4) Encontrar la respuesta al escalón de los sistemas con estas ecuaciones de planta

a) $G(s) = \dfrac{9}{s^2+2s+9} = \dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$\omega_n^2 = 9 \rightarrow \omega_n = \sqrt{9} = 3$

$2\xi\omega_n s = 2s \rightarrow \xi = 1/3 < 1$ Subamortiguado

✓ La respuesta al escalón es

$G(s) = \dfrac{1}{s}\left(\dfrac{9}{s^2+2s+9}\right) \rightarrow G(s) = \dfrac{9}{s^3+2s^2+9s}$

$G(s) = \dfrac{9}{(s+1-2{,}83j)(s+1+2{,}83j)(s)}$

✓ Al realizar ecuaciones parciales

$\dfrac{9}{(s+1-2{,}83j)(s+1+2{,}83j)(s)} = \dfrac{A}{(s+1-2{,}83j)} + \dfrac{B}{(s+1+2{,}83j)} + \dfrac{C}{(s)}$

$9 = A(s)(s+1+2{,}83j) + B(s)(s+1-2{,}83j) +$
$C(s+1+2{,}83j)(s+1-2{,}83j)$

✓ si $s=0$ $C(1+2{,}83j)(1-2{,}83j) = 9$

$C = 9/9 = 1$

✓ si $s = -1-2{,}83j$ $B(-1-2{,}83j)(-1-2{,}83j+1-2{,}83j) = 9$

$B(-16{,}0178+5{,}66j) = 9 \rightarrow B = -0{,}5-0{,}18j$

✓ Si $s = -1 + 2,83j$

$A(-1+2,83j)(-1+2,83j+1+2,83j) = 9$

$A(-16,0178 - 5,66j) = 9 \rightarrow A = -0,5 + 0,18j$

$$G(s) = \frac{(-0,5+0,18j)}{s+1-2,83j} + \frac{(-0,5-0,18j)}{s+1+2,83j} + \frac{1}{s}$$

✓ Para hallar la expresión en el tiempo

$$G(s) = (-0,5+0,18j)\frac{1}{s+1-2,83j} + (-0,5-0,18j)\frac{1}{s+1+2,83j}$$

$$+ \frac{1}{s}$$

✓ Al realizar la transformada inversa de Laplace

- $\dfrac{1}{s+\alpha-\beta j} = e^{-\alpha t}\cos(\beta t) + j e^{-\alpha t}\sin(\beta t)$

- $\dfrac{1}{s} = u(t) \rightarrow 1$

$$Y(t) = (-0,5+0,18j)(e^{-t}\cos(2,83t) + j e^{-t}\sin(2,83t)) + $$
$$(-0,5-0,18j)(e^{-t}\cos(-2,83t) + j e^{-t}\sin(-2,83t))$$
$$+ 1$$

$$Y(t) = -0,5(e^{-t}\cos(2,83t) + j e^{-t}\sin(2,83t)) +$$
$$0,18j(e^{-t}\cos(2,83t) + j e^{-t}\sin(2,83t))$$
$$-0,5(e^{-t}\cos(-2,83t) + j e^{-t}\sin(-2,83t)) +$$
$$-0,18j(e^{-t}\cos(-2,83t) + j e^{-t}\sin(-2,83t)) + 1$$

$$y(t) = -0{,}5\,e^{-t}\cos(2{,}83t) - 0{,}5j\,e^{-t}\sin(2{,}83t) +$$
$$0{,}18j\,e^{-t}\cos(2{,}83t) - 0{,}18\,e^{-t}\sin(2{,}83t) +$$
$$-0{,}18j\,e^{-t}\cos(-2{,}83t) + 0{,}18\,e^{-t}\sin(-2{,}83t) +$$
$$-0{,}5\,e^{-t}\cos(-2{,}83t) - 0{,}5j\,e^{-t}\sin(-2{,}83t) + 1$$

- Si consideramos $\cos(\alpha) = \cos(-\alpha)$ y $\sin(-\alpha) = -\sin(\alpha)$

$$Y(t) = -e^{-t}\cos(2{,}83t) - j e^{-t}\sin(-2{,}83t)$$
$$-0{,}36\,e^{-t}\sin(2{,}83t) + 1$$

- Al tomar la respuesta real $\mathbb{R}$

$$y(t) = -e^{-t}\cos(2{,}83t) - 0{,}36\,e^{-t}\sin(2{,}83t) + 1$$

b) $$G(s) = \frac{9}{s^2+9} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n^2 = 9 \rightarrow \omega_n = 3$

$2\zeta\omega_n s = 0 \rightarrow \zeta = 0 \rightarrow$ oscilador

✓ La respuesta al escalón es

$$G(s) = \frac{1}{s}\left(\frac{9}{s^2+9}\right) \rightarrow G(s) = \frac{9}{s^3+9s}$$

$$G(s) = \frac{9}{(s-3j)(s+3j)(s)}$$

✓ Al realizar ecuaciones Parciales

$$\frac{9}{(s-3j)(s+3j)(s)} = \frac{A}{(s-3j)} + \frac{B}{(s+3j)} + \frac{C}{s}$$

$$9 = A(s)(s+3j) + B(s-3j)(s) + C(s+3j)(s-3j)$$

✓ Si $s=0 \to 9 = C(3j)(-3j)$

$C = 9/9 = 1$

✓ Si $s=-3j$ $9 = B(-3j-3j)(-3j)$

$B = 9/-18 = -0,5$

✓ Si $s=3j$ $9 = A(3j)(3j+3j)$

$A = 9/-18 = -0,5$

$$G(s) = \frac{-0,5}{(s-3j)} - \frac{0,5}{(s+3j)} + \frac{1}{s}$$

✓ Para hallar la ecuación en el tiempo

$$G(s) = -0,5\left(\frac{1}{s-3j} + \frac{1}{s+3j}\right) + \frac{1}{s}$$

- Al realizar la transformada inversa.

- $\frac{1}{s+\alpha-Bj} = e^{-\alpha t}\cos(Bt) + je^{-\alpha t}\sin(Bt)$

- $\frac{1}{s} = u(t) = 1$

$$y(t) = -0{,}5\left((\cos(3t) + j\sin(3t)) + (\cos(-3t) + j\sin(-3t)\right)$$
$$+1$$

✓ Si $\cos(\alpha) = \cos(-\alpha)$ y $\sin(-\alpha) = -\sin(\alpha)$

$$y(t) = -0{,}5\,(2\cos(3t) + j\sin(3t) - j\sin(3t))$$
$$+1$$

$$y(t) = 1 - \cos(3t)$$

c) $G(s) = \dfrac{9}{s^2 + 9s + 9} = \dfrac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$

$w_n^2 = 9 \rightarrow w_n = 3$

$2\xi w_n s = 9s \rightarrow \xi = \dfrac{9}{6} > 1 \rightarrow$ sobreamortiguado

✓ La respuesta al escalón es

$$G(s) = \frac{1}{s}\left(\frac{9}{s^2 + 9s + 9}\right) \rightarrow \frac{9}{s^3 + 9s^2 + 9s} = G(s)$$

$$G(s) = \frac{9}{(s + 1{,}14)(s + 7{,}85)(s)}$$

✓ Al realizar ecuaciones Parciales

$$\frac{9}{(s+1,15)(s+7,85)(s)} = \frac{A}{(s+1,15)} + \frac{B}{(s+7,85)} + \frac{C}{s}$$

$$9 = A(s+7,85)(s) + B(s+1,15)(s) + C(s+1,15)(s+7,85)$$

✓ si $s = 0 \rightarrow 9 = C(1,15)(7,85)$

$C \approx 1$

✓ si $s = -7,85$

$9 = B(-7,85+1,15)(-7,85) \rightarrow 9 = B(52,6)$

$B = 9/52,6 = 0,171$

✓ si $s = -1,15$

$9 = A(-1,15)(-1,15+7,85) \rightarrow 9 = A(7,705)$

$A = 9/(7,705) = -1,168$

$$G(s) = \frac{-1,168}{(s+1,15)} + \frac{0,171}{(s+7,85)} + \frac{1}{s}$$

✓ Para hallar la ecuación en el tiempo

- $\frac{1}{s+\sigma} = e^{-\sigma t}$

$$Y(t) = -1,168\,e^{-1,15t} + 0,171\,e^{-7,85t} + 1$$

d) $G(s) = \dfrac{9}{s^2+6s+9} = \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\omega_n^2 = 9 \rightarrow \omega_n = 3$

$2\zeta\omega_n s = 6s \rightarrow \zeta = 1 \rightarrow$ Criticamente Amortiguado

✓ La respuesta al escalón es

$G(s) = \dfrac{1}{s}\left(\dfrac{9}{s^2+6s+9}\right) \rightarrow G(s) = \dfrac{9}{s^3+6s^2+9s}$

$G(s) = \dfrac{9}{(s+3)(s)(s+3)}$

✓ Al realizar fracciones Parciales

$\dfrac{9}{(s+3)(s+3)(s)} = \dfrac{A}{(s+3)} + \dfrac{B}{(s+3)^2} + \dfrac{C}{s}$

✓ si $s = 0 \rightarrow 9 = C(3)^2 \rightarrow C = 1$

✓ si $s = -3 \rightarrow 9 = B(-3) \rightarrow B = -3$

✓ $9 = A(s+3)(s) - 3(s) + (s+3)^2 \rightarrow A = -1$

$G(s) = \dfrac{-1}{s+3} \; \dfrac{-3}{s+3} + \dfrac{1}{s}$

- Para hallar la ecuación en el tiempo
- $\dfrac{1}{s+a} \rightarrow e^{-at}$: $y(t) = -e^{-3t} - 3e^{-3t} + 1$