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Contents

1 Template	3	5.3.2 *Floyd-Warshall's Algorithm	13
2 C++ Sintaxis	3	5.3.3 Bellman-Ford Algorithm	13
2.1 Compilation sentences	3	5.4 Minimum Spanning Tree (MST)	14
2.2 Custom comparators	3	5.4.1 *Prim's Algorithm	14
2.3 *BS & friends	4	5.4.2 *Kruskal's Algorithm	14
2.4 *STL DS Usage	4	5.5 *Bipartite Checking	14
2.5 *Strings methods	4	5.6 *Negative Cycles	14
2.6 *Pragmas	4	5.7 Topological Sort	14
2.7 Bit Manipulation Cheat Sheet	4	5.8 *Disjoint Set Union (DSU)	15
2.8 Comparing Floats	5	5.9 *Condensation Graph	15
2.9 Ceil	5	5.10 *Strongly Connected Components (SCC)	15
3 Miscellaneous	5	5.11 *2-SAT	15
3.1 Binary Search	5	5.12 *Bridges and point articulation	15
3.1.1 *Parallel Binary Search	6	5.13 Flood Fill	15
3.1.2 *Ternary Search	6	5.14 Lava Flow (Multi-source BFS)	16
3.2 Kadane's Algorithm	6	5.15 MaxFlow	18
4 Queries	7	5.15.1 Dinic's Algorihtm	18
4.1 *Prefix Sum 2D	7	5.15.2 *Ford-Fulkerson Algorithm	25
4.2 *Sparse Table	7	5.15.3 *Goldber-Tarjan Algorithm	25
4.3 *Sqrt Decomposition	7		
4.4 *Fenwick Tree	7		
4.5 *Fenwick Tree 2D	7		
4.6 Segment Tree	7		
4.6.1 *2D Segment Tree	9		
4.6.2 *Persistent Segment Tree	9		
4.6.3 *Lazy Propagation	9		
4.7 *Ordered Set	9		
4.7.1 Multi-Ordered Set	9		
4.8 *Treap	9		
4.9 *Trie	9		
5 Graph Theory	9		
5.1 Breadth-First Search (BFS)	9		
5.2 Deep-First Search (DFS)	10		
5.3 Shortest Path	12		
5.3.1 Dijkstra's Algorithm	12		
6 Trees	25		
6.1 Counting Childrens			25
6.2 *Tree Diameter			26
6.3 *Centroid Decomposition			26
6.4 *Euler Tour			26
6.5 *Lowest Common Ancestor (LCA)			26
6.6 *Heavy-Light Decomposition (HLD)			26
7 Strings	26		
7.1 *Knuth-Morris-Pratt Algorithm			26
7.2 *Suffix Array			26
7.3 *Rolling Hashing			26
7.4 *Z Function			26
7.5 *Aho-Corasick Algorithm			26
8 Dynamic Programming	26		
8.1 *Coins			26
8.2 *Longest Increasing Subsequence (LIS)			26
8.3 *Edit Distance			26
8.4 *Knapsack			26

8.5 *SOS DP	26
8.6 *Digit DP	26
8.7 *Bitmask DP	26
9 Mathematics	26
9.1 Number Theory	26
9.1.1 Greatest Common Divisor (GCD)	26
9.1.2 Gauss Sum	27
9.1.3 *Modular Theory	27
9.1.4 *Modulo Inverse	27
9.1.5 *Fermat's Little Theorem	27
9.1.6 *Chinese Remainder Theorem	27
9.1.7 Binpow	27
9.1.8 Matrix Exponentiation (Linear Recurrency)	27
9.1.9 Prime checking	29
9.1.10 Prime factorization	29
9.1.11 Sieve of Eratosthenes	29
9.1.12 Sum of Divisors	29
9.2 Combinatorics	30
9.2.1 Binomial Coefficients	30
9.2.2 Common combinatorics formulas	31
9.3 *Stars and Bars	31
9.4 Probability	31
9.5 Computational Geometry	31
9.5.1 *Cross Product	31
9.5.2 *Convex Hull	31
9.6 *Fast Fourier Transform (FFT)	31
10 Appendix	31
10.1 What to do against WA?	31
10.2 Primitive sizes	31
10.3 *ASCII table	32
10.4 *Numbers bit representation	32
10.5 How a <code>vector<vector<pair<int,int>>></code> looks like	32
10.6 How all neighbours of a grid looks like	33

1 Template

```

1 // Racso programmed here
2 #include <bits/stdc++.h>
3 using namespace std;
4 typedef long long ll;
5 typedef long double ld;
6 typedef __int128 sll;
7 typedef pair<int,int> ii;
8 typedef pair<ll,ll> pll;
9 typedef vector<int> vi;
10 typedef vector<ll> vll;
11 typedef vector<ii> vii;
12 typedef vector<vi> vvi;
13 typedef vector<vii> vvii;
14 typedef vector<pll> vpll;
15 typedef unsigned int uint;
16 typedef unsigned long long ull;
17 #define fi first
18 #define se second
19 #define pb push_back
20 #define all(v) v.begin(),v.end()
21 #define rall(v) v.rbegin(),v.rend()
22 #define sz(a) (int)(a.size())
23 #define fori(i,a,n) for(int i = a; i < n; i++)
24 #define endl '\n'
25 const int MOD = 1e9+7;
26 const int INFTY = INT_MAX;

```

```

27 const long long LLINF = LLONG_MAX;
28 const double EPS = DBL_EPSILON;
29 void printVector( auto& v ){ fori(i,0,sz(v)) cout << v[i] << " "; cout << endl; }
30 void fastIO() { ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0); }

```

Listing 1: Racso's template.

2 C++ Sintaxis

2.1 Compilation sentences

To compile and execute in C++:

```

g++ -Wall -o solucion.exe solucion.cpp
g++ -std=c++20 -Wall -o main a.cpp ./solucion.exe < input.txt > output.txt

```

To compile and execute in Java:

```

javac -Xlint Solucion.java
java Solucion < input.txt > output.txt

```

To execute in Python (two options):

```

python3 solucion.py < input.txt > output.txt
pypy3 solucion.py < input.txt > output.txt

```

2.2 Custom comparators

```

1 // Using function
2 bool cmpFunction(const pair<int,int> &a, const pair<
3   int,int> &b) {
4   return a.second < b.second; // ascending by second

```

```
4 }
5 sort(all(v), cmpFunction);
6 // Using functor
7 struct CmpFunctor {
8     bool operator()(const pair<int,int> &a, const pair
9         <int,int> &b) const {
10        return a.second < b.second; // ascending by
11        second
12    }
13 };
14 sort(all(v), CmpFunctor());
15 // Using lambda function
16 sort(all(v), [](const pair<int,int> &a, const pair<
17     int,int> &b) {
18     return a.second < b.second; // ascending by second
19 });
```

Listing 2: Template for custom sorting, using as example ordering a vii depending on second element.

2.3 *BS & friends

2.4 *STL DS Usage

2.5 *Strings methods

2.6 *Pragmas

2.7 Bit Manipulation Cheat Sheet

Bitwise operators: bits are 1.

- & (AND): Sets each bit to 1 if both
- | (OR): Sets each bit to 1 if al teast

- one bit is 1.
- \wedge (XOR): Sets each bit to 1 if bits are different.
- \sim (NOT): Inverts all bits.
- \ll (Left shift): Shifts bits left, fills with 0.
- \gg (Right shift): Shifts bits right.

Basic Bit Tasks:

- Get bit: $(n \& (1 \ll i)) != 0$
- Set bit: $n | (1 \ll i)$
- Clear bit: $n \& \sim(1 \ll i)$

Gray code: $G = B \oplus (B \gg 1)$

2.8 Comparing Floats

```

1 long double a, b, EPS = 1e-9;
2 if( abs(a - b) < EPS ) {
3     // 'a' equals 'b'
4 }
```

Listing 3: Check if two real numbers are equal using an epsilon scope.

2.9 Ceil

$$\lceil \frac{a}{b} \rceil = \frac{a+b-1}{b}$$

(Originally I thought $\lceil \frac{a}{b} \rceil = \frac{a-1}{b} + 1$, but this calculates the wrong way in the cases where $a = 0$)

```

1 int myCeil(long long a, long long b) {
2     return (a + b - 1)/b;
```

- Toggle bit: $n \wedge (1 \ll i)$
 - Clear LSB: $n \& (n - 1)$
 - Get LSB: $n \& \sim n$
- Set Operations:**
- Subset check: $(A \& B) == B$
 - Set union: $A | B$
 - Set intersection: $A \& B$
 - Set difference: $A \& \sim B$
 - Toggle subset: $A \hat{\&} B$

3 }

Listing 4: A way to do ceil operation between integers without making explicit conversions. a and b are `int`, but the operation $a+b-1$ can cause an overflow, so they must be casted into `long long` to avoid this. The result must be `int` anyway.

3 Miscellaneous**3.1 Binary Search**

```

1 int binary_search( vector<int>& list, int n, int
2                     target ) {
3     int x0 = 0, x1 = n-1, mid;
4     while( x0 <= x1 ) {
5         mid = (x0 + x1) / 2; // (x1 - x0) / 2 + x0;
6         if( list[mid] == target )
7             return mid;
8         list[mid] < target ? x0 = mid + 1 : x1 = mid
9             - 1;
10    }
11 }
```

Listing 5: Classic (Vanilla) implementation of Binary Search. Returns the index where `target` was found. Binary Search works on $O(\log_2(n))$, let n be the size of the container.

```

1 int binary_search( vector<int>& list, int n, int
2                     target ) {
3     int ans = 0;
```

```

3   function<bool(int)> check = [&](int idx) -> bool {
4       return idx < n && list[idx] <= target;
5   };
6   for( int i = 31; i >= 0; i-- ) {
7       if( check( ans + (1 << i) ) )
8           ans += 1 << i;
9   }
10  return list[ans] == target ? ans : -1;
11 }
```

Listing 6: Logarithmic jumps implementation of Binary Search. Returns the index where `target` was found. Binary Search works on $O(\log_2(n))$, let n be the size of the container.

```

1 int binary_search( vector<int>& list, int n, int
2     target ) {
3     int left = 1, right = n + 1, mid;
4     while(right - left >= 1) {
5         mid = left + (right - left) / 2;
6         if( list[mid] >= target && target > list[mid -
7             1] ) {
8             //----- TODO logic here -----//
9             break;
10        }
11        else
12            if( list[mid] < target )
13                left = mid + 1;
14            else
15                right = mid;
```

```

14    }
15 }
```

Listing 7: Binary Search implementation for searching an element in the interval $(numbers_{i-1}, numbers_i]$. Originally used on Codeforces problem 474 - B (Worms). Returns the index where `target` was found. Binary Search works on $O(\log_2(n))$, let n be the size of the container.

```

1 function<bool(ll)> check = [&](ll t) -> bool {
2     ll products = 0;
3     for(ll machine : v) {
4         products += t / machine;
5         if( products >= target )
6             return true;
7     }
8     return products >= target;
9 };
10 for( int i = 0; i < 70; i++ ) {
11     mid = (x0 + x1) / 2;
12     check(mid) ? x1 = mid : x0 = mid + 1;
13 }
```

Listing 8: Implementation of Binary Search in the Answer. Originally used on CSES problem *Factory Machines*. Binary Search works on $O(\log_2(n))$, let n be the size of the container.

3.1.1 *Parallel Binary Search

3.1.2 *Ternary Search

3.2 Kadane's Algorithm

```

1 arns = v[0], maxSum = 0;
2 for(i,0,n)
3 {
4     maxSum += v[i];
5     arns = max(arns, maxSum);
6     maxSum = max(0LL,maxSum);
7 }

```

Listing 9: Uses Kadane's Algorithm to find maximum subarray sum in $O(n)$.

4 Queries

- 4.1 *Prefix Sum 2D
- 4.2 *Sparse Table
- 4.3 *Sqrt Decomposition
- 4.4 *Fenwick Tree
- 4.5 *Fenwick Tree 2D
- 4.6 Segment Tree

```

1 typedef long long ll;
2 typedef vector<ll> vll;
3 typedef vector<int> vi;
4 const int INF = INT_MAX;
5
6 class Segment_tree
7 {
8     public: vll t;
9     Segment_tree( int n = 1e5+10 ) {

```

```

10         t.assign(n*4,INF);
11     }
12
13     void update(int node, int index, int tl, int tr,
14     int val) {
15         if( tr < index || tl > index ) return;
16         if( tr == tl ) t[node] = val;
17         else {
18             int mid = tl + ((tr-tl)>>1);
19             int lft = node << 1;
20             int rght = lft + 1;
21             update(lft,index,tl,mid,val);
22             update(rght,index,mid+1,tr,val);
23             t[node] = min(t[lft],t[rght]);
24         }
25     }
26
27     ll query(int node, int l, int r, int tl, int tr)
28     {
29         if( tl > r || tr < l ) return INF;
30         if( tl >= l and tr <= r ) return t[node];
31         else {
32             int mid = tl + ((tr-tl)>>1);
33             int lft = node << 1;
34             int rght = lft + 1;
35             ll q1 = query(lft,l,r,tl,mid);
36             ll q2 = query(rght,l,r,mid+1,tr);
37             return min(q1,q2);
38         }
39     }

```

```

36 void build(vi &v, int node, int tl, int tr) {
37     if( tl == tr ) t[node] = v[tl];
38     else {
39         int mid = tl + ((tr-tl)>>1);
40         int lft = node << 1;
41         int rght = lft + 1;
42         build(v,lft,tl,mid);
43         build(v,rght,mid+1,tr);
44         t[node] = min(t[lft],t[rght]);
45     }
46 }
47 };
48 Segment_tree st(n);
49 st.build(v,1,0,n-1);
50 st.update(1,a-1,0,n-1,b);
51 st.query(1,a-1,b-1,0,n-1));

```

```

10     }
11 }
12 void update(int node, int l, int r, int idx, int val) {
13     if(l == r){
14         v[idx] = val;
15         sex[node] = val;
16     }
17     else{
18         int mid = (l+r)/2;
19         if(l <= idx && idx <= mid) update(2*node, l,
mid, idx, val);
20         else update(2*node + 1, mid+1, r, idx, val);
21         sex[node] = sex[2*node] + sex[2*node + 1];
22     }
23 }
24 int query(int node, int tl, int tr, int l, int r){
25     if(r < tl || tr < l) return 0;
26     if(l <= tl && tr <= r) return sex[node];
27     int tm = (tl+tr)/2;
28     return query(2*node, tl, tm, l, r) + query(2*
node + 1, tm+1, tr, l, r);
29 }
30 v.resize(n);
31 sex.resize(4 * n);
32 build(1, 0, n - 1);
33 query(1, 0, n-1, l - 1, r - 1)

```

Listing 10: Segment Tree for Dynamic Range **Minimum** Queries. Racso's Implementation.

```

1 vector<long long> v, sex;
2 int n;
3 void build(int node, int l, int r){
4     if(l == r) sex[node] = v[l];
5     else{
6         int mid = (l+r)/2;
7         build(2*node, l, mid);
8         build(2*node + 1, mid+1, r);
9         sex[node] = sex[2*node] + sex[2*node + 1];

```

Listing 11: Segment Tree for Dynamic Range Sum Queries. Zum's Implementation.

- 4.6.1 *2D Segment Tree
- 4.6.2 *Persistent Segment Tree
- 4.6.3 *Lazy Propagation
- 4.7 *Ordered Set**
- 4.7.1 Multi-Ordered Set
- 4.8 *Treap
- 4.9 *Trie

5 Graph Theory

5.1 Breadth-First Search (BFS)

```

1 vector<vector<int>> graph;
2 vector<bool> visited;
3 graph.assign(n, vector<int>()); // <--- main
4 visited.assign(n, false); // <--- main
5
6 void bfs( int s ) {
7     queue<int> q;
8     q.push( s );
9     visited[ s ] = true;
10    while( ! q.empty() ) {
11        int u = q.front();
12        q.pop();
13        for( auto v : graph[ u ] ) {
14            if( ! visited[ u ] ) {

```

```

15                visited[ u ] = true;
16                q.push( v );
17                // --- ToDo logic here ---
18            }
19        }
20    }
21    return;
22}

```

Listing 12: Iterative implementation of BFS graph traversal over a graph represented as a AdjacencyList on vector of vectors. BFS runs in $O(|V| + |E|)$.

```

1 int n, m;
2 string arns = " ";
3 cin >> n >> m;
4 vector<vector<bool>> visited(n, vector<bool>(m, false));
5 vector<string> path(n, string(m, '0'));
6 vector<string> grid(n);
7 vii dirs = {{0,1},{0,-1},{1,0}, {-1,0}};
8 string commands = "LRUD";
9 queue<ii> q;
10 ii start, end, curr;
11 function<bool(int,int)> valid = [&](int i, int j) ->
12     bool {
13         return ( i >= 0 && i < n && j >= 0 && j < m &&
14             grid[i][j] != '#' && ! visited[i][j] );
15     };
14 for(i,0,n) cin >> grid[i];

```

```

15 for(i,0,n) {
16     for(j,0,m)
17         if( grid[i][j] == 'A' ) {
18             visited[i][j] = true;
19             q.push( {i,j} );
20         }
21     }
22 while( ! q.empty() ) {
23     curr = q.front();
24     q.pop();
25     int i = curr.fi;
26     int j = curr.se;
27     if( grid[i][j] == 'B' ) {
28         end.fi = i;
29         end.se = j;
30         break;
31     }
32     int newI, newJ;
33     for(I,0,4) {
34         newI = i + dirs[I].fi;
35         newJ = j + dirs[I].se;
36         if( valid(newI,newJ) ) {
37             visited[newI][newJ] = true;
38             q.push( {newI,newJ} );
39             path[newI][newJ] = commands[I];
40         }
41     }
42 }

43 while( path[ end.fi ][ end.se ] != '0' ) {
44     for(i,0,4) {
45         if( path[ end.fi ][ end.se ] == commands[i] )
46             {
47                 arns += i & 1 ? commands[i-1] : commands[i+1];
48                 end.fi -= dirs[i].fi;
49                 end.se -= dirs[i].se;
50             }
51     }
52 reverse(all(arns));
53 if( arns == "" ) cout << "NO" << endl;
54 else cout << "YES" << endl << arns.size() << endl << arns << endl;

```

Listing 13: BFS on Grid to find shortest path from an starting point *A* to an end *B*. Once the path is found, it reconstruct it with movements *LRUD*. Works in $O(n \cdot m)$. Originally used on problem *Labyrinth* from CSES.

5.2 Deep-First Search (DFS)

```

1 vector<vector<int>> graph;
2 vector<bool> visited;
3 graph.assign(n, vector<int>()); // <--- main
4 visited.assign(n, false); // <--- main
5
6 void dfs( int s ) {
7     if( visited[s] == true ) return;

```

```

8     visited[s] = true;
9     vector<int>::iterator i;
10    for( i = graph[s].begin(); i < graph[s].end();
11        ++i) {
12        if( ! visited[*i] ) {
13            // --- ToDo logic here ---
14            dfs(*i);
15        }
16    }

```

Listing 14: Recursive implementation of DFS graph traversal over a graph represented as a AdjacencyList on vector of vectors. DFS runs in $O(|V| + |E|)$.

```

1 vector<vector<int>> graph;
2 vector<bool> visited;
3 void dfs( int s ) {
4     stack<int> stk;
5     stk.push(s);
6     while ( !stk.empty() ) {
7         int u = stk.top();
8         stk.pop();
9         if ( visited[u] ) continue;
10        visited[u] = true;
11        // --- ToDo logic here ---
12        for(auto it = graph[u].rbegin(); it != graph
13            [u].rend(); ++it)
14            if ( !visited[*it])
15                stk.push(*it);

```

```

15    }
16 }

```

Listing 15: Iterative implementation of DFS graph traversal over a graph represented as a AdjacencyList on vector of vectors. DFS runs in $O(|V| + |E|)$.

```

1 typedef long long ll;
2 vector<vector<ll>> adj;
3 vector<bool> visited;
4 function<void(ll)> dfs = [&](ll u) -> void {
5     if( visited[u] ) return;
6     visited[u] = true;
7     for( ll v : adj[u] )
8         dfs(v);
9 };
10 dfs(n);

```

Listing 16: DFS implementation with a lambda function (adjacency list and visited don't need to be passed thorough argument). DFS runs in $O(|V| + |E|)$.

```

1 typedef long long ll;
2 typedef vector<ll> vll;
3 map<ll,vll> adj;
4 set<ll> visited;
5 function<void(ll)> dfs = [&](ll u) -> void {
6     if( visited.count(u) ) return;
7     visited.insert(u);
8     for( ll v : adj[u] )
9         dfs(v);
10 };

```

```
11 dfs(n);
```

Listing 17: DFS implementation with a lambda function implemented with a map instead of vector of vectors, and a set to track visited nodes. DFS runs in $O(|V| + |E|)$.

5.3 Shortest Path

5.3.1 Dijkstra's Algorithm

```
1 typedef long long ll;
2 typedef pair<ll,ll> pll;
3
4 vector<vector<ll>> graph;
5 vector<ll> visited;
6 graph.assign(n, vector<ll>()); // <--- main
7 visited.assign(n, false); // <--- main
8
9 vector<ll> dijkstra( int n, int source, vector<
    vector<pll>> &graph ) {
10    vector<ll> dist( n, INFTY );
11    priority_queue<pll, vector<pll>, greater<pll>> pq;
12    dist[ source ] = 0;
13    pq.push( {0, source} );
14    while( ! pq.empty() ) {
15        ll d = pq.top().first;
16        ll u = pq.top().second;
17        pq.pop();
18        if( d > dist[ u ] ) continue;
19        for( auto &edge : graph[ u ] ) {
```

```
20            ll v = edge.first;
21            ll weight = edge.second;
22            if( dist[ u ] + weight < dist[ v ] ) {
23                dist[ v ] = dist[ u ] + weight;
24                pq.push( {dist[ v ], v} );
25            }
26        }
27    }
28    return dist;
29}
```

Listing 18: Iterative implementation of Dijkstra's Algorithm for shortest path over a graph represented as a AdjacencyList on vector of vectors. Returns a vector with the shortest path to every other vertex in the graph. $O(|E| \times \log_2(|v|))$. doesn't work with negative weights.

```
1 vvppll graph(n+1,vpll());
2 vector<bool> visited(n+1,false);
3 function<vll(int)> dijkstra = [&](int source) -> vll
    {
4     vll dist(n+1,INF);
5     priority_queue<pll,vpll,greater<pll>> pq;
6     dist[source] = 0;
7     pq.push({0,source});
8     while( ! pq.empty() ) {
9         ll d = pq.top().fi;
10        ll u = pq.top().se;
11        pq.pop();
12        if( d > dist[u] ) continue;
```

```

13     for(pll edge : graph[u]) {
14         ll v = edge.fi;
15         ll w = edge.se;
16         if( dist[u] + w < dist[v] ) {
17             dist[v] = dist[u] + w;
18             pq.push({dist[v],v});
19         }
20     }
21 }
22 return dist;
23 };

```

Listing 19: Iterative implementation of Dijkstra's Algorithm as a Lambda Function for shortest path over a graph represented as a AdjacencyList on vector of vectors. Returns a vector with the shortest path to every other vertex in the graph. $O(|E| \times \log_2(|V|))$. Doesn't work with negative weights.

5.3.2 *Floyd-Warshall's Algorithm

5.3.3 Bellman–Ford Algorithm

```

1 int V, E;
2 cin >> V >> E;
3 vvi edges(E, vi(3,0));
4
5 for(int i = 1; i <= E; i++)
6     cin >> edges[i][0] >> edges[i][1] >> edges[i][2];
7
8 function<vi(int)> bellman_ford = [&](int src) -> vi
9 {

```

```

10    vi dist(V, INF);
11    dist[src] = 0;
12    for(int i = 0; i < V; i++)
13    {
14        for( vi edge : edges )
15        {
16            int u = edge[0];
17            int v = edge[1];
18            int w = edge[2];
19            if( dist[u] != INF and dist[u] + w < dist[v] )
20            )
21            {
22                if( i == V-1 )
23                    return = {-1};
24                dist[v] = dist[u] + w;
25            }
26        }
27    }
28    return dist;
29 };

```

Listing 20: Finds the shortest route from a source vertex, to every other one in the graph. Works over a list of edges. Runs in $O(|V| \times |E|)$. Can be used to find negative cycles.

5.4 Minimum Spanning Tree (MST)

5.4.1 *Prim's Algorithm

5.4.2 *Kruskal's Algorithm

5.5 *Bipartite Checking

5.6 *Negative Cycles

5.7 Topological Sort

```

1 vi topo;
2 vvi graph(V+1,vi());
3 vector<bool> visited(V+1,false);
4 function<void(int)> dfs = [&](int u) -> void {
5     visited[u] = true;
6     for(int v : graph[u])
7         if( ! visited[v] )
8             dfs(v);
9     topo.pb(u);
10};
11 function<void()> topological_sort = [&]() -> void {
12     for(int i = 1; i <= V; i++)
13         if( ! visited[i] )
14             dfs(i);
15     reverse(all(topo));
16 };
17 topological_sort();
18 for(int i = 0; i < V; i++) cout << topo[i] << " ";

```

Listing 21: Recursive toposort implementation for unweighted DAG through vvi with DFS with inverted postorder. Runs in $O(|V| \times |E|)$.

```

1 vi indegree(V+1,0);
2 vvi graph(V+1,vi());
3 vector<bool> visited(V+1,false);
4 for(i,0,E) {
5     graph[u].pb(v);
6     indegree[v]++;
7 }
8 function<vi()> topological_sort = [&]() -> vi {
9     vi order, deg = indegree; // copy
10    queue<int> q;
11    for(int i = 1; i <= V; i++)
12        if( deg[i] == 0 )
13            q.push(i);
14    while( ! q.empty() ) {
15        int u = q.front(); q.pop();
16        order.pb(u);
17        for(int v : graph[u]) {
18            deg[v]--;
19            if(deg[v] == 0)
20                q.push(v);
21        }
22    }
23    return order;
24 };
25 vi topo = topological_sort();
26 if( (int)(topo.size()) != V ) cout << "IMPOSSIBLE"
<< endl;

```

Listing 22: Kahn's Algorithm for Topological Sorting using BFS and indegree vertex analysis (nodes in a cycle will never have indegree zero). Works over unweighted directed graphs containing cycles through vvi. Runs in $O(|V| \times |E|)$.

5.8 *Disjoint Set Union (DSU)

5.9 *Condensation Graph

5.10 *Strongly Connected Components (SCC)

5.11 *2-SAT

5.12 *Bridges and point articulation

5.13 Flood Fill

```

1 vector<string> grid(n);
2 vii dirs = {{0,1},{0,-1},{1,0}, {-1,0}};
3 ii start;
4 int arns = 0;
5 function<void(int,int)> traverse = [&](int i, int j)
6     -> void {
7     if( grid[i][j] == '#' ) return;
8     int newI, newJ;
9     if( grid[i][j] != '.' ) arns += grid[i][j] - '0';
10    grid[i][j] = '#';
11    for( ii move : dirs ) {
12        newI = i + move.fi; newJ = j + move.se;
13        if( newI >= 0 && newI < n && newJ >= 0 && newJ <
14            m && grid[newI][newJ] == 'T' )
15            return;
16    }
17 }
```

```

14    }
15    for( ii move : dirs ) {
16        newI = i + move.fi; newJ = j + move.se;
17        if( newI >= 0 && newI < n && newJ >= 0 && newJ <
18            m )
19            traverse(newI, newJ);
20    }
21 };
```

```

21 fori(i,0,n)
22     cin >> grid[i];
23 fori(i,0,n) {
24     fori(j,0,m) {
25         if( grid[i][j] == 'S' ) {
26             grid[i][j] = '.';
27             start.fi = i;
28             start.se = j;
29         }
30     }
31 }
32 traverse(start.fi, start.se);
33 cout << arns << endl;
```

Listing 23: Traverse a matrix of ' n ' x ' m ' on grid representation. The matrix is composed of '?' for a valid space (empty), '#' for a wall, 'T' for a trap, and a number for a treasure. This implementation takes the sum of every treasure in the maze. The condition for moving to the next location is that there are no Traps nearby (up, down, left, right), so the player will never be killed while traversing. It also implements a way to read numerous test cases, but without knowing beforehand how many there are. Runs in $O(n \cdot m)$. Originally used on the problem *Treasures* from 2024-2025 ICPC Bolivia Pre-National Contest.

5.14 Lava Flow (Multi-source BFS)

```

1  typedef array<int,3> iii;
2
3  vii dirs = {{1,0},{0,1},{-1,0},{0,-1}};
4  map<int,string> path = {{0,"D"},{1,"R"},{2,"U"},{3,"L"}};
5  int n, m;
6  string arns = "";
7  bool escaped = false;
8  cin >> n >> m;
9  vector<string> grid(n);
10 vvi times(n,vi(m,INF)), prev(n,vi(m,-1));
11 vector<vector<bool>> visited(n,vector<bool>(m,false));
12 queue<iii> q;
13 ii start, end;
14 for(int i = 0; i < n; i++) {

```

```

15    cin >> grid[i];
16    for(int j = 0; j < m; j++) {
17      if( grid[i][j] == 'M' ) {
18        q.push({i,j,0});
19        times[i][j] = 0;
20      }
21      else if( grid[i][j] == 'A' )
22        start = {i,j};
23    }
24  }
25  function<bool(int,int)> valid = [&](int I, int J) ->
26    bool {
27    return (I >= 0) and (I < n) and (J >= 0) and (J <
28      m) and (grid[I][J] != '#') and (times[I][J] ==
29      INF);
30  };
31  function<bool(int,int)> valid_player = [&](int I,
32    int J) -> bool {
33    return (I >= 0) and (I < n) and (J >= 0) and (J <
34      m) and (!visited[I][J]) and (grid[I][J] != '#');
35  };
36  function<bool(int,int)> is_border = [&](int I, int J
37    ) -> bool {
38    return I == 0 || I == n-1 || J == 0 || J == m-1;
39  };
40  // Corner cases
41  if( is_border(start.fi,start.se) ) {
42    cout << "YES" << endl << "0" << endl;
43}

```

```
37     return 0;
38 }
39 // Multi-Source BFS
40 while( ! q.empty() ) {
41     iii u = q.front();
42     q.pop();
43     for(ii dir : dirs) {
44         int newI = u[0] + dir.fi;
45         int newJ = u[1] + dir.se;
46         int w = u[2] + 1;
47         if( valid(newI,newJ) ) {
48             times[newI][newJ] = w;
49             q.push({newI,newJ,w});
50         }
51     }
52 }
53 // Player BFS
54 q.push({start.fi,start.se,0});
55 visited[start.fi][start.se] = true;
56 while( ! q.empty() and !escaped ) {
57     iii u = q.front();
58     q.pop();
59     for(int i = 0; i < 4; i++) {
60         int newI = u[0] + dirs[i].fi;
61         int newJ = u[1] + dirs[i].se;
62         int w = u[2] + 1;
63         if( valid_player(newI,newJ) and w < times[newI][
64             newJ] ) {
65             visited[newI][newJ] = true;
66             prev[newI][newJ] = i;
67             q.push({newI,newJ,w});
68             if( is_border(newI,newJ) ) {
69                 end.fi = newI;
70                 end.se = newJ;
71                 escaped = true;
72             }
73         }
74     }
75 }
76 if( !escaped ) {
77     cout << "NO" << endl;
78     return 0;
79 }
80 // Path reconstruction
81 cout << "YES" << endl;
82 int i = end.fi;
83 int j = end.se;
84 while( prev[i][j] != -1 ) {
85     int oldI = i;
86     arns += path[ prev[i][j] ];
87     i -= dirs[prev[i][j]].fi;
88     j -= dirs[prev[oldI][j]].se;
89 }
90 reverse(all(arns));
91 cout << sz(arns) << endl << arns << endl;
```

Listing 24: Classic Lava Flow problem implementation, where the timer from the starting point A needs to be less than every other in the MS-BFS starting in M places. Once one edge is reached, the path is reconstructed from the output. Runs in BFS complexity $O(|V| + |E|)$. Originally used in the CSES problem *Monsters*.

5.15 MaxFlow

5.15.1 Dinic's Algorihtm

```

1 const ll INF = 1e17;
2 /**
3 * @brief Represents a directed edge in a flow
4 * network.
5 * @details Stores the edge's source, destination,
6 * capacity, and current flow.
7 *          Used in max-flow algorithms like Dinic
8 *          or Ford-Fulkerson. */
9 struct flowEdge {
10     int u; // Source node
11     int v; // Destination node
12     ll cap; // Maximum flow capacity of the edge
13     ll flow = 0; // Current flow through the edge (
14     // initially 0)
15     flowEdge( int u, int v, ll cap ) : u(u), v(v), cap
16     (cap) {};
17 };
18 /**
19 */

```

```

14     * @brief Implementation of Dinic's max-flow
15     * algorithm.
16     * @details Manages a flow network with BFS (Level
17     * Graph) and DFS (Blocking Flow) optimizations. */
18     struct Dinic {
19         vector<flowEdge> edges; // All edges in the flow
20         network (including reverse edges)
21         vector<vi> adj;
22         int n; // Total number of nodes in the graph
23         int s; // Source node
24         int t; // Sink node (destination of flow)
25         int id = 0; // Counter for edge indexing
26         vi level; // Stores the level (distance from 's')
27         of each node during BFS
28         vi next; // Optimization for DFS: tracks the next
29         edge to explore for each node
30         queue<int> q; // Queue for BFS traversal
31         /**
32          * @brief Constructs a Dinic solver for a flow
33          * network.
34          * @param n Number of nodes.
35          * @param s Source node.
36          * @param t Sink node. */
37         Dinic( int n, int s, int t ) : n(n), s(s), t(t) {
38             adj.resize(n); // Initialize adjacency list for
39             'n' nodes.
40             level.resize(n); // Prepare level array for BFS.
41         }
42     };
43 
```

```
34     next.resize(n); // Prepare next-edge array for
35     DFS.
36
37     fill(all(level), -1); // Mark all levels as
38     unvisited (-1).
39
40     level[s] = 0; // The source has level 0.
41     q.push(s); // Start BFS from the source.
42 }
43 /**
44 * @brief Adds a directed edge and its residual
45 * reverse edge to the flow network. */
46 void addEdge( int u, int v, ll cap ) {
47     edges.emplace_back(u,v,cap); // Original edge: u
48     -> v
49     edges.emplace_back(v,u,0); // Residual edge: v
50     -> u
51
52     adj[u].pb(id++);
53     adj[v].pb(id++);
54 }
55 /**
56 * @brief Performs BFS to construct the level
57 * graph (Layered Network) from source 's' to sink
58 * 't'.
59 * @details Assigns levels (minimum distances
60 * from 's') to all nodes and checks if 't' is
61 * reachable.
62 *
63 *          Levels are used to guide the DFS
64 * phase in Dinic's algorithm.
65
66     * @return bool True if the sink 't' is reachable
67     * (i.e., there exists an augmenting path), false
68     * otherwise. */
69     bool bfs() {
70         while( ! q.empty() ) {
71             int curr = q.front();
72             q.pop();
73
74             for( auto e : adj[curr] ) {
75                 if( edges[e].cap - edges[e].flow < 1 ) // Skip
76                 saturated edges (no residual capacity).
77                 continue;
78
79                 if( level[edges[e].v] != -1 ) // Skip
80                 already visited nodes (level assigned).
81                 continue;
82
83                 // Assign level to the neighbor node.
84                 level[edges[e].v] = level[edges[e].u] +
85                 1; // Next level = current + 1.
86
87                 q.push(edges[e].v); // Add neighbor to the
88                 queue for further BFS.
89             }
90         }
91
92         return level[t] != -1; // Return whether the
93         sink 't' was reached (level[t] != -1).
94     }
95
96 /**
97 * @brief Finds a blocking flow using DFS in the
98 * level graph constructed by BFS.
99 * @param u Current node being processed.
```

```
71     * @param flow Maximum flow that can be sent from
72     *      'u' to the sink 't'.
73
74     * @return ll The amount of flow successfully
75     *      sent to 't'. */
76
77     ll dfs( int u, ll flow ) {
78
79         if( flow == 0 ) // No remaining flow to send.
80             return 0;
81
82         if( u == t ) // Reached the sink; return
83             accumulated flow.
84
85         return flow;
86
87         // Explore edges from 'u' using 'next[u]' to
88         // avoid revisiting processed edges.
89
90         for( int& cid = next[u]; cid < sz(adj[u]); cid++ )
91             {
92
93                 int e = adj[u][cid]; // Index of the edge in
94                 // 'edges'.
95
96                 int v = edges[e].v; // Destination node of
97                 // the edge.
98
99                 // Skip invalid edges:
100
101                // 1. Not in the level graph (level[u] + 1 != level[v]). Just edges in exactly one level ahead (ensures shortest paths).
102
103                // 2. No residual capacity (cap - flow < 1).
104
105                if( level[edges[e].u] + 1 != level[v] || edges[e].cap - edges[e].flow < 1 )
106
107                    continue;
108
109                    ll f = dfs( v, min(flow, edges[e].cap - edges[e].flow) ); // Recursively send flow to 'v'.
110
111                    if( f == 0 ) // No flow could be sent via this
112                        edge.
113
114                        continue;
115
116                        // Update residual capacities:
117
118                        edges[e].flow += f; // Increase flow in
119                        // the original edge.
120
121                        edges[ e ^ 1 ].flow -= f; // Decrease flow in
122                        // the reverse edge. (All reverse edges have
123                        // distinct parity)
124
125                        return f; // Return the flow
126
127                        sent.
128
129
130                    }
131
132                    return 0; // No augmenting path found from 'u'.
133
134
135    }
136
137
138    /**
139     * @brief Computes the maximum flow from source 's' to sink 't' using Dinic's algorithm.
140     * @details Iterates through BFS and DFS phases
141     *          to find the maximum flow.
142
143     * Accumulates flow while there exists
144     * augmentation paths in the residual graph.
145
146     * Restart auxiliary structures for every new
147     * phase.
148
149     * @return ll The maximum flow value. */
150
151     ll maxFlow() {
152
153         ll flow = 0; // Tracks the total flow sent.
154
155         while( bfs() ) { // While there are augmenting
156
157             paths:
```

```
106     fill(all(next),0); // Reset 'next' for DFS.  
107     for( ll f = dfs(s,INF); f != 0ll; f = dfs(s,  
108         INF) ) // Send blocking flow in the level graph:  
109         flow += f;  
110         // Reset for next BFS phase:  
111         fill(all(level),-1);  
112         level[s] = 0;  
113         q.push(s);  
114     }  
115     return flow;  
116 }  
117 /**  
118 * @brief Finds edges belonging to the minimum  
119 cut after maxFlow().  
120 * @details First, it marks all the reachable  
121 nodes from 's' with an augmentation path after  
122 obtained the max flow  
123     and all the saturated edges coming out from  
124 any of the nodes who belong to the min-cut.  
125     For 'minCut()' to work, 'maxFlow()' must be  
126 first executed to get the min-cut.  
127     If only is needed the value, is enough  
128 returning the value of 'maxFlow()'.  
129     * @return vii List of edges (u, v) in the min-  
130 cut. Its size is the minimum number of 'roads' to  
131 close. */  
132 vii minCut() {  
133     vii ans;  
134     fill(all(level),-1); // Reset levels.  
135     level[s] = 0;         // Mark source as reachable  
136     .  
137     q.push(s);  
138     while( ! q.empty() ) { // BFS to mark nodes  
139         reachable from 's' in the residual graph.  
140         int curr = q.front();  
141         q.pop();  
142         for( int id = 0; id < sz(adj[curr]); id++ ) {  
143             // For every edge going out from 'curr'.  
144             int e = adj[curr][id];  
145             // If 'v' is has not been visited yet, and  
146             // the edge have residual capacity.  
147             if( level[edges[e].v] == -1 && edges[e].cap  
148 - edges[e].flow > 0 ) {  
149                 q.push(edges[e].v);  
150                 level[edges[e].v] = level[edges[e].u] + 1;  
151             }  
152         }  
153     }  
154     for( int i = 0; i < sz(level); i++ ) {  
155         if( level[i] != -1 ) {  
156             for( int id = 0; id < sz(adj[i]); id++ ) {  
157                 int e = adj[i][id];  
158                 if( level[edges[e].v] == -1 && edges[e].  
159 cap - edges[e].flow == 0 )  
160                     ans.emplace_back(edges[e].u,edges[e].v);  
161             }  
162         }  
163     }  
164 }
```

```
147     }
148 }
149 return ans;
150 }
151 /**
152 * @brief Reconstructs the maximum bipartite
153 matching after running 'maxFlow()'.
```

* @details Every edge that belong to the original graph and have flow greater than zero, belongs to the matching.

```
154     For 'maximumMatching()' to work, 'maxFlow()'.
155
156     * @return vii List of matched pairs (boy, girl).
157     */
158 vii maximumMatching() {
159     vii ans;
160     fill(all(level), -1); // Reset levels.
161     level[s] = 0;          // Mark source as reachable
162     .
163     q.push(s);
164     while( ! q.empty() ) { // BFS to mark nodes
165         reachable via saturated edges with flow greater
166         than zero.
167         int curr = q.front();
168         q.pop();
169         for( int id = 0; id < sz(adj[curr]); id++ ) {
170             int e = adj[curr][id];
171             if( level[edges[e].v] == -1 && edges[e].cap
172 - edges[e].flow == 0 && edges[e].flow != 011 ) {
173                 q.push(edges[e].v);
174                 level[edges[e].v] = level[edges[e].u] + 1;
175             }
176         }
177         for( int i = 0; i < sz(level); i++ ) { // Collect original edges (boy -> girl) that are
178             saturated and have flow > 0.
179             if( level[i] != -1 ) {
180                 for( int id = 0; id < sz(adj[i]); id++ ) {
181                     int e = adj[i][id];
182                     if( edges[e].u != s && edges[e].v != t
183 && edges[e].cap - edges[e].flow == 0 && edges[e].
184 flow != 011 )
185                         ans.emplace_back(edges[e].u, edges[e].v
186 );
187                 }
188             }
189         }
190     }
191     return ans;
192 }
```

Listing 25: Commented template for solving MaxFlow problems with Dinic's algorithm. Works in complexity $O(|V|^2 \times |E|)$. In bipartite graphs and graphs with unitary max capacity the complexity turns $O(|E| \times \sqrt{|V|})$.

Download Speed How does a particular flow problem looks like? Following is the statement for the problem CSES 1694 (Download Speed):

Consider a network consisting of n computers and m connections. Each connection specifies how fast a computer can send data to another computer.

Kotivalo wants to download some data from a server. What is the maximum speed he can do this, using the connections in the network?

Input

The first input line has two integers n and m : the number of computers and connections. The computers are numbered $1, 2, \dots, n$. Computer 1 is the server and computer n is Kotivalo's computer.

After this, there are m lines describing the connections. Each line has three integers a , b , and c : computer a can send data to computer b at speed c .

Output

Print one integer: the maximum speed Kotivalo can download data.

```

1 int main()
2 {
3     fastIO();
4
5     int n, m, u, v, w;
6
7     cin >> n >> m;
8
9     Dinic flow(n+1, 1, n); // size n+1 to fix 0-
```

indexed indexes, 1 is the source (server), 'n' is the sink (Kotivalo)

```

10
11     for(i, 0, m)
12     {
13         cin >> u >> v >> w;
14         flow.addEdge(u, v, w);
15     }
16
17     cout << flow.maxFlow() << endl;
18
19     return 0;
20 }
```

Listing 26: Main method for solving CSES 1697 Download Speed using MaxFlow template.

Police Chase Max Flow-Min Cut Theorem: MaxFlow = MinCut. Following is the statement for the problem CSES 1695 (Police Chase):

Kaaleppi has just robbed a bank and is now heading to the harbor. However, the police wants to stop him by closing some streets of the city.

What is the minimum number of streets that should be closed so that there is no route between the bank and the harbor?

Input

The first input line has two integers n and m : the number of crossings and streets. The crossings are numbered $1, 2, \dots, n$. The bank is located at crossing 1, and the harbor is located at crossing n .

After this, there are m lines that describing the streets. Each line has two integers a and b : there is a street between crossings a and b . All streets are

two-way streets, and there is at most one street between two crossings.

Output

First print an integer k : the minimum number of streets that should be closed. After this, print k lines describing the streets. You can print any valid solution.

```

1 int main()
2 {
3     fastIO();
4
5     int n, m, u, v;
6     vii minCut;
7
8     cin >> n >> m;
9
10    Dinic flow(n+1, 1, n); // size n+1 to fix 0-
11        indexed indexes, 1 is the source (bank), 'n' is
12        the sink (harbor)
13
14    for(i, 0, m)
15    {
16        cin >> u >> v;
17        flow.addEdge(u, v, 1);
18        flow.addEdge(v, u, 1);
19
20    }
21
22    flow.maxFlow();
23    minCut = flow.minCut();
24
25    cout << (sz(minCut)/2) << endl;
26
27 }
```

```

23     for(int i = 0; i < sz(minCut); i += 2)
24         cout << minCut[i].fi << " " << minCut[i].se
25         << endl;
26
27     return 0;
28 }
```

Listing 27: Main method for solving CSES 1695 Police Chase using MaxFlow template.

School Dance

MaxFlow = MinCut = MaxMatching.

Following is the statement for the problem CSES 1696 (School Dance):

There are n boys and m girls in a school. Next week a school dance will be organized. A dance pair consists of a boy and a girl, and there are k potential pairs.

Your task is to find out the maximum number of dance pairs and show how this number can be achieved.

Input

The first input line has three integers n , m and k : the number of boys, girls, and potential pairs. The boys are numbered $1, 2, \dots, n$, and the girls are numbered $1, 2, \dots, m$.

After this, there are k lines describing the potential pairs. Each line has two integers a and b : boy a and girl b are willing to dance together.

Output

First print one integer r : the maximum number of dance pairs. After this, print r lines describing the pairs. You can print any valid solution.

```

1 int main()
2 {
3     fastIO();
```

```

4
5     int n, m, k, a, b;
6     ll maxPairs;
7     vii pairs;
8
9     cin >> n >> m >> k;
10
11    Dinic flow(n+m+2,0,n+m+1);
12
13    fori(boy,0,n+1)
14        flow.addEdge(0,boy,1);
15
16    fori(girl,n+1,n+m+1)
17        flow.addEdge(girl,n+m+1,1);
18
19    fori(i,0,k)
20    {
21        cin >> a >> b;
22        flow.addEdge(a,n+b,1);
23    }
24
25    maxPairs = flow.maxFlow();
26    pairs = flow.maximumMatching();
27
28    cout << maxPairs << endl;
29    fori(i,0,sz(pairs))
30        cout << pairs[i].fi << " " << (pairs[i].se -
n) << endl;;

```

```

31
32     return 0;
33 }
```

Listing 28: Main method for solving CSES 1696 School Dancing using MaxFlow template.

5.15.2 *Ford-Fulkerson Algorithm

5.15.3 *Goldber-Tarjan Algorithm

6 Trees

6.1 Counting Childrens

```

1 vi childrens(n+1,0);
2 vvi graph(n+1);
3 vector<bool> visited(n+1, false);
4 fori(i,2,n+1) {
5     cin >> tmp;
6     graph[tmp].pb(i);
7     graph[i].pb(tmp);
8 }
9 function<int(int)> dfs = [&](int u) -> int {
10    visited[u] = true;
11    for(int v : graph[u]) {
12        if( !visited[v] )
13            childrens[u] += dfs(v);
14    }
15    return childrens[u] + 1;
16};
```

17 | `dfs(1);`

Listing 29: Algorithm that counts how many children does every node have, from 2..n in a rooted tree (root = 1).

- 6.2 *Tree Diameter
- 6.3 *Centroid Decomposition
- 6.4 *Euler Tour
- 6.5 *Lowest Common Ancestor (LCA)
- 6.6 *Heavy-Light Decomposition (HLD)

7 Strings

- 7.1 *Knuth-Morris-Pratt Algorithm
- 7.2 *Suffix Array
- 7.3 *Rolling Hashing
- 7.4 *Z Function
- 7.5 *Aho-Corasick Algorithm

8 Dynamic Programming

- 8.1 *Coins
- 8.2 *Longest Increasing Subsequence (LIS)
- 8.3 *Edit Distance
- 8.4 *Knapsack
- 8.5 *SOS DP
- 8.6 *Digit DP
- 8.7 *Bitmask DP

9 Mathematics

- 9.1 Number Theory
 - 9.1.1 Greatest Common Divisor (GCD)

```

1 int gcd(int a, int b) {
2     if (a == 0) return b;
3     if (b == 0) return a;
4     if (a == b) return a;
5     if (a > b)
6         return gcd(a - b, b);
7     return gcd(a, b - a);
8 }
```

Listing 30: Implementation of handmade GCD, because using `gcd()` runs slow with long long, also `_gcd()`.

9.1.2 Gauss Sum

The sum of the first n natural numbers in $O(1)$.

$$S = \frac{n(n+1)}{2} \quad (1)$$

$$n = \sqrt{2S + \frac{1}{4}} - \frac{1}{2} \quad (2)$$

```

1 int S = (1LL * n * (1LL * n + 1LL))/2;
2 int n = (int)(sqrt( 2 * S + 0.25 ) - 0.5 )
```

Listing 31: Implementation of the Gauss Sum.

- 9.1.3 *Modular Theory
- 9.1.4 *Modulo Inverse
- 9.1.5 *Fermat's Little Theorem
- 9.1.6 *Chinese Remainder Theorem
- 9.1.7 Binpow

```

1 const int MOD = 1e9+7;
2 int binpow( long long a, long long b ) { // a^b
3     long long sol = 1;
4     a %= MOD;
5     while( b > 0 ) {
6         if( b & 1 )
7             sol = ( 1LL * sol * a ) % MOD;
8         a = ( 1LL * a * a ) % MOD;
9         b >>= 1;
10    }
11    return sol % MOD;
12 }
```

Listing 32: Applying binary exponentiation to a problem requiring $a^b \bmod(10^9 + 7)$ in $O(\log_2(b))$.

9.1.8 Matrix Exponentiation (Linear Recurrency)

```

1 template <typename T> void matmul(vector<vector<T>>
2 &a, const vector<vector<T>>& b) {
3     size_t n = a.size(), m = a[0].size(), p = b[0].
4     size();
5     assert(m == b.size());
```

```
4     vector<vector<T>> c(n, vector<T>(p));
5     for(size_t i = 0; i < n; i++)
6         for(size_t j = 0; j < p; j++)
7             for(size_t k = 0; k < m; k++)
8                 c[i][j] = (c[i][j] + a[i][k] * b[k][j])
9                 % MOD;
10    a = c;
11 }
12 template <typename T> struct Matrix {
13     vector<vector<T>> mat;
14     Matrix() {}
15     Matrix(vector<vector<T>> a) { mat = a; }
16     Matrix(int n, int m) {
17         mat.resize(n);
18         for(int i = 0; i < n; i++) {mat[i].resize(m);}
19     }
20     int rows() const { return mat.size(); }
21     int cols() const { return mat[0].size(); }
22     void makeIden() {
23         for(int i = 0; i < rows(); i++)
24             for(int j = 0; j < cols(); j++)
25                 mat[i][j] = (i == j ? 1 : 0);
26     }
27     Matrix operator*=(const Matrix &b) {
28         matmul(mat, b.mat);
29         return *this;
30     }
31     void print() {
32         for(int i = 0; i < rows(); i++) {
33             for(int j = 0; j < cols(); j++)
34                 cout << mat[i][j] << " ";
35         }
36     }
37     Matrix operator*(const Matrix &b) { return Matrix
38         (*this) *= b; }
39 }
40 int main() {
41     Matrix<ll> A( {{1,1},{1,0}} );
42     Matrix<ll> ini(2,1);
43     ini.mat[0][0] = 0;
44     ini.mat[1][0] = 1;
45     Matrix<ll> iden(2,2);
46     iden.makeIden();
47     ll n;
48     cin >> n;
49     while(n > 0) {
50         if( n & 1 ) iden *= A;
51         A *= A;
52         n >>= 1;
53     }
54     Matrix<ll> res = iden * ini;
55     cout << res.mat[0][0] << endl;
56 }
```

Listing 33: Template to pow a matrix of size n to a certain exponent with logarithmic time (using binpow), and multiply it to another matrix, with modulo operation, as well as how to use it. Full implementation for calculating n -th Fibonacci term with linear recurrency.

9.1.9 Prime checking

```

1 bool prime( int n ){
2     if( n == 2 )
3         return true;
4     if( n % 2 == 0 || n <= 1 )
5         return false;
6     for( int i = 3; i * i <= n; i += 2 )
7         if( ( n % i ) == 0 )
8             return false;
9     return true;
10 }
```

Listing 34: Returns if n is a prime number in $O(\sqrt{n})$. Avoids overflow $\forall n \leq 10^6$ ($\approx INT_MAX$).

9.1.10 Prime factorization

```

1 void prime_factorization(vll& factorization, ll n) {
2     for(long long d = 2; d*d <= n; d++) {
3         while(n % d == 0) {
4             factorization.push_back(d);
5             n /= d;
```

```

6         }
7     }
8     if( n > 1 )
9         factorization.push_back(n);
10 }
```

Listing 35: Returns prime factorization of the number n using *trial division*, simplest way. Runs in $O(\sqrt{n})$. e.g. for 12 the result is 2x2x3.

9.1.11 Sieve of Eratosthenes

```

1 void sieve_of_eratosthenes(vector<bool>& is_prime,
2                             int n) {
3     is_prime.assign(n+1,true);
4     is_prime[0] = is_prime[1] = false;
5     for(int i = 2; i <= n; i++) {
6         if( is_prime[i] && (long long)i * i <= n ) {
7             for(int j = i*i; j <= n; j += i)
8                 is_prime[j] = false;
9         }
10 }
```

Listing 36: Calculates every prime number up to n with sieve of eratosthenes in a boolean 1-indexed vector. Runs in $O(n \log \log n)$.

9.1.12 Sum of Divisors

```

1 ll sum_of_divisors(ll n) {
2     ll sum = 1;
```

```

3   for (long long i = 2; i * i <= n; i++) {
4     if(n % i == 0) {
5       int e = 0;
6       do {
7         e++;
8         n /= i;
9       } while (n % i == 0);
10    ll s = 0, pow = 1;
11    do {
12      s += pow;
13      pow *= i;
14    } while (e-- > 0);
15    sum *= s;
16  }
17 }
18 if(n > 1)
19   sum *= (1 + n);
20 return sum;
21 }
```

Listing 37: Calculates the sum of all divisors of number n . e.g.
 $\text{sum_of_divisors}(12) = 18$. Runs in $O(\sqrt{n})$.

```

1 void sum_of_divisors_sieve( vll& sigma, int n ) {
2   sigma.assign(n+1,0);
3   for(int i = 1; i <= n; i++)
4     for(int j = i; j <= n; j+=i)
5       sigma[j] += i;
6 }
```

Listing 38: Calculates the sum of all divisors of all numbers from 1 to n . Runs in $O(n \log(n))$.

9.2 Combinatorics

9.2.1 Binomial Coefficients

```

1 const int MAXN = 1e6+1;
2 vll fact(MAXN+1), inv(MAXN+1);
3 int binpow( ll a, ll b ) { // a^b
4   ll sol = 1;
5   a %= MOD;
6   while( b > 0 ) {
7     if( b & 1 )
8       sol = ( 1LL * sol * a ) % MOD;
9     a = ( 1LL * a * a ) % MOD;
10    b >>= 1;
11  }
12  return sol % MOD;
13 }
14 void combi() {
15   fact[0] = inv[0] = 1;
16   for(i,1,MAXN+1) {
17     fact[i] = fact[i-1] * i % MOD;
18     inv[i] = binpow( fact[i], MOD - 2 );
19   }
20 }
21 ll nCr( ll n, ll r ) {
```

```

22     return fact[n] * inv[r] % MOD * inv[n-r] % MOD;
23 }
24 combi();
25 nCr(a, b);

```

Listing 39: Template for calculating binomial coefficients $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Precalculate *fact* and *inv* runs in $O(MAXN \cdot \log_2(MOD))$ ($\log_2(MOD) \approx 30$).

So, in general case when $NMAX = 10^6$ and $MOD = 10^9 + 7$ can be generalized to $O(n \cdot \log(n))$, $n \leq 10^6$.

9.2.2 Common combinatorics formulas

$$\binom{n}{2} = \frac{n(n-1)}{2} \quad (3)$$

$$\sum_{k=0}^n \binom{n}{k} = 2^n \quad (4)$$

$$\sum_{k=0}^n \binom{n}{k} \binom{n}{n-k} = \binom{2n}{n} \quad (5)$$

$$\sum_{k=0}^n k \binom{n}{k} = n2^{n-1} \quad (6)$$

$$\sum_{k=0}^{\infty} \binom{2k}{k} \binom{2n-2k}{n-k} = 4^n \quad (7)$$

(8)

9.3 *Stars and Bars

9.4 Probability

9.5 Computational Geometry

9.5.1 *Cross Product

9.5.2 *Convex Hull

9.6 *Fast Fourier Transform (FFT)

10 Appendix

10.1 What to do against WA?

1. Have you done the correct complexity analysis?
2. Have you understood well the statement?
3. Have you corroborated yet the trivial test cases?
4. Have you checked all the corner cases?
5. Have you proposed a lot of non-trivial test cases?
6. Isn't there any possibility of overflow? (Multiplying two `int` needs to be fitted into a `long long`)
7. Have you done a desktop test?
8. Have you red all the variables? (`tc` variable on `main`)
9. Every part of your code works as it's meant to?

10.2 Primitive sizes

Data type	[B]	Minimun value it takes	Maximum value it takes
bool	1	0	1
signed char	1	0	255
unsigned char	1	-128	127
signed int	4	$-2,147,483,648 \approx -2 \times 10^9$	$2,147,483,647 \approx 2 \times 10^9$
unsigned int	4	0	$4,294,967,295 \approx 4 \times 10^9$
signed short	2	-32,768	32,767
unsigned short	2	0	65,535
signed long long int	8	$-9,223,372,036,854,775,808 \approx -9 \times 10^{18}$	$9,223,372,036,854,775,807 \approx 9 \times 10^{18}$
unsigned long long int	8	0	$18,446,744,073,709,551,615 \approx 18 \times 10^{18}$
float	4	1.1×10^{-38}	3.4×10^{38}
double	8	2.2×10^{-308}	1.7×10^{308}
long double	12	3.3×10^{-4932}	1.1×10^{4932}

Table 1: Capacity of primitive data types in C++.

10.3 *ASCII table

10.4 *Numbers bit representation

10.5 How a `vector<vector<pair<int,int>>` looks like

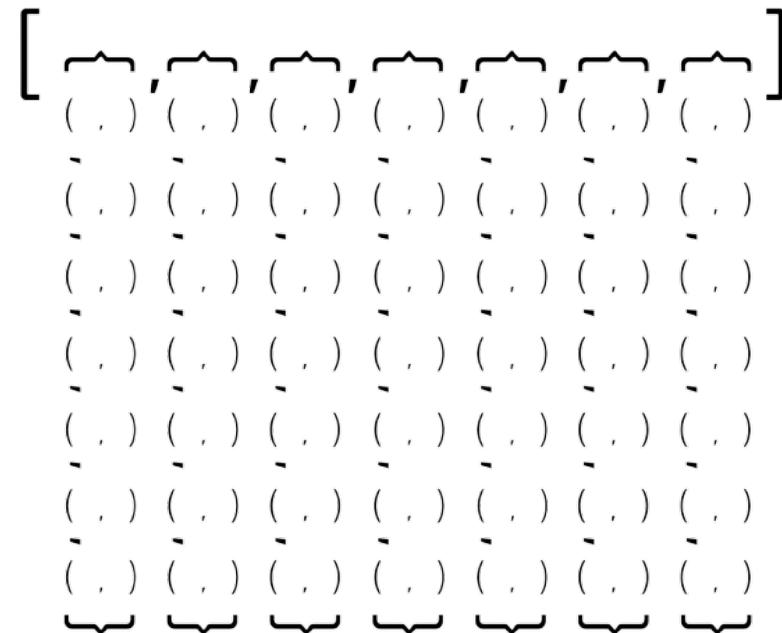


Figure 1: Visual representation of a vector of vector of pairs.

10.6 How all neighbours of a grid looks like

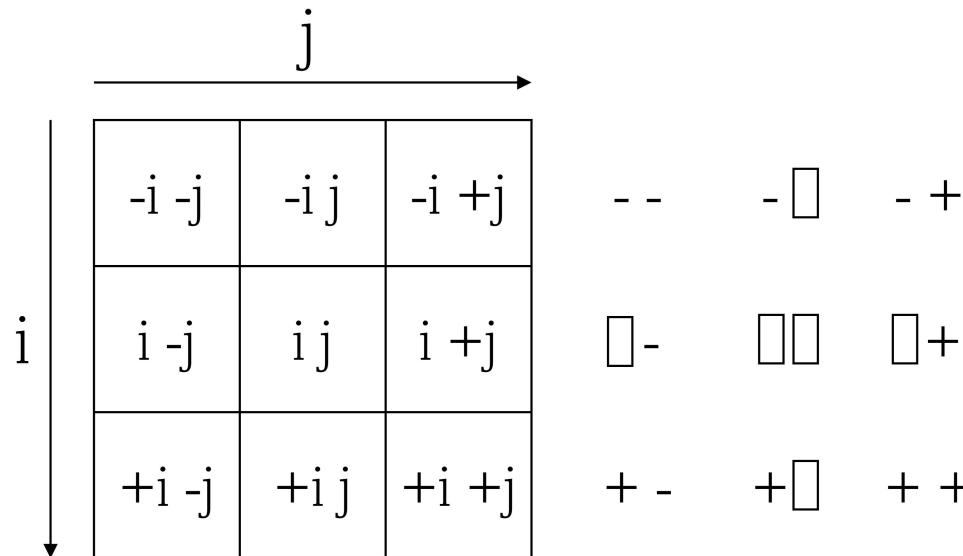


Figure 2: Visual representation of how all adjacent cells in a grid looks like.