Radar Exercises: Doppler, Sampling, Pulse Compression

1. The Poker Flat ISR operates at 450 MHz. The echo from the ionosphere is produced by reflection from ion sound waves. A typical phase speed for these waves is 3 km/s. What is the Doppler frequency shift in Hz caused by reflection off these waves? This represents the approximate width of the Doppler spectrum.

2. The Nyquist theorem states that we must sample a signal at a rate of at least twice its highest frequency in order to recover it. For part 1, this so-called “Nyquist rate” is about 20 kHz, meaning we need samples of I and Q from the target at a rate of 20kHz. What is the maximum target range at which we can obtain independent samples of I and Q at this rate? How does this compare with the altitude of the ionosphere?

3. Range resolution is controlled by the length of the transmitted pulse. The optimal detection strategy involves correlating the received signal with a replica of the signal we transmitted (called “Matched Filtering”). In the script given, the two vectors of 1’s represent identical uncoded radar pulses. Running the script plots the so-called “range ambiguity function” for the pulse, which is computed by correlating the pulse with itself. The origin represents the target location, but there is also received power at ranges other than 0, hence there is “range ambiguity” associated with any single detection. Try the following:

   a) First let’s try a shorter pulse. Replace pulse2 with the following
   pulse2 = [0,0,0,0,1,1,1,1,1,0,0,0]
   This represents a pulse that is 40% shorter. Rerun the script. What effects do you see compared to the original pulse?

   b) Now replace pulse2 with the folloing coded version of the pulse,
   pulse2 = [1,1,1,1,1,-1,-1,1,1,-1,1,-1,1]
   Each element represents a “bit” or “baud” whose signe we can control. This code is called a “13-baud Barker code.” The sign changes can be implemented in hardware by flipping the phase of the transmitted signal 180 degrees for these bauds. Rerun the script now. What have we achieved with this coding?

   c) The range ambiguity is generally defined as the “full-width at half-maximum” (FWHM) of the main peak of the matched filter output. Compare the range ambiguity of the uncoded and coded pulses based on this definition. The ratio of these quantities is referred to as the “pulse compression ratio”. What costs have we paid for the improved in range resolution from pulse coding?

Concept of a "Doppler Spectrum"

Superposition of targets moving with different velocities within the radar volume

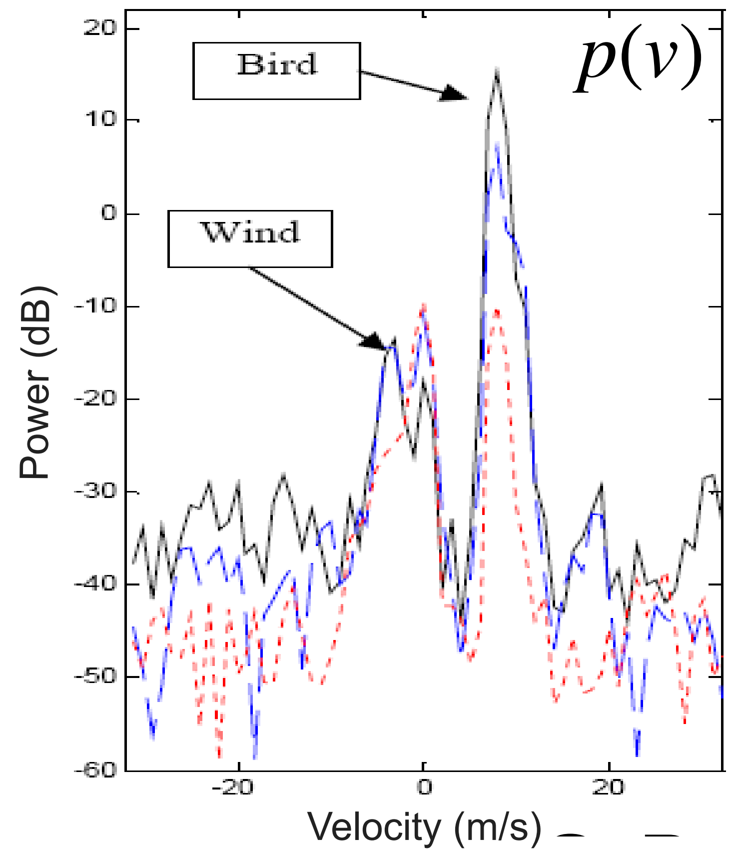
Two key concepts:

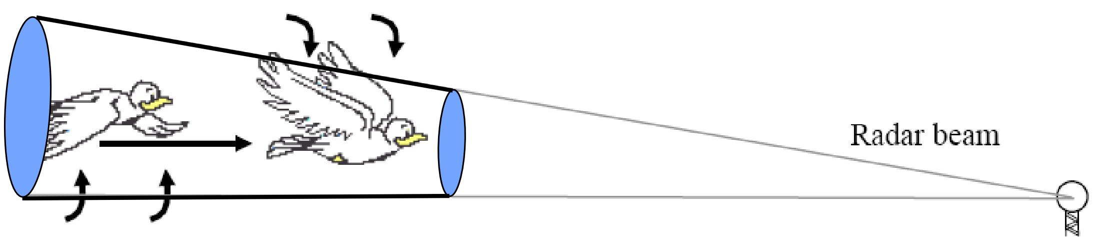
Time ⟺ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⟺ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$





Processing: $p(R, f_D) \rightarrow p(R, v)$

If there is a distribution of targets withdifferent velocities (e.g., bird, flapping wings,wind) then there is no single Doppler shift but, rather, a Doppler spectrum.

Distributed "beam filling" Target

A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

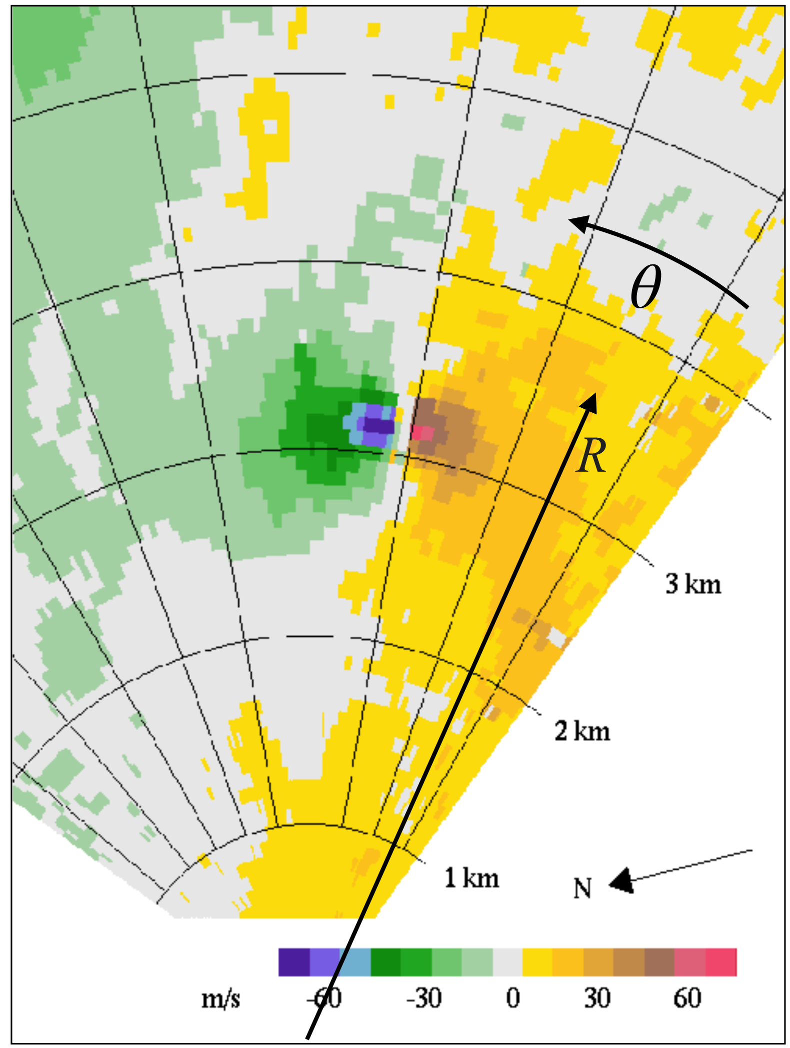
Two key concepts:

Time $\Leftrightarrow$ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency $\Leftrightarrow$ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$



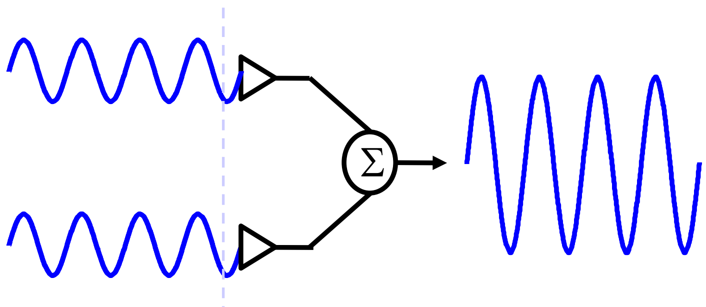
Processing:

$$p(R, f_D, t) \Rightarrow f_D(R, t) \Rightarrow v(R, \theta)$$

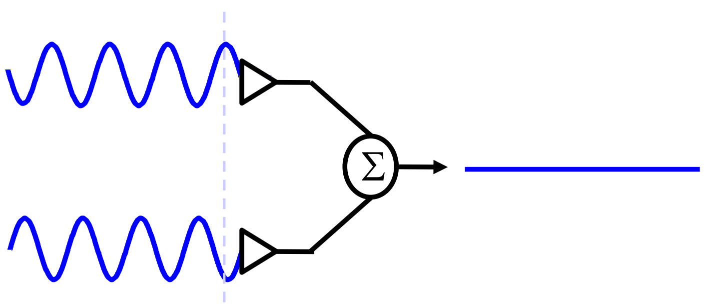
For a beam-filling target (like water droplets in a tornado), the radar can be used to construct insightful images of velocity relative to the radar.

Wave Interference and Bragg Scatter

Consider two waves with the same frequency but different phase.

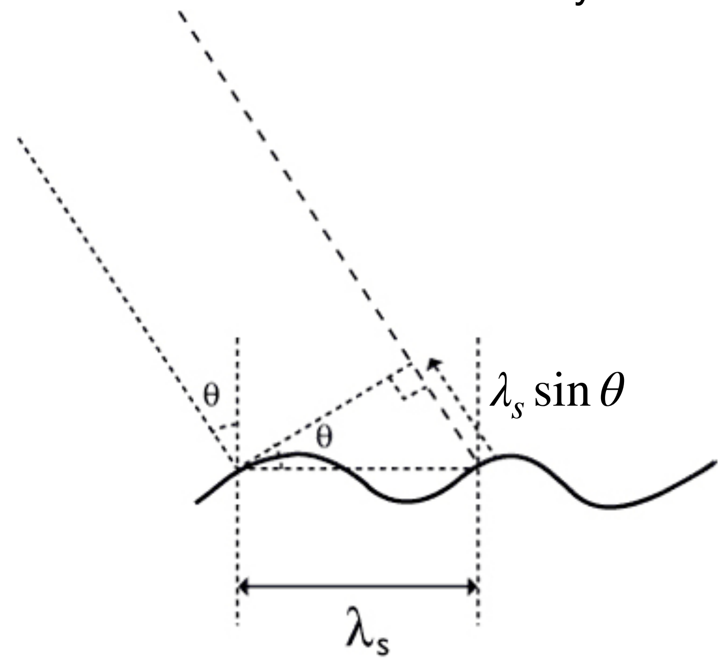


Constructive (in phase)



Destructive (180° out of phase)

Consider a wave along the interface between a dielectric and a conducting (reflective) medium, as depicted below. This is representative of an air-ocean boundary.

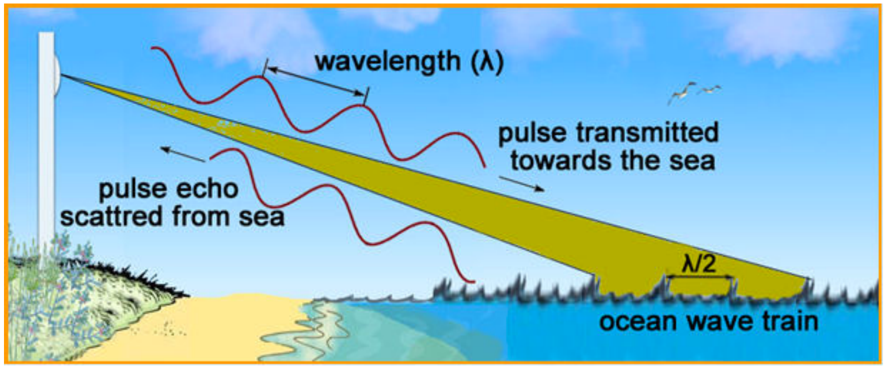


Suppose waves are observed at angle θ using a radar with wavelength λ_o. The condition for maximum constructive interference is

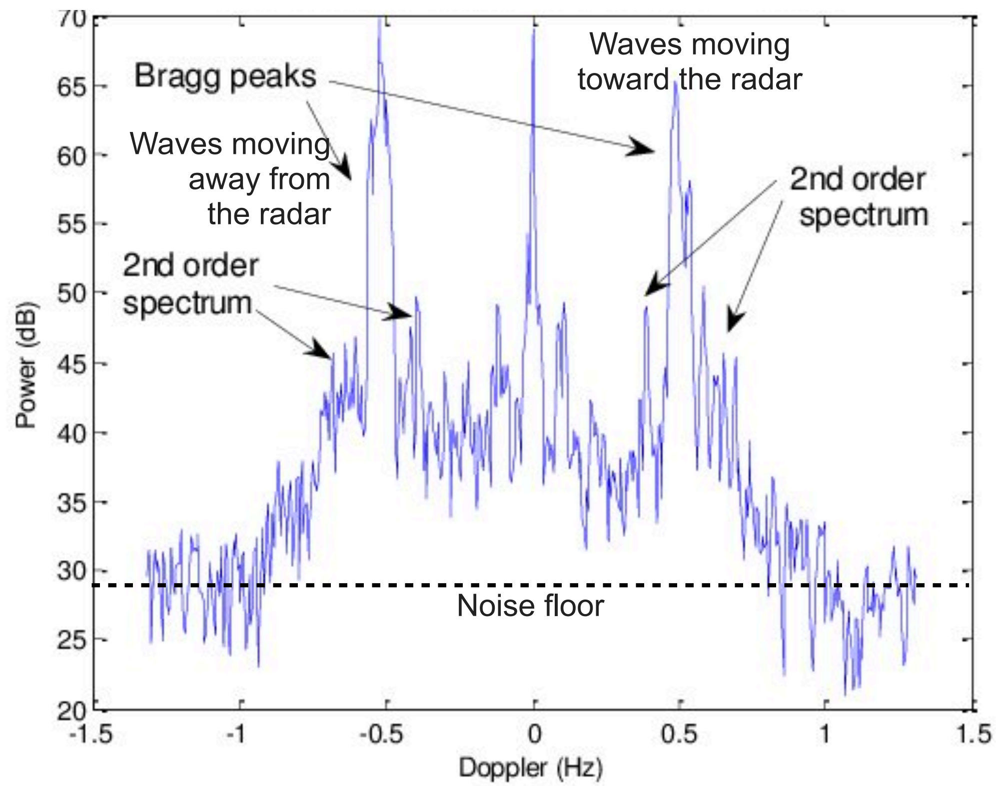
$$n\lambda_0 = 2\lambda_s \sin\theta$$

If $\theta = 90^\circ$ (or if these waves are propagating isotropically), then the Bragg condition is met for $n\lambda_0 = 2\lambda_s$

Doppler spectrum of ocean waves



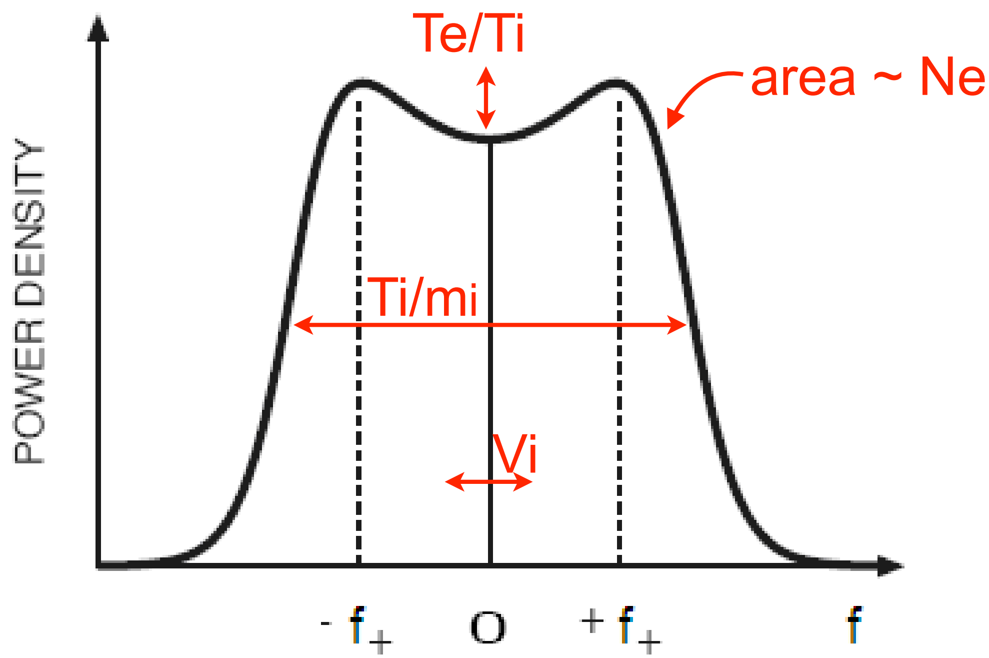
Backscatter from the ocean at low aspect angle shows peaks in the Doppler spectrum from the subset of waves matching the Bragg condition for the radar (spacing $\simeq$ half the radar wavelength)



Important points:

The target is distributed over the entire radar beam width.
The scattering is from free electrons in the conducting sea water.
The Doppler spectrum has peaks due to Bragg scatter from waves in the medium.
The frequency of the peaks tells us the velocity and direction of the waves.
The height of the peaks tells us something about the amplitude and density of the waves.
The width of the peaks tells us something about the spread in velocity of the waves

Standard parameters found by fitting the



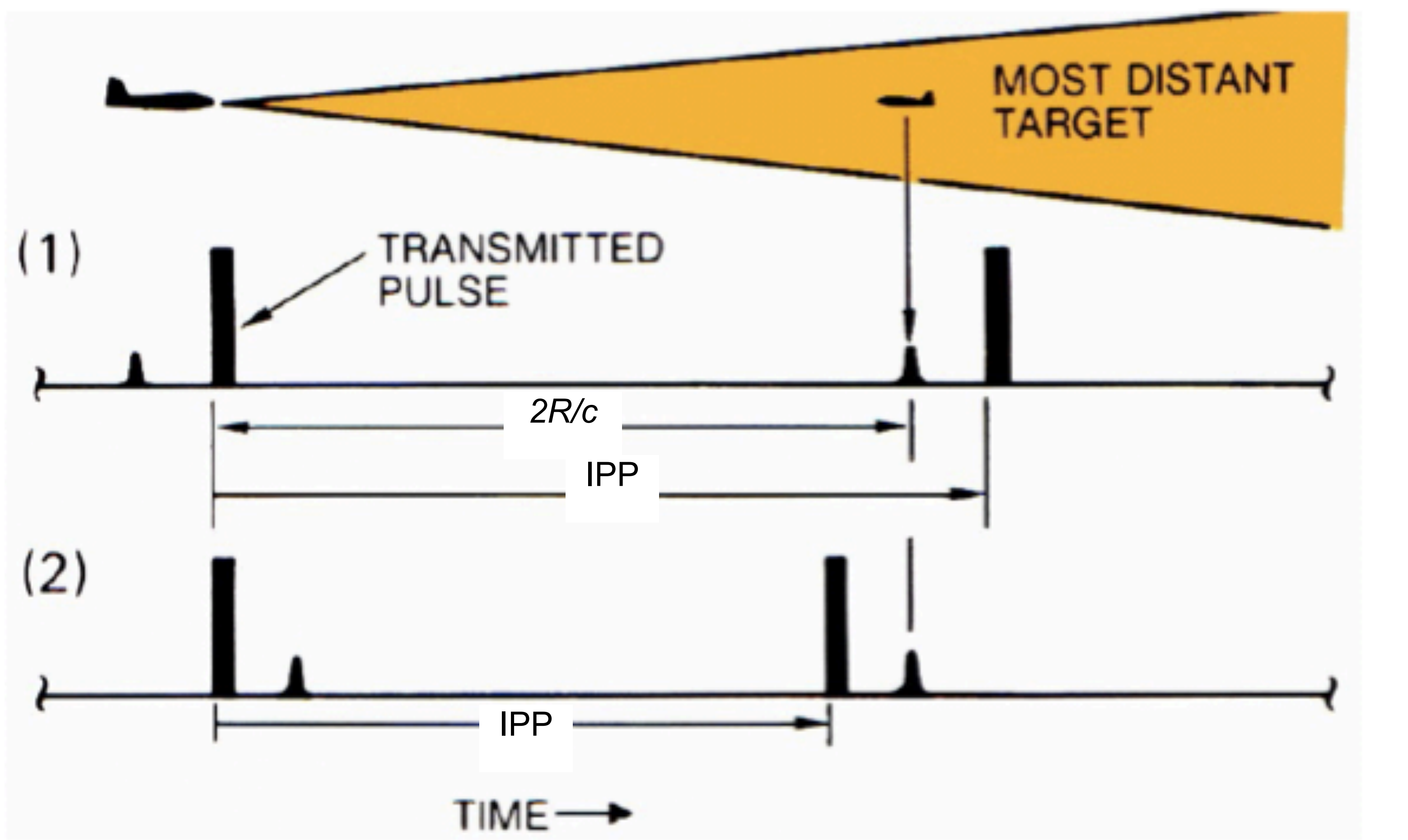
Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra

Electron to ion temperature ratio (Te/Ti) from “peak_to_valley” ratio

Electron (= ion) density from total area (corrected for temperatures)

Line-of-sight ion velocity (Vi) from the Doppler shift

Maximum Unambiguous Range

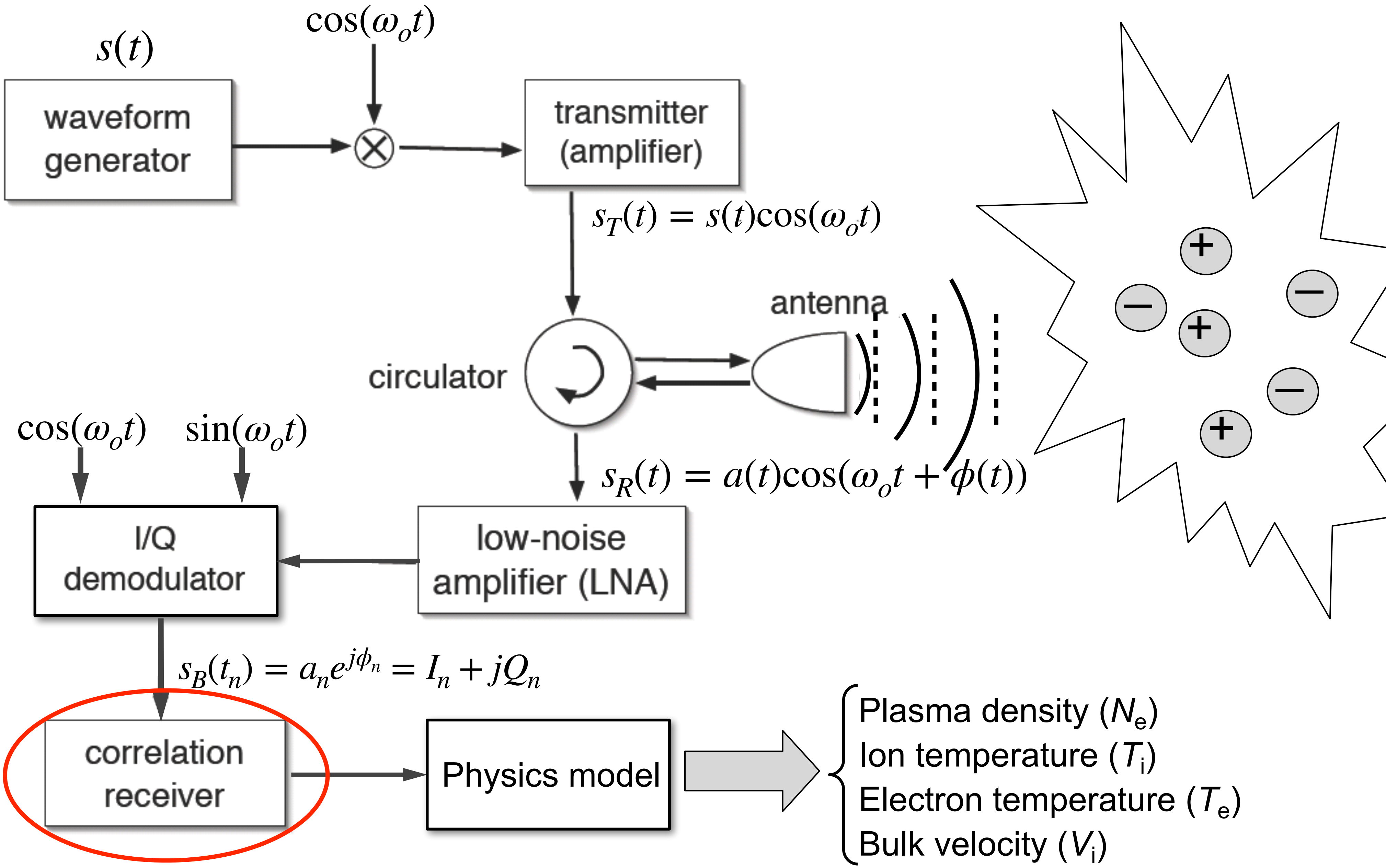


IPP = Inter-pulse Period

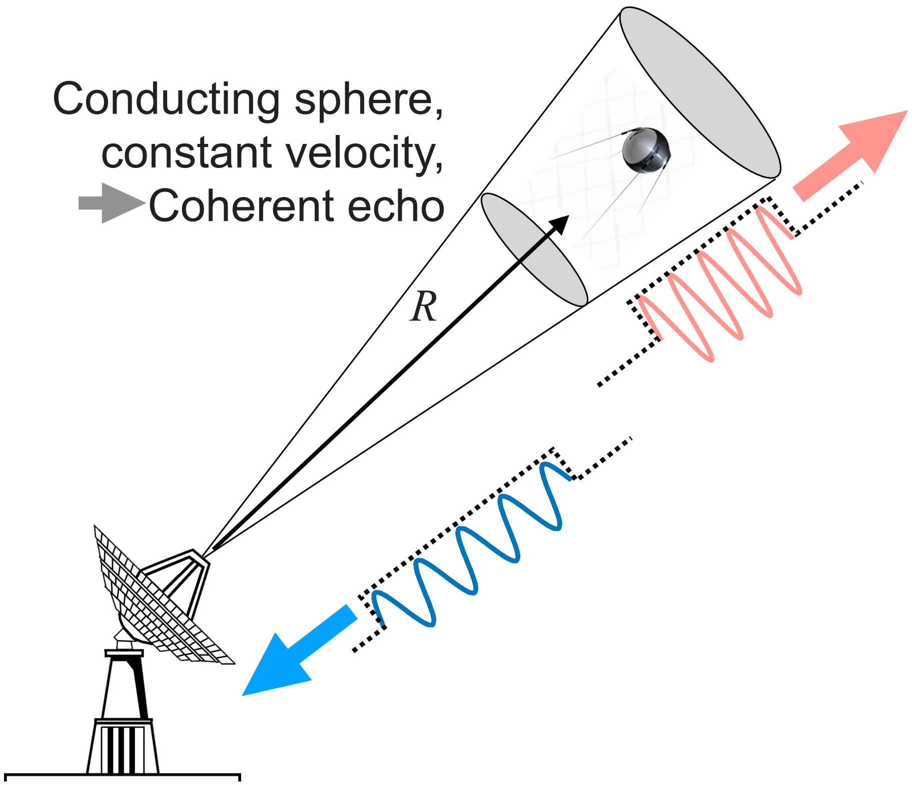
Combining the above relations, the maximum range we can reach for a given IPP is:

Max range = (IPP x c)/2

Components of a Pulsed Doppler Radar



Measuring Velocity



Assume a transmitted signal: $s(t)\cos(2\pi f_o t)$

After return from target: $a(t)\cos\left[2\pi f_o\left(t - \frac{2R(t)}{c}\right)\right]$

Let's assume target moves with constant velocity with respect to the radar during the measurement,

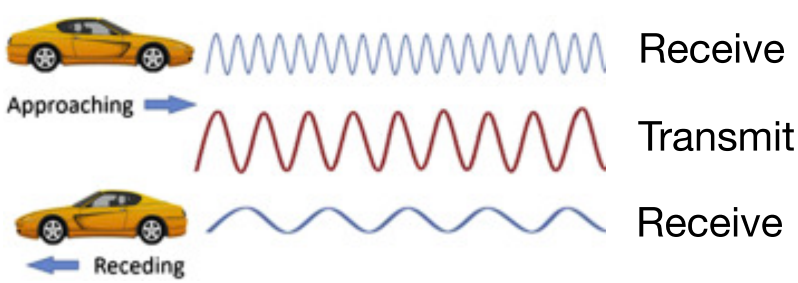
$$R = R_o + v_o t$$

Substituting we obtain:

$$a(t)\cos\left[\underbrace{2\pi f_o t}_{\omega_o t} \underbrace{- 2\pi f_D t - \frac{4\pi f_o R_o}{c}}_{\phi(t)}\right] \qquad f_D = -\frac{2f_o}{c}v_o$$

$$a(t)\cos\left[\omega_o t + \phi(t)\right] \qquad \boxed{\omega_D = 2\pi f_D = \frac{d\phi}{dt}}$$

$f_o \sim 500$ MHz, $\quad f_D \sim 50$ kHz $= 0.0001 f_o$



Two issues:

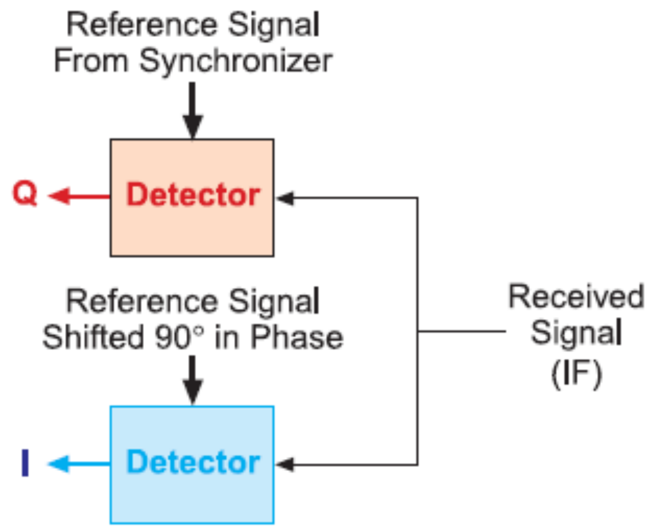
1) How do we discriminate positive from negative f_D?

2) How do we remove f_o , and just sample $a(t)\cos[\phi(t)]$?

I and Q Demodulation

Consider radar transmission of a simple RF pulse. The reflected signal from the target will be the original pulse with some time varying amplitude and phase applied to it:

$$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$$

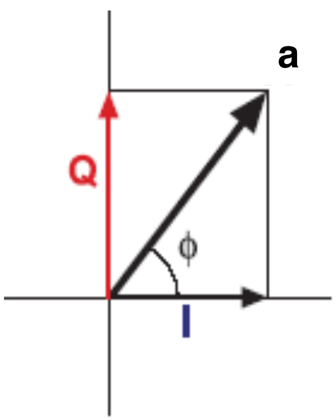


Multiplying by $\cos(\omega_0 t)$ gives the **"in-phase" (I) signal:**

$$s_R(t)\cos(\omega_o t) = a(t)\cos(\omega_o t + \phi(t))[\cos(\omega_o t)]$$
$$= a(t)\frac{1}{2}\left(\underbrace{\cos[2\omega_o t + \phi(t)]}_{\text{filter out}} + \cos[\phi(t)]\right)$$

Multiplying by $-\sin(\omega_0 t)$ gives the **"quadrature" (Q) signal:**

$$s_R(t)[-\sin(\omega_o t)] = a(t)\cos(\omega_o t + \phi(t))[-\sin(\omega_o t)]$$
$$= a(t)\frac{1}{2}\left(\underbrace{-\sin[2\omega_o t + \phi(t)]}_{\text{filter out}} + \sin[\phi(t)]\right)$$



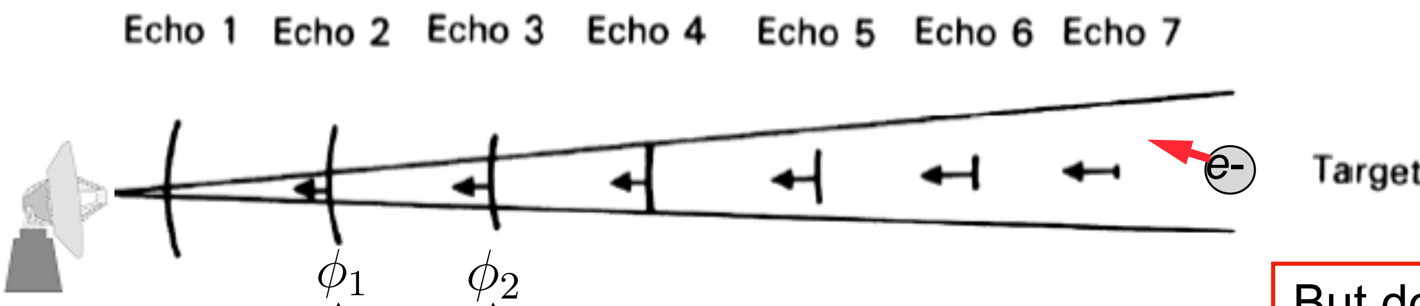
If we include a gain of 2, we retain the original signal energy. Using Euler's identity we construct the analytic baseband signal:

$$\boxed{s_B(t) = a(t)e^{j\phi(t)} = a(t)\cos\phi(t) + ja(t)\sin\phi(t) = I + jQ}$$

I/Q demodulation produces a time-series of complex voltage samples (I_n, Q_n) from which we can construct a discrete representation of $a(t) = \sqrt{I^2 + Q^2}$ and $\phi(t) = \tan^{-1}(Q/I)$.

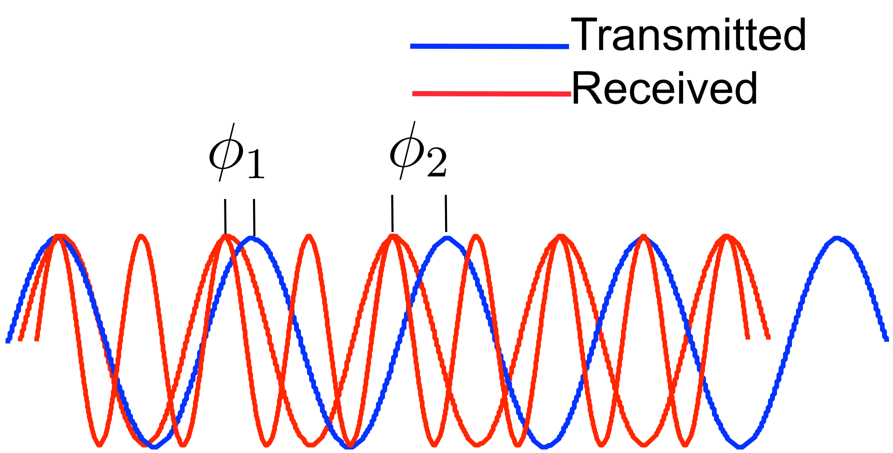
Doppler Detection: Intuition

Closing on target – positive Doppler shift



Inter-pulse period (IPP)

But do we expect an electron to maintain a constant velocity between pulses?

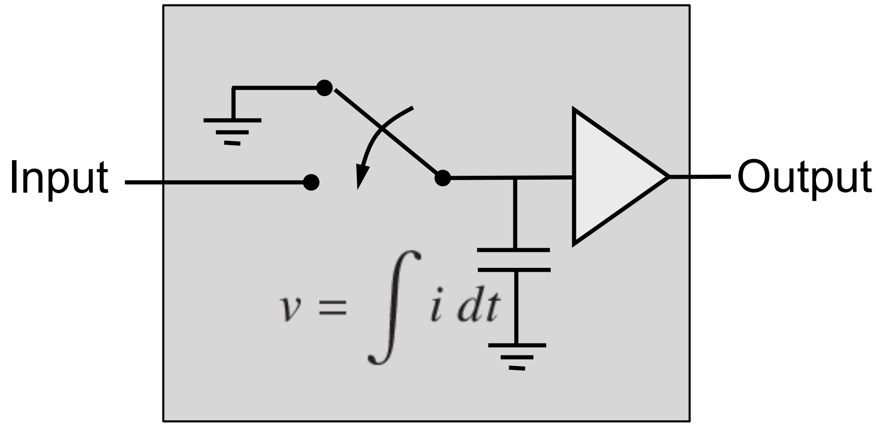


Target's Doppler frequency shows up as a pulse-to-pulse shift in phase.

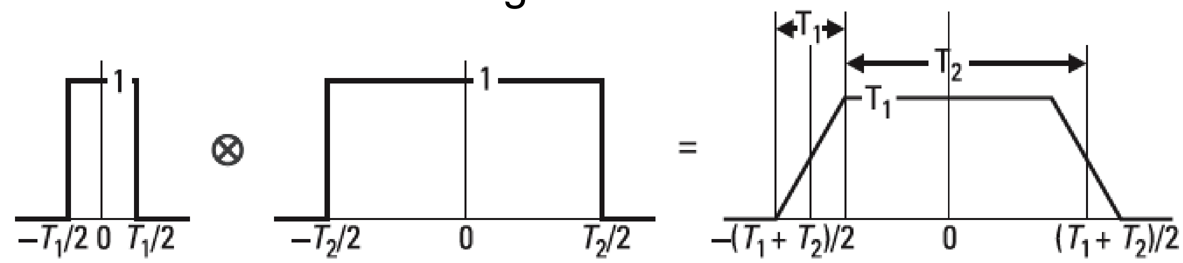
What is the maximum Doppler shift that can be unambiguously measured?

Sampling a signal require time-integration

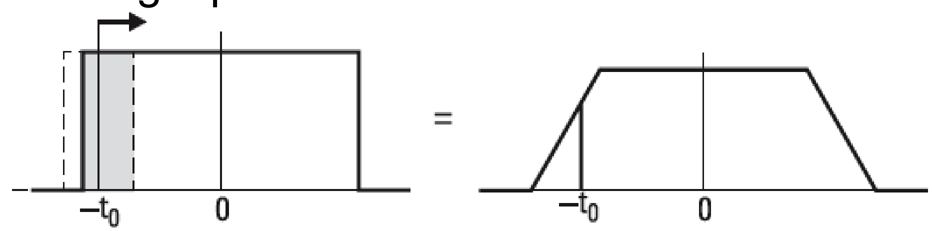
We send a pulse of duration τ. How should we listen for the echo?



Convolution of two rectangle functions

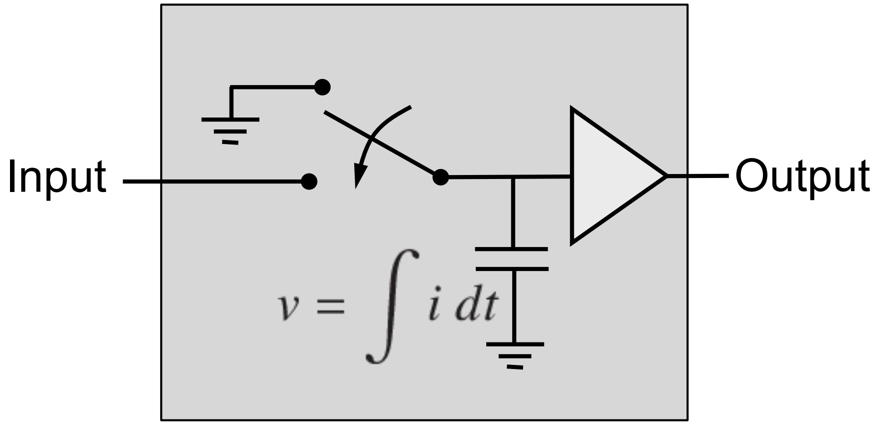


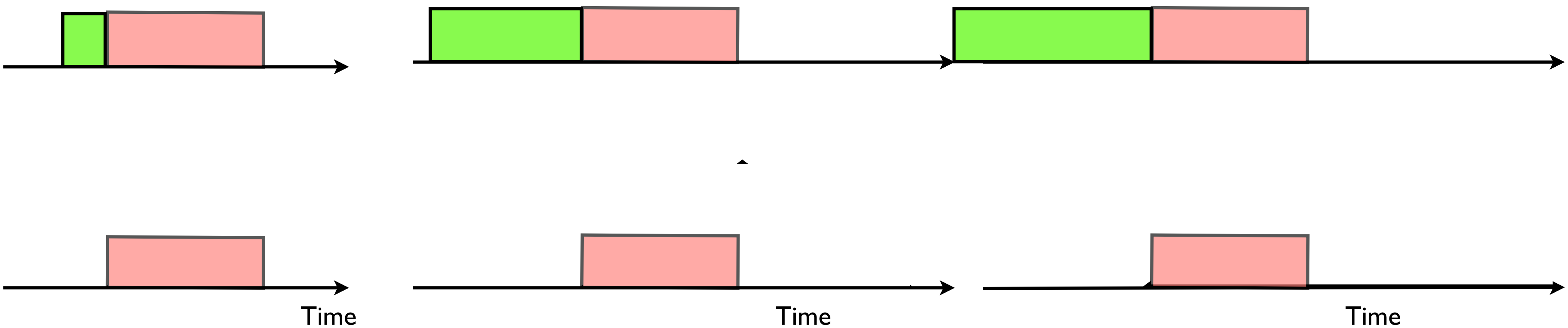
Value at a single point



- To determine range to our target, we only need to find the rising edge of the pulse we sent. So make $T_1 << T_2$.
- But that means large receiver bandwidth, lots of noise power, poor SNR.
- Could make $T_1 >> T_2$, then we're integrating noise in time domain.
- So how long should we close the switch?

Sampling the received signal





Exercises

https://hub.gke2.mybinder.org/user/isrsummerschool-upplements_2021-zrsxnm1q/tree/radar_intro