Bonificación 1

1) $G(s) = \dfrac{4}{s^3 + 2s^2 + s + 3}$

$$\frac{Y(s)}{U(s)} = \frac{4}{s^3 + 2s^2 + s + 3} \rightarrow Y(s)(s^3 + 2s^2 + s + 3) = 4\,U(s)$$

$$s^3 Y(s) + 2s^2 Y(s) + s\,Y(s) + 3Y(s) = 4U(s)$$

Aplicamos transformada inversa:

$$\dddot{y} + 2\ddot{y} + \dot{y} + 3y = 4u$$

$q_1 = x$

$q_2 = \dot{y} = \dot{q}_1$

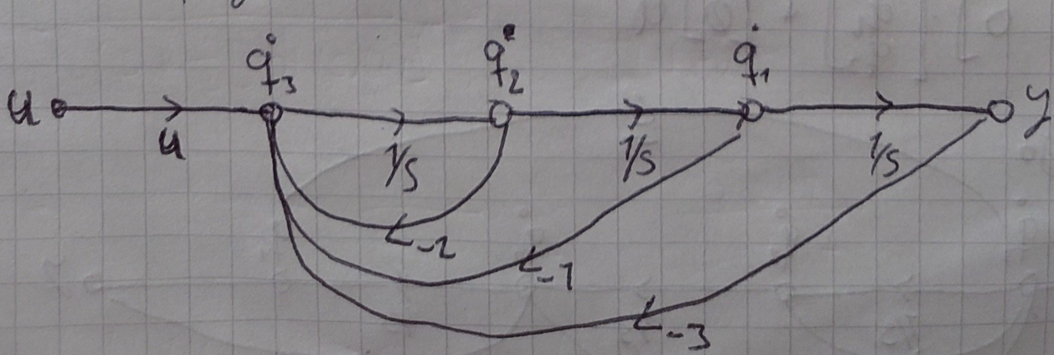
$q_3 = \ddot{y} = \dot{q}_2$

$\Rightarrow \dot{q}_3 = -2q_3 - q_2 - 3q_1 + 4u$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -1 & -2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} [u]$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + [0]\,u$$

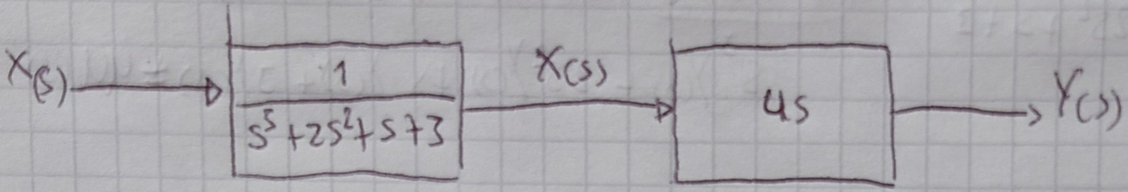
Diagrama de flujo



2) $G(s) = \dfrac{4s}{s^3 + 2s^2 + s + 3} = \dfrac{Y(s)}{X(s)}$

$\rightarrow Y(s)\left(s^3 + 2s^2 + s + 3\right) = 4s\,U(s)$

$\rightarrow Y(s)s^3 + 2Y(s)s^2 + Y(s)s + 3Y(s) = 4U(s)$



$\dfrac{X(s)}{U(s)} = \dfrac{1}{s^3 + 2s^2 + s + 3} \Rightarrow X(s)\left(s^3 + 2s^2 + s + 3\right) = U(s)$

$\Rightarrow \dddot{X}(s) + 2\ddot{x} + \dot{x} + 3x = u$

$q_1 = x$

$q_2 = \dot{x} = \dot{q}_1$

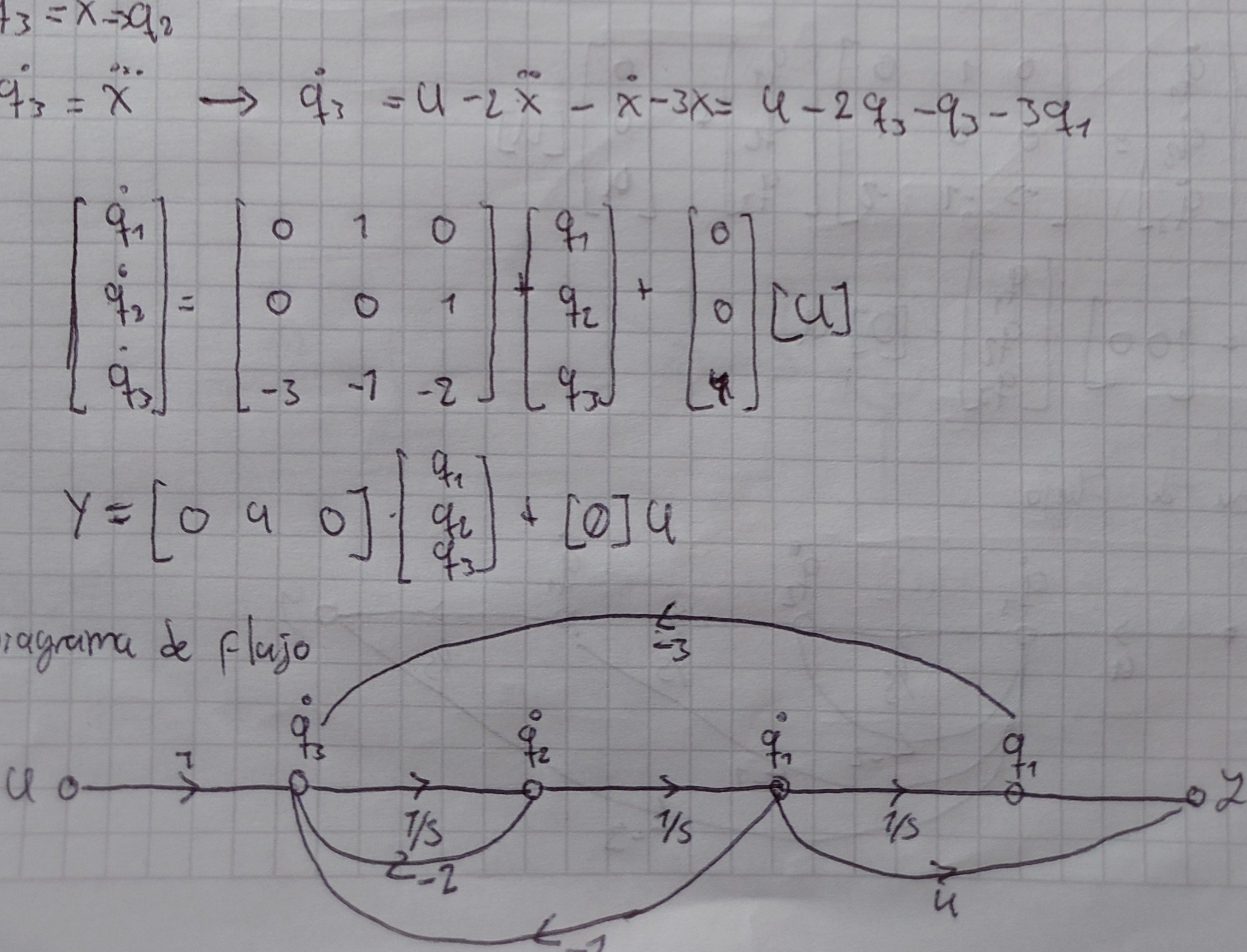
$q_3 = \ddot{x} = \dot{q}_2$

$\dot{q}_3 = \dddot{x} \longrightarrow \dot{q}_3 = u - 2\ddot{x} - \dot{x} - 3x = u - 2q_3 - q_3 - 3q_1$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -1 & -2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} [u]$$

$$Y = \begin{bmatrix} 0 & 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + [0]\,u$$

Diagrama de flujo



3)

$$G(s) = \frac{6s^2 + 4s + 2}{s^4 - s^3 + 2s + 3}$$

$$\Rightarrow U(s) \longrightarrow \boxed{\frac{1}{s^4 - s^3 + 2s + 3}} \xrightarrow{X(s)} \boxed{6s^2 + 4s + 2} \longrightarrow Y(s)$$

$$\Rightarrow \frac{X(s)}{U(s)} = \frac{1}{s^4 - s^3 + 2s + 3} \Rightarrow X(s)(s^4 - s^3 + 2s + 3) = U(s)$$

$$\Rightarrow \ddot{\ddot{x}} - \dddot{x} + 2\dot{x} + 3x = u$$

$q_1 = x$

$q_2 = \dot{q}_1 = \dot{x}$

$q_3 = \dot{q}_2 = \ddot{x}$

$q_4 = \dot{q}_3 = \dddot{x} \quad \Rightarrow \quad \dot{q}_4 = q_4 - 2q_2 + 3q_1 + u$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ +3 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [u]$$

$Y(s) = X(s) \cdot (6s^2 + 4s + 2)$

$Y(s) = X(s) \cdot 6s^2 + 4X(s)\, s + 2X(s) = 6\ddot{x} + 4\dot{x} + 2x \Rightarrow 6q_3 + 4q_2 + 2q_1$

$$y = [2 \; 4 \; 6 \; 0] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Diagrama de flujo.