Investigating Exoplanetary Properties with Simulated Transit Light Curves

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This study used a simulated time-series dataset to analyze the normalized relative brightness of a star as an exoplanet passes in front of it. The light curve revealed a periodic 'dip' in luminosity, corresponding to the times when the planet is directly obscuring the light from the star. Assuming prior knowledge of stellar properties and exoplanet mass, the analysis of the data included the determination of the period of the exoplanet, the depth of the transit, the radius of the exoplanet, the duration of the transit, the semi-major axis of the exoplanet and the star, the velocity of the exoplanet and the star, the inclination, the density of the exoplanet, and the equilibrium temperature of the exoplanet. To improve the measurements, the data was sliced and folded. The results showed that the period of the exoplanet is 27.51 ± 0.01 Days, the exoplanet covers 0.152 ± 0.002 % of the relative brightness from the star, the radius of the exoplanet is $R_B = 2.3 \pm 0.1\ R_\oplus$, and the composition is similar to that of Mars. These values were put into context by comparing to NASA's exoplanet archive. While the data used is simulated, this work serves as an informative guideline for analyzing simple observational transits.

I. Introduction

The detection of planets beyond our solar system has redefined our perception of our place in the universe, updated our models of planetary formation, and initiated a search for potentially habitable worlds. Despite the significant progress in investigating these exoplanets, the observation of an exoplanet typically results in very limited information on their physical properties, such as mass, radius, and atmospheric composition. The four main methods by which astronomers detect exoplanets are direct detection, astrometric method, Doppler method, and transit method [1]. However, it is very difficult to directly detect exoplanets as resolution limits and relative luminosities (compared to their host star) hinder our observational power. For this reason, astronomers often rely on the study of exoplanetary transits to characterize an exoplanet's size, velocity, orbital period, inclination, and semi-major axis. The transit method can infer these properties by analyzing the light curve produced from an exoplanet passing in front of its host star, 'eclipsing' part of the light relative to the Earth's point of view.

The transit method has proved to be the most reliable method of detecting exoplanets, constituting 4018 detections out of 5332 total confirmed exoplanets (as of April 2023) [2]. However, the transit method does not provide any information on the motion of the star; without this information, it is not possible to estimate the mass of the planet. Nevertheless, by combining observations from different techniques and analyzing their data, astronomers can extract valuable insights into the nature of exoplanets and their host systems, such as its atmosphere's composition and even the possibility of hosting life.

II. Methods

The data used in this analysis is a simulated time-series dataset of the normalized relative brightness from a star measured as an exoplanet passes in front of it. At first glance, a plot of this light curve reveals a periodic 'dip' in luminosity corresponding to the times when the planet is directly obscuring the light from the star.

A. Period

By measuring the time between these dips, the period of the exoplanet can be determined since this is the time required for it to complete one orbit around its host star. To do so, the time-series data was manually split into chunks centered around each dip, and the time corresponding to the minimum in brightness of each dip was recorded. The average time between these minimums corresponds to the period, and the discrepancies in the times between the minimums is used to calculate an error, resulting in a calculation of *27.51 ± 0.01 Days.*

Since the time-series data contains noise, measurements can be improved by folding the transits over and using the data from all the 'dips' rather than just from one. This was done by overplotting the slices centered around their minima, and then manually rolling the data until the transits were aligned.

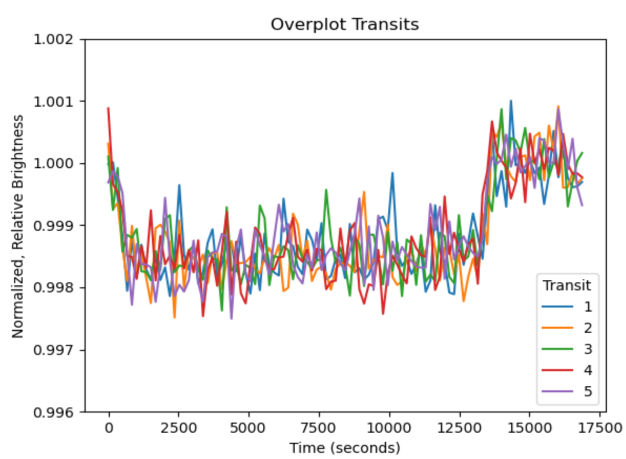

FIG 1. Overplot Transits – Five transits were plotted over the same time to find the depth more accurately – a key variable for finding planetary properties, such as size. The transit lasts *3.8 Hours ± 2.8 Minutes* with a depth of *0.152 ± 0.002 %* of the normalized, relative brightness.

B. Depth

In order to calculate the depth of the transit, the data was sliced to include only its 'full' depth (i.e., when the entire exoplanet is in front of the star as opposed to partially blocking light). To find the 'baseline' relative brightness, the average was taken from the times without any transit, resulting in a value of 1, agreeing with the fact that our data is normalized. Thus, for the depth calculation, the mean of the 'full' depth was subtracted from 1, with an error given by the standard error of the mean using scipy.stats.sem [3]. The exoplanet covers *0.152 ± 0.002 %* of the relative brightness from the star.

C. Radius

Using the given values of $M_{Star} = 0.568\ M_{\odot}$, $R_{Star} = 0.545\ R_{\odot}$, and $M_{Planet} = 9.243\ M_{\oplus}$ (all with corresponding 5% error), along with the relation between flux and the relative sizes of the exoplanet and host star from [4],

$$\frac{R_B}{R_A} = \left(\frac{\delta F}{F}\right)^{1/2}$$

the radius of the exoplanet is solved to be $R_B = 2.3 \pm 0.1\ R_{\odot}$, where the error is given by the rules of error propagation in section (c) from [5].

D. Transit Time

The duration of the transit can be calculated using the plot of the folded transits by visually finding the bin at which the relative brightness returns to its normal value. The transit thus occurs between bins 0 and 81 of the plot, which corresponds to a time of *3.8 Hours ± 2.8 Minutes.* The error was calculated by assuming that our calculation is accurate to one bin (as is done in measurements with physical instruments).

E. Semi Major Axis – Exoplanet

The semi-major axis of the exoplanet can be found using Keplar's third law, $p^2 = a^3$. Using our value for the period and solving for a, along with adding the fractional uncertainties in quadrature, a_B was calculated to be *0.0178 ± 0.0003 AU.*

F. Semi Major Axis – Star

While the calculation of a_B used keplar's third law, it is not possible to extend this method to a_A as there is no inherent information on the period of the star's orbit. Thus, the calculation will have to rely on extrapolating from knowledge of the center of mass of the system. Using *(12.12)* from [4],

$$\frac{v_A}{v_B} = \frac{a_A}{a_B} = \frac{M_B}{M_A}$$

a_A is calculated to *1e-05 ± 0.071 AU.* It is worth noting the high relative uncertainty in this measurement, which originates almost entirely from the 5% errors on the masses of the star and exoplanet.

G. Velocity – Exoplanet

In the case where the exoplanet is much less massive than the star, a_B is given by Keplar's laws and its speed v_B is thus given by *(12.25)* from [4] (assuming a circular orbit):

$$v_B \approx \frac{2\pi a_B}{P} \approx 30 \text{ km s}^{-1} \left(\frac{M_A}{1M_\odot}\right)^{1/3} \left(\frac{P}{1 \text{ yr}}\right)^{-1/3}$$

Thus, the velocity of the exoplanet, v_B is found to be *58.83 ± 0.02 km/s*, where the error is found with the rules of [5].

H. Velocity – Star

Much like the semi-major axis of the star, v_A cannot be calculated with the methods described for the exoplanet. However, using the same *(12.12)* from above, v_A can be found with the ratio of masses to be 0.003 ± *0.07 km/s*. It is interesting to note that this value can also be found using the ratio of the semi-major axis; however, the error will be unacceptably high as a_A is not well constrained.

I. Inclination

For exoplanets detected via the transit method, the angle that the earth-exoplanet-star system makes needs to be extremely small, given by *(12.31)* from [4]:

$$\cos i \lesssim \frac{R_A + R_B}{a}$$

Unless the planet is only a couple of stellar radii from its host star, i must be very close to 90 degrees. For the sample exoplanet, i need be at least *89.974 ± 0.001 degrees.*

J. Density

Given the mass and radius of the exoplanet, the density (and therefore composition) can be inferred by assuming a uniformly distributed sphere. Converting values to SI units, the density of the planet is 4020 ± *286 Kg/m³,* corresponding to a composition similar to that of Mars.

K. Temperature

The temperature of the exoplanet is constrained by its composition and the temperature of its host star. Without knowing these values, any inference of temperature is loosely defined. Nonetheless, T_B can be estimated with the equilibrium blackbody temperature. Given that the exoplanet is so close to the star (0.178 AU), it is likely tidally locked in an orbital resonance and is thus a slow rotator. Assuming this, *(8.13)* from [4] gives the equilibrium temperature:

$$T_p = \left(\frac{R_\odot}{r}\right)^{1/2} (1 - A)^{1/4} T_\odot$$

However, the temperature of the star is not defined. Nonetheless, given that the radius and the mass of the star are ~50% of the sun's, it is likely between a K and M type star. For this reason, a temperature of ~4000 K is assumed. Without any further information, calculations are done on an array of albedos from A = 0.1 to 0.5, which follow those of similar composition planets in the solar system (barring greenhouse gas effects). These values result in a temperature of ~400 K, with a loosely defined error. Using the fast rotator equation returns a temperature of ~320 K.

III. Results

The methods defined above returned the following properties for the exoplanet:

Property	Value	Error	Units
Period	27.51	0.01	Days
Radius	2.3	0.1	R_{Earth}
a_B	0.178	0.0003	AU
a_A	1e-05	0.071	AU
v_B	58.83	0.02	km / s
v_A	0.003	0.07	km / s
i	89.974	0.001	Degrees
Density	4020	286	Kg / m^3
Transit Time	3.80	0.05	Hours
Depth	0.152	0.002	% Blocked
Temperature	~400	Not well defined	K

When comparing to [2], this simulated data is most similar to that of the Keplar-267 d system. This exoplanet was detected in 2014 by the transit method, with

- Period: 28.46381 ± 0.00028 Days
- Orbit Semi-Major Axis: 0.15 AU
- Planet Radius: 2.31 ± 0.63 $R_\odot$
- Equilibrium Temperature: 337 K
- Stellar T_{eff} 4081.00±200.00 K
- Stellar Radius 0.5270±0.0290 $R_\odot$,
- Stellar Mass 0.5520±0.0380 $M_\odot$

It is interesting to note that while the stellar effective temperature agrees with our calculation in section K, the equilibrium temperature seems to align most with a fast rotator. Overall, the methodology used to infer these exoplanetary properties align with those used on Keplar data.

IV. Conclusion

While this data was created by moving a fully dark circle in front of a uniformly lit circle, the information extracted with the code in the appendix below can be extrapolated to real observations. However, it is important to note the limitations of the study, particularly in the use of simulated data. Real data will typically have higher noise since exoplanets are not fully dark and observations of stars return non-uniform surfaces as atmospheric conditions and inherent calibration errors add to the noise. Further, this data is assuming that there is only one planet transiting in front of the star; in real systems with multiple planets, further work (such as a Fourier analysis) is needed to separate the transits to find the periods. Nonetheless, the appendix provided can be extended as a template to interactively analyze the properties of exoplanets, while reminding astronomers that exoplanetary science can be conducted in a simple and accessible way.

References

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