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# ICPC 2026 Reference

*CPCFI UNAM*

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RacsoFractal, zum, edwardsal17

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# 1 Template

```

1 // Racso programmed here
2 #include <bits/stdc++.h>
3 using namespace std;
4 typedef long long ll;
5 typedef long double ld;
6 typedef __int128 sll;
7 typedef pair<int,int> ii;
8 typedef pair<ll,ll> pll;
9 typedef vector<int> vi;
10 typedef vector<ll> vll;
11 typedef vector<ii> vii;
12 typedef vector<vi> vvi;
13 typedef vector<vii> vvii;
14 typedef vector<pll> vpll;
15 typedef unsigned int uint;
16 typedef unsigned long long ull;
17 #define fi first
18 #define se second
19 #define pb push_back
20 #define all(v) v.begin(),v.end()
21 #define rall(v) v.rbegin(),v.rend()
22 #define sz(a) (int)(a.size())
23 #define fori(i,a,n) for(int i = a; i < n; i++)
24 #define endl '\n'
25 const int MOD = 1e9+7;
26 const int INFTY = INT_MAX;

```

```

27 const long long LLINF = LLONG_MAX;
28 const double EPS = DBL_EPSILON;
29 void printVector( auto& v ){ fori(i,0,sz(v)) cout << v[i] << " "; cout << endl; }
30 void fastIO() { ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0); }

```

Listing 1: Racso's template.

## 2 C++ Sintaxis

### 2.1 Compilation sentences

To compile and execute in C++:

```

g++ -Wall -o solucion.exe solucion.cpp
g++ -std=c++20 -Wall -o main a.cpp ./solucion.exe < input.txt > output.txt

```

To compile and execute in Java:

```

javac -Xlint Solucion.java
java Solucion < input.txt > output.txt

```

To execute in Python (two options):

```

python3 solucion.py < input.txt > output.txt
pypy3 solucion.py < input.txt > output.txt

```

### 2.2 Custom comparators

```

1 // Using function
2 bool cmpFunction(const pair<int,int> &a, const pair<
3   int,int> &b) {
4   return a.second < b.second;
}

```

```

4 }
5 sort(all(v), cmpFunction);
6 // Using functor
7 struct CmpFunctor {
8     bool operator()(const pair<int,int> &a, const pair
9         <int,int> &b) const {
10        return a.second < b.second;
11    }
12 };
13 sort(all(v), CmpFunctor());
14 // Using lambda function
15 sort(all(v), [](const pair<int,int> &a, const pair<
16        int,int> &b) {
17        return a.second < b.second;
18 });

```

Listing 2: Template for custom sorting, using as example ordering a vii ascending by second element.

### 2.3 \*STL DS Usage

### 2.4 \*Strings methods

### 2.5 Pragmas

```

1 #pragma GCC optimize("O3")
2 #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
3 #pragma GCC optimize("unroll-loops")

```

Listing 3: Common pragmas.

## 2.6 Bit Manipulation Cheat Sheet

### Bitwise operators:

- **&** (AND): Sets each bit to 1 if both bits are 1.
- **|** (OR): Sets each bit to 1 if at least one bit is 1.
- **$\wedge$**  (XOR): Sets each bit to 1 if bits are different.
- **$\sim$**  (NOT): Inverts all bits.
- **$\ll$**  (Left shift): Shifts bits left, fills with 0.
- **$\gg$**  (Right shift): Shifts bits right.

### Basic Bit Tasks:

- Get bit:  $(n \& (1 \ll i)) != 0$
- Set bit:  $n | (1 \ll i)$
- Clear bit:  $n \& \sim(1 \ll i)$
- Toggle bit:  $n \wedge (1 \ll i)$
- Clear LSB:  $n \& (n - 1)$
- Get LSB:  $n \& -n$

### Set Operations:

- Subset check:  $(A \& B) == B$
- Set union:  $A | B$
- Set intersection:  $A \& B$
- Set difference:  $A \& \sim B$
- Toggle subset:  $A \hat{B}$

### Equations:

Properties of bitwise:

- $a | b = a \oplus b + a \& b$

- $a \oplus (a \& b) = (a \mid b) \oplus b$
- $b \oplus (a \& b) = (a \mid b) \oplus a$
- $(a \& b) \oplus (a \mid b) = a \oplus b$

In addition and subtraction:

- $a + b = (a \mid b) + (a \& b)$
- $a + b = a \oplus b + 2(a \& b)$
- $a - b = (a \oplus (a \& b)) - ((a \mid b) \oplus a)$
- $a - b = ((a \mid b) \oplus b) - ((a \mid b) \oplus a)$
- $a - b = (a \oplus (a \& b)) - (b \oplus (a \& b))$
- $a - b = ((a \mid b) \oplus b) - (b \oplus (a \& b))$

**Gray code:**  $G = B \oplus (B \gg 1)$

**C++ built in functions:**

- `__builtin_popcount(x)` - Count the number of set bits in x.
- `__builtin_clz(x)` - Count the number of leading zeros in x.
- `__builtin_ctz(x)` - Count the number of trailing zeros in x.

## 2.7 Comparing Floats

```

1 long double a, b, EPS = 1e-9;
2 if( abs(a - b) < EPS ) {
3     // 'a' equals 'b'
4 }
```

Listing 4: Check if two real numbers are equal using an epsilon scope.

## 2.8 Ceil

$$\lceil \frac{a}{b} \rceil = \frac{a+b-1}{b}$$

(Originally I thought  $\lceil \frac{a}{b} \rceil = \frac{a-1}{b} + 1$ , but this calculates the wrong way in the cases where  $a = 0$ )

```

1 int myCeil(long long a, long long b) {
2     return (a + b - 1)/b;
3 }
```

Listing 5: A way to do ceil operation between integers without making explicit conversions. `a` and `b` are `int`, but the operation `a+b-1` can cause an overflow, so they must be casted into `long long` to avoid this. The result must be `int` anyway.

## 3 Miscellaneous

### 3.1 Binary Search

```

1 int binary_search( vector<int>& list, int n, int
2 target ) {
3     int x0 = 0, x1 = n-1, mid;
4     while( x0 <= x1 ) {
5         mid = (x0 + x1) / 2; // (x1 - x0) / 2 + x0;
6         if( list[mid] == target )
7             return mid;
8         list[mid] < target ? x0 = mid + 1 : x1 = mid
9             - 1;
10    }
11    return -1;
12 }
```

Listing 6: Classic (Vanilla) implementation of Binary Search. Returns the index where `target` was found. Binary Search works on  $O(\log_2(n))$ , let  $n$  be the size of the container.

```

1 int binary_search( vector<int>& list, int n, int
2     target ) {
3
4     int ans = 0;
5
6     function<bool(int)> check = [&](int idx)->bool {
7
8         return idx < n && list[idx] <= target;
9     };
10
11    for( int i = 31; i >= 0; i-- ) {
12        if( check( ans + (1 << i) ) )
13            ans += 1 << i;
14    }
15
16    return list[ans] == target ? ans : -1;
17 }
```

Listing 7: Logarithmic jumps implementation of Binary Search. Returns the index where `target` was found. Binary Search works on  $O(\log_2(n))$ , let  $n$  be the size of the container.

```

1 int binary_search( vector<int>& list, int n, int
2     target ) {
3
4     int left = 1, right = n + 1, mid;
5
6     while(right - left >= 1) {
7
8         mid = left + (right - left) / 2;
9
10        if( list[mid] >= target && target > list[mid -
11]) {
```

```

6         //----- TODO logic here -----//
7         break;
8     }
9     else
10    if( list[mid] < target )
11        left = mid + 1;
12    else
13        right = mid;
14 }
15 }
```

Listing 8: Binary Search implementation for searching an element in the interval  $(numbers_{i-1}, numbers_i]$ . Originally used on Codeforces problem 474 - B (Worms). Returns the index where `target` was found. Binary Search works on  $O(\log_2(n))$ , let  $n$  be the size of the container.

```

1 function<bool(ll)> check = [&](ll t) -> bool {
2
3     ll products = 0;
4
5     for(ll machine : v) {
6
7         products += t / machine;
8
9         if( products >= target )
10            return true;
11     }
12
13     return products >= target;
14 }
15
16 for(int i = 0; i < 70; i++) {
17
18     mid = (x0 + x1) / 2;
19
20     check(mid) ? x1 = mid : x0 = mid + 1;
21 }
```

Listing 9: Implementation of Binary Search in the Answer. Originally used on CSES problem *Factory Machines*. Binary Search works on  $O(\log_2(n))$ , let  $n$  be the size of the container.

### 3.1.1 \*Parallel Binary Search

### 3.1.2 \*Ternary Search

## 3.2 Kadane's Algorithm

```

1 arns = v[0], maxSum = 0;
2 for(i,0,n)
{
3     maxSum += v[i];
4     arns = max(arns, maxSum);
5     maxSum = max(0LL, maxSum);
6 }
7

```

Listing 10: Uses Kadane's Algorithm to find maximum subarray sum in  $O(n)$ .

## 4 Queries

### 4.1 Prefix Sum 2D

```

1 for(int i = 1; i <= n; i++)
2     for(int j = 1; j <= n; j++)
3     {
4         prefix[i][j] = prefix[i][j-1] + prefix[i-1][j] -
5             prefix[i-1][j-1];
6         prefix[i][j] += forest[i-1][j-1] == '*' ? 1 : 0;
7

```

```

6     }
7     for(int i = 0; i < q; i++)
8     {
9         pair<int,int> p1, p2;
10        cin >> p1.fi >> p1.se >> p2.fi >> p2.se;
11        int arns = prefix[p2.fi][p2.se];
12        arns -= prefix[p2.fi][p1.se-1] + prefix[p1.fi-1][
13            p2.se];
14        arns += prefix[p1.fi-1][p1.se-1];
15        cout << arns << endl;
16    }
17

```

Listing 11: Construction and querie of how many 1's are there in a matrix. Originally used on *Forest Queries* from CSES.

### 4.2 \*Sparse Table

### 4.3 \*Sqrt Decomposition

### 4.4 \*Fenwick Tree

### 4.5 \*Fenwick Tree 2D

### 4.6 Segment Tree

```

1 typedef long long ll;
2 typedef vector<ll> vll;
3 typedef vector<int> vi;
4 const int INF = INT_MAX;
5 class Segment_tree {
6     public: vll t;
7     Segment_tree( int n = 1e5+10 ) {

```

```

8     t.assign(n*4,INF);
9 }
10 void update(int node, int index, int tl, int tr,
11   int val) {
12   if( tr < index || tl > index ) return;
13   if( tr == tl ) t[node] = val;
14   else {
15     int mid = tl + ((tr-tl)>>1);
16     int lft = node << 1;
17     int rght = lft + 1;
18     update(lft,index,tl,mid,val);
19     update(rght,index,mid+1,tr,val);
20     t[node] = min(t[lft],t[rght]);
21   }
22 }
23 ll query(int node, int l, int r, int tl, int tr)
24 {
25   if( tl > r || tr < l ) return INF;
26   if( tl >= l and tr <= r ) return t[node];
27   else {
28     int mid = tl + ((tr-tl)>>1);
29     int lft = node << 1;
30     int rght = lft + 1;
31     ll q1 = query(lft,l,r,tl,mid);
32     ll q2 = query(rght,l,r,mid+1,tr);
33     return min(q1,q2);
34   }
35 }
36 void build(vi &v, int node, int tl, int tr) {
37   if( tl == tr ) t[node] = v[tl];
38   else {
39     int mid = tl + ((tr-tl)>>1);
40     int lft = node << 1;
41     int rght = lft + 1;
42     build(v,lft,tl,mid);
43     build(v,rght,mid+1,tr);
44     t[node] = min(t[lft],t[rght]);
45   }
46 }
47 Segment_tree st(n);
48 st.build(v,1,0,n-1);
49 st.update(1,a-1,0,n-1,b);
50 st.query(1,a-1,b-1,0,n-1));

```

Listing 12: Segment Tree for Dynamic Range **Minimum** Queries. Racso's Implementation.

```

1 vector<long long> v, sex;
2 int n;
3 void build(int node, int l, int r){
4   if(l == r) sex[node] = v[l];
5   else{
6     int mid = (l+r)/2;
7     build(2*node, l, mid);
8     build(2*node + 1, mid+1, r);
9     sex[node] = sex[2*node] + sex[2*node + 1];

```

```

10    }
11 }
12 void update(int node, int l, int r, int idx, int val
13 ) {
14     if(l == r) {
15         v[idx] = val;
16         sex[node] = val;
17     }
18     else{
19         int mid = (l+r)/2;
20         if(l <= idx && idx <= mid) update(2*node, l,
21         mid, idx, val);
22         else update(2*node +1, mid+1, r, idx, val);
23         sex[node] = sex[2*node] + sex[2*node + 1];
24     }
25 }
26 int query(int node, int tl, int tr, int l, int r){
27     if(r < tl || tr < l) return 0;
28     if(l <= tl && tr <= r) return sex[node];
29     int tm = (tl+tr)/2;
30     return query(2*node, tl, tm, l, r) + query(2*
31     node +1, tm+1, tr, l, r);
32 }
33 v.resize(n);
34 sex.resize(4 * n);
35 build(1, 0, n - 1);
36 query(1, 0, n-1, l - 1, r - 1)

```

Listing 13: Segment Tree for Dynamic Range **Sum** Queries. Zum's Implementation.

## 4.6.1 \*2D Segment Tree

## 4.6.2 \*Persistent Segment Tree

## 4.6.3 \*Lazy Propagation

## 4.7 Ordered Set

```

1 //<-- Header.
2 #include <bits/stdc++.h>
3 #include <ext/pb_ds/assoc_container.hpp>
4 #include <ext/pb_ds/tree_policy.hpp>
5 using namespace std;
6 using namespace __gnu_pbds;
7 template<typename T, typename Cmp = less<T>>
8 using ordered_set = tree<T,null_type,Cmp,rb_tree_tag
9 ,tree_order_statistics_node_update>;
10 //<-- Declaration.
11 ordered_set<int> oset;
12 //<-- Methods usage.
13 // K-th element in a set (counting from zero).
14 ordered_set<int>::iterator it = oset.find_by_order(k
15 );
16 // Number of items strictly smaller than k.
17 ordered_set<int>::iterator it = oset.order_of_key(k
18 );
19 // Every other std::set method.

```

Listing 14: Ordered set necessary includes in header, declaration of the object, and usage of its new methods.

#### 4.7.1 Multi-Ordered Set

```

1 //<-- Header.
2 #include <bits/stdc++.h>
3 #include <ext/pb_ds/assoc_container.hpp>
4 #include <ext/pb_ds/tree_policy.hpp>
5 using namespace std;
6 using namespace __gnu_pbds;
7 template<typename T, typename Cmp = less<T>>
8 using ordered_set = tree<T,null_type,Cmp,rb_tree_tag
    ,tree_order_statistics_node_update>;
9 //<-- Use in main.
10 ordered_set<pair<int,int>> multi_oiset;
11 map<int,int> cuenta;
12 function<void(int)> insertar = [&](int val) -> void
    {
13     multi_oiset.insert({val,++cuenta[val]});
14 };
15 function<void(int)> eliminar = [&](int val) -> void
    {
16     multi_oiset.erase({val,cuenta[val]--});
17 };

```

Listing 15: Declaration of multi-oiset structure.

#### 4.8 \*Treap

#### 4.9 \*Trie

## 5 Graph Theory

### 5.1 Breadth-First Search (BFS)

```

1 vector<vector<int>> graph;
2 vector<bool> visited;
3 graph.assign(n, vector<int>()); // <-- main
4 visited.assign(n, false); // <-- main
5
6 void bfs( int s ) {
7     queue<int> q;
8     q.push( s );
9     visited[ s ] = true;
10    while( ! q.empty() ) {
11        int u = q.front();
12        q.pop();
13        for( auto v : graph[ u ] ) {
14            if( ! visited[ u ] ) {
15                visited[ u ] = true;
16                q.push( v );
17                // --- ToDo logic here ---
18            }
19        }
20    }
21    return;
22 }

```

Listing 16: Iterative implementation of BFS graph traversal over a graph represented as an AdjacencyList on vector of vectors. BFS runs in  $O(|V| + |E|)$ .

```

1 int n, m;
2 string arns = "";
3 cin >> n >> m;
4 vector<vector<bool>> visited(n, vector<bool>(m, false));
5 vector<string> path(n, string(m, '0'));
6 vector<string> grid(n);
7 vii dirs = {{0,1},{0,-1},{1,0}, {-1,0}};
8 string commands = "LRUD";
9 queue<ii> q;
10 ii start, end, curr;
11 function<bool(int,int)> valid = [&](int i, int j) ->
12     bool {
13     return ( i >= 0 && i < n && j >= 0 && j < m &&
14     grid[i][j] != '#' && ! visited[i][j] );
15 };
16 fori(i,0,n) cin >> grid[i];
17 fori(i,0,n) {
18     fori(j,0,m)
19         if( grid[i][j] == 'A' ) {
20             visited[i][j] = true;
21             q.push( {i,j} );
22         }
23     }

```

```

22     while( ! q.empty() ) {
23         curr = q.front();
24         q.pop();
25         int i = curr.fi;
26         int j = curr.se;
27         if( grid[i][j] == 'B' ) {
28             end.fi = i;
29             end.se = j;
30             break;
31         }
32         int newI, newJ;
33         fori(I,0,4) {
34             newI = i + dirs[I].fi;
35             newJ = j + dirs[I].se;
36             if( valid(newI,newJ) ) {
37                 visited[newI][newJ] = true;
38                 q.push( {newI,newJ} );
39                 path[newI][newJ] = commands[I];
40             }
41         }
42     }
43     while( path[ end.fi ][ end.se ] != '0' ) {
44         fori(i,0,4) {
45             if( path[ end.fi ][ end.se ] == commands[i] )
46             {
47                 arns += i & 1 ? commands[i-1] : commands[i+1];
48                 end.fi -= dirs[i].fi;
49             }
50         }
51     }
52 }

```

```

48         end.se -= dirs[i].se;
49     }
50 }
51 reverse(all(arnts));
52 if( arnts == "" ) cout << "NO" << endl;
53 else cout << "YES" << endl << arnts.size() << endl <<
54     arnts << endl;

```

Listing 17: BFS on Grid to find shortest path from an starting point *A* to an end *B*. Once the path is found, it reconstruct it with movements *LRUD*. Works in  $O(n \cdot m)$ . Originally used on problem *Labyrinth* from CSES.

## 5.2 Deep-First Search (DFS)

```

1 vector<vector<int>> graph;
2 vector<bool> visited;
3 graph.assign(n, vector<int>()); // <--- main
4 visited.assign(n, false); // <--- main
5
6 void dfs( int s ) {
7     if( visited[s] == true ) return;
8     visited[s] = true;
9     vector<int>::iterator i;
10    for( i = graph[s].begin(); i < graph[s].end();
11        ++i ) {
12        if( ! visited[*i] ) {
13            // --- ToDo logic here ---
14            dfs(*i);
15        }
16    }
17}

```

```

14     }
15 }
16 }

```

Listing 18: Recursive implementation of DFS graph traversal over a graph represented as a AdjacencyList on vector of vectors. DFS runs in  $O(|V| + |E|)$ .

```

1 vector<vector<int>> graph;
2 vector<bool> visited;
3 void dfs( int s ) {
4     stack<int> stk;
5     stk.push(s);
6     while ( !stk.empty() ) {
7         int u = stk.top();
8         stk.pop();
9         if ( visited[u] ) continue;
10        visited[u] = true;
11        // --- ToDo logic here ---
12        for( auto it = graph[u].rbegin(); it != graph
13            [u].rend(); ++it)
14            if ( !visited[*it])
15                stk.push(*it);
16    }
17}

```

Listing 19: Iterative implementation of DFS graph traversal over a graph represented as a AdjacencyList on vector of vectors. DFS runs in  $O(|V| + |E|)$ .

```

1 typedef long long ll;
2 vector<vector<ll>> adj;

```

```
3 vector<bool> visited;
4 function<void(ll)> dfs = [&](ll u) -> void {
5     if( visited[u] ) return;
6     visited[u] = true;
7     for( ll v : adj[u] )
8         dfs(v);
9 };
10 dfs(n);
```

Listing 20: DFS implementation with a lambda function (adjacency list and visited don't need to be passed thorough argument). DFS runs in  $O(|V| + |E|)$ .

```
1  typedef long long ll;
2  typedef vector<ll> vll;
3  map<ll, vll> adj;
4  set<ll> visited;
5  function<void(ll)> dfs = [&](ll u) -> void {
6      if( visited.count(u) ) return;
7      visited.insert(u);
8      for( ll v : adj[u] )
9          dfs(v);
10 };
11 dfs(n);
```

Listing 21: DFS implementation with a lambda function implemented with a map instead of vector of vectors, and a set to track visited nodes. DFS runs in  $O(|V| + |E|)$ .

### 5.3 Shortest Path

### 5.3.1 Dijkstra's Algorithm

```
1  typedef long long ll;
2  typedef pair<ll, ll> pll;
3
4  vector<vector<ll>> graph;
5  vector<ll> visited;
6  graph.assign(n, vector<ll>() );
7  visited.assign(n, false);
8
9  vector<ll> dijkstra( int n, int source, vector<
10   vector<pll>> &graph ) {
11    vector<ll> dist( n, INFTY );
12    priority_queue<pll, vector<pll>, greater<pll>> pq;
13    dist[ source ] = 0;
14    pq.push( {0, source} );
15    while( ! pq.empty() ) {
16      ll d = pq.top().first;
17      ll u = pq.top().second;
18      pq.pop();
19      if( d > dist[ u ] ) continue;
20      for( auto &edge : graph[ u ] ) {
21        ll v = edge.first;
22        ll weight = edge.second;
23        if( dist[ u ] + weight < dist[ v ] ) {
24          dist[ v ] = dist[ u ] + weight;
25          pq.push( {dist[ v ], v} );
26        }
27      }
28    }
29  }
```

```

25     }
26 }
27
28 return dist;
29 }
```

Listing 22: Iterative implementation of Dijkstra's Algorithm for shortest path over a graph represented as a AdjacencyList on vector of vectors. Returns a vector with the shortest path to every other vertex in the graph.  $O(|E| \times \log_2(|V|))$ . doesn't work with negative weights.

```

1 vvp11 graph(n+1,vpll());
2 vector<bool> visited(n+1, false);
3 function<vll(int)> dijkstra = [&](int source) -> vll
4 {
5     vll dist(n+1, INF);
6     priority_queue<pll,vpll,greater<pll>> pq;
7     dist[source] = 0;
8     pq.push({0,source});
9     while( ! pq.empty() ) {
10         ll d = pq.top().fi;
11         ll u = pq.top().se;
12         pq.pop();
13         if( d > dist[u] ) continue;
14         for(pll edge : graph[u]) {
15             ll v = edge.fi;
16             ll w = edge.se;
17             if( dist[u] + w < dist[v] ) {
18                 dist[v] = dist[u] + w;
19                 pq.push({dist[v],v});
20             }
21         }
22     }
23     return dist;
24 };
```

```

18         pq.push({dist[v],v});
19     }
20 }
21
22 return dist;
23 };
```

Listing 23: Iterative implementation of Dijkstra's Algorithm as a Lambda Function for shortest path over a graph represented as a AdjacencyList on vector of vectors. Returns a vector with the shortest path to every other vertex in the graph.  $O(|E| \times \log_2(|V|))$ . Doesn't work with negative weights.

### 5.3.2 \*Floyd-Warshall's Algorithm

### 5.3.3 Bellman–Ford Algorithm

```

1 int V, E;
2 cin >> V >> E;
3 vvi edges(E,vi(3,0));
4
5 for(int i = 1; i <= E; i++)
6     cin >> edges[i][0] >> edges[i][1] >> edges[i][2];
7
8 function<vi(int)> bellman_ford = [&](int src) -> vi
9 {
10     vi dist(V,INF);
11     dist[src] = 0;
12     for(int i = 0; i < V; i++)
13     {
14         for( vi edge : edges )
15             if( dist[edge[0]] + edge[2] < dist[edge[1]] )
16                 dist[edge[1]] = dist[edge[0]] + edge[2];
17     }
18 }
```

```

15     {
16         int u = edge[0];
17         int v = edge[1];
18         int w = edge[2];
19         if( dist[u] != INF and dist[u] + w < dist[v]
20     )
21     {
22         if( i == V-1 )
23             return {-1};
24         dist[v] = dist[u] + w;
25     }
26 }
27 return dist;
28 };

```

Listing 24: Finds the shortest route from a source vertex, to every other one in the graph. Works over a list of edges. Runs in  $O(|V| \times |E|)$ . Can be used to find negative cycles.

```

2 vvi graph(V+1,vi());
3 vector<bool> visited(V+1,false);
4 function<void(int)> dfs = [&](int u) -> void {
5     visited[u] = true;
6     for(int v : graph[u])
7         if( ! visited[v] )
8             dfs(v);
9     topo.pb(u);
10 };
11 function<void()> topological_sort = [&]() -> void {
12     for(int i = 1; i <= V; i++)
13         if( ! visited[i] )
14             dfs(i);
15     reverse(all(topo));
16 };
17 topological_sort();
18 for(int i = 0; i < V; i++) cout << topo[i] << " ";

```

Listing 25: Recursive toposort implementation for unweighted DAG through vvi with DFS with inverted postorder. Runs in  $O(|V| \times |E|)$ .

## 5.4 Minimum Spanning Tree (MST)

### 5.4.1 \*Prim's Algorithm

### 5.4.2 \*Kruskal's Algorithm

## 5.5 \*Bipartite Checking

## 5.6 \*Negative Cycles

## 5.7 Topological Sort

```
1 vi topo;
```

d

```

1 vi indegree(V+1,0);
2 vvi graph(V+1,vi());
3 vector<bool> visited(V+1,false);
4 fori(i,0,E) {
5     graph[u].pb(v);
6     indegree[v]++;
7 }

```

```

8 function<vi()> topological_sort = [&]() -> vi {
9     vi order, deg = indegree; // copy
10    queue<int> q;
11    for(int i = 1; i <= V; i++) {
12        if( deg[i] == 0 )
13            q.push(i);
14    while( ! q.empty() ) {
15        int u = q.front(); q.pop();
16        order.pb(u);
17        for(int v : graph[u]) {
18            deg[v]--;
19            if(deg[v] == 0)
20                q.push(v);
21        }
22    }
23    return order;
24 };
25 vi topo = topological_sort();
26 if( (int)(topo.size()) != V ) cout << "IMPOSSIBLE"
27     << endl;

```

Listing 26: Kahn's Algorithm for Topological Sorting using BFS and indegree vertex analysis (nodes in a cycle will never have indegree zero). Works over unweighted directed graphs containing cycles through vvi. Runs in  $O(|V| \times |E|)$ .

## 5.8 Disjoint Set Union (DSU)

```

1 class DisjointSets {
2 private:

```

```

3     vector<int> parents;
4     public:
5         vector<int> sizes;
6         DisjointSets(int size) : parents(size), sizes(size
7             , 1) {
8             for (int i = 0; i < size; i++) { parents[i] = i;
9             }
10        }
11        /** @return the "representative" node in x's
12         * component */
13        int find( int x ) {
14            return parents[x] == x ? x : ( parents[x] =
15            find( parents[x] ) );
16        }
17        /** @return whether the merge changed connectivity
18         */
19        bool unite( int x, int y ) {
20            int x_root = find(x);
21            int y_root = find(y);
22            if (x_root == y_root) return false;
23            if ( sizes[x_root] < sizes[y_root] ) swap(x_root
24            ,y_root);
25            sizes[x_root] += sizes[y_root];
26            parents[y_root] = x_root;
27            return true;
28        }
29        /** @return whether x and y are in the same
30         * connected component */

```

```

24     bool connected( int x, int y ){
25         return find(x) == find(y);
26     }
27     void printLists() {
28         cout << "Printing parents..." << endl;
29         for(auto i : parents)
30             cout << i << " ";
31         cout << endl << "Printing sizes..." << endl;
32         for(auto i : sizes )
33             cout << i << " ";
34     }
35 };
36 DisjointSets dsu( V );
37 int number_of_components = V, largest_component = 1;
38 if( dsu.unite(x, y) ) {
39     largest_component = max( largest_component, dsu.
40         sizes[ dsu.find( x ) ] );
41     number_of_components--;
}

```

Listing 27: Template and usage of DSU with path compression. Complexity of  $O(m \alpha(n))$  for a sequence of  $m$  operations over  $n$  elements. Where  $\alpha$  denotes Ackerman function, where  $\alpha(n) \leq 4, \forall n \leq 10^{18}$ . Practically  $O(1)$ .

## 5.9 \*Condensation Graph

## 5.10 \*Strongly Connected Components (SCC)

## 5.11 2-SAT

```

1 struct TwoSatSolver {
2     int n_vars;
3     int n_vertices;
4     vector<vector<int>> adj, adj_t;
5     vector<bool> used;
6     vector<int> order, comp;
7     vector<bool> assignment;
8
9     TwoSatSolver(int _n_vars) : n_vars(_n_vars),
10      n_vertices(2 * n_vars), adj(n_vertices), adj_t(
11      n_vertices), used(n_vertices), order(), comp(
12      n_vertices, -1), assignment(n_vars) {
13         order.reserve(n_vertices);
14     }
15     void dfs1(int v) {
16         used[v] = true;
17         for (int u : adj[v]) {
18             if (!used[u])
19                 dfs1(u);
20         }
21         order.push_back(v);
22     }
23     void dfs2(int v, int cl) {
24         comp[v] = cl;
25         for (int u : adj_t[v]) {
26             if (comp[u] == -1)
27                 dfs2(u, cl);
28         }
29     }
30
31     void solve() {
32         for (int v : order) {
33             if (!used[v])
34                 dfs1(v);
35         }
36         for (int v : order) {
37             if (comp[v] == -1)
38                 dfs2(v, 0);
39         }
40         for (int v : order) {
41             if (comp[v] == 0)
42                 assignment[v] = 1;
43             else if (comp[v] == 1)
44                 assignment[v] = -1;
45         }
46     }
47
48     int getAssignment(int v) {
49         return assignment[v];
50     }
51
52     int getComp(int v) {
53         return comp[v];
54     }
55
56     void printAssignment() {
57         for (int v : order) {
58             cout << assignment[v] << " ";
59         }
60         cout << endl;
61     }
62
63     void printComp() {
64         for (int v : order) {
65             cout << comp[v] << " ";
66         }
67         cout << endl;
68     }
69
70     void printOrder() {
71         for (int v : order) {
72             cout << v << " ";
73         }
74         cout << endl;
75     }
76
77     void printUsed() {
78         for (int v : order) {
79             cout << used[v] << " ";
80         }
81         cout << endl;
82     }
83
84     void printAdj() {
85         for (int v : order) {
86             cout << v << ": ";
87             for (int u : adj[v]) {
88                 cout << u << " ";
89             }
90             cout << endl;
91         }
92     }
93
94     void printAdjT() {
95         for (int v : order) {
96             cout << v << ": ";
97             for (int u : adj_t[v]) {
98                 cout << u << " ";
99             }
100            cout << endl;
101        }
102    }
103
104    void printOrder() {
105        for (int v : order) {
106            cout << v << " ";
107        }
108        cout << endl;
109    }
110
111    void printComp() {
112        for (int v : order) {
113            cout << comp[v] << " ";
114        }
115        cout << endl;
116    }
117
118    void printAssignment() {
119        for (int v : order) {
120            cout << assignment[v] << " ";
121        }
122        cout << endl;
123    }
124
125    void printUsed() {
126        for (int v : order) {
127            cout << used[v] << " ";
128        }
129        cout << endl;
130    }
131
132    void printAdj() {
133        for (int v : order) {
134            cout << v << ": ";
135            for (int u : adj[v]) {
136                cout << u << " ";
137            }
138            cout << endl;
139        }
140    }
141
142    void printAdjT() {
143        for (int v : order) {
144            cout << v << ": ";
145            for (int u : adj_t[v]) {
146                cout << u << " ";
147            }
148            cout << endl;
149        }
150    }
151
152    void printOrder() {
153        for (int v : order) {
154            cout << v << " ";
155        }
156        cout << endl;
157    }
158
159    void printComp() {
160        for (int v : order) {
161            cout << comp[v] << " ";
162        }
163        cout << endl;
164    }
165
166    void printAssignment() {
167        for (int v : order) {
168            cout << assignment[v] << " ";
169        }
170        cout << endl;
171    }
172
173    void printUsed() {
174        for (int v : order) {
175            cout << used[v] << " ";
176        }
177        cout << endl;
178    }
179
180    void printAdj() {
181        for (int v : order) {
182            cout << v << ": ";
183            for (int u : adj[v]) {
184                cout << u << " ";
185            }
186            cout << endl;
187        }
188    }
189
190    void printAdjT() {
191        for (int v : order) {
192            cout << v << ": ";
193            for (int u : adj_t[v]) {
194                cout << u << " ";
195            }
196            cout << endl;
197        }
198    }
199
200    void printOrder() {
201        for (int v : order) {
202            cout << v << " ";
203        }
204        cout << endl;
205    }
206
207    void printComp() {
208        for (int v : order) {
209            cout << comp[v] << " ";
210        }
211        cout << endl;
212    }
213
214    void printAssignment() {
215        for (int v : order) {
216            cout << assignment[v] << " ";
217        }
218        cout << endl;
219    }
220
221    void printUsed() {
222        for (int v : order) {
223            cout << used[v] << " ";
224        }
225        cout << endl;
226    }
227
228    void printAdj() {
229        for (int v : order) {
230            cout << v << ": ";
231            for (int u : adj[v]) {
232                cout << u << " ";
233            }
234            cout << endl;
235        }
236    }
237
238    void printAdjT() {
239        for (int v : order) {
240            cout << v << ": ";
241            for (int u : adj_t[v]) {
242                cout << u << " ";
243            }
244            cout << endl;
245        }
246    }
247
248    void printOrder() {
249        for (int v : order) {
250            cout << v << " ";
251        }
252        cout << endl;
253    }
254
255    void printComp() {
256        for (int v : order) {
257            cout << comp[v] << " ";
258        }
259        cout << endl;
260    }
261
262    void printAssignment() {
263        for (int v : order) {
264            cout << assignment[v] << " ";
265        }
266        cout << endl;
267    }
268
269    void printUsed() {
270        for (int v : order) {
271            cout << used[v] << " ";
272        }
273        cout << endl;
274    }
275
276    void printAdj() {
277        for (int v : order) {
278            cout << v << ": ";
279            for (int u : adj[v]) {
280                cout << u << " ";
281            }
282            cout << endl;
283        }
284    }
285
286    void printAdjT() {
287        for (int v : order) {
288            cout << v << ": ";
289            for (int u : adj_t[v]) {
290                cout << u << " ";
291            }
292            cout << endl;
293        }
294    }
295
296    void printOrder() {
297        for (int v : order) {
298            cout << v << " ";
299        }
300        cout << endl;
301    }
302
303    void printComp() {
304        for (int v : order) {
305            cout << comp[v] << " ";
306        }
307        cout << endl;
308    }
309
310    void printAssignment() {
311        for (int v : order) {
312            cout << assignment[v] << " ";
313        }
314        cout << endl;
315    }
316
317    void printUsed() {
318        for (int v : order) {
319            cout << used[v] << " ";
320        }
321        cout << endl;
322    }
323
324    void printAdj() {
325        for (int v : order) {
326            cout << v << ": ";
327            for (int u : adj[v]) {
328                cout << u << " ";
329            }
330            cout << endl;
331        }
332    }
333
334    void printAdjT() {
335        for (int v : order) {
336            cout << v << ": ";
337            for (int u : adj_t[v]) {
338                cout << u << " ";
339            }
340            cout << endl;
341        }
342    }
343
344    void printOrder() {
345        for (int v : order) {
346            cout << v << " ";
347        }
348        cout << endl;
349    }
350
351    void printComp() {
352        for (int v : order) {
353            cout << comp[v] << " ";
354        }
355        cout << endl;
356    }
357
358    void printAssignment() {
359        for (int v : order) {
360            cout << assignment[v] << " ";
361        }
362        cout << endl;
363    }
364
365    void printUsed() {
366        for (int v : order) {
367            cout << used[v] << " ";
368        }
369        cout << endl;
370    }
371
372    void printAdj() {
373        for (int v : order) {
374            cout << v << ": ";
375            for (int u : adj[v]) {
376                cout << u << " ";
377            }
378            cout << endl;
379        }
380    }
381
382    void printAdjT() {
383        for (int v : order) {
384            cout << v << ": ";
385            for (int u : adj_t[v]) {
386                cout << u << " ";
387            }
388            cout << endl;
389        }
390    }
391
392    void printOrder() {
393        for (int v : order) {
394            cout << v << " ";
395        }
396        cout << endl;
397    }
398
399    void printComp() {
400        for (int v : order) {
401            cout << comp[v] << " ";
402        }
403        cout << endl;
404    }
405
406    void printAssignment() {
407        for (int v : order) {
408            cout << assignment[v] << " ";
409        }
410        cout << endl;
411    }
412
413    void printUsed() {
414        for (int v : order) {
415            cout << used[v] << " ";
416        }
417        cout << endl;
418    }
419
420    void printAdj() {
421        for (int v : order) {
422            cout << v << ": ";
423            for (int u : adj[v]) {
424                cout << u << " ";
425            }
426            cout << endl;
427        }
428    }
429
430    void printAdjT() {
431        for (int v : order) {
432            cout << v << ": ";
433            for (int u : adj_t[v]) {
434                cout << u << " ";
435            }
436            cout << endl;
437        }
438    }
439
440    void printOrder() {
441        for (int v : order) {
442            cout << v << " ";
443        }
444        cout << endl;
445    }
446
447    void printComp() {
448        for (int v : order) {
449            cout << comp[v] << " ";
450        }
451        cout << endl;
452    }
453
454    void printAssignment() {
455        for (int v : order) {
456            cout << assignment[v] << " ";
457        }
458        cout << endl;
459    }
460
461    void printUsed() {
462        for (int v : order) {
463            cout << used[v] << " ";
464        }
465        cout << endl;
466    }
467
468    void printAdj() {
469        for (int v : order) {
470            cout << v << ": ";
471            for (int u : adj[v]) {
472                cout << u << " ";
473            }
474            cout << endl;
475        }
476    }
477
478    void printAdjT() {
479        for (int v : order) {
480            cout << v << ": ";
481            for (int u : adj_t[v]) {
482                cout << u << " ";
483            }
484            cout << endl;
485        }
486    }
487
488    void printOrder() {
489        for (int v : order) {
490            cout << v << " ";
491        }
492        cout << endl;
493    }
494
495    void printComp() {
496        for (int v : order) {
497            cout << comp[v] << " ";
498        }
499        cout << endl;
500    }
501
502    void printAssignment() {
503        for (int v : order) {
504            cout << assignment[v] << " ";
505        }
506        cout << endl;
507    }
508
509    void printUsed() {
510        for (int v : order) {
511            cout << used[v] << " ";
512        }
513        cout << endl;
514    }
515
516    void printAdj() {
517        for (int v : order) {
518            cout << v << ": ";
519            for (int u : adj[v]) {
520                cout << u << " ";
521            }
522            cout << endl;
523        }
524    }
525
526    void printAdjT() {
527        for (int v : order) {
528            cout << v << ": ";
529            for (int u : adj_t[v]) {
530                cout << u << " ";
531            }
532            cout << endl;
533        }
534    }
535
536    void printOrder() {
537        for (int v : order) {
538            cout << v << " ";
539        }
540        cout << endl;
541    }
542
543    void printComp() {
544        for (int v : order) {
545            cout << comp[v] << " ";
546        }
547        cout << endl;
548    }
549
550    void printAssignment() {
551        for (int v : order) {
552            cout << assignment[v] << " ";
553        }
554        cout << endl;
555    }
556
557    void printUsed() {
558        for (int v : order) {
559            cout << used[v] << " ";
560        }
561        cout << endl;
562    }
563
564    void printAdj() {
565        for (int v : order) {
566            cout << v << ": ";
567            for (int u : adj[v]) {
568                cout << u << " ";
569            }
570            cout << endl;
571        }
572    }
573
574    void printAdjT() {
575        for (int v : order) {
576            cout << v << ": ";
577            for (int u : adj_t[v]) {
578                cout << u << " ";
579            }
580            cout << endl;
581        }
582    }
583
584    void printOrder() {
585        for (int v : order) {
586            cout << v << " ";
587        }
588        cout << endl;
589    }
590
591    void printComp() {
592        for (int v : order) {
593            cout << comp[v] << " ";
594        }
595        cout << endl;
596    }
597
598    void printAssignment() {
599        for (int v : order) {
600            cout << assignment[v] << " ";
601        }
602        cout << endl;
603    }
604
605    void printUsed() {
606        for (int v : order) {
607            cout << used[v] << " ";
608        }
609        cout << endl;
610    }
611
612    void printAdj() {
613        for (int v : order) {
614            cout << v << ": ";
615            for (int u : adj[v]) {
616                cout << u << " ";
617            }
618            cout << endl;
619        }
620    }
621
622    void printAdjT() {
623        for (int v : order) {
624            cout << v << ": ";
625            for (int u : adj_t[v]) {
626                cout << u << " ";
627            }
628            cout << endl;
629        }
630    }
631
632    void printOrder() {
633        for (int v : order) {
634            cout << v << " ";
635        }
636        cout << endl;
637    }
638
639    void printComp() {
640        for (int v : order) {
641            cout << comp[v] << " ";
642        }
643        cout << endl;
644    }
645
646    void printAssignment() {
647        for (int v : order) {
648            cout << assignment[v] << " ";
649        }
650        cout << endl;
651    }
652
653    void printUsed() {
654        for (int v : order) {
655            cout << used[v] << " ";
656        }
657        cout << endl;
658    }
659
660    void printAdj() {
661        for (int v : order) {
662            cout << v << ": ";
663            for (int u : adj[v]) {
664                cout << u << " ";
665            }
666            cout << endl;
667        }
668    }
669
670    void printAdjT() {
671        for (int v : order) {
672            cout << v << ": ";
673            for (int u : adj_t[v]) {
674                cout << u << " ";
675            }
676            cout << endl;
677        }
678    }
679
680    void printOrder() {
681        for (int v : order) {
682            cout << v << " ";
683        }
684        cout << endl;
685    }
686
687    void printComp() {
688        for (int v : order) {
689            cout << comp[v] << " ";
690        }
691        cout << endl;
692    }
693
694    void printAssignment() {
695        for (int v : order) {
696            cout << assignment[v] << " ";
697        }
698        cout << endl;
699    }
700
701    void printUsed() {
702        for (int v : order) {
703            cout << used[v] << " ";
704        }
705        cout << endl;
706    }
707
708    void printAdj() {
709        for (int v : order) {
710            cout << v << ": ";
711            for (int u : adj[v]) {
712                cout << u << " ";
713            }
714            cout << endl;
715        }
716    }
717
718    void printAdjT() {
719        for (int v : order) {
720            cout << v << ": ";
721            for (int u : adj_t[v]) {
722                cout << u << " ";
723            }
724            cout << endl;
725        }
726    }
727
728    void printOrder() {
729        for (int v : order) {
730            cout << v << " ";
731        }
732        cout << endl;
733    }
734
735    void printComp() {
736        for (int v : order) {
737            cout << comp[v] << " ";
738        }
739        cout << endl;
740    }
741
742    void printAssignment() {
743        for (int v : order) {
744            cout << assignment[v] << " ";
745        }
746        cout << endl;
747    }
748
749    void printUsed() {
750        for (int v : order) {
751            cout << used[v] << " ";
752        }
753        cout << endl;
754    }
755
756    void printAdj() {
757        for (int v : order) {
758            cout << v << ": ";
759            for (int u : adj[v]) {
760                cout << u << " ";
761            }
762            cout << endl;
763        }
764    }
765
766    void printAdjT() {
767        for (int v : order) {
768            cout << v << ": ";
769            for (int u : adj_t[v]) {
770                cout << u << " ";
771            }
772            cout << endl;
773        }
774    }
775
776    void printOrder() {
777        for (int v : order) {
778            cout << v << " ";
779        }
780        cout << endl;
781    }
782
783    void printComp() {
784        for (int v : order) {
785            cout << comp[v] << " ";
786        }
787        cout << endl;
788    }
789
790    void printAssignment() {
791        for (int v : order) {
792            cout << assignment[v] << " ";
793        }
794        cout << endl;
795    }
796
797    void printUsed() {
798        for (int v : order) {
799            cout << used[v] << " ";
800        }
801        cout << endl;
802    }
803
804    void printAdj() {
805        for (int v : order) {
806            cout << v << ": ";
807            for (int u : adj[v]) {
808                cout << u << " ";
809            }
810            cout << endl;
811        }
812    }
813
814    void printAdjT() {
815        for (int v : order) {
816            cout << v << ": ";
817            for (int u : adj_t[v]) {
818                cout << u << " ";
819            }
820            cout << endl;
821        }
822    }
823
824    void printOrder() {
825        for (int v : order) {
826            cout << v << " ";
827        }
828        cout << endl;
829    }
830
831    void printComp() {
832        for (int v : order) {
833            cout << comp[v] << " ";
834        }
835        cout << endl;
836    }
837
838    void printAssignment() {
839        for (int v : order) {
840            cout << assignment[v] << " ";
841        }
842        cout << endl;
843    }
844
845    void printUsed() {
846        for (int v : order) {
847            cout << used[v] << " ";
848        }
849        cout << endl;
850    }
851
852    void printAdj() {
853        for (int v : order) {
854            cout << v << ": ";
855            for (int u : adj[v]) {
856                cout << u << " ";
857            }
858            cout << endl;
859        }
860    }
861
862    void printAdjT() {
863        for (int v : order) {
864            cout << v << ": ";
865            for (int u : adj_t[v]) {
866                cout << u << " ";
867            }
868            cout << endl;
869        }
870    }
871
872    void printOrder() {
873        for (int v : order) {
874            cout << v << " ";
875        }
876        cout << endl;
877    }
878
879    void printComp() {
880        for (int v : order) {
881            cout << comp[v] << " ";
882        }
883        cout << endl;
884    }
885
886    void printAssignment() {
887        for (int v : order) {
888            cout << assignment[v] << " ";
889        }
890        cout << endl;
891    }
892
893    void printUsed() {
894        for (int v : order) {
895            cout << used[v] << " ";
896        }
897        cout << endl;
898    }
899
900    void printAdj() {
901        for (int v : order) {
902            cout << v << ": ";
903            for (int u : adj[v]) {
904                cout << u << " ";
905            }
906            cout << endl;
907        }
908    }
909
910    void printAdjT() {
911        for (int v : order) {
912            cout << v << ": ";
913            for (int u : adj_t[v]) {
914                cout << u << " ";
915            }
916            cout << endl;
917        }
918    }
919
920    void printOrder() {
921        for (int v : order) {
922            cout << v << " ";
923        }
924        cout << endl;
925    }
926
927    void printComp() {
928        for (int v : order) {
929            cout << comp[v] << " ";
930        }
931        cout << endl;
932    }
933
934    void printAssignment() {
935        for (int v : order) {
936            cout << assignment[v] << " ";
937        }
938        cout << endl;
939    }
940
941    void printUsed() {
942        for (int v : order) {
943            cout << used[v] << " ";
944        }
945        cout << endl;
946    }
947
948    void printAdj() {
949        for (int v : order) {
950            cout << v << ": ";
951            for (int u : adj[v]) {
952                cout << u << " ";
953            }
954            cout << endl;
955        }
956    }
957
958    void printAdjT() {
959        for (int v : order) {
960            cout << v << ": ";
961            for (int u : adj_t[v]) {
962                cout << u << " ";
963            }
964            cout << endl;
965        }
966    }
967
968    void printOrder() {
969        for (int v : order) {
970            cout << v << " ";
971        }
972        cout << endl;
973    }
974
975    void printComp() {
976        for (int v : order) {
977            cout << comp[v] << " ";
978        }
979        cout << endl;
980    }
981
982    void printAssignment() {
983        for (int v : order) {
984            cout << assignment[v] << " ";
985        }
986        cout << endl;
987    }
988
989    void printUsed() {
990        for (int v : order) {
991            cout << used[v] << " ";
992        }
993        cout << endl;
994    }
995
996    void printAdj() {
997        for (int v : order) {
998            cout << v << ": ";
999            for (int u : adj[v]) {
1000                cout << u << " ";
1001            }
1002            cout << endl;
1003        }
1004    }
1005
1006    void printAdjT() {
1007        for (int v : order) {
1008            cout << v << ": ";
1009            for (int u : adj_t[v]) {
1010                cout << u << " ";
1011            }
1012            cout << endl;
1013        }
1014    }
1015
1016    void printOrder() {
1017        for (int v : order) {
1018            cout << v << " ";
1019        }
1020        cout << endl;
1021    }
1022
1023    void printComp() {
1024        for (int v : order) {
1025            cout << comp[v] << " ";
1026        }
1027        cout << endl;
1028    }
1029
1030    void printAssignment() {
1031        for (int v : order) {
1032            cout << assignment[v] << " ";
1033        }
1034        cout << endl;
1035    }
1036
1037    void printUsed() {
1038        for (int v : order) {
1039            cout << used[v] << " ";
1040        }
1041        cout << endl;
1042    }
1043
1044    void printAdj() {
1045        for (int v : order) {
1046            cout << v << ": ";
1047            for (int u : adj[v]) {
1048                cout << u << " ";
1049            }
1050            cout << endl;
1051        }
1052    }
1053
1054    void printAdjT() {
1055        for (int v : order) {
1056            cout << v << ": ";
1057            for (int u : adj_t[v]) {
1058                cout << u << " ";
1059            }
1060            cout << endl;
1061        }
1062    }
1063
1064    void printOrder() {
1065        for (int v : order) {
1066            cout << v << " ";
1067        }
1068        cout << endl;
1069    }
1070
1071    void printComp() {
1072        for (int v : order) {
1073            cout << comp[v] << " ";
1074        }
1075        cout << endl;
1076    }
1077
1078    void printAssignment() {
1079        for (int v : order) {
1080            cout
```



```

73         assert(solver.assignment == expected);
74     }
75 }
```

Listing 28: 2-SAT implementation from [cp-algorithms.com](http://cp-algorithms.com). Each component added is a expression of the form  $a \vee b$ , which is equivalent to  $\neg a \Rightarrow b \wedge \neg b \Rightarrow a$  (if one of the variables is false, then the other one must be true). A directed graph is constructed based on these implication: For each  $x$ , there's two vertices  $v_x$  and  $v_{\neg x}$ . If there is an edge  $a \Rightarrow b$ , then there also is an edge  $\neg b \Rightarrow \neg a$ . For any  $x$ , if  $x$  is reachable from  $\neg x$  and  $\neg x$  is reachable from  $x$ , the problem has no solution. This means, each variable must be in a different SCC than their negative. This is verified by the method `solve_2SAT()`, which returns a boolean: `True` if it has a solution and `False` if it doesn't.

**Giant Pizza** How does a particular 2-SAT problem look like? Following is the statement for the problem CSES 1684 (Giant Pizza):

Uolevi's family is going to order a large pizza and eat it together. A total of  $n$  family members will join the order, and there are  $m$  possible toppings. The pizza may have any number of toppings. Each family member gives two wishes concerning the toppings of the pizza. The wishes are of the form "topping  $x$  is good/bad". Your task is to choose the toppings so that at least one wish from everybody becomes true (a good topping is included in the pizza or a bad topping is not included).

#### Input

The first input line has two integers  $n$  and  $m$ : the number of family members and toppings. The toppings are numbered  $1, 2, \dots, m$ . After this, there are  $n$  lines describing the wishes. Each line has two wishes of the form " $+x$ " (topping  $x$  is good) or " $-x$ " (topping  $x$  is bad).

#### Output

Print a line with  $m$  symbols: for each topping "+" if it is included and "-" if it is not included. You can print any valid solution. If there are no valid

solutions, print "IMPOSSIBLE".

```

1 int main(){
2     fastIO();
3     int n = nxt(), m = nxt();
4     TwoSatSolver TwoSat(m);
5
6     for(i, 0, n){
7         char type1, type2;
8         int top1, top2;
9         cin >> type1 >> top1 >> type2 >> top2;
10
11        top1--; top2--;
12        TwoSat.add_disjunction(top1, type1 == '+' ,
13                               top2, type2 == '+');
14    }
15
16    if(TwoSat.solve_2SAT()){
17        for(i, 0, m){
18            if(TwoSat.assignment[i])
19                cout << "+ ";
20            else
21                cout << "- ";
22        }
23        else cout << "IMPOSSIBLE";
24    }
25 }
```

Listing 29: Main method for solving CSES 1684 Giant Pizza using 2-SAT template.

## 5.12 \*Bridges and point articulation

### 5.13 Flood Fill

```

1 vector<string> grid(n);
2 vii dirs = {{0,1},{0,-1},{1,0}, {-1,0}};
3 ii start;
4 int arns = 0;
5 function<void(int,int)> traverse = [&](int i, int j)
6     -> void {
7     if( grid[i][j] == '#' ) return;
8     int newI, newJ;
9     if( grid[i][j] != '.' ) arns += grid[i][j] - '0';
10    grid[i][j] = '#';
11    for( ii move : dirs ) {
12        newI = i + move.fi; newJ = j + move.se;
13        if( newI >= 0 && newI < n && newJ >= 0 && newJ <
14            m && grid[newI][newJ] == 'T' )
15            return;
16    }
17    for( ii move : dirs ) {
18        newI = i + move.fi; newJ = j + move.se;
19        if( newI >= 0 && newI < n && newJ >= 0 && newJ <
20            m )
21            traverse(newI, newJ);
22    }
23 }
```

```

20    };
21    fori(i,0,n)
22        cin >> grid[i];
23    fori(i,0,n) {
24        fori(j,0,m) {
25            if( grid[i][j] == 'S' ) {
26                grid[i][j] = '.';
27                start.fi = i;
28                start.se = j;
29            }
30        }
31    }
32    traverse(start.fi, start.se);
33    cout << arns << endl;
```

Listing 30: Traverse a matrix of ' $n$ ' x ' $m$ ' on grid representation. The matrix is composed of '.' for a valid space (empty), '#' for a wall, 'T' for a trap, and a number for a treasure. This implementation takes the sum of every treasure in the maze. The condition for moving to the next location is that there are no Traps nearby (up, down, left, right), so the player will never be killed while traversing. It also implements a way to read numerous test cases, but without knowing beforehand how many there are. Runs in  $O(n \cdot m)$ . Originally used on the problem *Treasures* from *2024-2025 ICPC Bolivia Pre-National Contest*.

## 5.14 Lava Flow (Multi-source BFS)

```

1 typedef array<int,3> iii;
2
3 vii dirs = {{1,0},{0,1},{-1,0},{0,-1}};
```

```
4 map<int, string> path = {{0, "D"}, {1, "R"}, {2, "U"}, {3, "L"}};
5 int n, m;
6 string arns = "";
7 bool escaped = false;
8 cin >> n >> m;
9 vector<string> grid(n);
10 vvi times(n, vi(m, INF)), prev(n, vi(m, -1));
11 vector<vector<bool>> visited(n, vector<bool>(m, false));
12 queue<iiii> q;
13 ii start, end;
14 for(int i = 0; i < n; i++) {
15     cin >> grid[i];
16     for(int j = 0; j < m; j++) {
17         if( grid[i][j] == 'M' ) {
18             q.push({i, j, 0});
19             times[i][j] = 0;
20         }
21         else if( grid[i][j] == 'A' )
22             start = {i, j};
23     }
24 }
25 function<bool(int, int)> valid = [&](int I, int J) ->
26     bool {
27     return (I >= 0) and (I < n) and (J >= 0) and (J <
28         m) and (grid[I][J] != '#') and (times[I][J] ==
29             INF);
30 };
31 function<bool(int, int)> valid_player = [&](int I,
32     int J) -> bool {
33     return (I >= 0) and (I < n) and (J >= 0) and (J <
34         m) and (!visited[I][J]) and (grid[I][J] != '#');
35 };
36 function<bool(int, int)> is_border = [&](int I, int J
37     ) -> bool {
38     return I == 0 || I == n-1 || J == 0 || J == m-1;
39 };
40 // Corner cases
41 if( is_border(start.fi, start.se) ) {
42     cout << "YES" << endl << "0" << endl;
43     return 0;
44 }
45 // Multi-Source BFS
46 while( ! q.empty() ) {
47     iii u = q.front();
48     q.pop();
49     for(ii dir : dirs) {
50         int newI = u[0] + dir.fi;
51         int newJ = u[1] + dir.se;
52         int w = u[2] + 1;
53         if( valid(newI, newJ) ) {
54             times[newI][newJ] = w;
55             q.push({newI, newJ, w});
56         }
57     }
58 }
```

```

52 }
53 // Player BFS
54 q.push({start.fi,start.se,0});
55 visited[start.fi][start.se] = true;
56 while( ! q.empty() and !escaped ) {
57     iii u = q.front();
58     q.pop();
59     for(int i = 0; i < 4; i++) {
60         int newI = u[0] + dirs[i].fi;
61         int newJ = u[1] + dirs[i].se;
62         int w = u[2] + 1;
63         if( valid_player(newI,newJ) and w < times[newI][newJ] ) {
64             visited[newI][newJ] = true;
65             prev[newI][newJ] = i;
66             q.push({newI,newJ,w});
67             if( is_border(newI,newJ) ) {
68                 end.fi = newI;
69                 end.se = newJ;
70                 escaped = true;
71                 break;
72             }
73         }
74     }
75 }
76 if( !escaped ) {
77     cout << "NO" << endl;
78     return 0;

```

```

79 }
80 // Path reconstruction
81 cout << "YES" << endl;
82 int i = end.fi;
83 int j = end.se;
84 while( prev[i][j] != -1 ) {
85     int oldI = i;
86     arns += path[ prev[i][j] ];
87     i -= dirs[prev[i][j]].fi;
88     j -= dirs[prev[oldI][j]].se;
89 }
90 reverse(all(arns));
91 cout << sz(arns) << endl << arns << endl;

```

Listing 31: Classic Lava Flow problem implementation, where the timer from the starting point  $A$  needs to be less than every other in the MS-BFS starting in  $M$  places. Once one edge is reached, the path is reconstructed from the output. Runs in BFS complexity  $O(|V| + |E|)$ . Originally used in the CSES problem *Monsters*.

## 5.15 MaxFlow

### 5.15.1 Dinic's Algorihtm

```

1 const ll INF = 1e17;
2 /**
3 * @brief Represents a directed edge in a flow
4 * network.
5 * @details Stores the edge's source, destination,
6 * capacity, and current flow.

```

```
5 *           Used in max-flow algorithms like Dinic
6 or Ford-Fulkerson. */
7 struct flowEdge {
8     int u; // Source node
9     int v; // Destination node
10    ll cap; // Maximum flow capacity of the edge
11    ll flow = 0; // Current flow through the edge (
12        initially 0)
13    flowEdge( int u, int v, ll cap ) : u(u), v(v), cap
14        (cap) {};
15 };
16 /**
17 * @brief Implementation of Dinic's max-flow
18 algorithm.
19 * @details Manages a flow network with BFS (Level
20 Graph) and DFS (Blocking Flow) optimizations. */
21 struct Dinic {
22     vector<flowEdge> edges; // All edges in the flow
23         network (including reverse edges)
24     vector<vi> adj;
25     int n; // Total number of nodes in the graph
26     int s; // Source node
27     int t; // Sink node (destination of flow)
28     int id = 0; // Counter for edge indexing
29     vi level; // Stores the level (distance from 's')
30         of each node during BFS
31     vi next; // Optimization for DFS: tracks the next
32         edge to explore for each node
33
34     queue<int> q; // Queue for BFS traversal
35     /**
36      * @brief Constructs a Dinic solver for a flow
37      network.
38      * @param n Number of nodes.
39      * @param s Source node.
40      * @param t Sink node. */
41     Dinic( int n, int s, int t ) : n(n), s(s), t(t) {
42         adj.resize(n); // Initialize adjacency list for
43         'n' nodes.
44         level.resize(n); // Prepare level array for BFS.
45         next.resize(n); // Prepare next-edge array for
46         DFS.
47         fill(all(level),-1); // Mark all levels as
48         unvisited (-1).
49         level[s] = 0; // The source has level 0.
50         q.push(s); // Start BFS from the source.
51     }
52     /**
53      * @brief Adds a directed edge and its residual
54      reverse edge to the flow network. */
55     void addEdge( int u, int v, ll cap ) {
56         edges.emplace_back(u,v,cap); // Original edge: u
57             -> v
58         edges.emplace_back(v,u,0); // Residual edge: v
59             -> u
60         adj[u].pb(id++);
61         adj[v].pb(id++);
62     }
63 }
```

```
46 }  
47 /**  
48 * @brief Performs BFS to construct the level  
graph (Layered Network) from source 's' to sink '  
t'.  
49 * @details Assigns levels (minimum distances  
from 's') to all nodes and checks if 't' is  
reachable.  
50 *          Levels are used to guide the DFS  
phase in Dinic's algorithm.  
51 * @return bool True if the sink 't' is reachable  
(i.e., there exists an augmenting path), false  
otherwise. */  
52 bool bfs() {  
53     while( ! q.empty() ) {  
54         int curr = q.front();  
55         q.pop();  
56         for( auto e : adj[curr] ) {  
57             if( edges[e].cap - edges[e].flow < 1 ) //  
Skip saturated edges (no residual capacity).  
58                 continue;  
59             if( level[ edges[e].v ] != -1 ) // Skip  
already visited nodes (level assigned).  
60                 continue;  
61             // Assign level to the neighbor node.  
62             level[ edges[e].v ] = level[ edges[e].u ] +  
1; // Next level = current + 1.  
63         q.push( edges[e].v ); // Add neighbor to the  
queue for further BFS.  
64     }  
65 }  
66 return level[t] != -1; // Return whether the  
sink 't' was reached (level[t] != -1).  
67 }  
68 /**  
69 * @brief Finds a blocking flow using DFS in the  
level graph constructed by BFS.  
70 * @param u Current node being processed.  
71 * @param flow Maximum flow that can be sent from  
'u' to the sink 't'.  
72 * @return ll The amount of flow successfully  
sent to 't'. */  
73 ll dfs( int u, ll flow ) {  
74     if( flow == 0 ) // No remaining flow to send.  
        return 0;  
75     if( u == t ) // Reached the sink; return  
accumulated flow.  
76     return flow;  
77     // Explore edges from 'u' using 'next[u]' to  
avoid revisiting processed edges.  
78     for( int& cid = next[u]; cid < sz(adj[u]); cid++  
) {  
        int e = adj[u][cid]; // Index of the edge in '  
edges'.
```

```
81     int v = edges[e].v; // Destination node of
the edge.
82     // Skip invalid edges:
83     // 1. Not in the level graph (level[u] + 1 != level[v]). Just edges in exactly one level ahead (ensures shortest paths).
84     // 2. No residual capacity (cap - flow < 1).
85     if( level[edges[e].u] + 1 != level[v] || edges[e].cap - edges[e].flow < 1 )
86         continue;
87     ll f = dfs( v, min(flow, edges[e].cap - edges[e].flow) ); // Recursively send flow to 'v'.
88     if( f == 0 ) // No flow could be sent via this edge.
89         continue;
90     // Update residual capacities:
91     edges[e].flow += f; // Increase flow in the original edge.
92     edges[e ^ 1].flow -= f; // Decrease flow in the reverse edge. (All reverse edges have distinct parity)
93     return f; // Return the flow sent.
94 }
95 return 0; // No augmenting path found from 'u'.
96 }
97 /**
98 * @brief Computes the maximum flow from source 's' to sink 't' using Dinic's algorithm.
99 * @details Iterates through BFS and DFS phases to find the maximum flow.
100 Accumulates flow while there exists augmentation paths in the residual graph.
101 Restart auxiliary structures for every new phase.
102 * @return ll The maximum flow value. */
103 ll maxFlow() {
104     ll flow = 0; // Tracks the total flow sent.
105     while( bfs() ) { // While there are augmenting paths:
106         fill(all(next), 0); // Reset 'next' for DFS.
107         for( ll f = dfs(s, INF); f != 0LL; f = dfs(s,
108             INF) ) // Send blocking flow in the level graph:
109             flow += f;
110         // Reset for next BFS phase:
111         fill(all(level), -1);
112         level[s] = 0;
113         q.push(s);
114     }
115     return flow;
116 }
117 /**
118 * @brief Finds edges belonging to the minimum cut after maxFlow().
119 */
```

```
118     * @details First, it marks all the reachable
119     nodes from 's' with an augmentation path after
120     obtained the max flow
121
122     and all the saturated edges coming out from
123     any of the nodes who belong to the min-cut.
124
125     For 'minCut()' to work, 'maxFlow()' must be
126     first executed to get the min-cut.
127
128     If only is needed the value, is enough
129     returning the value of 'maxFlow()'.
```

```
122     * @return vii List of edges (u, v) in the min-
123     cut. Its size is the minimum number of 'roads' to
124     close. */
125 vii minCut() {
126     vii ans;
127     fill(all(level), -1); // Reset levels.
128     level[s] = 0; // Mark source as reachable
129     .
130     q.push(s);
131     while( ! q.empty() ) { // BFS to mark nodes
132         reachable from 's' in the residual graph.
133         int curr = q.front();
134         q.pop();
135         for( int id = 0; id < sz(adj[curr]); id++ ) {
136             // For every edge going out from 'curr'.
137             int e = adj[curr][id];
138             // If 'v' is has not been visited yet, and
139             // the edge have residual capacity.
140             if( level[edges[e].v] == -1 && edges[e].cap
141 - edges[e].flow > 0 ) {
142                 q.push(edges[e].v);
143                 level[edges[e].v] = level[edges[e].u] + 1;
144             }
145         }
146     }
147     return ans;
148 }
149 /**
150 * @brief Reconstructs the maximum bipartite
151 matching after running 'maxFlow()'.
```

```
152 * @details Every edge that belong to the
153 original graph and have flow greater than zero,
154 belongs to the matching.
155
156 For 'maximumMatching()' to work, 'maxFlow()'
157 .
```

```

155     * @return vii List of matched pairs (boy, girl).
156     */
157
158     vii maximumMatching() {
159         vii ans;
160         fill(all(level), -1); // Reset levels.
161         level[s] = 0; // Mark source as reachable
162         .
163         q.push(s);
164         while( ! q.empty() ) { // BFS to mark nodes
165             reachable via saturated edges with flow greater
166             than zero.
167             int curr = q.front();
168             q.pop();
169             for( int id = 0; id < sz(adj[curr]); id++ ) {
170                 int e = adj[curr][id];
171                 // If 'v' has not been visited yet, the edge
172                 is saturated and have flow greater than zero.
173                 if( level[edges[e].v] == -1 && edges[e].cap -
174                     edges[e].flow == 0 && edges[e].flow != 0ll ) {
175                     q.push(edges[e].v);
176                     level[edges[e].v] = level[edges[e].u] + 1;
177                 }
178             }
179             for( int i = 0; i < sz(level); i++ ) { //
180                 Collect original edges (boy -> girl) that are
181                 saturated and have flow > 0.
182                 if( level[i] != -1 ) {
183                     for( int id = 0; id < sz(adj[i]); id++ ) {
184                         int e = adj[i][id];
185                         if( edges[e].u != s && edges[e].v != t
186                             && edges[e].cap - edges[e].flow == 0 && edges[e].
187                             flow != 0ll )
188                             ans.emplace_back(edges[e].u,edges[e].v
189 );
190                     }
191                 }
192             }
193         }
194         return ans;
195     }
196 }
```

Listing 32: Commented template for solving MaxFlow problems with Dinic's algorithm. Works in complexity  $O(|V|^2 \times |E|)$ . In bipartite graphs and graphs with unitary max capacity the complexity turns  $O(|E| \times \sqrt{|V|})$ .

**Download Speed** How does a particular flow problem looks like? Following is the statement for the problem CSES 1694 (Download Speed):

Consider a network consisting of  $n$  computers and  $m$  connections. Each connection specifies how fast a computer can send data to another computer.

Kotivalo wants to download some data from a server. What is the maximum speed he can do this, using the connections in the network?

#### Input

The first input line has two integers  $n$  and  $m$ : the number of computers and connections. The computers are numbered  $1, 2, \dots, n$ . Computer 1 is the server and computer  $n$  is Kotivalo's computer.

After this, there are  $m$  lines describing the connections. Each line has three

integers  $a$ ,  $b$ , and  $c$ : computer  $a$  can send data to computer  $b$  at speed  $c$ .

#### Output

Print one integer: the maximum speed Kotivalo can download data.

```

1 int main()
2 {
3     fastIO();
4
5     int n, m, u, v, w;
6
7     cin >> n >> m;
8
9     Dinic flow(n+1,1,n); // size n+1 to fix 0-
10    indexed indexes, 1 is the source (server), 'n' is
11    the sink (Kotivalo)
12
13    for(i,0,m)
14    {
15        cin >> u >> v >> w;
16        flow.addEdge(u,v,w);
17    }
18
19    cout << flow.maxFlow() << endl;
20
21    return 0;
22 }
```

Listing 33: Main method for solving CSES 1697 Download Speed using MaxFlow template.

**Police Chase Max Flow-Min Cut Theorem:**  $\text{MaxFlow} = \text{MinCut}$ .

Following is the statement for the problem CSES 1695 (Police Chase):

Kaaleppi has just robbed a bank and is now heading to the harbor. However, the police wants to stop him by closing some streets of the city.

What is the minimum number of streets that should be closed so that there is no route between the bank and the harbor?

#### Input

The first input line has two integers  $n$  and  $m$ : the number of crossings and streets. The crossings are numbered  $1, 2, \dots, n$ . The bank is located at crossing 1, and the harbor is located at crossing  $n$ .

After this, there are  $m$  lines that describing the streets. Each line has two integers  $a$  and  $b$ : there is a street between crossings  $a$  and  $b$ . All streets are two-way streets, and there is at most one street between two crossings.

#### Output

First print an integer  $k$ : the minimum number of streets that should be closed. After this, print  $k$  lines describing the streets. You can print any valid solution.

```

1 int main()
2 {
3     fastIO();
4
5     int n, m, u, v;
6     vii minCut;
7
8     cin >> n >> m;
9
10    Dinic flow(n+1,1,n); // size n+1 to fix 0-
11    indexed indexes, 1 is the source (bank), 'n' is
12    the sink (harbor)
13
14    cout << minCut.size() << endl;
15
16    for(i,0,minCut.size())
17    {
18        cout << minCut[i].first << " " << minCut[i].second << endl;
19    }
20
21    return 0;
22 }
```

```

12     for(i,0,m)
13     {
14         cin >> u >> v;
15         flow.addEdge(u,v,1);
16         flow.addEdge(v,u,1);
17     }
18
19     flow.maxFlow();
20
21     minCut = flow.minCut();
22
23     cout << (sz(minCut)/2) << endl;
24     for(int i = 0; i < sz(minCut); i += 2)
25         cout << minCut[i].fi << " " << minCut[i].se
26     << endl;
27
28     return 0;
29 }
```

Listing 34: Main method for solving CSES 1695 Police Chase using MaxFlow template.

#### School Dance MaxFlow = MinCut = MaxMatching.

Following is the statement for the problem CSES 1696 (School Dance):

There are  $n$  boys and  $m$  girls in a school. Next week a school dance will be organized. A dance pair consists of a boy and a girl, and there are  $k$  potential pairs.

Your task is to find out the maximum number of dance pairs and show how this number can be achieved.

#### Input

The first input line has three integers  $n$ ,  $m$  and  $k$ : the number of boys, girls, and potential pairs. The boys are numbered  $1, 2, \dots, n$ , and the girls are numbered  $1, 2, \dots, m$ .

After this, there are  $k$  lines describing the potential pairs. Each line has two integers  $a$  and  $b$ : boy  $a$  and girl  $b$  are willing to dance together.

#### Output

First print one integer  $r$ : the maximum number of dance pairs. After this, print  $r$  lines describing the pairs. You can print any valid solution.

```

1 int main()
2 {
3     fastIO();
4
5     int n, m, k, a, b;
6     ll maxPairs;
7     vii pairs;
8
9     cin >> n >> m >> k;
10
11    Dinic flow(n+m+2,0,n+m+1);
12
13    fori(boy,0,n+1)
14        flow.addEdge(0,boy,1);
15
16    fori(girl,n+1,n+m+1)
17        flow.addEdge(girl,n+m+1,1);
18
19    fori(i,0,k)
20    {
```

```

21     cin >> a >> b;
22     flow.addEdge(a, n+b, 1);
23 }
24
25 maxPairs = flow.maxFlow();
26 pairs = flow.maximumMatching();
27
28 cout << maxPairs << endl;
29 for(i, 0, sz(pairs))
30     cout << pairs[i].fi << " " << (pairs[i].se -
n) << endl;;
31
32 return 0;
33 }
```

Listing 35: Main method for solving CSES 1696 School Dancing using MaxFlow template.

#### 5.15.2 \*Ford-Fulkerson Algorithm

#### 5.15.3 \*Goldber-Tarjan Algorithm

## 6 Trees

### 6.1 Counting Childrens

```

1 vi childrens(n+1, 0);
2 vvi graph(n+1);
3 vector<bool> visited(n+1, false);
4 for(i, 2, n+1) {
5     cin >> tmp;
```

```

6     graph[tmp].pb(i);
7     graph[i].pb(tmp);
8 }
9 function<int(int)> dfs = [&](int u) -> int {
10     visited[u] = true;
11     for(int v : graph[u]) {
12         if( !visited[v] )
13             childrens[u] += dfs(v);
14     }
15     return childrens[u] + 1;
16 };
17 dfs(1);
```

Listing 36: Algorithm that counts how many childrens does every node have, from 2..n in a rooted tree (root = 1).

### 6.2 \*Tree Diameter

### 6.3 \*Centroid Decomposition

### 6.4 \*Euler Tour

### 6.5 \*Lowest Common Ancestor (LCA)

### 6.6 \*Heavy-Light Decomposition (HLD)

## 7 Strings

### 7.1 Knuth-Morris-Pratt Algorithm (KMP)

```

1 // Longest Prefix-Suffix
2 vi compute_LPS(string s) {
3     size_t len = 0, i = 1, sz = s.size();
```

```
4     vi lps(sz,0);
5     while( i < sz ) {
6         if( s[i] == s[len] )
7             lps[i++] = ++len;
8         else
9             if( len != 0 )
10                 len = lps[len-1];
11             else
12                 lps[i++] = 0;
13     }
14     return lps;
15 }
16 // Get number of occurrences of a pattern p in a
17 // string s
18 int kmp(string s, string p) {
19     vi lps = compute_LPS(p);
20     size_t n = s.size(), m = p.size(), i = 0, j = 0;
21     int cnt = 0;
22     while( i < n ) {
23         if( p[j] == s[i] ) {
24             j++; i++;
25         }
26         if( j == m ) { // Full match
27             cnt++;
28             j = lps[j-1];
29         }
30         else if( i < n and p[j] != s[i] ) { // Mismatch
31             if( j != 0 )
32                 j = lps[j-1];
33             else
34                 i++;
35         }
36     }
37     return cnt;
38 }
```

Listing 37: KMP algorithm for counting how many times a pattern appear into a string. Runs in  $O(n + m)$ .

**7.2 \*Suffix Array****7.3 \*Rolling Hashing****7.4 \*Z Function****7.5 \*Aho-Corasick Algorithm****8 Dynamic Programming****8.1 \*Coins****8.2 \*Longest Increasing Subsequence (LIS)****8.3 \*Edit Distance****8.4 \*Knapsack****8.5 \*SOS DP****8.6 \*Digit DP****8.7 \*Bitmask DP****9 Mathematics****9.1 Number Theory****9.1.1 Greatest Common Divisor (GCD)**

```

1 int gcd(int a, int b) {
2     if (a == 0) return b;
3     if (b == 0) return a;
4     if (a == b) return a;
5     if (a > b)
6         return gcd(a - b, b);
7     return gcd(a, b - a);
8 }
```

Listing 38: Implementation of handmade GCD, because using `gcd()` runs slow with long long, also `--gcd()`.

**9.1.2 Gauss Sum**

The sum of the first  $n$  natural numbers in  $O(1)$ .

$$S = \frac{n(n+1)}{2} \quad (1)$$

$$n = \sqrt{2S + \frac{1}{4}} - \frac{1}{2} \quad (2)$$

```

1 int S = (1LL * n * (1LL * n + 1LL))/2;
2 int n = (int)( sqrt( 2 * S + 0.25 ) - 0.5 )
```

Listing 39: Implementation of the Gauss Sum.

**9.1.3 \*Modular Theory****9.1.4 \*Modulo Inverse****9.1.5 \*Fermat's Little Theorem****9.1.6 \*Chinese Remainder Theorem****9.1.7 Binpow**

```

1 const int MOD = 1e9+7;
2 int binpow( long long a, long long b ) { // a^b
3     long long sol = 1;
4     a %= MOD;
5     while( b > 0 ) {
6         if( b & 1 )
```

```
7         sol = ( 1LL * sol * a ) % MOD;
8
9         a = ( 1LL * a * a ) % MOD;
10
11        b >= 1;
12
13    }
14
15    return sol % MOD;
16
17 }
```

Listing 40: Applying binary exponentiation to a problem requiring  $a^b \bmod(10^9 + 7)$  in  $O(\log_2(b))$ .

```
15 Matrix(int n, int m) {
16     mat.resize(n);
17     for(int i = 0; i < n; i++) {mat[i].resize(m);}
18 }
19 int rows() const { return mat.size(); }
20 int cols() const { return mat[0].size(); }
21 void makeIdent() {
22     for(int i = 0; i < rows(); i++)
23         for(int j = 0; j < cols(); j++)
24             mat[i][j] = (i == j ? 1 : 0);
25 }
26 Matrix operator*=(const Matrix &b) {
27     matmul(mat, b.mat);
28     return *this;
29 }
30 void print() {
31     for(int i = 0; i < rows(); i++) {
32         for(int j = 0; j < cols(); j++)
33             cout << mat[i][j] << " ";
34         cout << endl;
35     }
36 }
37 Matrix operator*(const Matrix &b) { return Matrix
38     (*this) *= b; }
39 };
40 int main() {
41     Matrix<ll> A( {{1,1},{1,0}} );
```

### 9.1.8 Matrix Exponentiation (Linear Recurrency)

```
1 template <typename T> void matmul(vector<vector<T>>
2     &a, const vector<vector<T>>& b) {
3     size_t n = a.size(), m = a[0].size(), p = b[0].
4     size();
5     assert(m == b.size());
6     vector<vector<T>> c(n, vector<T>(p));
7     for(size_t i = 0; i < n; i++)
8         for(size_t j = 0; j < p; j++)
9             for(size_t k = 0; k < m; k++)
10                 c[i][j] = (c[i][j] + a[i][k] * b[k][j])
11                 % MOD;
12     a = c;
13 }
14 template <typename T> struct Matrix {
15     vector<vector<T>> mat;
16     Matrix() {}
17     Matrix(vector<vector<T>> a) { mat = a; }
```

```

41 Matrix<ll> ini(2,1);
42 ini.mat[0][0] = 0;
43 ini.mat[1][0] = 1;
44 Matrix<ll> iden(2,2);
45 iden.makeIden();
46 ll n;
47 cin >> n;
48 while(n > 0) {
49     if( n & 1 ) iden *= A;
50     A *= A;
51     n >>= 1;
52 }
53 Matrix<ll> res = iden * ini;
54 cout << res.mat[0][0] << endl;
55 return 0;
56 }
```

Listing 41: Template to pow a matrix of size  $n$  to a certain exponent with logarithmic time (using binpow), and multiply it to another matrix, with modulo operation, as well as how to use it. Full implementation for calculating  $n$ -th Fibonacci term with linear recurrency.

### 9.1.9 Prime checking

```

1 bool prime( int n ){
2     if( n == 2 )
3         return true;
4     if( n % 2 == 0 || n <= 1 )
5         return false;
```

```

6     for( int i = 3; i * i <= n; i += 2 )
7         if( ( n % i ) == 0 )
8             return false;
9     return true;
10 }
```

Listing 42: Returns if  $n$  is a prime number in  $O(\sqrt{n})$ . Avoids overflow  $\forall n \leq 10^6$  ( $\approx INT\_MAX$ ).

### 9.1.10 Prime factorization

```

1 void prime_factorization(vll& factorization, ll n) {
2     for(long long d = 2; d*d <= n; d++) {
3         while(n % d == 0) {
4             factorization.push_back(d);
5             n /= d;
6         }
7     }
8     if( n > 1 )
9         factorization.push_back(n);
10 }
```

Listing 43: Returns prime factorization of the number  $n$  using *trial division*, simplest way. Runs in  $O(\sqrt{n})$ . e.g. for 12 the result is 2x2x3.

### 9.1.11 Sieve of Eratosthenes

```

1 void sieve_of_eratosthenes(vector<bool>& is_prime,
2                             int n) {
3     is_prime.assign(n+1, true);
```

```

3     is_prime[0] = is_prime[1] = false;
4     for(int i = 2; i <= n; i++) {
5         if( is_prime[i] && (long long)i * i <= n ) {
6             for(int j = i*i; j <= n; j += i)
7                 is_prime[j] = false;
8         }
9     }
10 }
```

Listing 44: Calculates every prime number up to  $n$  with sieve of eratosthenes in a boolean 1-indexed vector. Runs in  $O(n \log \log n)$ .

### 9.1.12 Sum of Divisors

```

1 ll sum_of_divisors(ll n) {
2     ll sum = 1;
3     for (long long i = 2; i * i <= n; i++) {
4         if(n % i == 0) {
5             int e = 0;
6             do {
7                 e++;
8                 n /= i;
9             } while (n % i == 0);
10            ll s = 0, pow = 1;
11            do {
12                s += pow;
13                pow *= i;
14            } while (e-- > 0);
15            sum *= s;
16        }
17    }
18    if(n > 1)
19        sum *= (1 + n);
20    return sum;
21 }
```

```

16     }
17 }
18 if(n > 1)
19     sum *= (1 + n);
20 return sum;
21 }
```

Listing 45: Calculates the sum of all divisors of number  $n$ . e.g.  
 $\text{sum\_of\_divisors}(12) = 18$ . Runs in  $O(\sqrt{n})$ .

```

1 void sum_of_divisors_sieve( vll& sigma, int n ) {
2     sigma.assign(n+1,0);
3     for(int i = 1; i <= n; i++)
4         for(int j = i; j <= n; j+=i)
5             sigma[j] += i;
6 }
```

Listing 46: Calculates the sum of all divisors of all numbers from 1 to  $n$ . Runs in  $O(n \log(n))$ .

## 9.2 Combinatorics

### 9.2.1 Binomial Coefficients

```

1 const int MAXN = 1e6+1;
2 vll fact(MAXN+1), inv(MAXN+1);
3 int binpow( ll a, ll b ) { // a^b
4     ll sol = 1;
5     a %= MOD;
6     while( b > 0 ) {
```

```

7     if( b & 1 )
8         sol = ( 1LL * sol * a ) % MOD;
9         a = ( 1LL * a * a ) % MOD;
10        b >>= 1;
11    }
12
13    return sol % MOD;
14}
15
16 void combi() {
17     fact[0] = inv[0] = 1;
18     for(i,1,MAXN+1) {
19         fact[i] = fact[i-1] * i % MOD;
20         inv[i] = binpow( fact[i], MOD - 2 );
21     }
22
23 nCr( ll n, ll r ) {
24     return fact[n] * inv[r] % MOD * inv[n-r] % MOD;
25 }
26 combi();
27 nCr(a,b);

```

Listing 47: Template for calculating binomial coefficients  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ . Precalculate *fact* and *inv* runs in  $O(MAXN \cdot \log_2(MOD))$  ( $\log_2(MOD) \approx 30$ ). So, in general case when  $NMAX = 10^6$  and  $MOD = 10^9 + 7$  can be generalized to  $O(n \cdot \log(n))$ ,  $n \leq 10^6$ .

### 9.2.2 Common combinatorics formulas

$$\binom{n}{2} = \frac{n(n-1)}{2} \quad (3)$$

$$\sum_{k=0}^n \binom{n}{k} = 2^n \quad (4)$$

$$\sum_{k=0}^n \binom{n}{k} \binom{n}{n-k} = \binom{2n}{n} \quad (5)$$

$$\sum_{k=0}^n k \binom{n}{k} = n2^{n-1} \quad (6)$$

$$\sum_{k=0}^{\infty} \binom{2k}{k} \binom{2n-2k}{n-k} = 4^n \quad (7)$$

$$(8)$$

## 9.3 \*Stars and Bars

### 9.4 Probability

### 9.5 Computational Geometry

#### 9.5.1 \*Cross Product

#### 9.5.2 \*Convex Hull

#### 9.6 \*Fast Fourier Transform (FFT)

## 10 Appendix

### 10.1 What to do against WA?

1. Have you done the correct complexity analysis?
2. Have you understood well the statement?
3. Have you corroborated yet the trivial test cases?
4. Have you checked all the corner cases?
5. Have you proposed a lot of non-trivial test cases?

6. Isn't there any possibility of overflow? (Multiplying two `int` needs to be fitted into a `long long`)
7. Have you done a desktop test?
8. Have you red all the variables? (`tc` variable on `main`)
9. Every part of your code works as it's meant to?

## 10.2 Primitive sizes

Data type	[B]	Minimum value it takes	Maximum value it takes
<code>bool</code>	1	0	1
<code>signed char</code>	1	0	255
<code>unsigned char</code>	1	-128	127
<code>signed int</code>	4	$-2,147,483,648 \approx -2 \times 10^9$	$2,147,483,647 \approx 2 \times 10^9$
<code>unsigned int</code>	4	0	$4,294,967,295 \approx 4 \times 10^9$
<code>signed short</code>	2	-32,768	32,767
<code>unsigned short</code>	2	0	65,535
<code>signed long long int</code>	8	$-9,223,372,036,854,775,808 \approx -9 \times 10^{18}$	$9,223,372,036,854,775,807 \approx 9 \times 10^{18}$
<code>unsigned long long int</code>	8	0	$18,446,744,073,709,551,615 \approx 18 \times 10^{18}$
<code>float</code>	4	$1.1 \times 10^{-38}$	$3.4 \times 10^{38}$
<code>double</code>	8	$2.2 \times 10^{-308}$	$1.7 \times 10^{308}$
<code>long double</code>	12	$3.3 \times 10^{-4932}$	$1.1 \times 10^{4932}$

Table 1: Capacity of primitive data types in C++.

32	whitespace	58	:	65	A	97	a
33	!	59	;	66	B	98	b
34	"	60	i	67	C	99	c
35	#	61	=	68	D	100	d
36	\$	62	¸	69	E	101	e
37	%	63	?	70	F	102	f
38	&	64	@	71	G	103	g
39	'	91	[	72	H	104	h
40	(	92	\	73	I	105	i
41	)	93	]	74	J	106	j
42	*	94	^	75	K	107	k
43	+	95	-	76	L	108	l
44	,	96	,	77	M	109	m
45	-	126	~	78	N	110	n
46	.			79	O	111	o
47	/			80	P	112	p
48	0			81	Q	113	q
49	1			82	R	114	r
50	2			83	S	115	s
51	3			84	T	116	t
52	4			85	U	117	u
53	5			86	V	118	v
54	6			87	W	119	w
55	7			88	X	120	x
56	8			89	Y	121	y
57	9			90	Z	122	z

Table 2: Code and symbol of printable ASCII characters.

## 10.4 Numbers bit representation

1	00000001	31	00011111	61	00111101	91	01011011	121	01111001
2	00000010	32	00100000	62	00111110	92	01011100	122	01111010
3	00000011	33	00100001	63	00111111	93	01011101	123	01111011
4	00000100	34	00100010	64	01000000	94	01011110	124	01111100
5	00000101	35	00100011	65	01000001	95	01011111	125	01111101
6	00000110	36	00100100	66	01000010	96	01100000	126	01111110
7	00000111	37	00100101	67	01000011	97	01100001	127	01111111
8	00001000	38	00100110	68	01000100	98	01100010	128	10000000
9	00001001	39	00100111	69	01000101	99	01100011	129	10000001
10	00001010	40	00101000	70	01000110	100	01100100	130	10000010
11	00001011	41	00101001	71	01000111	101	01100101	131	10000011
12	00001100	42	00101010	72	01001000	102	01100110	132	10000100
13	00001101	43	00101011	73	01001001	103	01100111	133	10000101
14	00001110	44	00101100	74	01001010	104	01101000	134	10000110
15	00001111	45	00101101	75	01001011	105	01101001	135	10000111
16	00010000	46	00101110	76	01001100	106	01101010	136	10001000
17	00010001	47	00101111	77	01001101	107	01101011	137	10001001
18	00010010	48	00110000	78	01001110	108	01101100	138	10001010
19	00010011	49	00110001	79	01001111	109	01101101	139	10001011
20	00010100	50	00110010	80	01010000	110	01101110	140	10001100
21	00010101	51	00110011	81	01010001	111	01101111	141	10001101
22	00010110	52	00110100	82	01010010	112	01110000	142	10001110
23	00010111	53	00110101	83	01010011	113	01110001	143	10001111
24	00011000	54	00110110	84	01010100	114	01110010	144	10010000
25	00011001	55	00110111	85	01010101	115	01110011	145	10010001
26	00011010	56	00111000	86	01010110	116	01110100	146	10010010
27	00011011	57	00111001	87	01010111	117	01110101	147	10010011
28	00011100	58	00111010	88	01011000	118	01110110	148	10010100
29	00011101	59	00111011	89	01011001	119	01110111	149	10010101
30	00011110	60	00111100	90	01011010	120	01111000	150	10010110

## 10.5 How a `vector<vector<pair<int,int>>` looks like

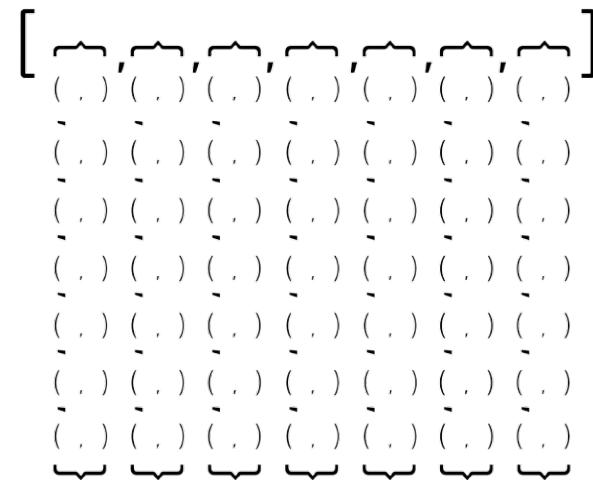


Figure 1: Visual representation of a vector of vector of pairs.

## 10.6 How all neighbours of a grid looks like

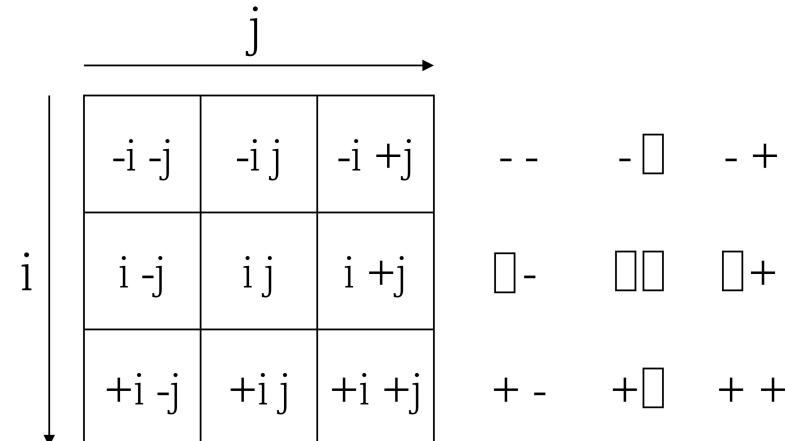


Figure 2: Visual representation of how all adjacent cells in a grid looks like.