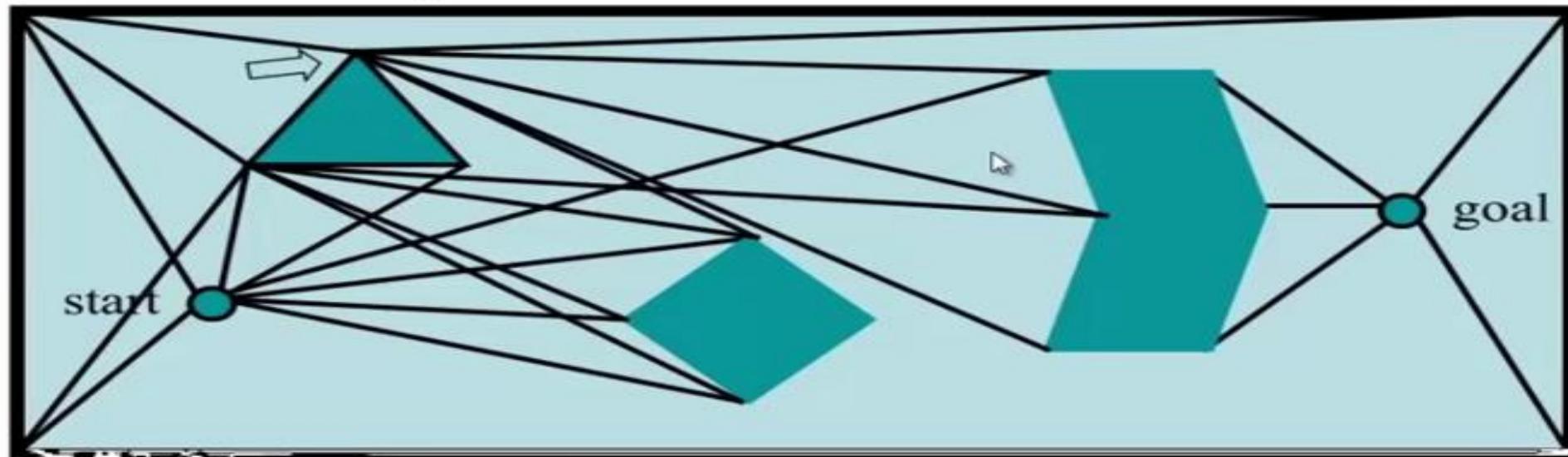
The Visibility Graph in Action (Part 2)

- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



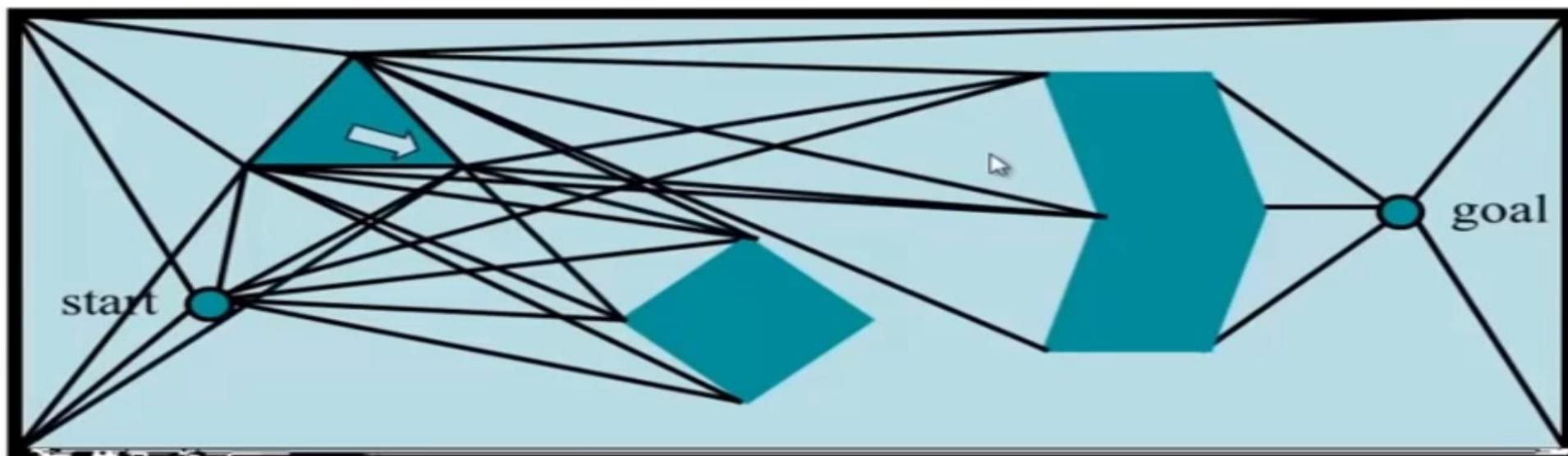
The Visibility Graph in Action (Part 3)

- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



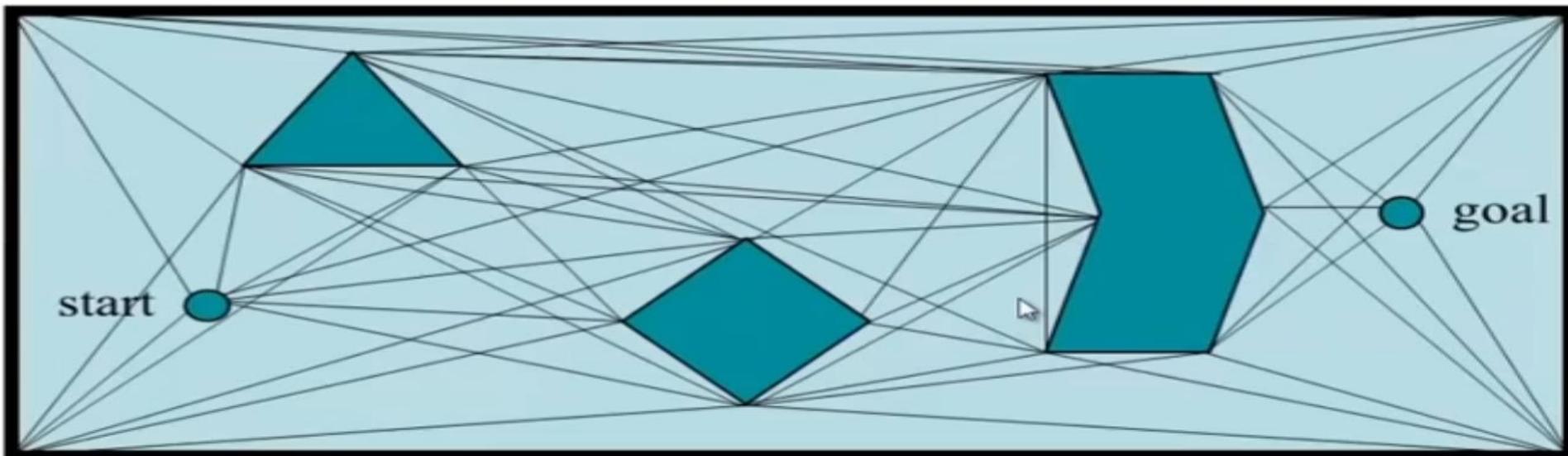
The Visibility Graph in Action (Part 4)

- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



The Visibility Graph (Done)

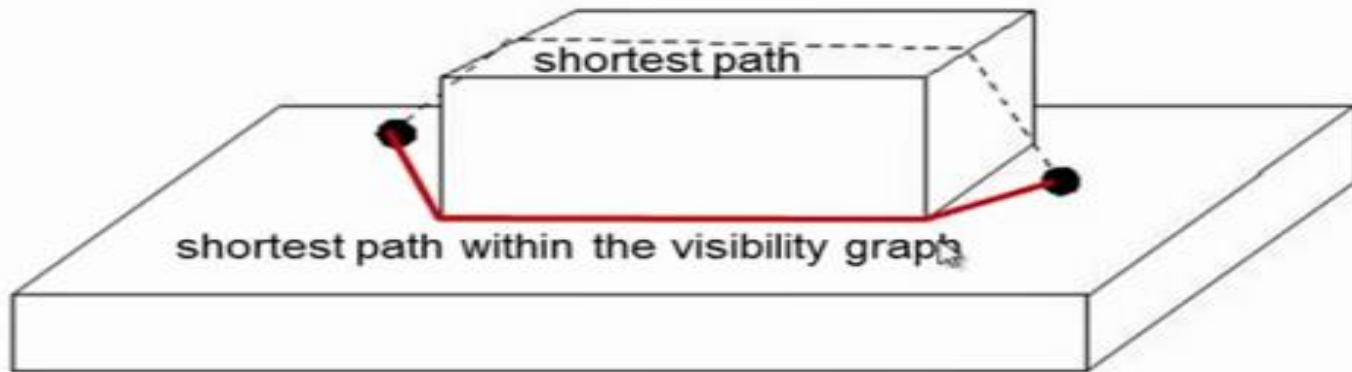
- Repeat until you're done.



Since the map was in C-space, each line potentially represents part of a path from the start to the goal.

Visibility graph drawbacks

Visibility graphs do not preserve their optimality in higher dimensions:



In addition, the paths they find are “semi-free,” i.e. in contact with obstacles.

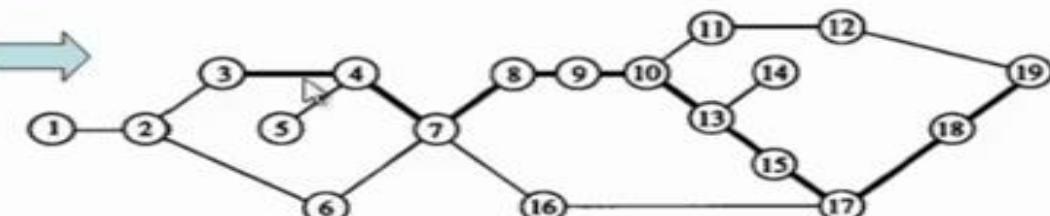
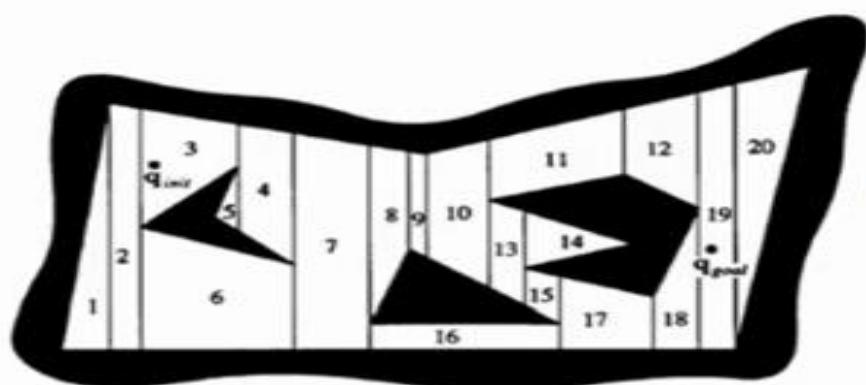
No clearance

Exact Cell Decomposition

Trapezoidal Decomposition:

Decomposition of the free space
into trapezoidal & triangular cells

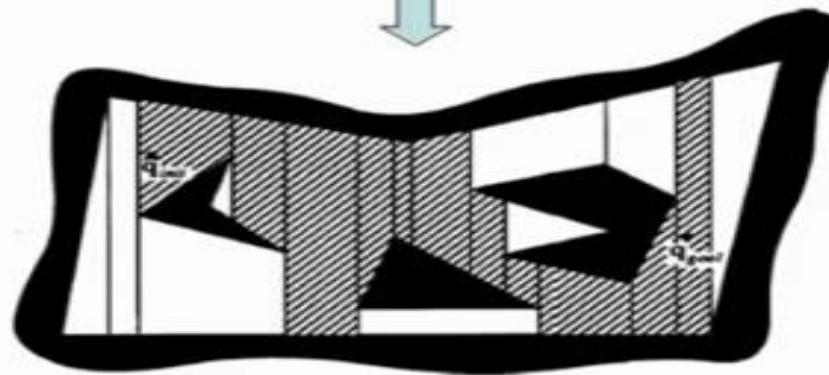
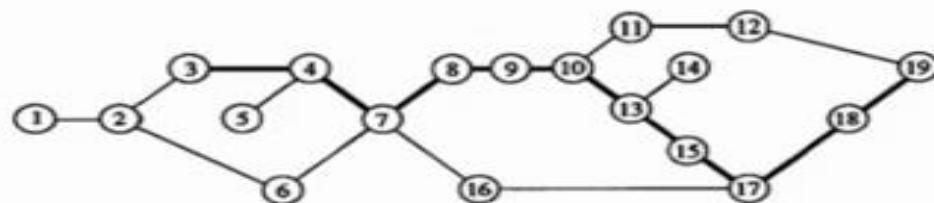
Connectivity graph representing the
adjacency relation between the cells



(Sweepline algorithm)

Exact Cell Decomposition

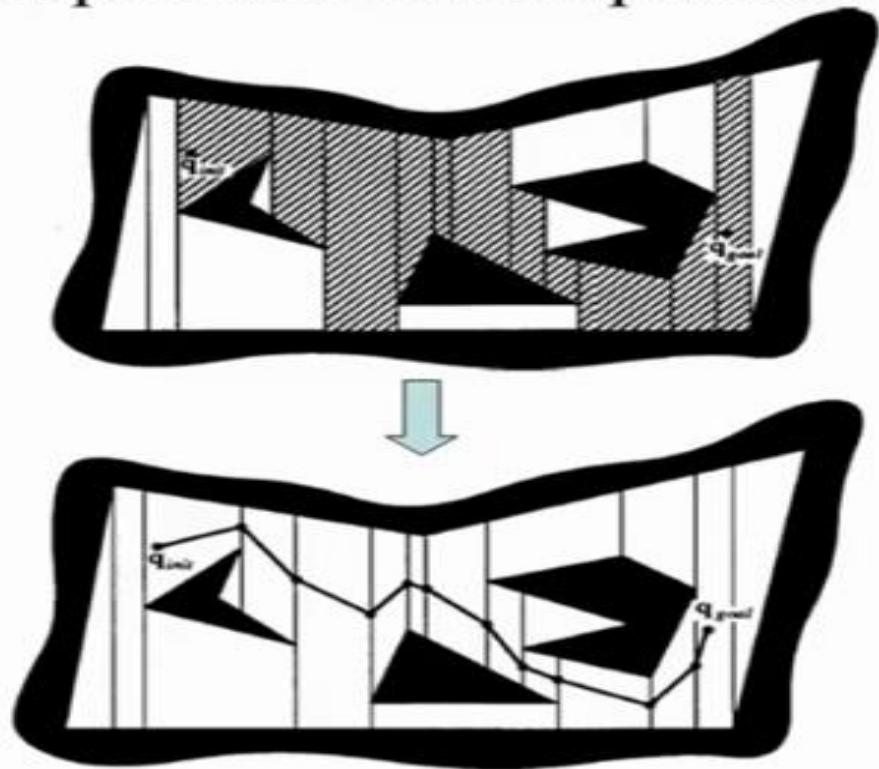
Trapezoidal Decomposition:



Search the graph for a path
(sequence of consecutive cells)

Exact Cell Decomposition

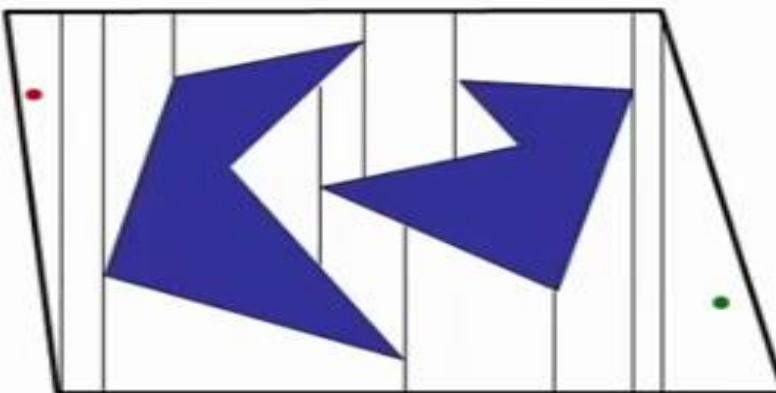
Trapezoidal Decomposition:



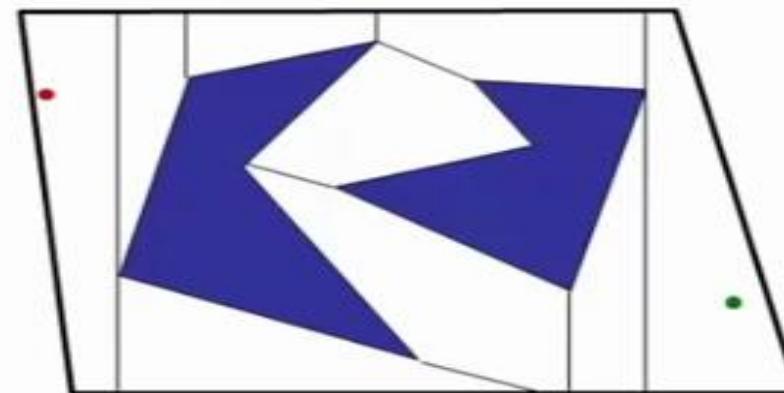
Transform the sequence of cells into a free path (e.g., connecting the mid-points of the intersection of two consecutive cells)

Optimality

Trapezoidal Decomposition:



15 cells



9 cells

Trapezoidal decomposition is exact and complete, but not optimal

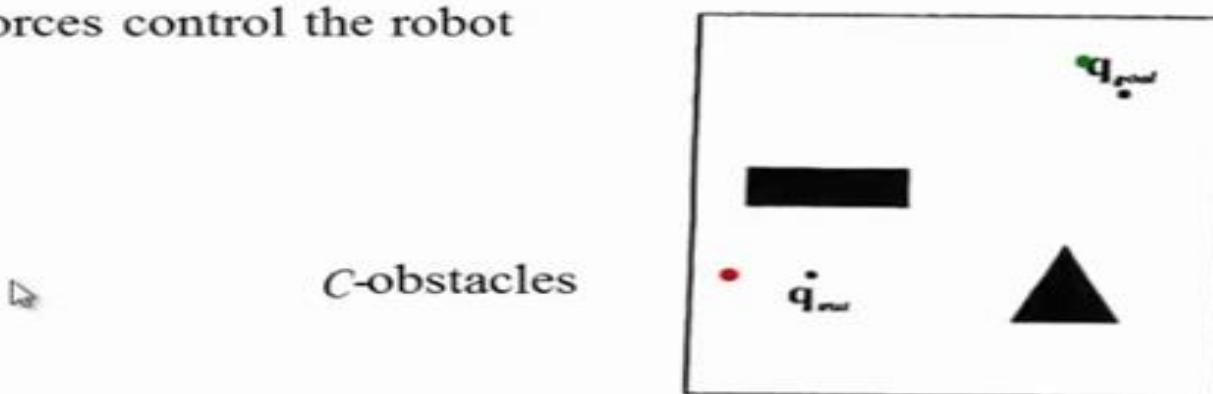
Obtaining the *minimum* number of convex cells is NP-complete.

there may be more details in the world than the task needs to worry about...

Potential Field Method

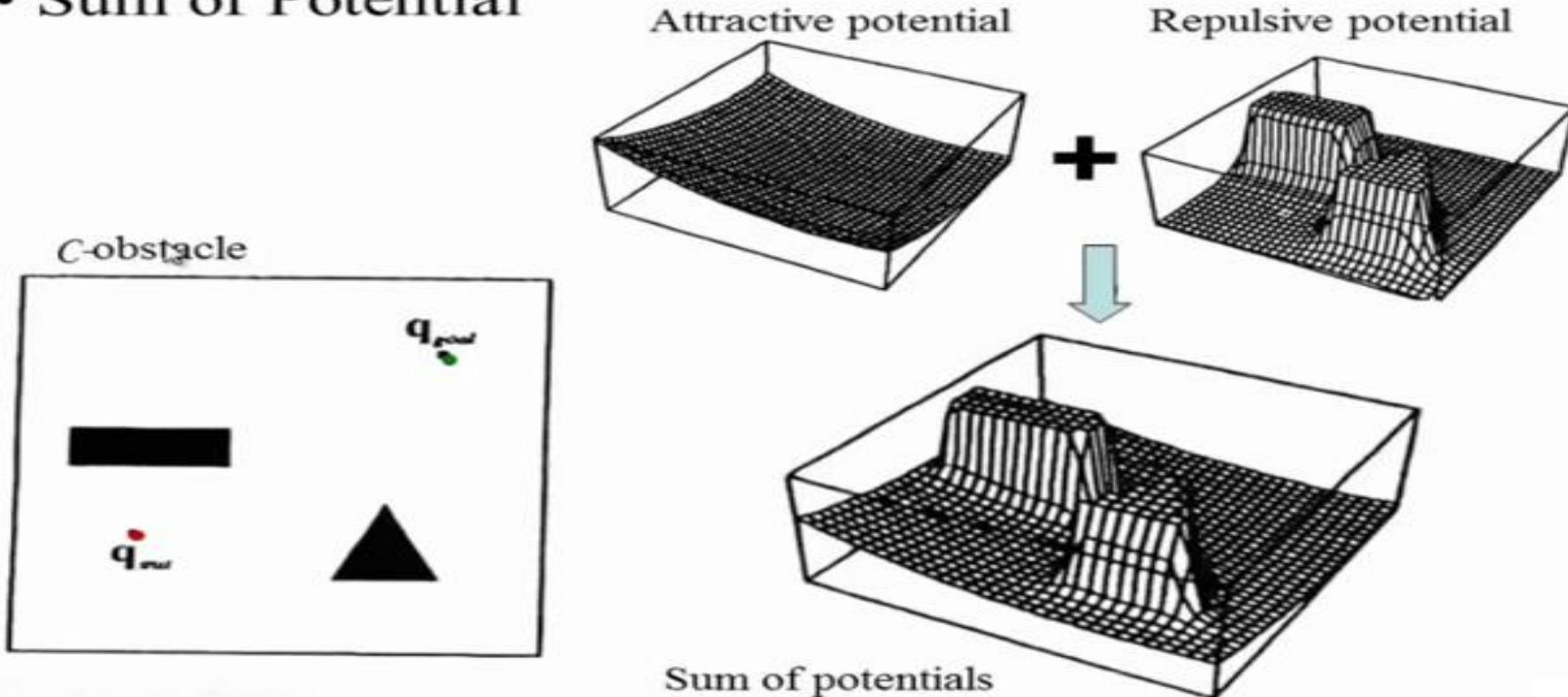
Potential Field (Working Principle)

- The goal location generates an **attractive potential** – pulling the robot towards the goal
- The obstacles generate a **repulsive potential** – pushing the robot far away from the obstacles
- The **negative gradient of the total potential** is treated as an artificial force applied to the robot
 - Let the sum of the forces control the robot



Potential Field Method

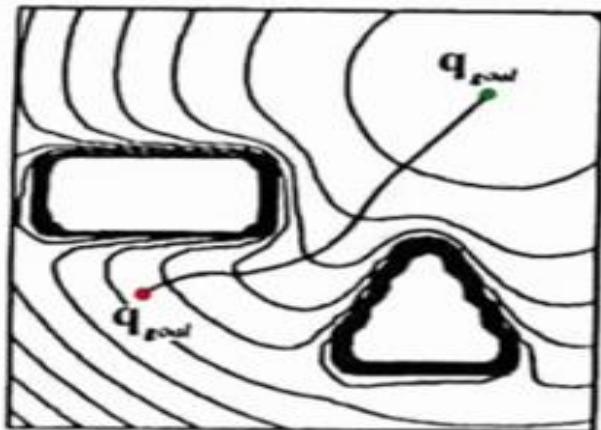
- Sum of Potential



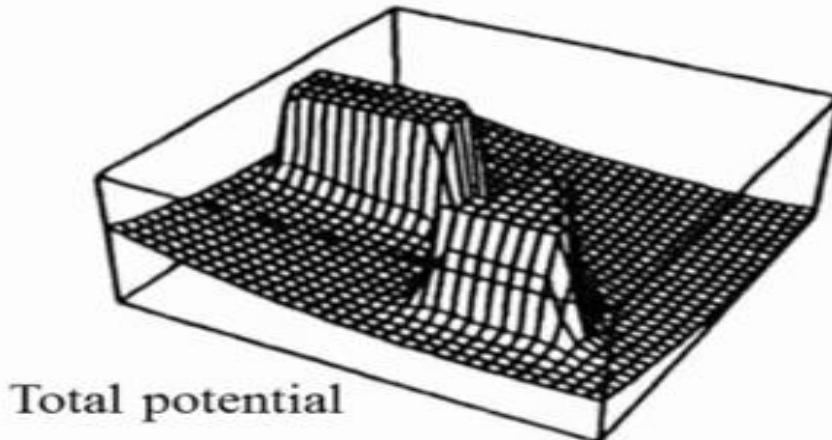
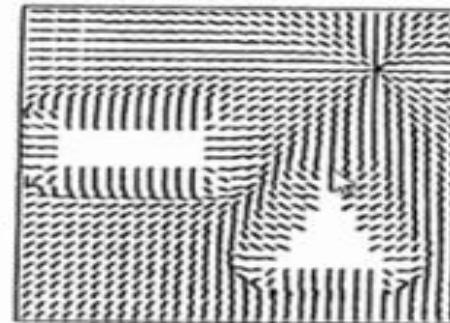
Potential Field Method

- After get total potential, generate force field (negative gradient)
- Let the sum of the forces control the robot

Equipotential contours



Negative gradient



Total potential

To a large extent, this is computable from sensor readings

Potential Field Method

Pros:

- Spatial paths are not preplanned and can be generated in real time
- Planning and control are merged into one function
- Smooth paths are generated
- Planning can be coupled directly to a control algorithm



Cons:

- Trapped in local minima in the potential field
- Because of this limitation, commonly used for local path planning
- Use random walk, backtracking, etc to escape the local minima

Motion Planning Methods

Roadmap approaches

- Visibility Graph

Cell decomposition

- Trapezoidal decomposition

Potential Fields

Bug algorithm

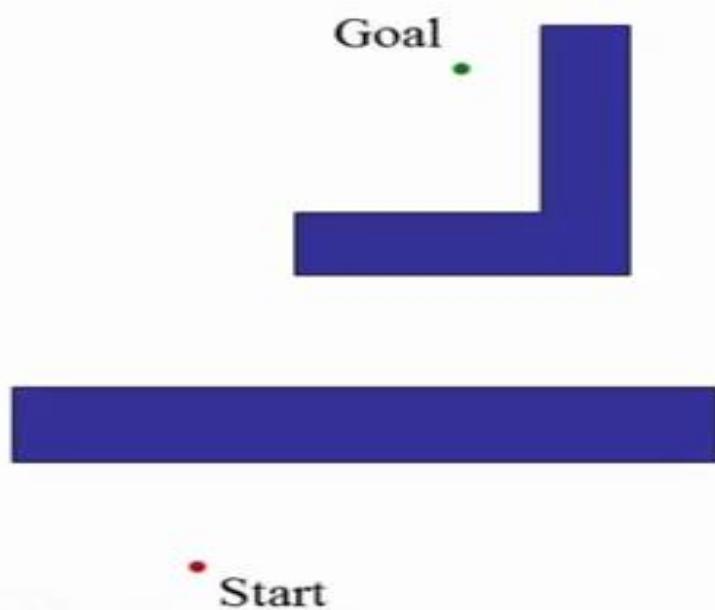
Full-knowledge motion planning

Limited-knowledge path planning

Bug Algorithms

Path planning with limited knowledge

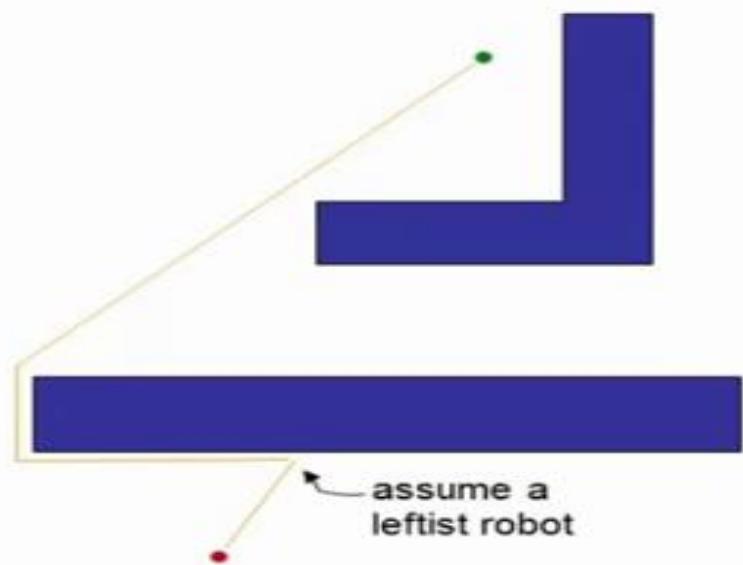
- Insect-inspired “bug” algorithms



- known direction to goal
- only local sensing
(walls/obstacles encoders)
- “reasonable” world
 - 1) finite obstacles in any finite range
 - 2) a line will intersect an obstacle finite times

Beginner Strategy

Insect-inspired “bug” algorithms



Switching between two simple behaviors:

1. Moving directly towards the goal
2. Circumnavigating an obstacle

“Bug” algorithm

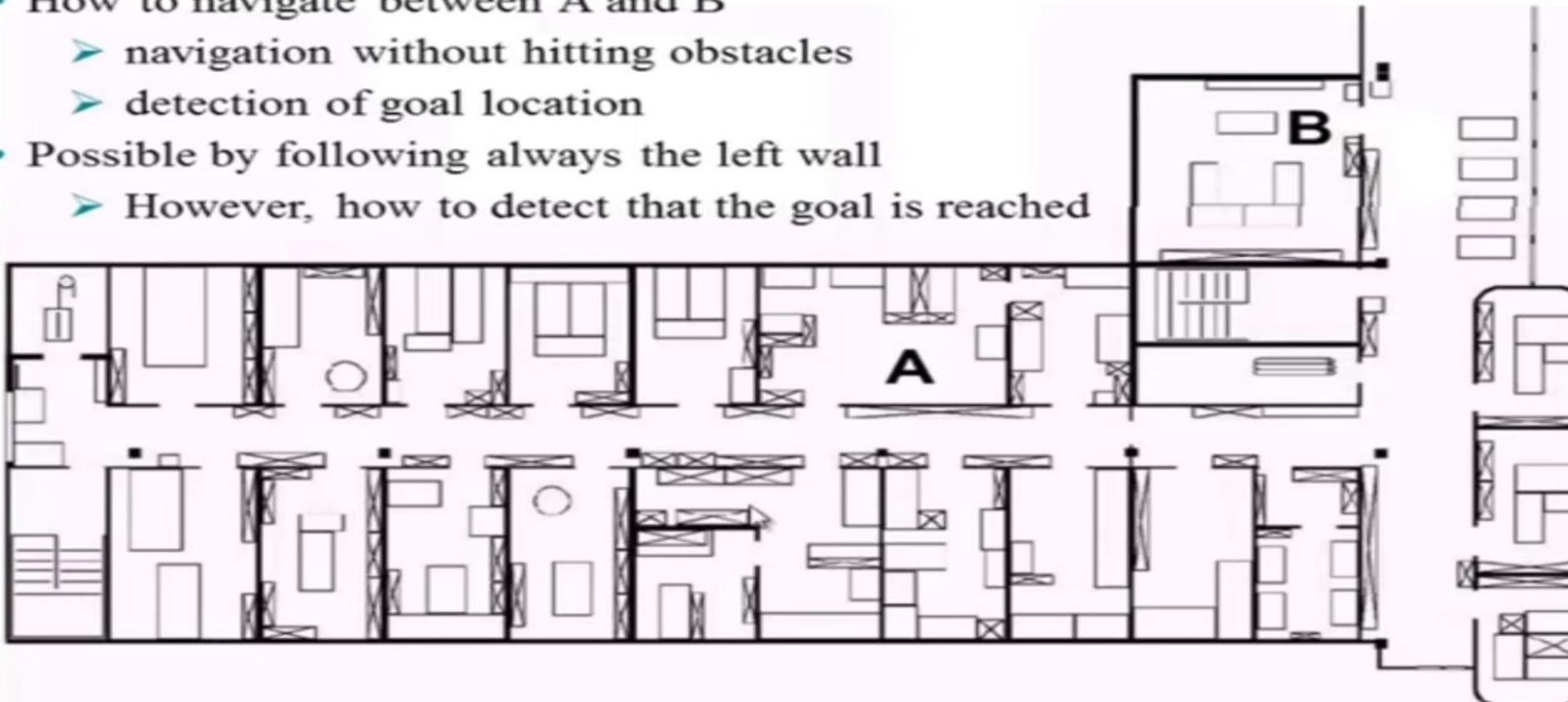
- 1) head toward goal
- 2) follow obstacles until you can head toward the goal again
- 3) continue

Control Cycle of Autonomous Robots



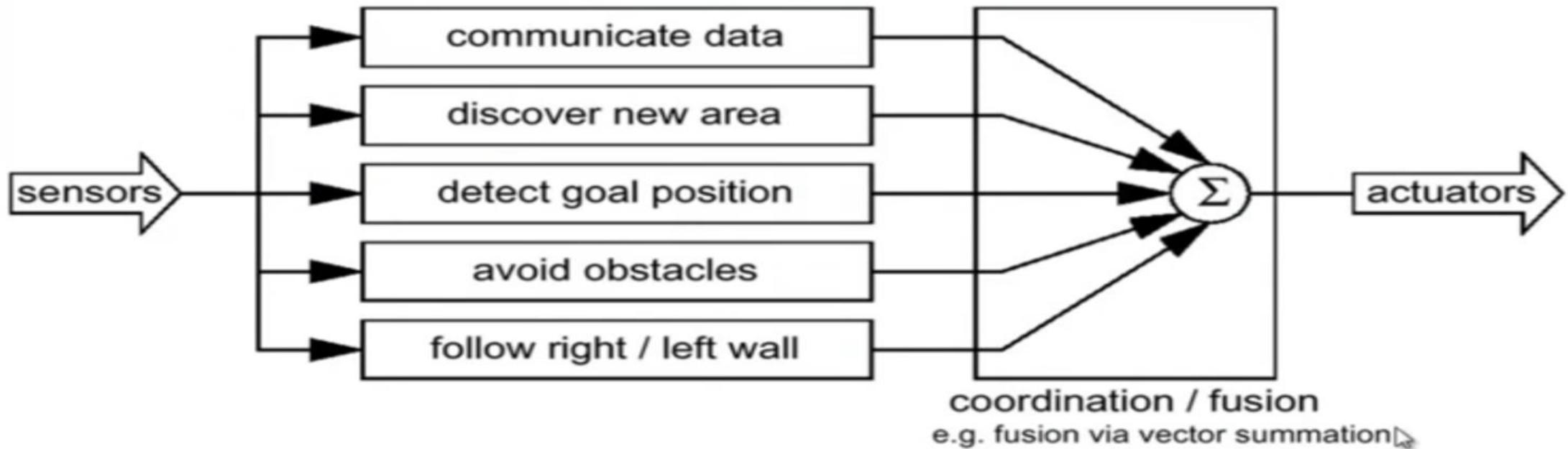
To localize or not?

- How to navigate between A and B
 - navigation without hitting obstacles
 - detection of goal location
- Possible by following always the left wall
 - However, how to detect that the goal is reached

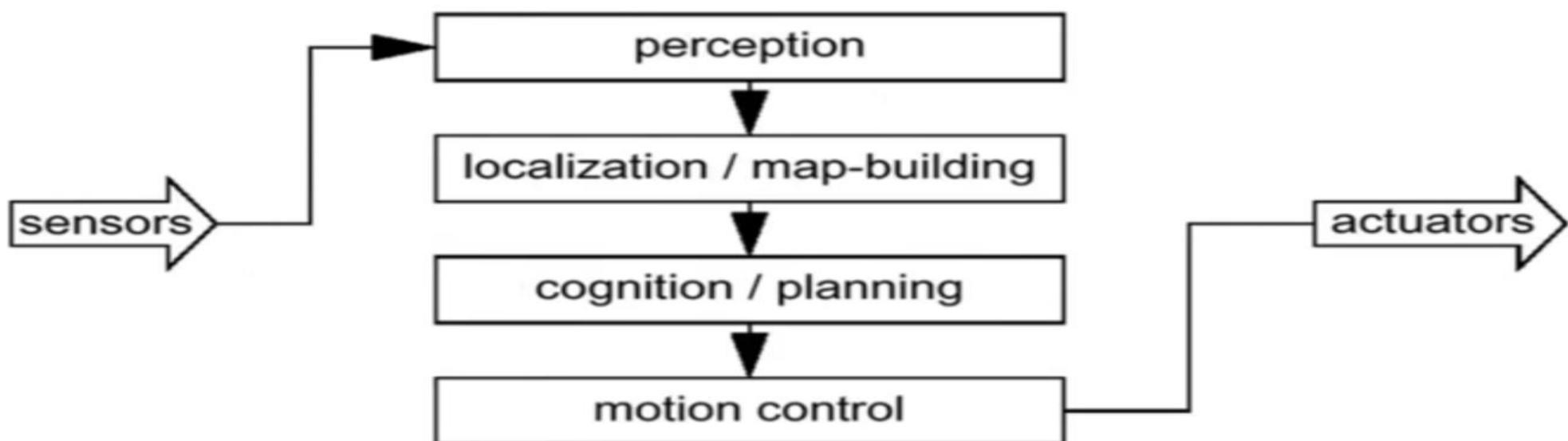




Behavior Based Navigation



Model Based Navigation



A taxonomy of probabilistic models

More general

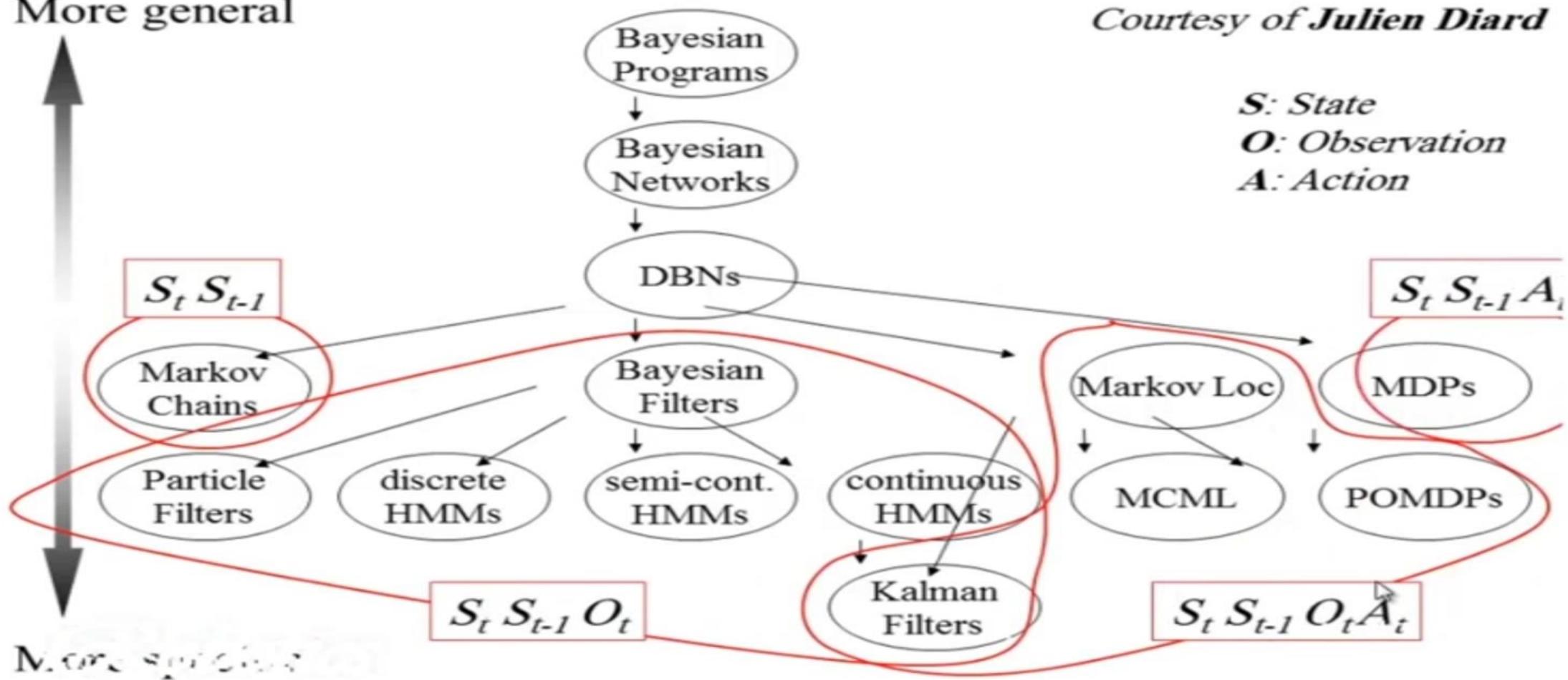


Courtesy of **Julien Diard**

S: State

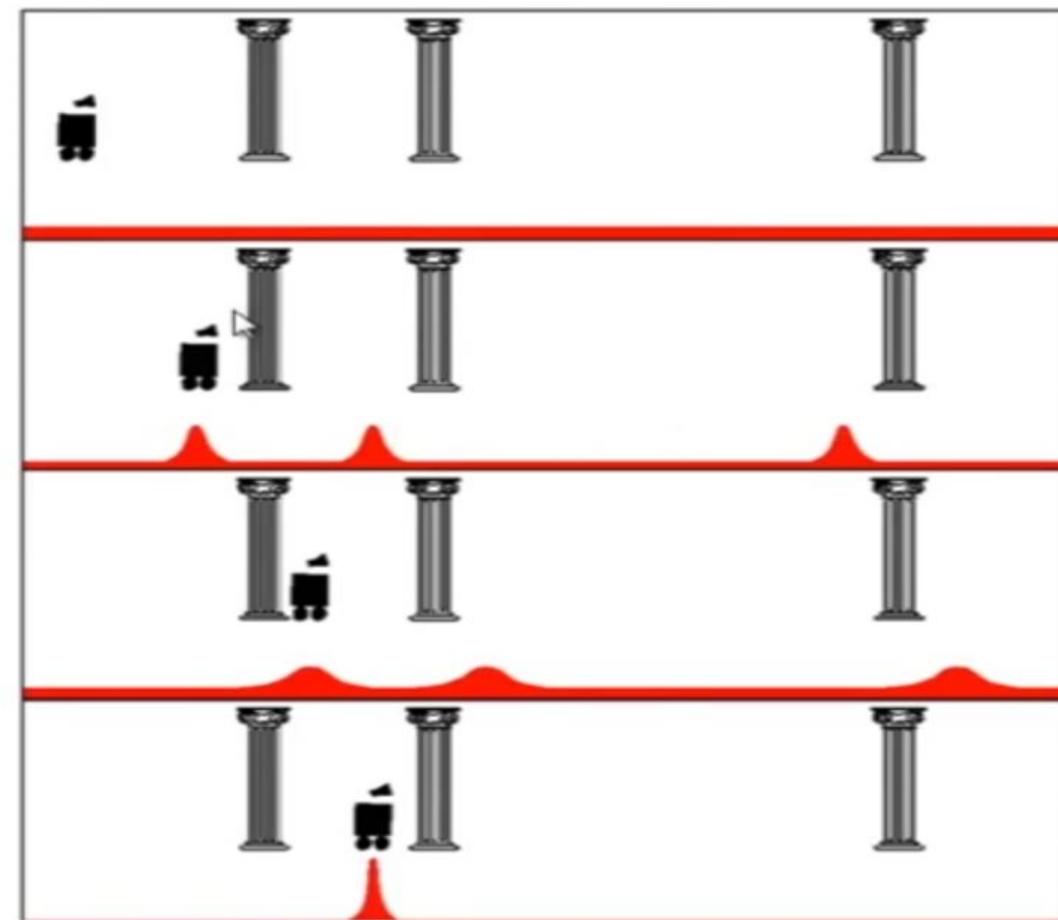
O: Observation

A: Action

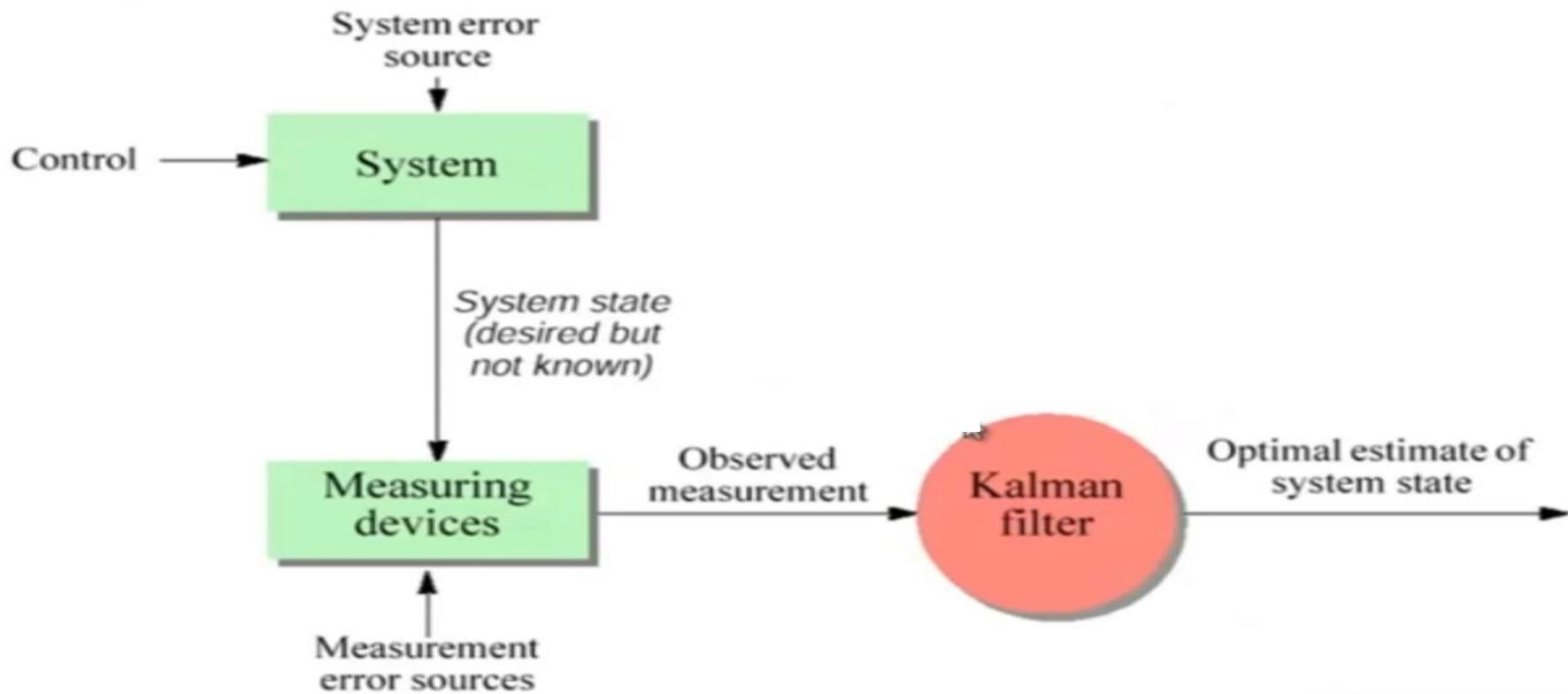


Markov Localization: Grid Map

- The 1D case
 1. Start
 - > No knowledge at start, thus we have an uniform probability distribution.
 2. Robot perceives first pillar
 - > Seeing only one pillar, the probability being at pillar 1, 2 or 3 is equal.
 3. Robot moves
 - > Action model enables to estimate the new probability distribution based on the previous one and the motion.
 4. Robot perceives second pillar
 - > Based on all prior knowledge the probability being at pillar 2 becomes dominant

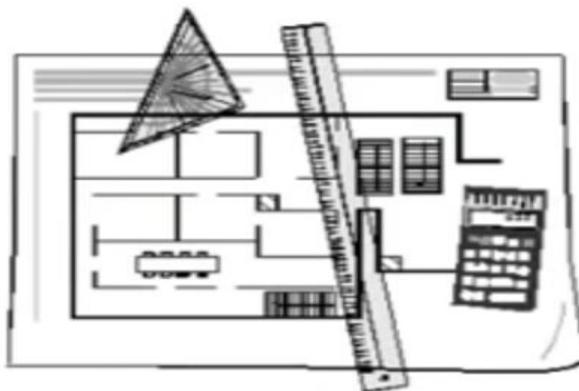


Kalman Filter Localization



How to Establish a Map

1. By Hand



2. Automatically: Map Building

The robot learns its environment

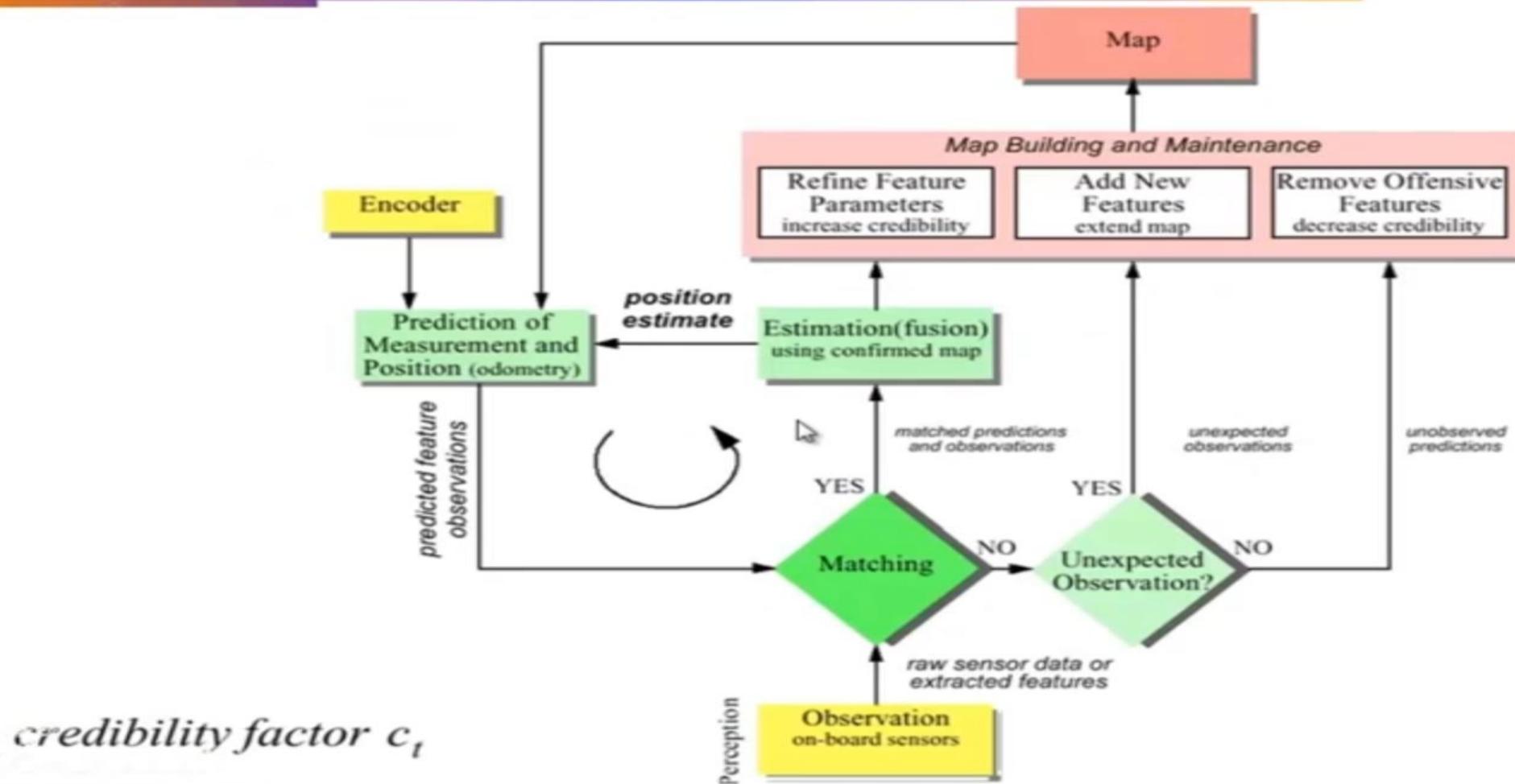
Motivation:

- by hand: hard and costly
- dynamically changing environment
- different look due to different perception

3. Basic Requirements of a Map:

- a way to incorporate newly sensed information into the existing world model
- information and procedures for estimating the robot's position
- information to do path planning and other navigation task (e.g. obstacle avoidance)
- Measure of Quality of a map
 - topological correctness
 - metrical correctness
- But: Most environments are a mixture of predictable and unpredictable features
→ hybrid approach
 - model-based vs. behaviour-based

General Map Building Schematics

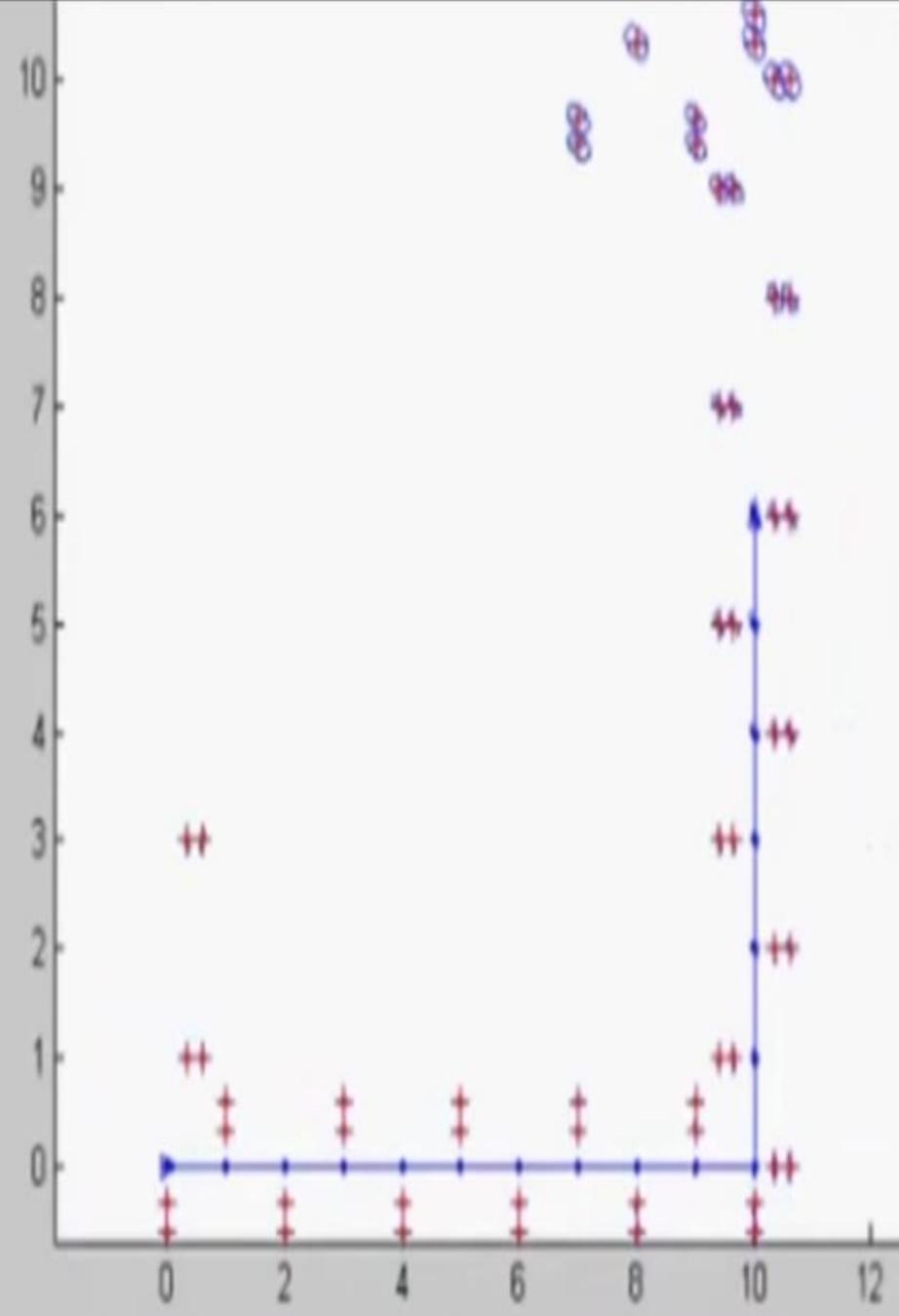
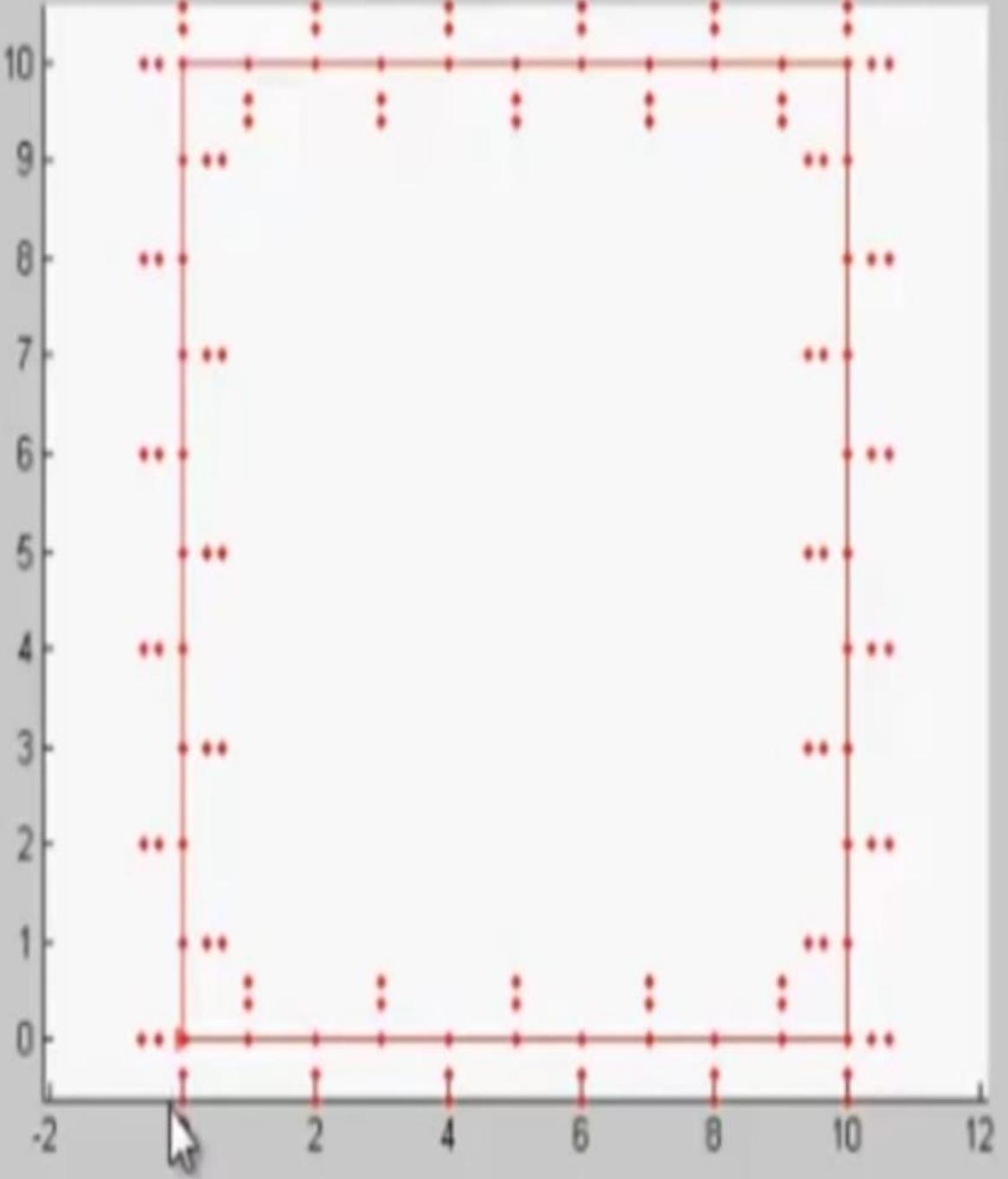


Autonomous Map Building

Starting from an arbitrary initial point,
a mobile robot should be able to autonomously explore the
environment with its on board sensors,
gain knowledge about it,
interpret the scene,
build an appropriate map
and localize itself relative to this map.

SLAM

The Simultaneous Localization and Mapping
Problem



Thank you!

