National Autonomous University of México
School of Engineering
Division of Electrical Engineering

ICPC 2026 Reference

CPCFI UNAM

>:)

RacsoFractal, zum, edwardsal17

Contents

1 Template

```cpp
// Racso programmed here
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef long double ld;
typedef __int128 sll;
typedef pair<int,int> ii;
typedef pair<ll,ll> pll;
typedef vector<int> vi;
typedef vector<ll> vll;
typedef vector<ii> vii;
typedef vector<vi> vvi;
typedef vector<vii> vvii;
typedef vector<pll> vpll;
typedef unsigned int uint;
typedef unsigned long long ull;
#define fi first
#define se second
#define pb push_back
#define all(v) v.begin(),v.end()
#define rall(v) v.rbegin(),v.rend()
#define sz(a) (int)(a.size())
#define fori(i,a,n) for(int i = a; i < n; i++)
#define endl '\n'
const int MOD = 1e9+7;
const int INFTY = INT_MAX;
const long long LLINF = LLONG_MAX;
const double EPS = DBL_EPSILON;
void printVector( auto& v ){ fori(i,0,sz(v)) cout <<
    v[i] << " "; cout << endl; }
void fastIO() { ios_base::sync_with_stdio(0); cin.
  tie(0); cout.tie(0); }
```

Listing 1: Racso's template.

2 C++ Sintaxis

2.1 Compilation sentences

To compile and execute in C++:

```
g++-13 -Wall -o solucion.exe solucion.cpp
g++ -std=c++20 -Wall -o main a.cpp ./solucion.exe < input.txt > output.txt
```

To compile and execute in Java:

```
javac -Xlint Solucion.java
java Solucion < input.txt > output.txt
```

To execute in Python (two options):

```
python3 solucion.py < input.txt > output.txt
pypy3 solucion.py < input.txt > output.txt
```

2.2 Custom comparators

```cpp
// Using function
bool cmpFunction(const pair<int,int> &a, const pair<
  int,int> &b) {
  return a.second < b.second;
```

```cpp
}
sort(all(v), cmpFunction);
// Using functor
struct CmpFunctor {
  bool operator()(const pair<int,int> &a, const pair
   <int,int> &b) const {
    return a.second < b.second;
  }
};
sort(all(v), CmpFunctor());
// Using lambda function
sort(all(v), [](const pair<int,int> &a, const pair<
    int,int> &b) {
  return a.second < b.second;
});
```

Listing 2: Template for custom sorting, using as example ordering a vii ascending by second element.

2.3 *STL DS Usage

2.4 *Strings methods

2.5 Pragmas

```cpp
#pragma GCC optimize("O3")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#pragma GCC optimize("unroll-loops")
```

Listing 3: Common pragmas.

2.6 Bit Manipulation Cheat Sheet

Bitwise operators:

- `&` (AND): Sets each bit to 1 if both bits are 1.
- `|` (OR): Sets each bit to 1 if al teast one bit is 1.
- ∧ (XOR): Sets each bit to 1 if bits are different.
- ∼ (NOT): Inverts all bits.
- `<<` (Left shift): Shifts bits left, fills with 0.
- `>>` (Right shift): Shifts bits right.

Basic Bit Tasks:

- Get bit: `(n & (1 << i)) != 0`
- Set bit: `n | (1 << i)`
- Clear bit: `n & ∼(1 << i)`
- Toggle bit: `n ∧ (1 << i)`
- Clear LSB: `n & (n - 1)`
- Get LSB: `n & -n`

Set Operations:

- Subset check: `(A & B) == B`
- Set union: `A | B`
- Set intersection: `A & B`
- Set difference: `A & ∼B`
- Toggle subset: `A B̂`

Equations:

Properties of bitwise:

- `a | b = a ⊕ b + a & b`

- a $\oplus$ (a & b) = (a | b) $\oplus$ b
- b $\oplus$ (a & b) = (a | b) $\oplus$ a
- (a & b) $\oplus$ (a | b) = a $\oplus$ b

In addition and substraction:

- a + b = (a | b) + (a & b)
- a + b = a $\oplus$ b + 2 (a & b)
- a - b = (a $\oplus$ (a & b)) - ((a | b) $\oplus$ a))
- a - b = ((a | b) $\oplus$ b)) - ((a | b) $\oplus$ a)
- a - b = (a $\oplus$ (a & b)) - (b $\oplus$ (a & b))
- a - b = ((a | b) $\oplus$ b)) - (b $\oplus$ (a & b))

Gray code: G = B $\oplus$ (B >> 1)

C++ built in functions:

- `__builtin_popcount(x)` - Count the number of set bits in x.
- `__builatin_clz(x)` - Count the number of leading zeros in x.
- `__builtint_ctz(x)` - Count the number of trailing zeros in x.

2.7 Comparing Floats

```
long double a, b, EPS = 1e-9;
if( abs(a - b) < EPs ) {
    // 'a' equals 'b'
}
```

Listing 4: Check if two real numbers are equal using an épsilon scope.

2.8 Ceil

$$\left\lceil \frac{a}{b} \right\rceil = \frac{a+b-1}{b}$$

(Originally I though $\lceil \frac{a}{b} \rceil = \frac{a-1}{b} + 1$, but this calculates the wrong way in the cases where $a = 0$)

```
int myCeil(long long a, long long b) {
    return (a + b - 1)/b;
}
```

Listing 5: A way to do ceil operation between integers without making explicit conversions. `a` and `b` are `int`, but the operation `a+b-1` can cause an overflow, so they must be casted into `long long` to avoid this. The result must be `int` anyway.

3 Miscellaneous

3.1 Binary Search

```
int binary_search( vector<int>& list, int n, int
    target ) {
    int x0 = 0, x1 = n-1, mid;
    while( x0 <= x1 ) {
        mid = (x0 + x1) / 2; // (x1 - x0) / 2 + x0;
        if( list[mid] == target )
            return mid;
        list[mid] < target ? x0 = mid + 1 : x1 = mid
    - 1;
    }
    return -1;
}
```

Listing 6: Classic (Vanilla) implementation of Binary Search. Returns the index where `target` was found. Binary Search works on $O(log_2(n))$, let n be the size of the container.

```
int binary_search( vector<int>& list, int n, int target ) {
    int ans = 0;
    function<bool(int)> check = [&](int idx)->bool {
        return idx < n && list[idx] <= target;
    };
    for( int i = 31; i >= 0; i-- ) {
        if( check( ans + (1 << i) ) )
            ans += 1 << i;
    }
    return list[ans] == target ? ans : -1;
}
```

Listing 7: Logarithmic jumps implementation of Binary Search. Returns the index where `target` was found. Binary Search works on $O(log_2(n))$, let n be the size of the container.

```
int binary_search( vector<int>& list, int n, int target ) {
  int  left = 1, right = n + 1, mid;
  while(right - left >= 1) {
    mid = left + (right - left) / 2;
    if( list[mid] >= target && target > list[mid - 1]) {
      //--------- TODO logic here ---------//
      break;
    }
    else
      if( list[mid] < target )
        left = mid + 1;
      else
        right = mid;
  }
}
```

Listing 8: Binary Search implementation for searching an element in the interval $(numbers_{i-1}, numbers_i]$. Originally used on Codeforces problem 474 - B (Worms). Returns the index where `target` was found. Binary Search works on $O(\log_2(n))$, let n be the size of the container.

```
function<bool(ll)> check = [&](ll t) -> bool {
  ll products = 0;
  for(ll machine : v) {
    products += t / machine;
    if( products >= target )
      return true;
  }
  return products >= target;
};
for(int i = 0; i < 70; i++) {
  mid = (x0 + x1) / 2;
  check(mid) ? x1 = mid : x0 = mid + 1;
}
```

Listing 9: Implementation of Binary Search in the Answer. Originally used on CSES problem *Factory Machines*. Binary Search works on $O(\log_2(n))$, let n be the size of the container.

3.1.1 *Parallel Binary Search

3.1.2 *Ternary Search

3.2 Kadane's Algorithm

```
ll arns = v[0], maxSum = 0;
fori(i,0,n)
{
  maxSum += v[i];
  arns = max(arns, maxSum);
  maxSum = max(0LL,maxSum);
}
```

Listing 10: Uses Kadane's Algorithm to find maximum subarray sum in $O(n)$.

4 Queries

4.1 Prefix Sum 2D

```
for(int i = 1; i <= n; i++)
  for(int j = 1; j <= n; j++)
  {
    prefix[i][j] = prefix[i][j-1] + prefix[i-1][j] - prefix[i-1][j-1];
    prefix[i][j] += forest[i-1][j-1] == '*' ? 1 : 0;
```

```
  }
for(int i = 0; i < q; i++)
{
  pair<int,int> p1, p2;
  cin >> p1.fi >> p1.se >> p2.fi >> p2.se;
  int arns = prefix[p2.fi][p2.se];
  arns -= prefix[p2.fi][p1.se-1] + prefix[p1.fi-1][p2.se];
  arns += prefix[p1.fi-1][p1.se-1];
  cout << arns << endl;
}
```

Listing 11: Construction and querie of how many 1's are there in a matrix. Originally used on *Forest Queries* from CSES.

4.2 *Sparse Table

4.3 *Sqrt Decomposition

4.4 *Fenwick Tree

4.5 *Fenwick Tree 2D

4.6 Segment Tree

```
typedef long long ll;
typedef vector<ll> vll;
typedef vector<int> vi;
const int INF = INT_MAX;
class Segment_tree {
   public: vll t;
   Segment_tree( int n = 1e5+10 ) {
```

```
        t.assign(n*4,INF);
    }
    void update(int node, int index, int tl, int tr,
    int val) {
        if( tr < index || tl > index ) return;
        if( tr == tl ) t[node] = val;
        else {
            int mid = tl + ((tr-tl)>>1);
            int lft = node << 1;
            int rght = lft + 1;
            update(lft,index,tl,mid,val);
            update(rght,index,mid+1,tr,val);
            t[node] = min(t[lft],t[rght]);
        }
    }
    ll query(int node, int l, int r, int tl, int tr)
    {
        if( tl > r || tr < l ) return INF;
        if( tl >= l and tr <= r ) return t[node];
        else {
            int mid = tl + ((tr-tl)>>1);
            int lft = node << 1;
            int rght = lft + 1;
            ll q1 = query(lft,l,r,tl,mid);
            ll q2 = query(rght,l,r,mid+1,tr);
            return min(q1,q2);
        }
    }
    void build(vi &v, int node, int tl, int tr) {
        if( tl == tr ) t[node] = v[tl];
        else {
            int mid = tl + ((tr-tl)>>1);
            int lft = node << 1;
            int rght = lft + 1;
            build(v,lft,tl,mid);
            build(v,rght,mid+1,tr);
            t[node] = min(t[lft],t[rght]);
        }
    }
};
Segment_tree st(n);
st.build(v,1,0,n-1);
st.update(1,a-1,0,n-1,b);
st.query(1,a-1,b-1,0,n-1));
```

Listing 12: Segment Tree for Dynamic Range **Minimum** Queries. Racso's Implementation.

```
vector<long long> v, sex;
int n;
void build(int node, int l, int r){
    if(l == r) sex[node] = v[l];
    else{
        int mid = (l+r)/2;
        build(2*node, l, mid);
        build(2*node + 1, mid+1, r);
        sex[node] = sex[2*node] + sex[2*node +1];
```

```
    }
}
void update(int node, int l, int r, int idx, int val
  ){
    if(l == r){
        v[idx] = val;
        sex[node] = val;
    }
    else{
        int mid = (l+r)/2;
        if(l <= idx && idx <= mid) update(2*node, l,
    mid, idx, val);
        else update(2*node +1, mid+1, r, idx, val);
        sex[node] = sex[2*node] + sex[2*node + 1];
    }
}
int query(int node, int tl, int tr, int l, int r){
    if(r < tl || tr < l) return 0;
    if(l <= tl && tr <= r) return sex[node];
    int tm = (tl+tr)/2;
    return query(2*node, tl, tm, l, r) + query(2*
  node +1, tm+1, tr, l, r);
}
v.resize(n);
sex.resize(4 * n);
build(1, 0, n - 1);
query(1, 0, n-1, l - 1, r - 1)
```

Listing 13: Segment Tree for Dynamic Range **Sum** Queries. Zum's Implementation.

4.6.1 *2D Segment Tree

4.6.2 *Persistent Segment Tree

4.6.3 *Lazy Propagation

4.7 Ordered Set

```
//<-- Header.
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template<typename T, typename Cmp = less<T>>
using ordered_set = tree<T,null_type,Cmp,rb_tree_tag
    ,tree_order_statistics_node_update>;
//<-- Declaration.
ordered_set<int> oset;
//<-- Methods usage.
// K-th element in a set (counting from zero).
ordered_set<int>::iterator it = oset.find_by_order(k
    );
// Number of items strictly smaller than k.
ordered_set<int>::iterator it = oset.order_of_key(k)
    ;
// Every other std::set method.
```

Listing 14: Ordered set necessary includes in header, declaration of the object, and usage of its new methods.

4.7.1 Multi-Ordered Set

```
//<-- Header.
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template<typename T, typename Cmp = less<T>>
using ordered_set = tree<T,null_type,Cmp,rb_tree_tag,tree_order_statistics_node_update>;
//<-- Use in main.
ordered_set<pair<int,int>> multi_oset;
map<int,int> cuenta;
function<void(int)> insertar = [&](int val) -> void {
  multi_oset.insert({val,++cuenta[val]});
};
function<void(int)> eliminar = [&](int val) -> void {
  multi_oset.erase({val,cuenta[val]--});
};
```

Listing 15: Declaration of multi-oset structure.

4.8 *Treap

4.9 *Trie

5 Graph Theory

5.1 Breadth-First Search (BFS)

```
vector<vector<int>> graph;
vector<bool> visited;
graph.assign(n, vector<int>() ); // <--- main
visited.assign(n, false); // <--- main

void bfs( int s ) {
  queue<int> q;
  q.push( s );
  visited[ s ] = true;
  while( ! q.empty() ) {
    int u = q.front();
    q.pop();
    for( auto v : graph[ u ] ) {
      if( ! visited[ u ] ) {
        visited[ u ] = true;
        q.push( v );
        // --- ToDo logic here ---
      }
    }
  }
  return;
}
```

Listing 16: Iterative implementation of BFS graph traversal over a graph represented as a AdjacencyList on vector of vectors. BFS runs in $O(|V| + |E|)$.

```cpp
int n, m;
string arns = "";
cin >> n >> m;
vector<vector<bool>> visited(n,vector<bool>(m,false));
vector<string> path(n,string(m,'0'));
vector<string> grid(n);
vii dirs = {{0,1},{0,-1},{1,0},{-1,0}};
string commands = "LRUD";
queue<ii> q;
ii start, end, curr;
function<bool(int,int)> valid = [&](int i, int j) -> bool {
    return ( i >= 0 && i < n && j >= 0 && j < m && grid[i][j] != '#' && ! visited[i][j] );
};
fori(i,0,n) cin >> grid[i];
fori(i,0,n) {
    fori(j,0,m)
        if( grid[i][j] == 'A' ) {
            visited[i][j] = true;
            q.push( {i,j} );
        }
}
while( ! q.empty() ) {
    curr = q.front();
    q.pop();
    int i = curr.fi;
    int j = curr.se;
    if( grid[i][j] == 'B' ) {
        end.fi = i;
        end.se = j;
        break;
    }
    int newI, newJ;
    fori(I,0,4) {
        newI = i + dirs[I].fi;
        newJ = j + dirs[I].se;
        if( valid(newI,newJ) ) {
            visited[newI][newJ] = true;
            q.push( {newI,newJ} );
            path[newI][newJ] = commands[I];
        }
    }
}
while( path[ end.fi ][ end.se ] != '0' ) {
    fori(i,0,4) {
        if( path[ end.fi ][ end.se ] == commands[i] ) {
            arns += i & 1 ? commands[i-1] : commands[i+1];
            end.fi -= dirs[i].fi;
```

```
            end.se -= dirs[i].se;
        }
    }
}
reverse(all(arns));
if( arns == "" ) cout << "NO" << endl;
else cout << "YES" << endl << arns.size() << endl << arns << endl;
```

Listing 17: BFS on Grid to find shortest path from an starting point A to an end B. Once the path is found, it reconstruct it with movements *LRUD*. Works in $O(n \cdot m)$. Originally used on problem *Labyrinth* from CSES.

5.2 Deep-First Search (DFS)

```
vector<vector<int>> graph;
vector<bool> visited;
graph.assign(n, vector<int>() ); // <--- main
visited.assign(n, false); // <--- main

void dfs( int s ) {
    if( visited[s] == true ) return;
    visited[s] = true;
    vector<int>::iterator i;
    for( i = graph[s].begin(); i < graph[s].end(); ++i) {
        if( ! visited[*i] ) {
            // --- ToDo logic here ---
            dfs(*i);
        }
    }
}
```

Listing 18: Recursive implementation of DFS graph traversal over a graph represented as a AdjacencyList on vector of vectors. DFS runs in $O(|V| + |E|)$.

```
vector<vector<int>> graph;
vector<bool> visited;
void dfs( int s ) {
    stack<int> stk;
    stk.push(s);
    while (!stk.empty()) {
        int u = stk.top();
        stk.pop();
        if ( visited[u] ) continue;
        visited[u] = true;
        // --- ToDo logic here ---
        for(auto it = graph[u].rbegin(); it != graph[u].rend(); ++it)
            if (!visited[*it])
                stk.push(*it);
    }
}
```

Listing 19: Iterative implementation of DFS graph traversal over a graph represented as a AdjacencyList on vector of vectors. DFS runs in $O(|V| + |E|)$.

```
typedef long long ll;
vector<vector<ll>> adj;
```

```
vector<bool> visited;
function<void(ll)> dfs = [&](ll u) -> void {
    if( visited[u] ) return;
    visited[u] = true;
    for( ll v : adj[u] )
        dfs(v);
};
dfs(n);
```

Listing 20: DFS implementation with a lambda function (adjacency list and visited don't need to be passed thorough argument). DFS runs in $O(|V| + |E|)$.

```
typedef long long ll;
typedef vector<ll> vll;
map<ll,vll> adj;
set<ll> visited;
function<void(ll)> dfs = [&](ll u) -> void {
    if( visited.count(u) ) return;
    visited.insert(u);
    for( ll v : adj[u] )
        dfs(v);
};
dfs(n);
```

Listing 21: DFS implementation with a lambda function implemented with a map instead of vector of vectors, and a set to track visited nodes. DFS runs in $O(|V| + |E|)$.

5.3 Shortest Path

5.3.1 Dijkstra's Algorithm

```
typedef long long ll;
typedef pair<ll,ll> pll;

vector<vector<ll>> graph;
vector<ll> visited;
graph.assign(n, vector<ll>() ); // <--- main
visited.assign(n, false); // <--- main

vector<ll> dijkstra( int n, int source, vector<
    vector<pll>> &graph ) {
  vector<ll> dist( n, INFTY );
  priority_queue<pll, vector<pll>, greater<pll>> pq;
  dist[ source ] = 0;
  pq.push( {0, source} );
  while( ! pq.empty() ) {
    ll d = pq.top().first;
    ll u = pq.top().second;
    pq.pop();
    if( d > dist[ u ] ) continue;
    for( auto &edge : graph[ u ] ) {
      ll v = edge.first;
      ll weight = edge.second;
      if( dist[ u ] + weight < dist[ v ] ) {
        dist[ v ] = dist[ u ] + weight;
        pq.push( {dist[ v ], v} );
```

```cpp
            }
        }
    }
    return dist;
}
```

Listing 22: Iterative implementation of Dijkstra's Algorithm for shortest path over a graph represented as a AdjacencyList on vector of vectors. Returns a vector with the shortest path to every other vertex in the graph. $O(|E| \times log_2(|v|))$. doesn't work with negative weights.

```cpp
vvpll graph(n+1,vpll());
vector<bool> visited(n+1,false);
function<vll(int)> dijkstra = [&](int source) -> vll
    {
    vll dist(n+1,INF);
    priority_queue<pll,vpll,greater<pll>> pq;
    dist[source] = 0;
    pq.push({0,source});
    while( ! pq.empty() ) {
        ll d = pq.top().fi;
        ll u = pq.top().se;
        pq.pop();
        if( d > dist[u] ) continue;
        for(pll edge : graph[u]) {
            ll v = edge.fi;
            ll w = edge.se;
            if( dist[u] + w < dist[v] ) {
                dist[v] = dist[u] + w;
                pq.push({dist[v],v});
            }
        }
    }
    return dist;
};
```

Listing 23: Iterative implementation of Dijkstra's Algorithm as a Lambda Function for shortest path over a graph represented as a AdjacencyList on vector of vectors. Returns a vector with the shortest path to every other vertex in the graph. $O(|E| \times log_2(|V|))$. Doesn't work with negative weights.

5.3.2 *Floyd-Warshall's Algorithm

5.3.3 Bellman–Ford Algorithm

```cpp
int V, E;
cin >> V >> E;
vvi edges(E,vi(3,0));

for(int i = 1; i <= E; i++)
  cin >> edges[i][0] >> edges[i][1] >> edges[i][2];

function<vi(int)> bellman_ford = [&](int src) -> vi
{
  vi dist(V,INF);
  dist[src] = 0;
  for(int i = 0; i < V; i++)
  {
    for( vi edge : edges )
```

```
    {
        int u = edge[0];
        int v = edge[1];
        int w = edge[2];
        if( dist[u] != INF and dist[u] + w < dist[v]
    )
        {
            if( i == V-1 )
                return = {-1};
            dist[v] = dist[u] + w;
        }
    }
}
return dist;
};
```

Listing 24: Finds the shortest route from a source vertex, to every other one in the graph. Works over a list of edges. Runs in $O(|V| \times |E|)$. Can be used to find negative cycles.

5.4 Minimum Spanning Tree (MST)

5.4.1 *Prim's Algorithm

5.4.2 *Kruskal's Algorithm

5.5 *Bipartite Checking

5.6 *Negative Cycles

5.7 Topological Sort

```
vi topo;
vvi graph(V+1,vi());
vector<bool> visited(V+1,false);
function<void(int)> dfs = [&](int u) -> void {
    visited[u] = true;
    for(int v : graph[u])
        if( ! visited[v] )
            dfs(v);
    topo.pb(u);
};
function<void()> topological_sort = [&]() -> void {
    for(int i = 1; i <= V; i++)
        if( ! visited[i] )
            dfs(i);
    reverse(all(topo));
};
topological_sort();
for(int i = 0; i < V; i++) cout << topo[i] << " ";
```

Listing 25: Recursive toposort implementation for unweighted DAG through vvi with DFS with inverted postorder. Runs in $O(|V| \times |E|)$.

d

```
vi indegree(V+1,0);
vvi graph(V+1,vi());
vector<bool> visited(V+1,false);
fori(i,0,E) {
    graph[u].pb(v);
    indegree[v]++;
}
```

```
function<vi()> topological_sort = [&]() -> vi {
  vi order, deg = indegree; // copy
  queue<int> q;
  for(int i = 1; i <= V; i++)
    if( deg[i] == 0 )
      q.push(i);
  while( ! q.empty() ) {
    int u = q.front(); q.pop();
    order.pb(u);
    for(int v : graph[u]) {
      deg[v]--;
      if(deg[v] == 0)
        q.push(v);
    }
  }
  return order;
};
vi topo = topological_sort();
if( (int)(topo.size()) != V ) cout << "IMPOSSIBLE"
   << endl;
```

Listing 26: Kahn's Algorithm for Topological Sorting using BFS and indegree vertex analysis (nodes in a cycle will never have indegree zero). Works over unweighted directed graphs containing cycles through vvi. Runs in $O(|V| \times |E|)$.

5.8 Disjoint Set Union (DSU)

```
class DisjointSets {
private:
  vector<int> parents;
public:
  vector<int> sizes;
  DisjointSets(int size) : parents(size), sizes(size
   , 1) {
    for (int i = 0; i < size; i++) { parents[i] = i;
    }
  }
  /** @return the "representative" node in x's
   component */
  int find( int x ) {
      return parents[x] == x ? x : ( parents[x] =
   find( parents[x] ) );
  }
  /** @return whether the merge changed connectivity
    */
  bool unite( int x, int y ) {
    int x_root = find(x);
    int y_root = find(y);
    if (x_root == y_root) return false;
    if ( sizes[x_root] < sizes[y_root] ) swap(x_root
   ,y_root);
    sizes[x_root] += sizes[y_root];
    parents[y_root] = x_root;
    return true;
  }
  /** @return whether x and y are in the same
   connected component */
```

```cpp
  bool connected( int x, int y ){
    return find(x) == find(y);
  }
  void printLists() {
      cout << "Printing parents..." << endl;
      for(auto i : parents)
          cout << i << " ";
      cout << endl << "Printing sizes..." << endl;
      for(auto i : sizes )
          cout << i << " ";
  }
};
DisjointSets dsu( V );
int number_of_components = V, largest_component = 1;
if( dsu.unite(x, y) ) {
  largest_component = max( largest_component, dsu.sizes[ dsu.find( x ) ] );
  number_of_components--;
}
```

Listing 27: Template and usage of DSU with path compression. Complexity of $O(m\ \alpha(n))$ for a sequence of m operations over n elements. Where α denotes Ackerman function, where $\alpha(n) \leq 4, \forall n \leq 10^{18}$. Practically $O(1)$.

5.9 *Condensation Graph

5.10 *Strongly Connected Components (SCC)

5.11 2-SAT

```cpp
struct TwoSatSolver {
    int n_vars;
    int n_vertices;
    vector<vector<int>> adj, adj_t;
    vector<bool> used;
    vector<int> order, comp;
    vector<bool> assignment;

    TwoSatSolver(int _n_vars) : n_vars(_n_vars), n_vertices(2 * n_vars), adj(n_vertices), adj_t(n_vertices), used(n_vertices), order(), comp(n_vertices, -1), assignment(n_vars) {
        order.reserve(n_vertices);
    }
    void dfs1(int v) {
        used[v] = true;
        for (int u : adj[v]) {
            if (!used[u])
                dfs1(u);
        }
        order.push_back(v);
    }

    void dfs2(int v, int cl) {
        comp[v] = cl;
        for (int u : adj_t[v]) {
            if (comp[u] == -1)
                dfs2(u, cl);
```

```
        }
    }

    bool solve_2SAT() {
        order.clear();
        used.assign(n_vertices, false);
        for (int i = 0; i < n_vertices; ++i) {
            if (!used[i])
                dfs1(i);
        }

        comp.assign(n_vertices, -1);
        for (int i = 0, j = 0; i < n_vertices; ++i) {
            int v = order[n_vertices - i - 1];
            if (comp[v] == -1)
                dfs2(v, j++);
        }

        assignment.assign(n_vars, false);
        for (int i = 0; i < n_vertices; i += 2) {
            if (comp[i] == comp[i + 1])
                return false;
            assignment[i / 2] = comp[i] > comp[i + 1];
        }
        return true;
    }

    void add_disjunction(int a, bool na, int b, bool nb) {
        // na and nb signify whether a and b are to be negated
        a = 2 * a ^ na;
        b = 2 * b ^ nb;
        int neg_a = a ^ 1;
        int neg_b = b ^ 1;
        adj[neg_a].push_back(b);
        adj[neg_b].push_back(a);
        adj_t[b].push_back(neg_a);
        adj_t[a].push_back(neg_b);
    }

    static void example_usage() {
        TwoSatSolver solver(3); // a, b, c
        solver.add_disjunction(0, false, 1, true);  //     a  v  not b
        solver.add_disjunction(0, true, 1, true);   // not a  v  not b
        solver.add_disjunction(1, false, 2, false); //     b  v      c
        solver.add_disjunction(0, false, 0, false); //     a  v      a
        assert(solver.solve_2SAT() == true);
        auto expected = vector<bool>(True, False, True);
```

```
        assert(solver.assignment == expected);
    }
};
```

Listing 28: 2-SAT implementation from `cp-algorithms.com`. Each component added is a expression of the form $a \vee b$, which is equivalent to $\neg a \Rightarrow b \wedge \neg b \Rightarrow a$ (if one of the variables is false, then the other one must be true). A directed graph is constructed based on these implication: For each x, there's two vertices v_x and $v_{\neg x}$. If there is an edge $a \Rightarrow b$, then there also is an edge $\neg b \Rightarrow \neg a$. For any x, if x is reachable from $\neg x$ and $\neg x$ is reachable from x, the problem has no solution. This means, each variable must be in a different SCC than their negative. This is verified by the method `solve_2SAT()`, which returns a boolean: `True` if it has a solution and `False` if it doesn't.

Giant Pizza How does a particular 2-SAT problem look like? Following is the statement for the problem CSES 1684 (Giant Pizza):

> Uolevi's family is going to order a large pizza and eat it together. A total of n family members will join the order, and there are m possible toppings. The pizza may have any number of toppings. Each family member gives two wishes concerning the toppings of the pizza. The wishes are of the form "topping x is good/bad". Your task is to choose the toppings so that at least one wish from everybody becomes true (a good topping is included in the pizza or a bad topping is not included).
>
> **Input**
>
> The first input line has two integers n and m: the number of family members and toppings. The toppings are numbered $1, 2, \ldots, m$. After this, there are n lines describing the wishes. Each line has two wishes of the form "$+x$" (topping x is good) or "$-x$" (topping x is bad).
>
> **Output**
>
> Print a line with m symbols: for each topping "+" if it is included and "-" if it is not included. You can print any valid solution. If there are no valid solutions, print "IMPOSSIBLE".

```
int main(){
    fastIO();
    int n = nxt(), m = nxt();
    TwoSatSolver TwoSat(m);

    fori(i, 0, n){
        char type1, type2;
        int top1, top2;
        cin >> type1 >> top1 >> type2 >> top2;

        top1--; top2--;
        TwoSat.add_disjunction(top1, type1 == '-',
    top2, type2 == '-');
    }

    if(TwoSat.solve_2SAT()){
        fori(i, 0, m){
            if(TwoSat.assignment[i])
                cout << "+ ";
            else
                cout << "- ";
        }
    }
    else cout << "IMPOSSIBLE";
    return 0;
}
```

Listing 29: Main method for solving CSES 1684 Giant Pizza using 2-SAT template.

5.12 *Bridges and point articulation

5.13 Flood Fill

```
vector<string> grid(n);
vii dirs = {{0,1},{0,-1},{1,0},{-1,0}};
ii start;
int arns = 0;
function<void(int,int)> traverse = [&](int i, int j) -> void {
  if( grid[i][j] == '#' ) return;
  int newI, newJ;
  if( grid[i][j] != '.' ) arns += grid[i][j] - '0';
  grid[i][j] = '#';
  for( ii move : dirs ) {
    newI = i + move.fi; newJ = j + move.se;
    if( newI >= 0 && newI < n && newJ >= 0 && newJ < m && grid[newI][newJ] == 'T' )
      return;
  }
  for( ii move : dirs ) {
    newI = i + move.fi; newJ = j + move.se;
    if( newI >= 0 && newI < n && newJ >= 0 && newJ < m )
      traverse(newI, newJ);
  }
};
fori(i,0,n)
  cin >> grid[i];
fori(i,0,n) {
  fori(j,0,m) {
    if( grid[i][j] == 'S' ) {
      grid[i][j] = '.';
      start.fi = i;
      start.se = j;
    }
  }
}
traverse(start.fi, start.se);
  cout << arns << endl;
```

Listing 30: Traverse a matrix of 'n' x 'm' on grid representation. The matrix is composed of '.' for a valid space (empty), '#' for a wall, 'T' for a trap, and a number for a treasure. This implementation takes the sum of every treasure in the maze. The condition for moving to the next location is that there are no Traps nearby (up, down, left, right), so the player will never be killed while traversing. It also implements a way to read numerous test cases, but without knowing beforehand how many there are. Runs in $O(n \cdot m)$. Originally used on the problem *Treasures* from *2024-2025 ICPC Bolivia Pre-National Contest.*

5.14 Lava Flow (Multi-source BFS)

```
typedef array<int,3> iii;

vii dirs = {{1,0},{0,1},{-1,0},{0,-1}};
```

```cpp
map<int,string> path = {{0,"D"},{1,"R"},{2,"U"},{3,"
    L"}};
int n, m;
string arns = "";
bool escaped = false;
cin >> n >> m;
vector<string> grid(n);
vvi times(n,vi(m,INF)), prev(n,vi(m,-1));
vector<vector<bool>> visited(n,vector<bool>(m,false)
    );
queue<iii> q;
ii start, end;
for(int i = 0; i < n; i++) {
  cin >> grid[i];
  for(int j = 0; j < m; j++) {
    if( grid[i][j] == 'M' ) {
      q.push({i,j,0});
      times[i][j] = 0;
    }
    else if( grid[i][j] == 'A' )
      start = {i,j};
  }
}
function<bool(int,int)> valid = [&](int I, int J) ->
    bool {
  return (I >= 0) and (I < n) and (J >= 0 ) and (J <
    m) and (grid[I][J] != '#') and (times[I][J] ==
    INF);
};
function<bool(int,int)> valid_player = [&](int I,
    int J) -> bool {
  return (I >= 0) and (I < n) and (J >= 0) and (J <
    m) and (!visited[I][J]) and (grid[I][J] != '#');
};
function<bool(int,int)> is_border = [&](int I, int J
    ) -> bool {
  return I == 0 || I == n-1 || J == 0 || J == m-1;
};
// Corner cases
if( is_border(start.fi,start.se) ) {
  cout << "YES" << endl << "0" << endl;
  return 0;
}
// Multi-Source BFS
while( ! q.empty() ) {
  iii u = q.front();
  q.pop();
  for(ii dir : dirs) {
    int newI = u[0] + dir.fi;
    int newJ = u[1] + dir.se;
    int w = u[2] + 1;
    if( valid(newI,newJ) ) {
      times[newI][newJ] = w;
      q.push({newI,newJ,w});
    }
  }
```

```
}
// Player BFS
q.push({start.fi,start.se,0});
visited[start.fi][start.se] = true;
while( ! q.empty() and !escaped ) {
  iii u = q.front();
  q.pop();
  for(int i = 0; i < 4; i++) {
    int newI = u[0] + dirs[i].fi;
    int newJ = u[1] + dirs[i].se;
    int w = u[2] + 1;
    if( valid_player(newI,newJ) and w < times[newI][newJ] ) {
      visited[newI][newJ] = true;
      prev[newI][newJ] = i;
      q.push({newI,newJ,w});
      if( is_border(newI,newJ) ) {
        end.fi = newI;
        end.se = newJ;
        escaped = true;
        break;
      }
    }
  }
}
if( !escaped ) {
  cout << "NO" << endl;
  return 0;
}
// Path reconstruction
cout << "YES" << endl;
int i = end.fi;
int j = end.se;
while( prev[i][j] != -1 ) {
  int oldI = i;
  arns += path[ prev[i][j] ];
  i -= dirs[prev[i][j]].fi;
  j -= dirs[prev[oldI][j]].se;
}
reverse(all(arns));
cout << sz(arns) << endl << arns << endl;
```

Listing 31: Classic Lava Flow problem implementation, where the timer from the starting point A needs to be less than every other in the MS-BFS starting in M places. Once one edge is reached, the path is reconstructed from the output. Runs in BFS complexity $O(|V| + |E)$. Originally used in the CSES problem *Monsters*.

5.15 MaxFlow

5.15.1 Dinic's Algorihtm

```
const ll INF = 1e17;
/**
 * @brief Represents a directed edge in a flow network.
 * @details Stores the edge's source, destination, capacity, and current flow.
```

```cpp
 *          Used in max-flow algorithms like Dinic or Ford-Fulkerson. */
struct flowEdge {
  int u; // Source node
  int v; // Destination node
  ll cap; // Maximum flow capacity of the edge
  ll flow = 0; // Current flow through the edge (initially 0)
  flowEdge( int u, int v, ll cap ) : u(u), v(v), cap(cap) {};
};
/**
 * @brief Implementation of Dinic's max-flow algorithm.
 * @details Manages a flow network with BFS (Level Graph) and DFS (Blocking Flow) optimizations. */
struct Dinic {
  vector<flowEdge> edges; // All edges in the flow network (including reverse edges)
  vector<vi> adj;
  int n; // Total number of nodes in the graph
  int s; // Source node
  int t; // Sink node (destination of flow)
  int id = 0; // Counter for edge indexing
  vi level; // Stores the level (distance from 's') of each node during BFS
  vi next; // Optimization for DFS: tracks the next edge to explore for each node
  queue<int> q; // Queue for BFS traversal
  /**
   * @brief Constructs a Dinic solver for a flow network.
   * @param n Number of nodes.
   * @param s Source node.
   * @param t Sink node. */
  Dinic( int n, int s, int t ) : n(n), s(s), t(t) {
    adj.resize(n); // Initialize adjacency list for `n` nodes.
    level.resize(n); // Prepare level array for BFS.
    next.resize(n); // Prepare next-edge array for DFS.
    fill(all(level),-1); // Mark all levels as unvisited (-1).
    level[s] = 0; // The source has level 0.
    q.push(s); // Start BFS from the source.
  }
  /**
   * @brief Adds a directed edge and its residual reverse edge to the flow network. */
  void addEdge( int u, int v, ll cap ) {
    edges.emplace_back(u,v,cap); // Original edge: u -> v
    edges.emplace_back(v,u,0); // Residual edge: v -> u
    adj[u].pb(id++);
    adj[v].pb(id++);
```

```
}
/**
  * @brief Performs BFS to construct the level graph (Layered Network) from source 's' to sink 't'.
  * @details Assigns levels (minimum distances from 's') to all nodes and checks if 't' is reachable.
  *          Levels are used to guide the DFS phase in Dinic's algorithm.
  * @return bool True if the sink 't' is reachable (i.e., there exists an augmenting path), false otherwise. */
bool bfs() {
  while( ! q.empty() ) {
    int curr = q.front();
    q.pop();
    for( auto e : adj[curr] ) {
      if( edges[e].cap - edges[e].flow < 1 ) // Skip saturated edges (no residual capacity).
        continue;
      if( level[ edges[e].v ] != -1 ) // Skip already visited nodes (level assigned).
        continue;
      // Assign level to the neighbor node.
      level[ edges[e].v ] = level[ edges[e].u ] + 1; // Next level = current + 1.
      q.push( edges[e].v ); // Add neighbor to the queue for further BFS.
    }
  }
  return level[t] != -1; // Return whether the sink 't' was reached (level[t] != -1).
}
/**
  * @brief Finds a blocking flow using DFS in the level graph constructed by BFS.
  * @param u Current node being processed.
  * @param flow Maximum flow that can be sent from 'u' to the sink 't'.
  * @return ll The amount of flow successfully sent to 't'. */
ll dfs( int u, ll flow ) {
  if( flow == 0 ) // No remaining flow to send.
    return 0;
  if( u == t )    // Reached the sink; return accumulated flow.
    return flow;
  // Explore edges from 'u' using 'next[u]' to avoid revisiting processed edges.
  for( int& cid = next[u]; cid < sz(adj[u]); cid++ ) {
    int e = adj[u][cid]; // Index of the edge in 'edges'.
```

```
      int v = edges[e].v;  // Destination node of the edge.
      // Skip invalid edges:
      // 1. Not in the level graph (level[u] + 1 != level[v]). Just edges in exactly one leve ahead (ensures shortest paths).
      // 2. No residual capacity (cap - flow < 1).
      if( level[edges[e].u] + 1 != level[v] || edges[e].cap - edges[e].flow < 1 )
        continue;
      ll f = dfs( v, min(flow, edges[e].cap - edges[e].flow ) ); // Recursively send flow to 'v'.
      if( f == 0 ) // No flow could be sent via this edge.
        continue;
      // Update residual capacities:
      edges[e].flow += f;       // Increase flow in the original edge.
      edges[ e ^ 1 ].flow -= f; // Decrease flow in the reverse edge. (All reverse edges have distinct parity)
      return f;                 // Return the flow sent.
    }
    return 0; // No augmenting path found from 'u'.
  }
  /**
   * @brief Computes the maximum flow from source 's' to sink 't' using Dinic's algorithm.
   * @details Iterates through BFS and DFS phases to find the maximum flow.
        Accumulates flow while there exists augmention paths in the residual graph.
        Restart auxiliary structures for every new phase.
   * @return ll The maximum flow value. */
  ll maxFlow() {
    ll flow = 0; // Tracks the total flow sent.
    while( bfs() ) { // While there are augmenting paths:
      fill(all(next),0); // Reset 'next' for DFS.
      for( ll f = dfs(s,INF); f != 0ll; f = dfs(s,INF) ) // Send blocking flow in the level graph:
        flow += f;
      // Reset for next BFS phase:
      fill(all(level),-1);
      level[s] = 0;
      q.push(s);
    }
    return flow;
  }
  /**
   * @brief Finds edges belonging to the minimum cut after maxFlow().
```

```
 * @details First, it marks all the reachable nodes from 's' with an augmention path after obtained the max flow
     and all the saturated edges coming out from any of the nodes who belong to the min-cut.
     For 'minCut()' to work, 'maxFlow()' must be first executed to get the min-cut.
     If only is needed the value, is enough returning the value of 'maxFlow()'.
 * @return vii List of edges (u, v) in the min-cut. Its size is the minimum number of 'roads' to close. */
vii minCut() {
  vii ans;
  fill(all(level),-1); // Reset levels.
  level[s] = 0;        // Mark source as reachable.
  q.push(s);
  while( ! q.empty() ) { // BFS to mark nodes reachable from 's' in the residual graph.
    int curr = q.front();
    q.pop();
    for( int id = 0; id < sz(adj[curr]); id++ ) { // For every edge going out from 'curr'.
      int e = adj[curr][id];
      // If 'v' is has not been visited yet, and the edge have residual capacity.
      if( level[edges[e].v] == -1 && edges[e].cap - edges[e].flow > 0 ) {
        q.push(edges[e].v);
        level[edges[e].v] = level[edges[e].u] + 1;
      }
    }
  }
  for( int i = 0; i < sz(level); i++ ) {
    if( level[i] != -1 ) {
      for( int id = 0; id < sz(adj[i]); id++ ) {
        int e = adj[i][id];
        if( level[edges[e].v] == -1 && edges[e].cap - edges[e].flow == 0 )
          ans.emplace_back(edges[e].u,edges[e].v);
      }
    }
  }
  return ans;
}
/**
 * @brief Reconstructs the maximum bipartite matching after running 'maxFlow()'.
 * @details Every edge that belong to the original graph and have flow greater than zero, belongs to the matching.
     For 'maximumMatching()' to work, 'maxFlow()'.
```

```
   * @return vii List of matched pairs (boy, girl).
   */
  vii maximumMatching() {
    vii ans;
    fill(all(level),-1); // Reset levels.
    level[s] = 0;        // Mark source as reachable.
    q.push(s);
    while( ! q.empty() ) { // BFS to mark nodes reachable via saturated edges with flow greater than zero.
      int curr = q.front();
      q.pop();
      for( int id = 0; id < sz(adj[curr]); id++ ) {
        int e = adj[curr][id];
        // If 'v' has not been visited yet, the edge is saturated and have flow greater than zero.
        if( level[edges[e].v] == -1 && edges[e].cap - edges[e].flow == 0 && edges[e].flow != 0ll ) {
          q.push(edges[e].v);
          level[edges[e].v] = level[edges[e].u] + 1;
        }
      }
    }
    for( int i = 0; i < sz(level); i++ ) { // Collect original edges (boy -> girl) that are saturated and have flow > 0.
      if( level[i] != -1 ) {
        for( int id = 0; id < sz(adj[i]); id++ ) {
          int e = adj[i][id];
          if( edges[e].u != s && edges[e].v != t && edges[e].cap - edges[e].flow == 0 && edges[e].flow != 0ll )
            ans.emplace_back(edges[e].u,edges[e].v);
        }
      }
    }
    return ans;
  }
};
```

Listing 32: Commented template for solving MaxFlow problems with Dinic's algorithm. Works in complexity $O(|V|^2 \times |E|)$. In bipartite graphs and graphs with unitary max capacity the complexity turns $O(|E| \times \sqrt{|V|})$.

Download Speed How does a particular flow problem looks like? Following is the statement for the problem CSES 1694 (Download Speed):

> Consider a network consisting of n computers and m connections. Each connection specifies how fast a computer can send data to another computer.
>
> Kotivalo wants to download some data from a server. What is the maximum speed he can do this, using the connections in the network?
>
> **Input**
>
> The first input line has two integers n and m: the number of computers and connections. The computers are numbered $1, 2, \ldots, n$. Computer 1 is the server and computer n is Kotivalo's computer.
>
> After this, there are m lines describing the connections. Each line has three

integers a, b, and c: computer a can send data to computer b at speed c.

Output

Print one integer: the maximum speed Kotivalo can download data.

```
int main()
{
    fastIO();

    int n, m, u, v, w;

    cin >> n >> m;

    Dinic flow(n+1,1,n); // size n+1 to fix 0-indexed indexes, 1 is the source (server), 'n' is the sink (Kotivalo)

    fori(i,0,m)
    {
        cin >> u >> v >> w;
        flow.addEdge(u,v,w);
    }

    cout << flow.maxFlow() << endl;

    return 0;
}
```

Listing 33: Main method for solving CSES 1697 Download Speed using MaxFlow template.

Police Chase **Max Flow-Min Cut Theorem:** MaxFlow = MinCut.

Following is the statement for the problem CSES 1695 (Police Chase):

Kaaleppi has just robbed a bank and is now heading to the harbor. However, the police wants to stop him by closing some streets of the city.

What is the minimum number of streets that should be closed so that there is no route between the bank and the harbor?

Input

The first input line has two integers n and m: the number of crossings and streets. The crossings are numbered $1, 2, \ldots, n$. The bank is located at crossing 1, and the harbor is located at crossing n.

After this, there are m lines that describing the streets. Each line has two integers a and b: there is a street between crossings a and b. All streets are two-way streets, and there is at most one street between two crossings.

Output

First print an integer k: the minimum number of streets that should be closed. After this, print k lines describing the streets. You can print any valid solution.

```
int main()
{
    fastIO();

    int n, m, u, v;
    vii minCut;

    cin >> n >> m;

    Dinic flow(n+1,1,n); // size n+1 to fix 0-indexed indexes, 1 is the source (bank), 'n' is the sink (harbor)

```

```
    fori(i,0,m)
    {
        cin >> u >> v;
        flow.addEdge(u,v,1);
        flow.addEdge(v,u,1);
    }

    flow.maxFlow();
    minCut = flow.minCut();

    cout << (sz(minCut)/2) << endl;
    for(int i = 0; i < sz(minCut); i += 2)
        cout << minCut[i].fi << " " << minCut[i].se
    << endl;

    return 0;
}
```

Listing 34: Main method for solving CSES 1695 Police Chase using MaxFlow template.

School Dance MaxFlow = MinCut = MaxMatching.
Following is the statement for the problem CSES 1696 (School Dance):

There are n boys and m girls in a school. Next week a school dance will be organized. A dance pair consists of a boy and a girl, and there are k potential pairs.

Your task is to find out the maximum number of dance pairs and show how this number can be achieved.

Input

The first input line has three integers n, m and k: the number of boys, girls, and potential pairs. The boys are numbered $1, 2, \ldots, n$, and the girls are numbered $1, 2, \ldots, m$.

After this, there are k lines describing the potential pairs. Each line has two integers a and b: boy a and girl b are willing to dance together.

Output

First print one integer r: the maximum number of dance pairs. After this, print r lines describing the pairs. You can print any valid solution.

```
int main()
{
    fastIO();

    int n, m, k, a, b;
    ll maxPairs;
    vii pairs;

    cin >> n >> m >> k;

    Dinic flow(n+m+2,0,n+m+1);

    fori(boy,0,n+1)
        flow.addEdge(0,boy,1);

    fori(girl,n+1,n+m+1)
        flow.addEdge(girl,n+m+1,1);

    fori(i,0,k)
    {
```

```cpp
        cin >> a >> b;
        flow.addEdge(a,n+b,1);
    }

    maxPairs = flow.maxFlow();
    pairs = flow.maximumMatching();

    cout << maxPairs << endl;
    fori(i,0,sz(pairs))
        cout << pairs[i].fi << " " << (pairs[i].se - n) << endl;;

    return 0;
}
```

Listing 35: Main method for solving CSES 1696 School Dancing using MaxFlow template.

5.15.2 *Ford-Fulkerson Algorithm

5.15.3 *Goldber-Tarjan Algorithm

6 Trees

6.1 Counting Childrens

```cpp
vi childrens(n+1,0);
vvi graph(n+1);
vector<bool> visited(n+1, false);
fori(i,2,n+1) {
    cin >> tmp;
    graph[tmp].pb(i);
    graph[i].pb(tmp);
}
function<int(int)> dfs = [&](int u) -> int {
    visited[u] = true;
    for(int v : graph[u]) {
        if( !visited[v] )
            childrens[u] += dfs(v);
    }
    return childrens[u] + 1;
};
dfs(1);
```

Listing 36: Algorithm that counts how many childrens does every node have, from 2..n in a rooted tree (root = 1).

6.2 *Tree Diameter

6.3 *Centroid Decomposition

6.4 *Euler Tour

6.5 *Lowest Common Ancestor (LCA)

6.6 *Heavy-Light Decomposition (HLD)

7 Strings

7.1 Knuth-Morris-Pratt Algorithm (KMP)

```cpp
// Longest Prefix-Suffix
vi compute_LPS(string s) {
  size_t len = 0, i = 1, sz = s.size();
```

```
  vi lps(sz,0);
  while( i < sz ) {
    if( s[i] == s[len] )
      lps[i++] = ++len;
    else
      if( len != 0 )
        len = lps[len-1];
      else
        lps[i++] = 0;
  }
  return lps;
}
// Get number of occurrences of a pattern p in a string s
int kmp(string s, string p) {
  vi lps = compute_LPS(p);
  size_t n = s.size(), m = p.size(), i = 0, j = 0;
  int cnt = 0;
  while( i < n ) {
    if( p[j] == s[i] ) {
      j++; i++;
    }
    if( j == m ) { // Full match
      cnt++;
      j = lps[ j - 1];
    }
    else if( i < n and p[j] != s[i] ) { // Mismatch after j matches
      if( j != 0 )
        j = lps[ j - 1 ];
      else
        i++;
    }
  }
  return cnt;
}
```

Listing 37: KMP algorithm for counting how many times a pattern appear into a string. Runs in $O(n + m)$.

7.2 *Suffix Array

7.3 *Rolling Hashing

7.4 *Z Function

7.5 *Aho-Corasick Algorithm

8 Dynamic Programming

8.1 *Coins

8.2 *Longest Increasing Subsequence (LIS)

8.3 *Edit Distance

8.4 *Knapsack

8.5 *SOS DP

8.6 *Digit DP

8.7 *Bitmask DP

9 Mathematics

9.1 Number Theory

9.1.1 Greatest Common Divisor (GCD)

```
int gcd(int a, int b) {
  if (a == 0) return b;
  if (b == 0) return a;
  if (a == b) return a;
  if (a > b)
    return gcd(a - b, b);
  return gcd(a, b - a);
}
```

Listing 38: Implementation of handmade GCD, because using `gcd()` runs slow with long long, also `__gcd()`.

9.1.2 Gauss Sum

The sum of the first n natural numbers in $O(1)$.

$$S = \frac{n\,(n+1)}{2} \tag{1}$$

$$n = \sqrt{2S + \frac{1}{4}} - \frac{1}{2} \tag{2}$$

```
int S = (1LL * n * (1LL * n + 1LL))/2;
int n = (int)( sqrt( 2 * S + 0.25 ) - 0.5 )
```

Listing 39: Implementation of the Gauss Sum.

9.1.3 *Modular Theory

9.1.4 *Modulo Inverse

9.1.5 *Fermat's Little Theorem

9.1.6 *Chinese Remainder Theorem

9.1.7 Binpow

```
const int MOD = 1e9+7;
int binpow( long long a, long long b ) { // a^b
    long long sol = 1;
    a %= MOD;
    while( b > 0 ) {
        if( b & 1 )
```

```
            sol = ( 1LL * sol * a ) % MOD;
        a = ( 1LL * a * a ) % MOD;
        b >>= 1;
    }
    return sol % MOD;
}
```

Listing 40: Applying binary exponentiation to a problem requiring $a^b\, mod(10^9+7)$ in $O(log_2(b))$.

9.1.8 Matrix Exponentiation (Linear Recurrency)

```
template <typename T> void matmul(vector<vector<T>> &a, const vector<vector<T>>& b) {
   size_t n = a.size(), m = a[0].size(), p = b[0].size();
   assert(m == b.size());
   vector<vector<T>> c(n,vector<T>(p));
   for(size_t i = 0; i < n; i++)
      for(size_t j = 0; j < p; j++)
         for(size_t k = 0; k < m; k++)
            c[i][j] = (c[i][j] + a[i][k] * b[k][j]) % MOD;
   a = c;
}
template <typename T> struct Matrix {
   vector<vector<T>> mat;
   Matrix() {}
   Matrix(vector<vector<T>> a) { mat = a; }
   Matrix(int n, int m) {
      mat.resize(n);
      for(int i = 0; i < n; i++) {mat[i].resize(m); }
   }
   int rows() const { return mat.size(); }
   int cols() const { return mat[0].size(); }
   void makeIden() {
      for(int i = 0; i < rows(); i++)
         for(int j = 0; j < cols(); j++)
            mat[i][j] = (i == j ? 1 : 0);
   }
   Matrix operator*=(const Matrix &b) {
      matmul(mat, b.mat);
      return *this;
   }
   void print() {
      for(int i = 0; i < rows(); i++) {
         for(int j = 0; j < cols(); j++)
            cout << mat[i][j] << " ";
         cout << endl;
      }
   }
   Matrix operator*(const Matrix &b) { return Matrix(*this) *= b; }
};
int main() {
   Matrix<ll> A( {{1,1},{1,0}} );
```

```
    Matrix<ll> ini(2,1);
    ini.mat[0][0] = 0;
    ini.mat[1][0] = 1;
    Matrix<ll> iden(2,2);
    iden.makeIden();
    ll n;
    cin >> n;
    while(n > 0) {
        if( n & 1 ) iden *= A;
        A *= A;
        n >>= 1;
    }
    Matrix<ll> res = iden * ini;
    cout << res.mat[0][0] << endl;
    return 0;
}
```

Listing 41: Template to pow a matrix of size n to a certain exponent with logarithmic time (using binpow), and multiply it to another matrix, with modulo operation, as well as how to use it. Full implementation for calculating `n-th` Fibonacci term with linear recurrency.

9.1.9 Prime checking

```
bool prime( int n ){
    if( n == 2 )
        return true;
    if( n % 2 == 0 || n <= 1 )
        return false;
    for( int i = 3; i * i <= n; i += 2 )
        if( ( n % i ) == 0 )
            return false;
    return true;
}
```

Listing 42: Returns if n is a prime number in $O(\sqrt{n})$. Avoids overflow $\forall\ n \leq 10^6$ $(\approx INT\_MAX)$.

9.1.10 Prime factorization

```
void prime_factorization(vll& factorization, ll n) {
    for(long long d = 2; d*d <= n; d++) {
        while(n % d == 0) {
            factorization.push_back(d);
            n /= d;
        }
    }
    if( n > 1 )
        factorization.push_back(n);
}
```

Listing 43: Returns prime factorization of the number n using *trial division*, simplest way. Runs in $O(\sqrt{n})$. e.g. for 12 the result is 2x2x3.

9.1.11 Sieve of Eratosthenes

```
void sieve_of_eratosthenes(vector<bool>& is_prime,
    int n) {
    is_prime.assign(n+1,true);
```

```cpp
    is_prime[0] = is_prime[1] = false;
    for(int i = 2; i <= n; i++) {
        if( is_prime[i] && (long long)i * i <= n ) {
            for(int j = i*i; j <= n; j += i)
            is_prime[j] = false;
        }
    }
}
```

Listing 44: Calculates every prime number up to n with sieve of eratosthenes in a boolean 1-indexed vector. Runs in $O(n\ log\ log\ n)$.

9.1.12 Sum of Divisors

```cpp
ll sum_of_divisors(ll n) {
  ll sum = 1;
  for (long long i = 2; i * i <= n; i++) {
    if(n % i == 0) {
      int e = 0;
      do {
        e++;
        n /= i;
      } while (n % i == 0);
      ll s = 0, pow = 1;
      do {
        s += pow;
        pow *= i;
      } while (e-- > 0);
      sum *= s;
    }
  }
  if(n > 1)
    sum *= (1 + n);
  return sum;
}
```

Listing 45: Calculates the sum of all divisors of number n. e.g. $sum\_of\_divisors(12) = 18$. Runs in $O(\sqrt{n})$.

```cpp
void sum_of_divisors_sieve( vll& sigma, int n ) {
    sigma.assign(n+1,0);
    for(int i = 1; i <= n; i++)
        for(int j = i; j <= n; j+=i)
            sigma[j] += i;
}
```

Listing 46: Calculates the sum of all divisors of all numbers from 1 to n. Runs in $O(\,n\,log\,(n))$.

9.2 Combinatorics

9.2.1 Binomial Coefficients

```cpp
const int MAXN = 1e6+1;
vll fact(MAXN+1), inv(MAXN+1);
int binpow( ll a, ll b ) { // a^b
    ll sol = 1;
    a %= MOD;
    while( b > 0 ) {
```

```
        if( b & 1 )
            sol = ( 1LL * sol * a ) % MOD;
        a = ( 1LL * a * a ) % MOD;
        b >>= 1;
    }
    return sol % MOD;
}
void combi() {
    fact[0] = inv[0] = 1;
    fori(i,1,MAXN+1) {
        fact[i] = fact[i-1] * i % MOD;
        inv[i] = binpow( fact[i], MOD - 2 );
    }
}
ll nCr( ll n, ll r ) {
    return fact[n] * inv[r] % MOD * inv[n-r] % MOD;
}
combi();
nCr(a,b);
```

Listing 47: Template for calculating binomial coefficients $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$. Precalculate *fact* and *inv* runs in $O(MAXN \cdot log_2(MOD))$ ($log_2(MOD) \approx 30$). So, in general case when $NMAX = 10^6$ and $MOD = 10^9 + 7$ can be generalizad to $O(n \cdot log(n))$, $n \leq 10^6$.

9.2.2 Common combinatorics formulas

$$\binom{n}{2} = \frac{n\,(n-1)}{2} \tag{3}$$

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n \tag{4}$$

$$\sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} = \binom{2n}{n} \tag{5}$$

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1} \tag{6}$$

$$\sum_{k=0}^{\infty} \binom{2k}{k}\binom{2n-2k}{n-k} = 4^n \tag{7}$$

$$\tag{8}$$

9.3 *Stars and Bars

9.4 Probability

9.5 Computational Geometry

9.5.1 *Cross Product

9.5.2 *Convex Hull

9.6 *Fast Fourier Transform (FFT)

10 Appendix

10.1 What to do against WA?

1. Have you done the correct complexity analysis?
2. Have you understood well the statement?
3. Have you corroborated yet the trivial test cases?
4. Have you checked all the corner cases?
5. Have you proposed a lot of non-trivial test cases?

6. Isn't there any posisbilitie of overflow? (Multiplying two `int` needs to be fitted into a `long long`)
7. Have you done a desktop test?
8. Have you red all the variables? (`tc` variable on `main`)
9. Every part of your code works as it's meant to?

10.2 Primitive sizes

Data type	[B]	**Minimun value it takes**	**Maximum value it takes**
bool	1	0	1
signed char	1	0	255
unsigned char	1	-128	127
signed int	4	-2,147,483,648 $\approx -2 \times 10^9$	2,147,483,647 $\approx 2 \times 10^9$
unsigned int	4	0	4,294,967,295 $\approx 4 \times 10^9$
signed short	2	-32,768	32,767
unsigned short	2	0	65,535
signed long long int	8	-9,223,372,036,854,775,808 $\approx -9 \times 10^{18}$	9,223,372,036,854,775,807 $\approx 9 \times 10^{18}$
unsigned long long int	8	0	18,446,744,073,709,551,615 $\approx 18 \times 10^{18}$
float	4	1.1×10^{-38}	3.4×10^{38}
double	8	2.2×10^{-308}	1.7×10^{308}
long double	12	3.3×10^{-4932}	1.1×10^{4932}

Table 1: Capacity of primitive data types in C++.

10.3 Printable ASCII characters

32	whitespace	58	:	65	A	97	a
33	!	59	;	66	B	98	b
34	”	60	¡	67	C	99	c
35	#	61	=	68	D	100	d
36	$	62	¿	69	E	101	e
37	%	63	?	70	F	102	f
38	&	64	@	71	G	103	g
39	'	91	[	72	H	104	h
40	(	92	\	73	I	105	i
41	)	93	]	74	J	106	j
42	*	94	∧	75	K	107	k
43	+	95	_	76	L	108	l
44	,	96	’	77	M	109	m
45	-	126	∼	78	N	110	n
46	.			79	O	111	o
47	/			80	P	112	p
48	0			81	Q	113	q
49	1			82	R	114	r
50	2			83	S	115	s
51	3			84	T	116	t
52	4			85	U	117	u
53	5			86	V	118	v
54	6			87	W	119	w
55	7			88	X	120	x
56	8			89	Y	121	y
57	9			90	Z	122	z

Table 2: Code and symbol of printable ASCII characters.

10.4 Numbers bit representation

1	00000001	31	00011111	61	00111101	91	01011011	121	01111001
2	00000010	32	00100000	62	00111110	92	01011100	122	01111010
3	00000011	33	00100001	63	00111111	93	01011101	123	01111011
4	00000100	34	00100010	64	01000000	94	01011110	124	01111100
5	00000101	35	00100011	65	01000001	95	01011111	125	01111101
6	00000110	36	00100100	66	01000010	96	01100000	126	01111110
7	00000111	37	00100101	67	01000011	97	01100001	127	01111111
8	00001000	38	00100110	68	01000100	98	01100010	128	10000000
9	00001001	39	00100111	69	01000101	99	01100011	129	10000001
10	00001010	40	00101000	70	01000110	100	01100100	130	10000010
11	00001011	41	00101001	71	01000111	101	01100101	131	10000011
12	00001100	42	00101010	72	01001000	102	01100110	132	10000100
13	00001101	43	00101011	73	01001001	103	01100111	133	10000101
14	00001110	44	00101100	74	01001010	104	01101000	134	10000110
15	00001111	45	00101101	75	01001011	105	01101001	135	10000111
16	00010000	46	00101110	76	01001100	106	01101010	136	10001000
17	00010001	47	00101111	77	01001101	107	01101011	137	10001001
18	00010010	48	00110000	78	01001110	108	01101100	138	10001010
19	00010011	49	00110001	79	01001111	109	01101101	139	10001011
20	00010100	50	00110010	80	01010000	110	01101110	140	10001100
21	00010101	51	00110011	81	01010001	111	01101111	141	10001101
22	00010110	52	00110100	82	01010010	112	01110000	142	10001110
23	00010111	53	00110101	83	01010011	113	01110001	143	10001111
24	00011000	54	00110110	84	01010100	114	01110010	144	10010000
25	00011001	55	00110111	85	01010101	115	01110011	145	10010001
26	00011010	56	00111000	86	01010110	116	01110100	146	10010010
27	00011011	57	00111001	87	01010111	117	01110101	147	10010011
28	00011100	58	00111010	88	01011000	118	01110110	148	10010100
29	00011101	59	00111011	89	01011001	119	01110111	149	10010101
30	00011110	60	00111100	90	01011010	120	01111000	150	10010110

10.5 How a `vector<vector<pair<int,int>>>` looks like

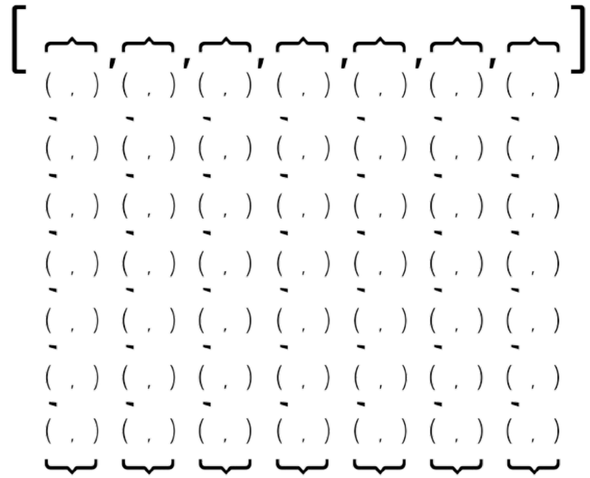

Figure 1: Visual representation of a vector of vector of pairs.

10.6 How all neighbours of a grid looks like

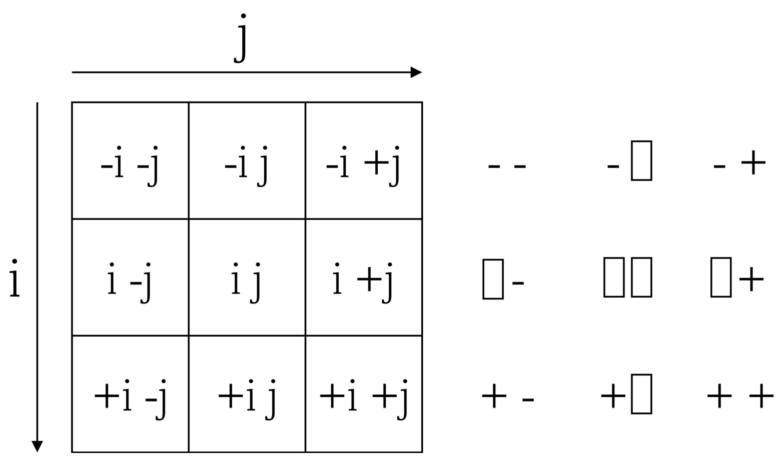

Figure 2: Visual representation of how all adjacent cells in a grid looks like.