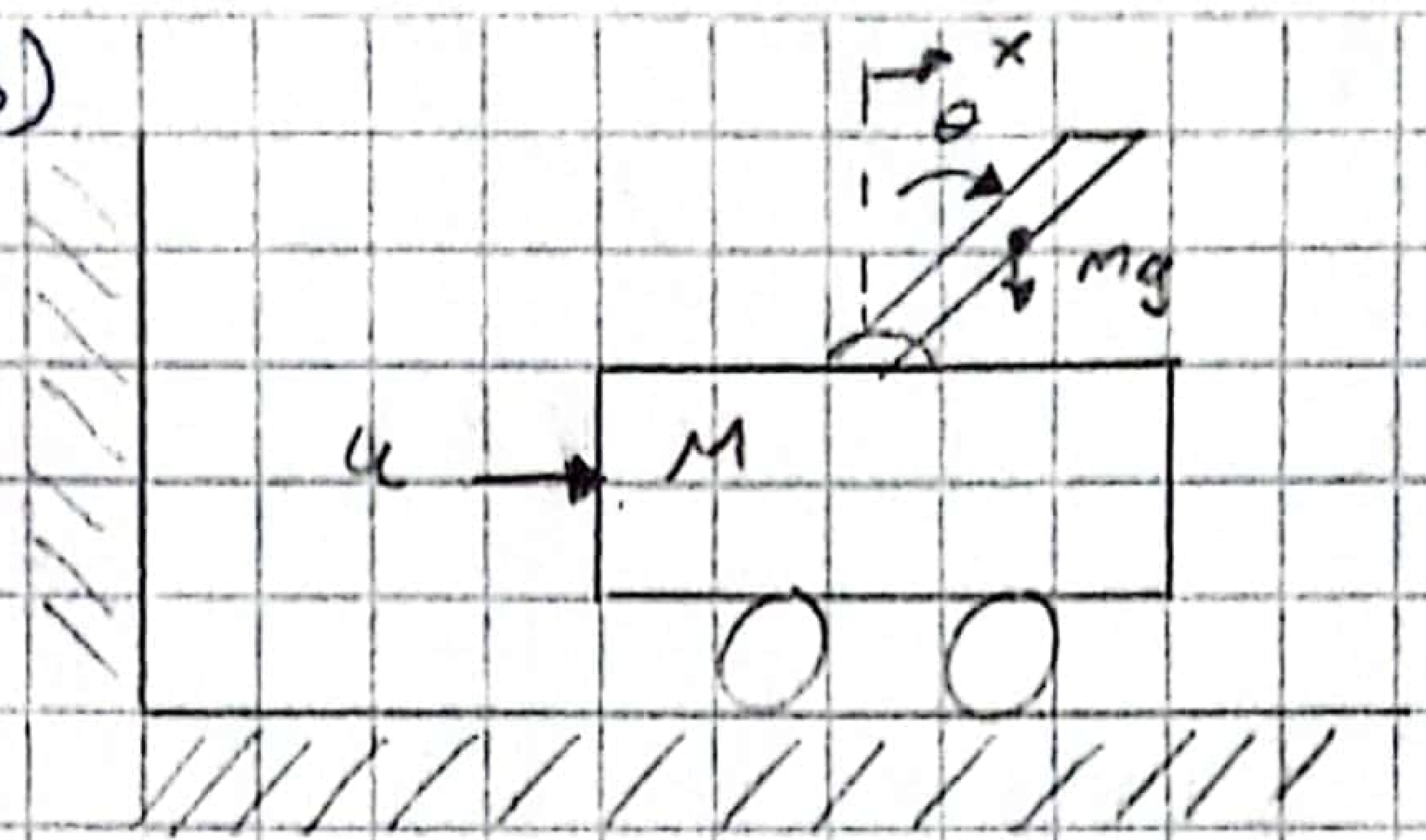
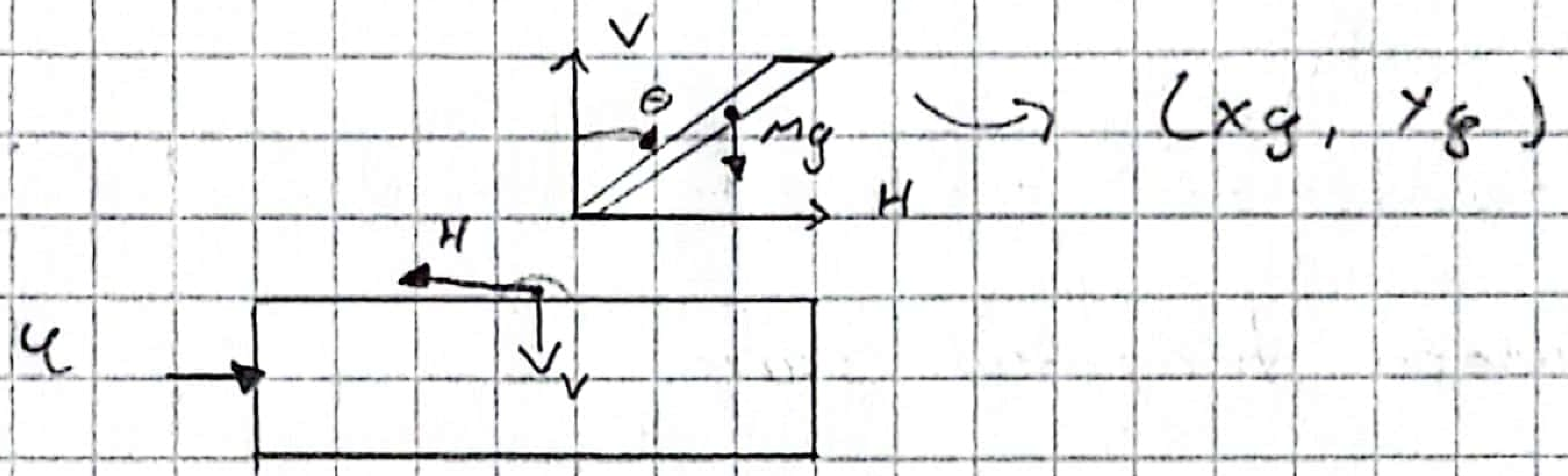
3)





$$x_g = x + l\sin(\theta)$$
$$y_g = l\cos(\theta)$$

✓ Movimiento Rotacional

$$I\ddot{\theta} = V l \sin(\theta) - H l \cos(\theta) \qquad (1)$$

✓ Movimiento horizontal

$$H = m\frac{d^2(x + l\sin(\theta))}{dt^2}$$

$$H = m\ddot{x} + \frac{m\, d^2(l\sin(\theta))}{dt^2}$$

$$H = m\ddot{x} + m\frac{d(l\cos(\theta)\dot{\theta})}{dt}$$

$$H = m\ddot{x} + ml(-\sin(\theta)\dot{\theta}\dot{\theta} + \cos(\theta)\ddot{\theta})$$

$$H = m\ddot{x} - ml\sin(\theta)\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta} \qquad (2)$$

✓ Movimiento Vertical

$$m \frac{d^2(\ell\cos(\theta))}{dt} = V - mg \qquad (3)$$

$$m \frac{d(\ell - \sin(\theta)\dot{\theta})}{dt} = V - mg$$

$$m\ell(-\cos(\theta)\dot{\theta}^2 - \sin(\theta)\ddot{\theta}) = V - mg$$

✓ Movimiento horizontal carro

$$M\ddot{x} = u - H \qquad (4)$$

→ Si θ muy pequeño

$$\sin(\theta) \to \theta \quad ; \quad \cos(\theta) \to 1 \quad ; \quad \theta\dot{\theta} \to 0$$

✓ De (1)

$$I\ddot{\theta} = V\ell\theta - H\ell \qquad (5)$$

✓ De (2)

$$m\ddot{x} - m\ell\theta\dot{\theta}^2 + m\ell\ddot{\theta} = H$$

$$m\ddot{x} + m\ell\ddot{\theta} = H \to m(\ddot{x} + \ell\ddot{\theta}) = H \qquad (6)$$

✓ De (3)

$$0 = V - mg \to V = mg \qquad (7)$$

✓ Reemplazando H de (4) y (6)

$$M\ddot{x} = u - m(\ddot{x} + \ell\ddot{\theta})$$

$$\ddot{x}(M+m) + m\ell\ddot{\theta} = u \qquad (8)$$

Reemplazando (5) y (6) y (7)

$$I\ddot{\theta} = Vl\theta - Hl$$

$$I\ddot{\theta} = (mg)l\theta - (m(\ddot{x} + l\ddot{\theta}))l$$

$$I\ddot{\theta} = mgl\theta - ml\ddot{x} - ml^2\ddot{\theta}$$

$$\ddot{\theta}(I + ml^2) + ml\ddot{x} = mgl\theta \qquad (9)$$

Al despejar $\ddot{x}$ de (8) y reemplazar en (9)

$$\ddot{x}(M+m) + ml\ddot{\theta} = u$$

$$\ddot{x} = \frac{u - ml\ddot{\theta}}{M+m}$$

$$\ddot{\theta}(I + ml^2) + ml\ddot{x} = mgl\theta$$

$$\ddot{\theta}(I + ml^2) + ml\left(\frac{u - ml\ddot{\theta}}{M+m}\right) = mgl\theta$$

$$\ddot{\theta}(I + ml^2) + \frac{mlu}{M+m} - \frac{m^2l^2\ddot{\theta}}{M+m} = mgl\theta$$

$$\ddot{\theta}\left(\frac{I(M+m) + Mml^2}{M+m}\right) + \frac{mlu}{M+m} = mgl\theta$$

$$\ddot{\theta} = \left(\frac{(M+m)}{I(M+m) + Mml^2}\right) mgl\theta - \frac{mlu}{I(M+m) + Mml^2} \qquad (10)$$

✓ Al despejar $\ddot{\theta}$ de (8) y reemplazar en (9)

$$\ddot{x}(M+m) + ml\ddot{\theta} = u$$

$$\ddot{\theta} = \frac{u - \ddot{x}(M+m)}{ml}$$

$$\ddot{\theta}(I + ml^2) + ml\ddot{x} = mgl\theta$$

$$\left(\frac{u - \ddot{x}(M+m)}{ml}\right)(I+ml^2) + ml\ddot{x} = mgl\theta$$

$$u\left(\frac{I+ml^2}{ml}\right) - \ddot{x}\left(\frac{(M+m)(I+ml^2)}{ml}\right) + ml\ddot{x} = mgl\theta$$

$$-\ddot{x}\left(\frac{I(M+m) + Mml^2}{ml}\right) + u\left(\frac{I+ml^2}{ml}\right) = mgl\theta$$

$$-\ddot{x} = m^2l^2g\theta\left(\frac{1}{I(M+m)+Mml^2}\right) - u\left(\frac{I+ml^2}{I(M+m)+Mml^2}\right)$$

$$\ddot{x} = -m^2l^2g\theta\left(\frac{1}{I(M+m)+Mml^2}\right) + u\left(\frac{I+ml^2}{I(M+m)+Mml^2}\right) \quad (11)$$

✓ Al considerar las variables de estado así:

$q_1 = x$; $q_2 = \dot{x}$; $\dot{q}_2 = \ddot{x}$

$q_3 = \theta$; $q_4 = \dot{\theta}$; $\dot{q}_4 = \ddot{\theta}$

✓ Al reemplazar las variables de estado de (10) y (11)

$$\dot{q}_4 = \left(\frac{(M+m)}{I(M+m)+Mml^2} \right) mgl\, q_3 - \frac{ml}{I(M+m)+Mml^2}\, u$$

$$\dot{q}_2 = \frac{-m^2 l^2 g}{I(M+m)+Mml^2}\, q_3 + \frac{I+ml^2}{I(M+m)+Mml^2}\, u$$

✓ La Matriz de variables de estado es así:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & (-m^2 l^2 g)/(I(M+m)+Mml^2) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & ((M+m)mgl)/(I(M+m)+Mml^2) & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ (I+ml^2)/(I(M+m)+Mml^2) \\ 0 \\ -ml/(I(M+m)+Mml^2) \end{bmatrix} u$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$