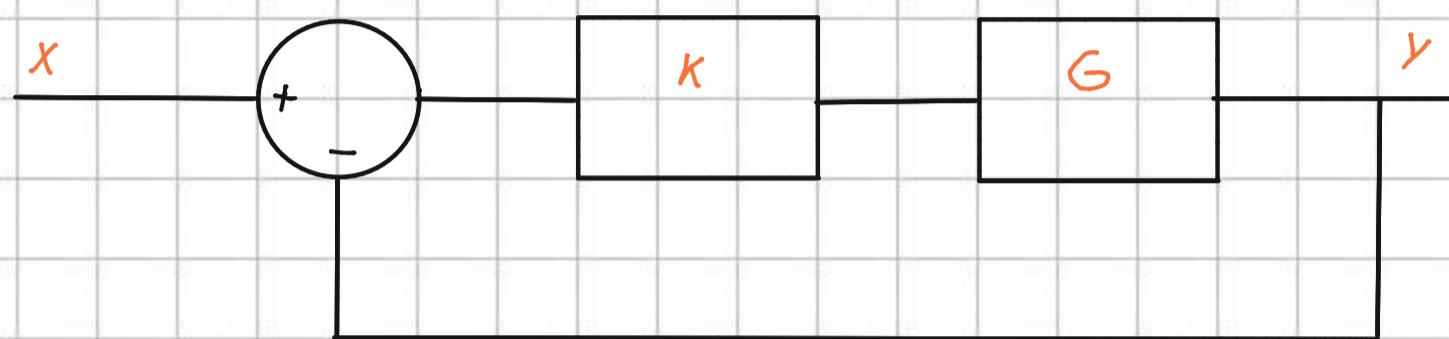


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Taller 1 - Control

- Considera el siguiente sistema el cual posee una planta G , una entrada X , una salida Y y un control K como se observa en la figura.



Considera la respuesta en el tiempo del sistema con la entrada al escalón y tenga en cuenta las siguientes consideraciones.

$$Y = \frac{KG}{X} ; G = \frac{1}{s(s+3)(s+6)}$$

Al reemplazar G se tiene:

$$\frac{Y}{X} = \frac{K}{s(s+3)(s+6)} + \frac{K}{s(s+3)(s+6)}$$

$$\frac{Y}{X} = \frac{K}{s(s+3)(s+6) + K}$$

a) $K = 0,7$; $X = 1/s \rightarrow$ Entrada escalón

$$\frac{Y}{X} = \frac{0,7}{s(s+3)(s+6) + 0,7} \cdot \frac{1}{s}$$

$$\frac{Y}{X} = \frac{0,7}{s^3 + 9s^2 + 18s + 0,7} \cdot \frac{1}{s}$$

$$\frac{Y}{X} = \frac{0,7}{(s+6,0382)(s+0,0397)(s+2,922)} \cdot \frac{1}{s}$$

Al realizar fracciones parciales se tiene lo siguiente:

$$\frac{0,7}{(s+6,0382)(s+0,0397)(s+2,922)s} = \frac{A}{(s+6,0382)} + \frac{B}{(s+0,0397)} + \frac{C}{(s+2,922)} + \frac{D}{s}$$

$$0,7 = A((s+0,0397)(s+2,922)s) + B((s+6,0382)(s+2,922)s) + C((s+6,0382)(s+0,0397)s) + D((s+6,0382)(s+0,0397)(s+2,922))$$

✓ Si $s = -0,0397$

$$0,7 = B((-0,0397+6,0382)(-0,0397+2,922)(-0,0397))$$

$$0,7 = B(-0,68639)$$

$$B = -1,0198$$

✓ Si $s = -6,0382$

$$0,7 = A(-6,0382(-6,0382+2,922)(-6,0382+0,0397))$$

$$0,7 = A(-112,8692)$$

$$A = -0,0062$$

✓ Si $s = -2,922$

$$0,7 = C(-2,922(-2,922+6,0382)(-2,922+0,0397))$$

$$0,7 = C(26,2449)$$

$$C = 0,0267$$

✓ Si $s = 0$

$$0,7 = D((6,0382)(2,922)(0,0397))$$

$$0,7 = D(0,70045)$$

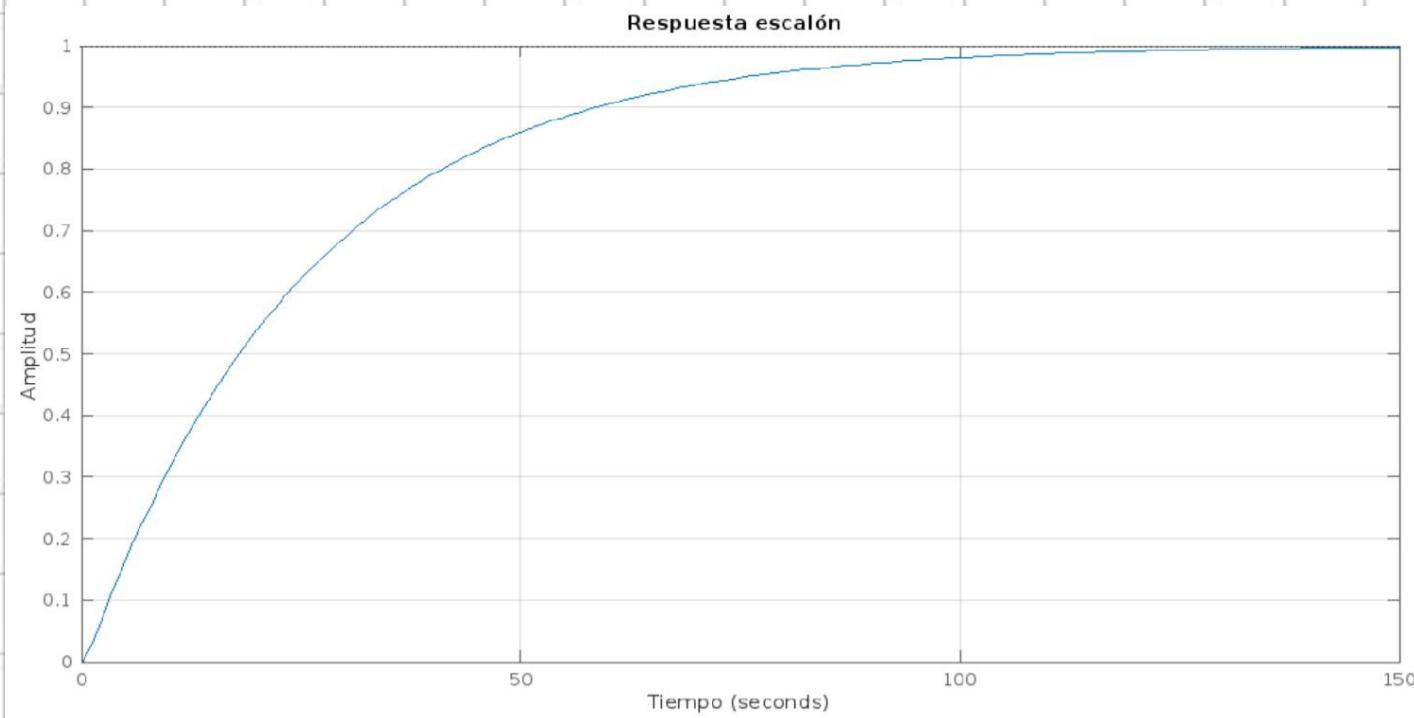
$$D = 1$$

$$\frac{Y}{X} = \frac{-0,0062}{(s+6,0382)} + \frac{-1,0198}{(s+0,0397)} + \frac{0,0267}{(s+2,922)} + \frac{1}{s}$$

La respuesta del sistema en el tiempo es:

$$y(t) = -0,0062 e^{-6,0382 t} - 1,0198 e^{-0,0397 t} + 0,0267 e^{-2,922 t} + u(t)$$

✓ Respuesta al escalón



Gráfica 1. Respuesta sobre amortiguada

$$b) K = 35$$

$$\frac{Y}{X} = \frac{35}{s^3 + 9s^2 + 18s + 35} \cdot \frac{1}{s}$$

$$\frac{Y}{X} = \frac{35}{(s+7,17)(s+0,9148-j2,011)(s+0,9148+j2,011)} s$$

Al realizar fracciones parciales

$$\frac{35}{(s+7,17)(s+0,9148-j2,011)(s+0,9148+j2,011)} s = \frac{A}{s} + \frac{B}{(s+7,17)} + \frac{C}{(s+0,9148-j2,011)} + \frac{D}{(s+0,9148+j2,011)}$$

$$35 = A((s+0,9148-j2,011)(s+0,9148+j2,011)(s)) + B((s+7,17)(s+0,9148+j2,011)(s)) + C((s+7,17)(s+0,9148-j2,011)(s)) + D((s+7,17)(s+0,9148-j2,011)(s+0,9148+j2,011))$$

$$\checkmark \quad \text{Si } s = 0$$

$$35 = D(7,17)(0,9148-j2,011)(0,9148+j2,011)$$

$$35 = D(34,997)$$

$$D = 1$$

$$\checkmark \quad \text{Si } s = -0,9148 - j2,011$$

$$35 = C((-0,9148-j2,011+7,17)(-0,9148-j2,011) \\ (-0,9148-j2,011+0,9148-j2,011))$$

$$35 = C(-43,194 + j39,28)$$

$$C = -0,4435 - j0,4033$$

$$\checkmark \quad \text{Si } s = -0,9148 + j2,011$$

$$35 = B((-0,9148+j2,011+7,17)(-0,9148+j2,011) \\ (-0,9148+j2,011+0,9148+j2,011))$$

$$35 = B(-43,194 - j39,28)$$

$$B = -0,4435 + j0,4033$$

✓ Reemplazando los valores conocidos

$$35 = A((s+0,9148-j2,011)(s+0,9148+j2,011)(s)) + \\ -0,4435 + j0,4033 ((s+7,17)(s+0,9148+j2,011)(s)) + \\ -0,4435 - j0,4033 ((s+7,17)(s+0,9148-j2,011)(s)) + \\ ((s+7,17)(s+0,9148-j2,011)(s+0,9148+j2,011))$$

$$35 = A(s^3 + 1,8296s^2 + 4,8809s) + \\ -0,4435s^3 - 4,3966s^2 - 8,7241s) + \\ (0,4033s^3 + 2,3687s^2 - 3,7495s)j \\ -0,4435s^3 - 4,3966s^2 - 8,7241s) \\ (-0,4033s^3 - 2,3687s^2 + 3,7495s)j + \\ s^3 + 9s^2 + 18s + 35$$

$$35 = A(s^3 + 1,8296s^2 + 4,8809s) + \\ 0,113s^3 + 0,2067s^2 + 0,5518s + 35$$

$$35 = s^3(0,113 + A) + s^2(1,8296A + 0,2067) + \\ s(4,8809A + 0,5518s) + 35$$

✓ Igualando términos

$$0 = 0,113 + A \rightarrow A = -0,113$$

$$\frac{Y}{X} = \frac{-0,113}{s+7,17} + \frac{-0,4435 + j0,4033}{s+0,9148-2,011j} + \\ \frac{-0,4435 - j0,4033}{s+0,9148+2,011j} + \frac{1}{s}$$

A) realizar la transformada inversa Laplace

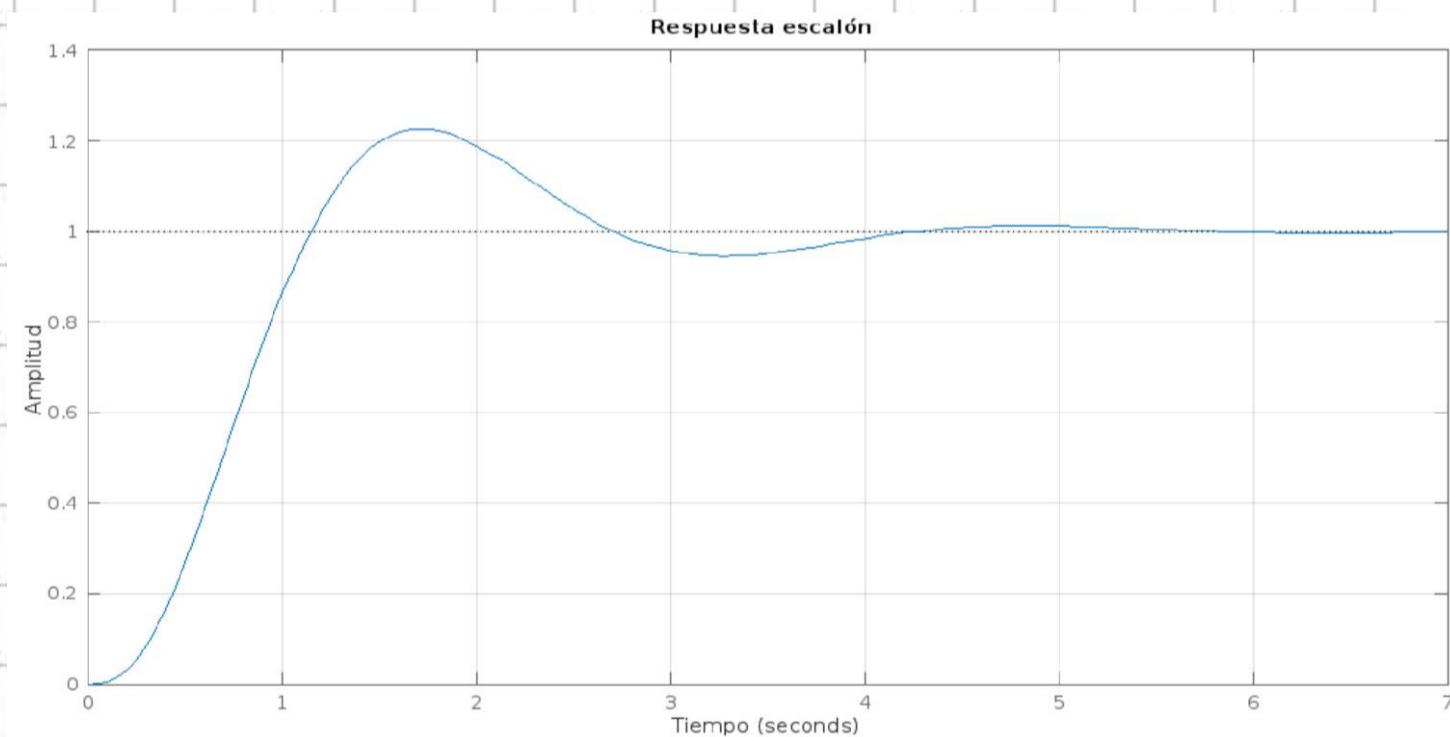
$$\frac{1}{s+\alpha - \beta j} = e^{-\alpha t} \cos(\beta t) + j e^{-\alpha t} \sin(\beta t)$$

$$\frac{Y}{X} = -0,113 e^{-7,17t} + (-0,4435 + j 0,4033)^* \\ (e^{-0,9148t} \cos(2,011t) + j e^{-0,9148t} \sin(2,011t)) \\ + (-0,4435 - j 0,4033)^* \\ (e^{-0,9148t} \cos(-2,011t) + j e^{-0,9148t} \sin(-2,011t))$$

Considerando $\cos(\alpha) = \cos(-\alpha)$
 $\sin(\alpha) = -\sin(-\alpha)$

La respuesta en el tiempo es:

$$\frac{Y}{X} = -0,113 e^{-7,17t} - 0,887 e^{-0,9148t} \cos(2,011t) \\ - 0,8066 e^{-0,9148t} \sin(2,011t) + u(t)$$



Gráfica 2. Respuesta Subamortiguada

c) $K = 162$

$$\frac{Y}{X} = \frac{162}{s^3 + 9s^2 + 18s + 162} \cdot \frac{1}{s}$$

$$\frac{Y}{X} = \frac{162}{(s+9)(s-j4,243)(s+j4,243)} \cdot \frac{1}{s}$$

Al realizar Fracciones Parciales

$$\frac{Y}{X} = \frac{162}{(s+9)(s-j4,243)(s+j4,243)s} = \frac{A}{(s+9)} + \frac{B}{(s-j4,243)} + \frac{C}{(s+j4,243)} + \frac{D}{s}$$

$$162 = A(s-j4,243)(s+j4,243)(s) + \\ B(s+9)(s+j4,243)(s) + \\ C(s+9)(s-j4,243)(s) + \\ D(s+9)(s-j4,243)(s+j4,243)$$

/ si $s=0$

$$162 = D(9)(-j4,243)(j4,243)$$

$$162 = D(162)$$

$$D = 1$$

✓ si $s=-4,243j$

$$162 = C(-j4,243+9)(-j4,243-j4,243)(-j4,243)$$

$$162 = C(-324,0549 + j152,7739j)$$

$$C = -0,409 - j0,1928$$

✓ si $s=j4,243$

$$162 = B(j4,243+9)(j4,243+j4,243)(j4,243)$$

$$162 = B(-324,0549 + j152,7739)$$

$$B = -0,409 + j0,1928$$

$$162 = A(s+j4,243)(s-j4,243)(s) + \\ (-0,409 + j0,1928)(s+9)(s+j4,243)(s) + \\ (-0,409 - j0,1928)(s+9)(s-j4,243)(s) + \\ (s+j4,243)(s+9)(s+j4,243)$$

$$162 = A(s^3 + 18s) +$$

$$(-0,409s^3 - 4,4990s^2 - 7,3624s) +$$

$$(0,1928s^3 - 0,0002s^2 - 15,6184s)j +$$

$$(-0,409s^3 - 4,4990s^2 - 7,3624s) +$$

$$(-0,1928s^3 + 0,0002s^2 + 15,6184s)j +$$

$$s^3 + 9s^2 + 18s + 162$$

$$162 = A(s^3 + 18s) + 0,182s^3 + 0,0019s^2 + 3,275s + 162$$

• Al igualar términos

$$0 = A + 0,182 \rightarrow A = -0,182$$

$$\frac{Y}{X} = \frac{-0,182}{s+9} + \frac{-0,409+j0,1928}{s-j4,243} + \frac{-0,409-j0,1928}{s+j4,243} + \frac{1}{s}$$

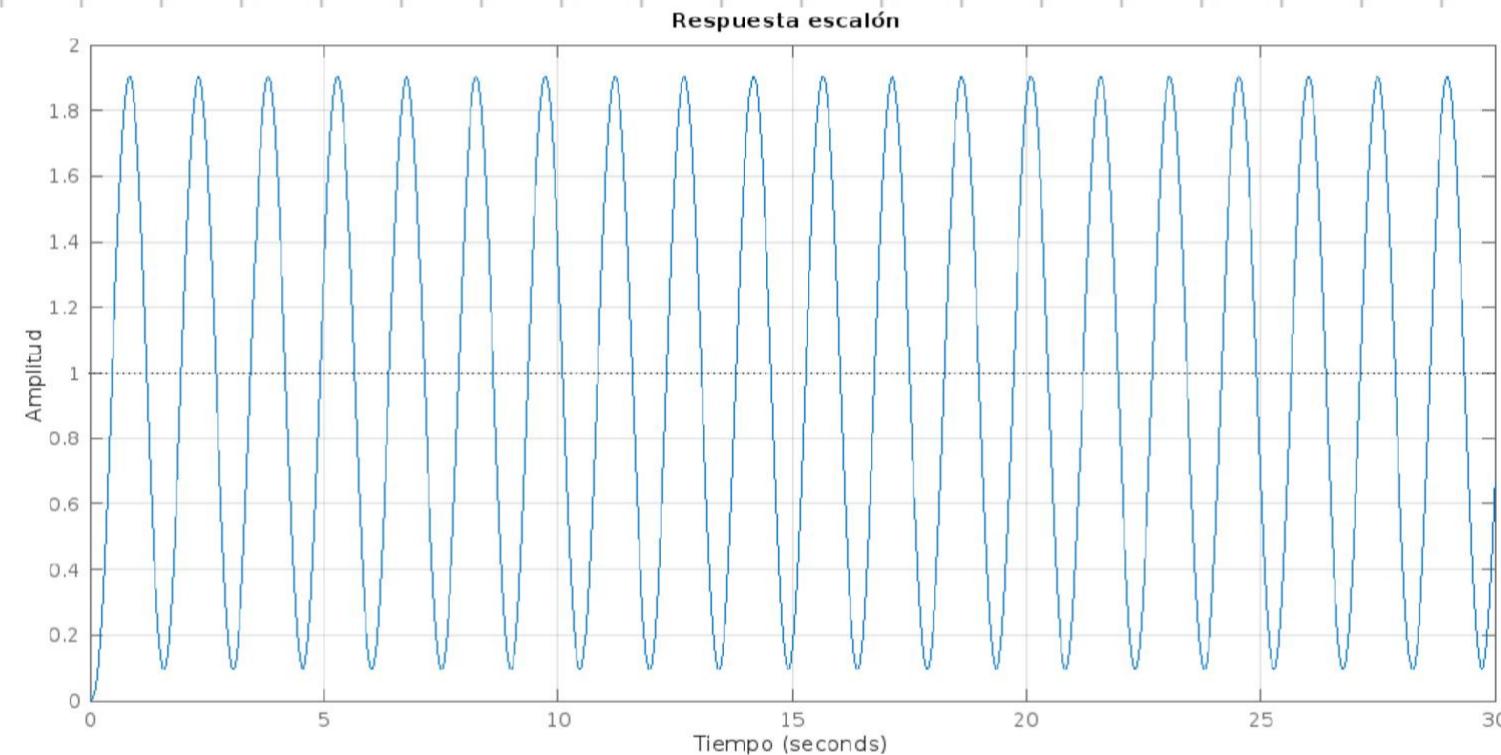
Al considerar la respuesta en el tiempo del sistema

- $\frac{1}{s+d-\beta j} = e^{-\alpha t} \cos(\beta t) + j e^{-\alpha t} \sin(\beta t)$

$$\frac{Y}{X} = -0,182 e^{-9t} + (-0,409 + j0,1928)(\cos(4,243t) + j \sin(4,243t)) + (-0,409 - j0,1928)(\cos(-4,243t) + j \sin(-4,243t)) + u(t)$$

- Considerando $\cos(\alpha) = \cos(-\alpha)$
 $\sin(\alpha) = -\sin(-\alpha)$

$$\frac{Y}{X} = -0,182 e^{-9t} - 0,818 \cos(4,243t) - 0,3856 \sin(4,243t) + u(t)$$



Gráfica 3. Respuesta sin amortiguamiento

d) $K = 1350$

$$\frac{Y}{X} = \frac{1350}{s^3 + 9s^2 + 18s + 1350} \cdot \frac{1}{s}$$

$$Y = \frac{1350}{X(s+14,3235)(s-2,6617-j9,3363)(s-2,6617+j9,3363)s}$$

Al realizar fracciones Parciales

$$1350 = \frac{A}{(s+14,3235)} + \frac{B}{(s-2,6617-j9,3363)} + \frac{C}{(s-2,6617+j9,3363)} + \frac{D}{s}$$

$$1350 = A(s-2,6617-j9,3363)(s-2,6617+j9,3363)(s) + B(s+14,3235)(s-2,6617+j9,3363)(s) + C(s+14,3235)(s-2,6617-j9,3363)(s) + D(s+14,3235)(s-2,6617-j9,3363)(s-2,6617+j9,3363)$$

✓ Si $s=0$

$$1350 = D(14,3235)(-2,6617-j9,3363)(-2,6617+j9,3363)$$

$$1350 = D(1350)$$

$$D = 1$$

✓ Si $s = 2,6617 - j9,3363$

$$1350 = C(2,6617 - j9,3363 + 14,3235)^*$$

$$(2,6617 - j9,3363 - 2,6617 - j9,3363)^*$$

$$(2,6617 - j9,3363)$$

$$1350 = C(-3425,1029 + j783,4461)$$

$$C = -0,3745 - 0,0857j$$

✓ Si $s = 2,6617 + j9,3363$

$$1350 = B(2,6617 + j9,3363 + 14,3235)^*$$

$$(2,6617 + j9,3363 - 2,6617 + j9,3363)^*$$

$$(2,6617 + j9,3363)$$

$$1350 = B(-3425,1029 - j783,4461)$$

$$B = -0,3745 + j0,0857$$

✓ Reemplazando los valores conocidos

$$1350 = A(s-2,6617-j9,3363)(s-2,6617+j9,3363)(s) + -0,3745 - j0,0857(s+14,3235)(s-2,6617+j9,3363)(s) + -0,3745 + j0,0857(s+14,3235)(s-2,6617-j9,3363)(s) + (s+14,3235)(s-2,6617-j9,3363)(s-2,6617+j9,3363)$$

$$1350 = A(s^3 - 5,3235s^2 + 94,2511s) +$$

$$(-0,3745s^3 - 3,5672s^2 + 25,7383s) +$$

$$(-0,0857s^3 - 4,4959s^2 - 46,8140s)j +$$

$$(-0,3745s^3 - 3,5672s^2 + 25,7383s) +$$

$$(0,0857s^3 + 4,4959s^2 + 46,8140s)j +$$

$$(s^3 + 9s^2 + 18s + 1350)$$

$$1350 = A(s^3 - 5,3234s^2 + 94,2511s) + (0,251s^3 + 1,8656s^2 + 69,4766s + 1350)$$

✓ Igualando términos

$$0 = (A + 0,251) \rightarrow A = -0,251$$

$$\frac{Y}{X} = \frac{-0,251}{(s + 14,3234)} \frac{-0,3745 + j0,0857}{(s - 2,6617 - j9,3363)} + \frac{1}{s}$$

$$\frac{-0,3745 - j0,0857}{(s - 2,6617 + j9,3363)} + \frac{1}{s}$$

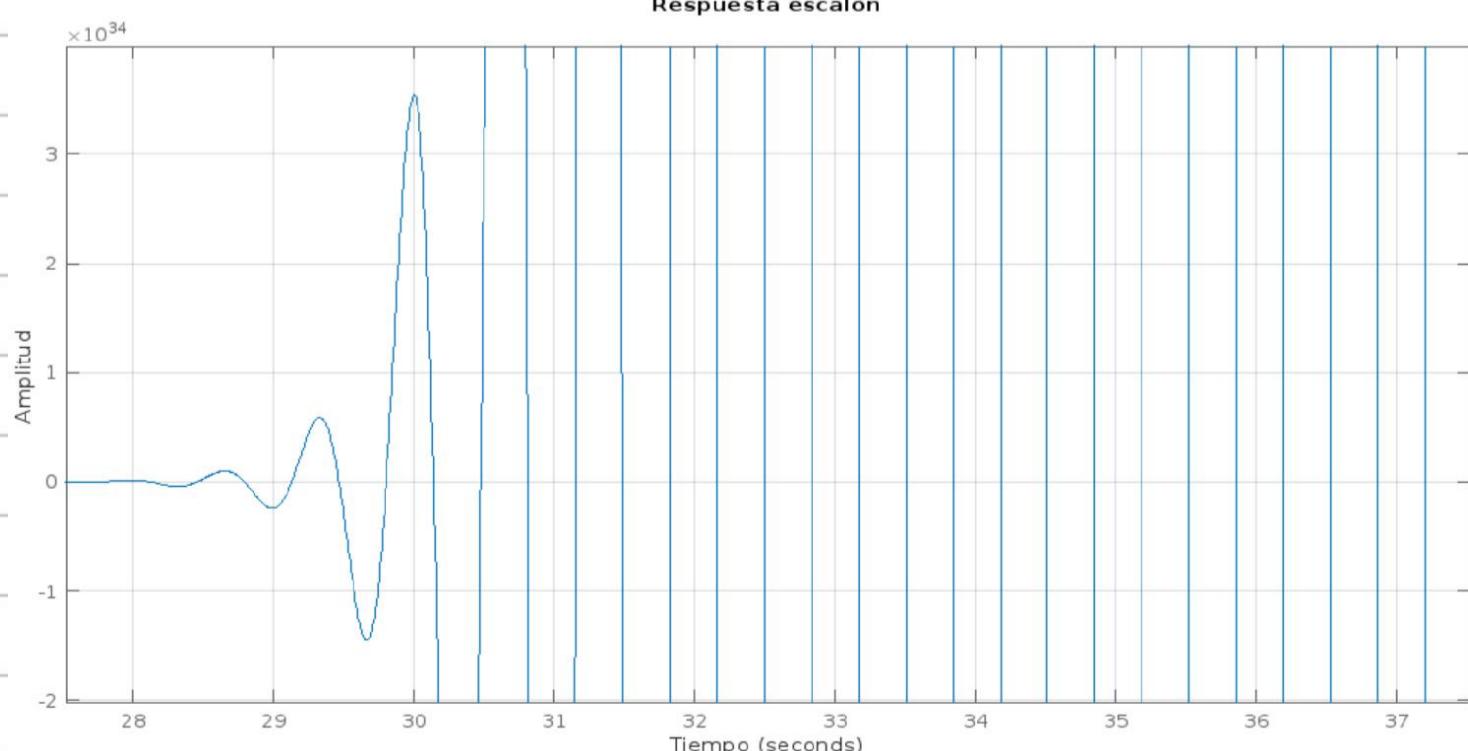
Al considerar la respuesta en el tiempo del sistema

- $\frac{1}{s + \alpha - \beta j} = e^{-\alpha t} \cos(\beta t) + j e^{-\alpha t} \sin(\beta t)$

$$\frac{Y}{X} = -0,251 e^{-14,3234t} + (-0,3745 + j0,0857)^* \\ (e^{2,6617t} \cos(9,3363t) + j e^{2,6617t} \sin(9,3363t)) \\ + (-0,3745 - j0,0857)^* \\ (e^{2,6617t} \cos(-9,3363t) + j e^{2,6617t} \sin(-9,3363t)) \\ + u(t)$$

- Considerando $\cos(\alpha) = \cos(-\alpha)$
 $\sin(\alpha) = -\sin(-\alpha)$

$$\frac{Y}{X} = -0,251 e^{-14,3234t} - 0,749 e^{2,6617t} \cos(9,3363t) \\ - 0,1714 e^{2,6617t} \sin(9,3363t)$$



Gráfica 4. Respuesta amplificada sin amortiguación.