Nombre Oscar David Poblador Parra
Codigo 20211005116

Sistema de control por realimentación de estados

✓ Para un sistema con

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)} \quad \begin{cases} 0 s\% & 9{,}5\% \\ ts & 0{,}74\ sg \end{cases}$$

✓ Tomando el sistema en bloques

$$u(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \longrightarrow Y(s)$$

✓ Considerando el primer bloque

$$\frac{X_1(s)}{u(s)} = \frac{1}{s^3+5s^2+4s} \rightarrow X_1(s)(s^3+5s^2+4s) = u(s)$$

✓ Realizando Laplace

$$\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x} = u$$

✓ Considerando el segundo bloque

$$(20s+100)X_1(s) = Y(s) \rightarrow Y = 20\dot{x}_1 + 100x_1$$

✓ Tomando las variable de estado

$$x_1 = x_1 \ ; \quad x_2 = \dot{x} \ ; \quad x_3 = \dot{x}_2 = \ddot{x}_1 \ ; \quad \dot{x}_3 = \dddot{x}_1$$

✓ Reemplazando

$$\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u \rightarrow \dot{x}_3 + 5x_3 + 4x_2 = u \rightarrow \dot{x}_3 = u - 5x_3 - 4x_2$$

$$Y = 20x_2 + 100x_1$$

✓ Tomando la matriz de variables de estado

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

✓ Al considerar su amortiguamiento

$\%Os = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100$

$0,095 = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 \rightarrow \ln(0,095) = -\frac{\zeta\pi}{\sqrt{1-\zeta^2}}$

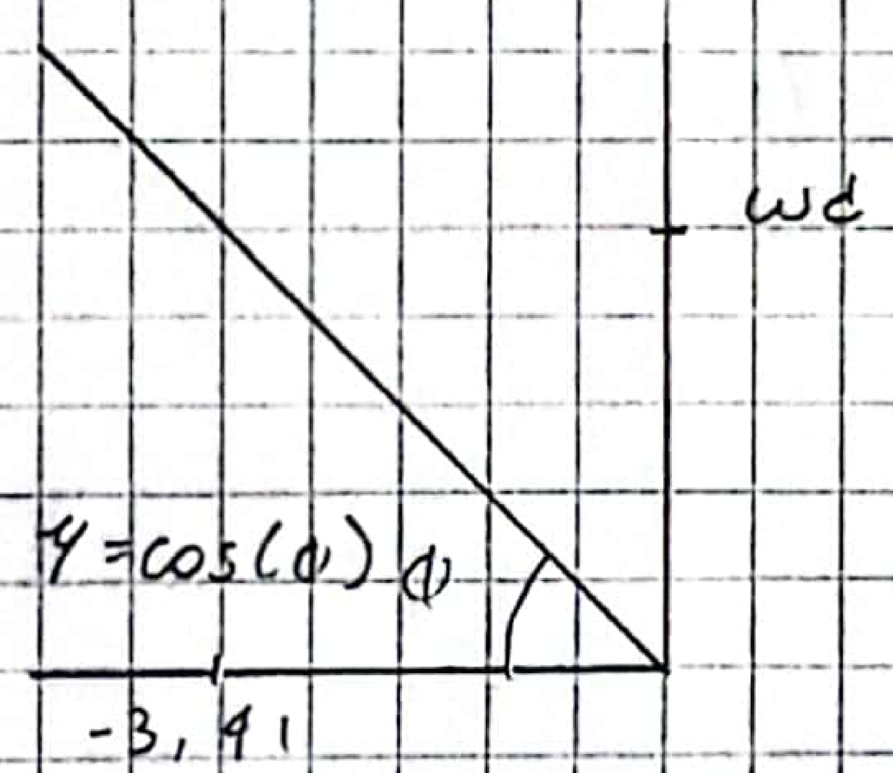
$-2,3539\left(\sqrt{1-\zeta^2}\right) = -\zeta\pi \rightarrow (2,3539)^2(1-\zeta^2) = (\zeta\pi)^2$

$5,5407 - 5,5407\zeta^2 - \zeta^2\pi^2 = 0$

$5,5407 = \zeta^2(\pi^2 + 5,5407) \rightarrow \zeta^2 = \frac{5,5407}{\pi^2 + 5,5407}$

$\zeta = \left(\frac{5,5407}{\pi^2 + 5,5407}\right)^{1/2} \rightarrow \zeta = 0,5996$

→ Plano S



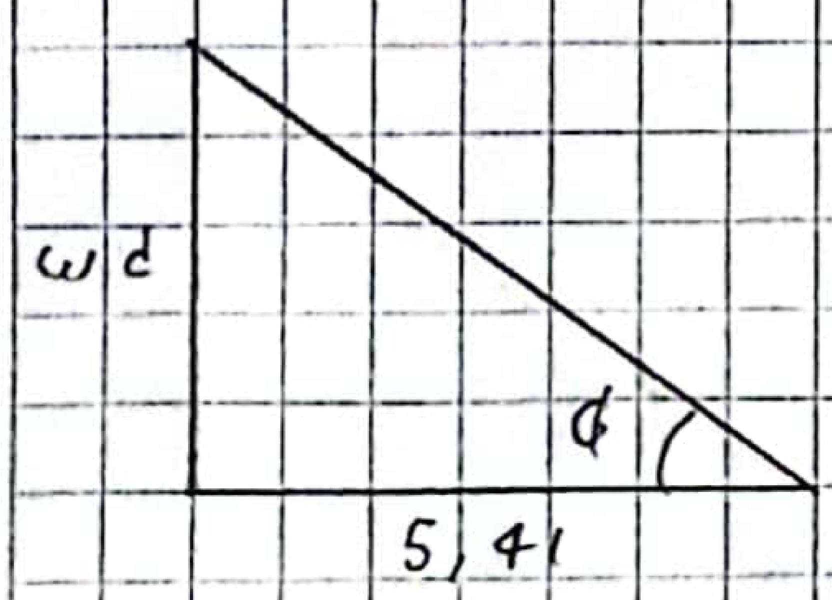
• $s = \sigma + j\omega d \rightarrow \sigma = \zeta\omega n$

$\phi = \cos^{-1}(0,5996) = 53,16°$

$\frac{t}{s} = \frac{4}{\sigma} \rightarrow 0,74 = \frac{4}{\sigma} \rightarrow \sigma = 5,405$

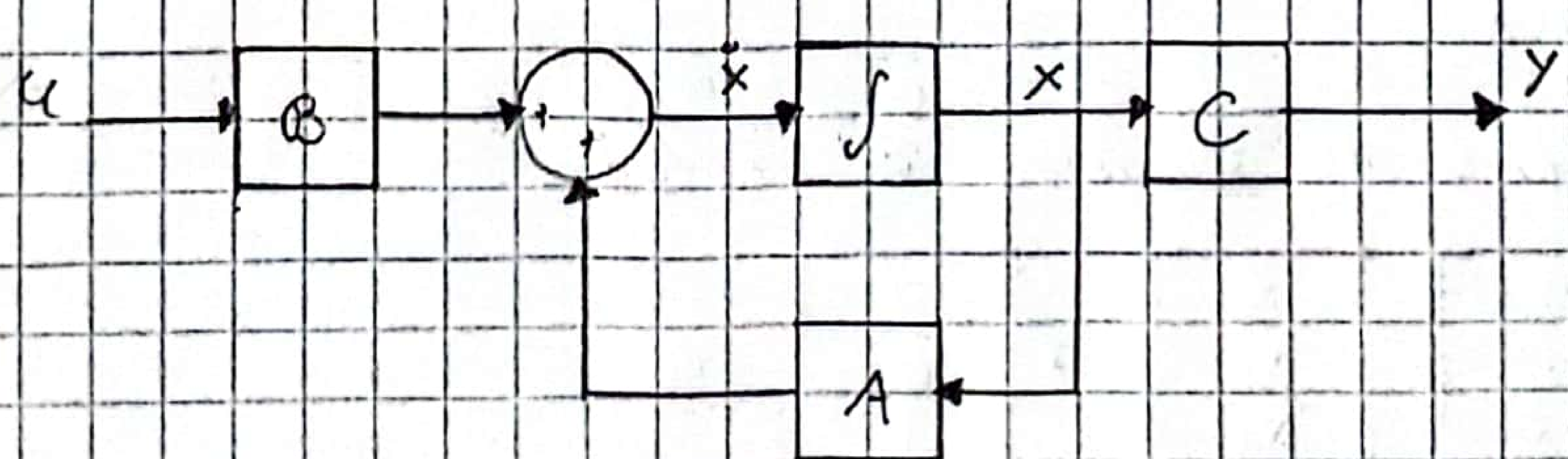
$\sigma = \zeta\omega n \rightarrow \omega n = \frac{\sigma}{\zeta} \rightarrow \omega n = 9,02$ rad/s

$\tan^{-1}(\phi) = \frac{\omega d}{5,41}$

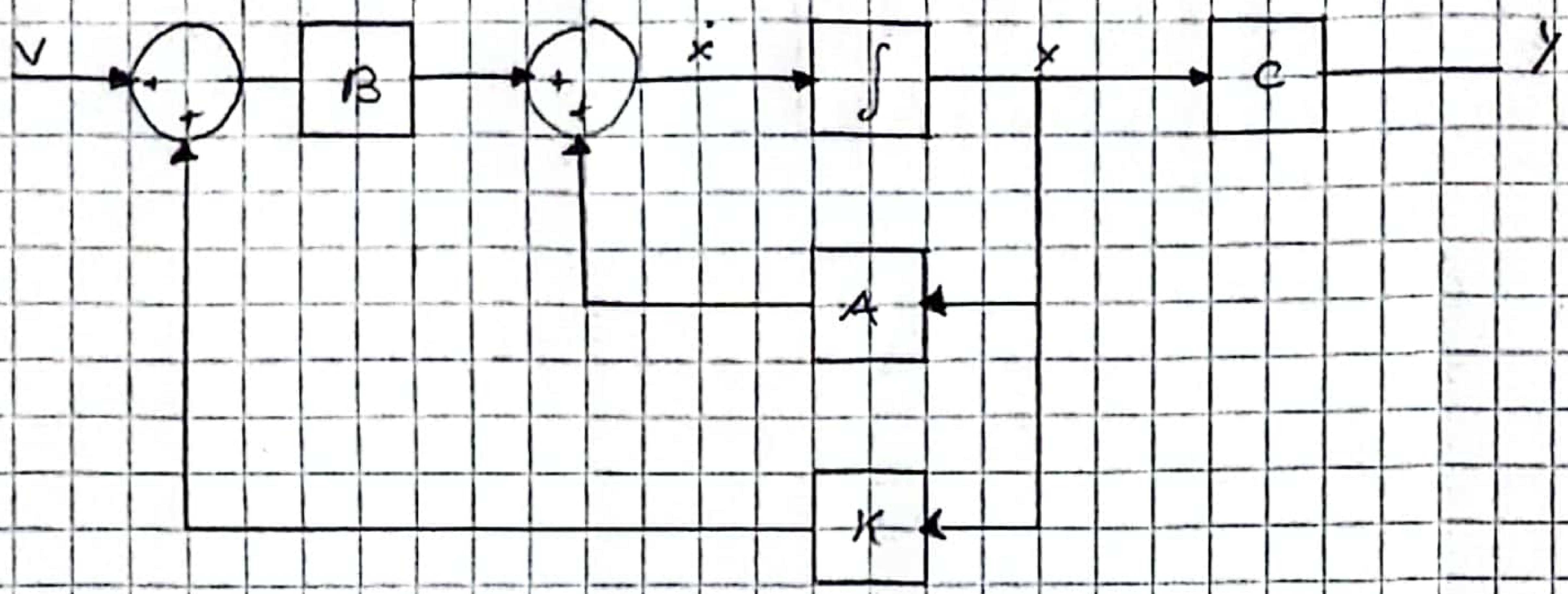


$\omega d = \tan^{-1}(53,16°)(5,41) = \omega d = 7,2146$

→ Realimentación en espacio de Estados



$\dot{x} = Ax + Bx$; $y = CX$



$\dot{x} = Ax + Bu \rightarrow \dot{x} = Ax + B(-Kx + v) \rightarrow \dot{x} = Ax - KxB + Bv$

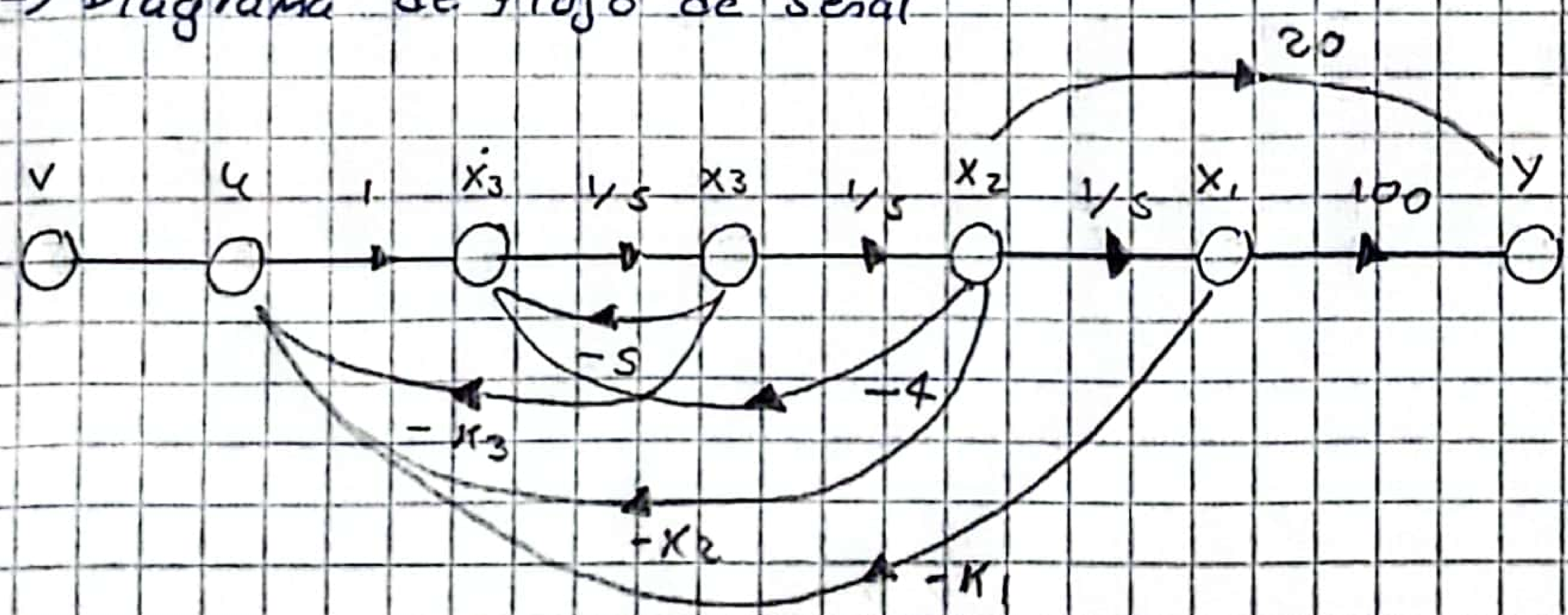
$\dot{x} = (A - B)x + v$

✓ Para la Matriz de variables de estado

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

→ Diagrama de flujo de señal



$\dot{x}_3 = -4x_2 - 5x_3 + u \rightarrow \dot{x}_3 = -4x_2 - 5x_3 + (-K_3x_3 - K_2x_2 - K_1x_1) + v$

$\dot{x}_3 = -4x_2 - 5x_3 - K_3x_3 - K_2x_2 - K_1x_1 + v$

$\dot{x}_3 = -K_1x_1 - (4 + K_2)x_2 - (5 + K_3)x_3 + v$

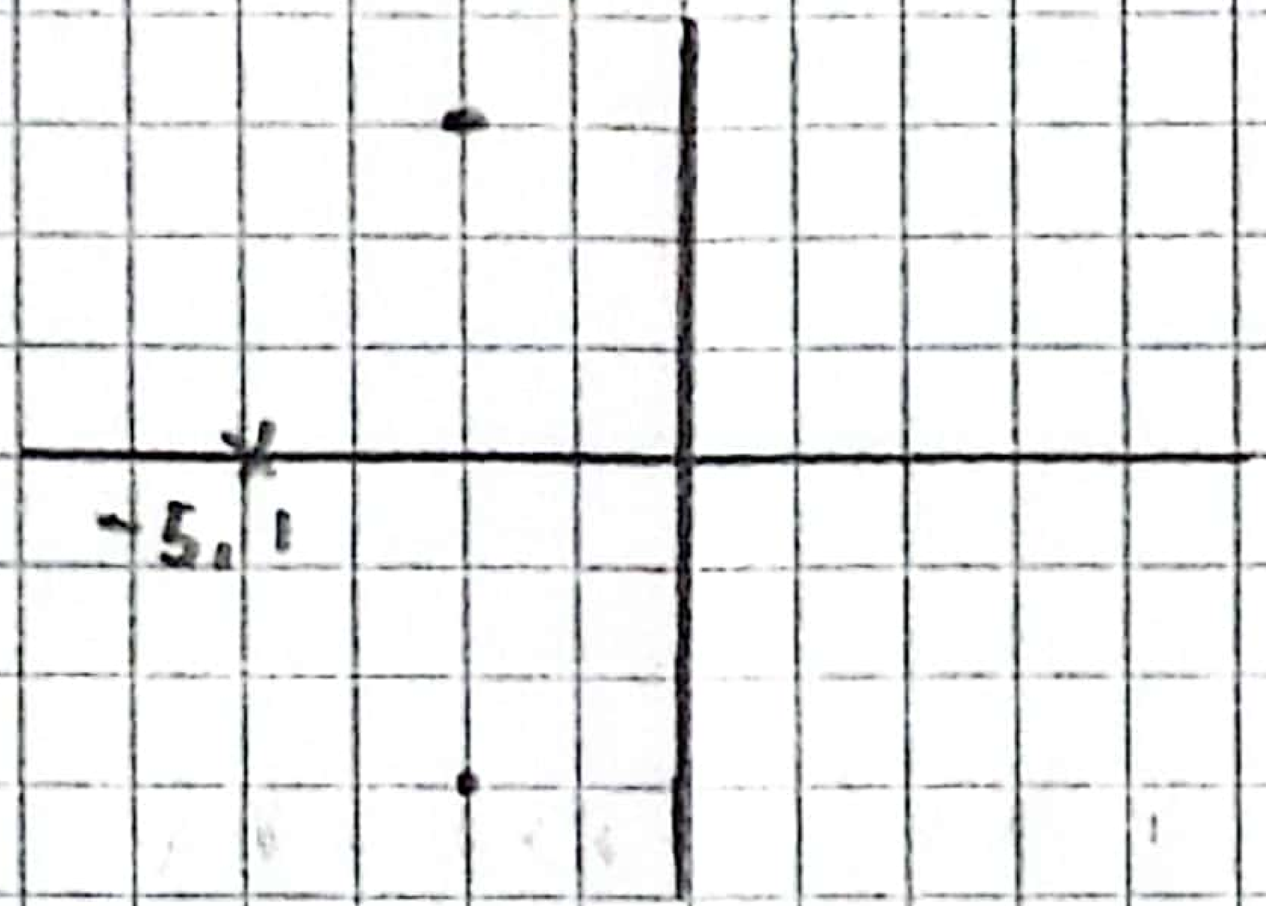
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_1 & -(4+K_2) & -(5+K_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$ ✓

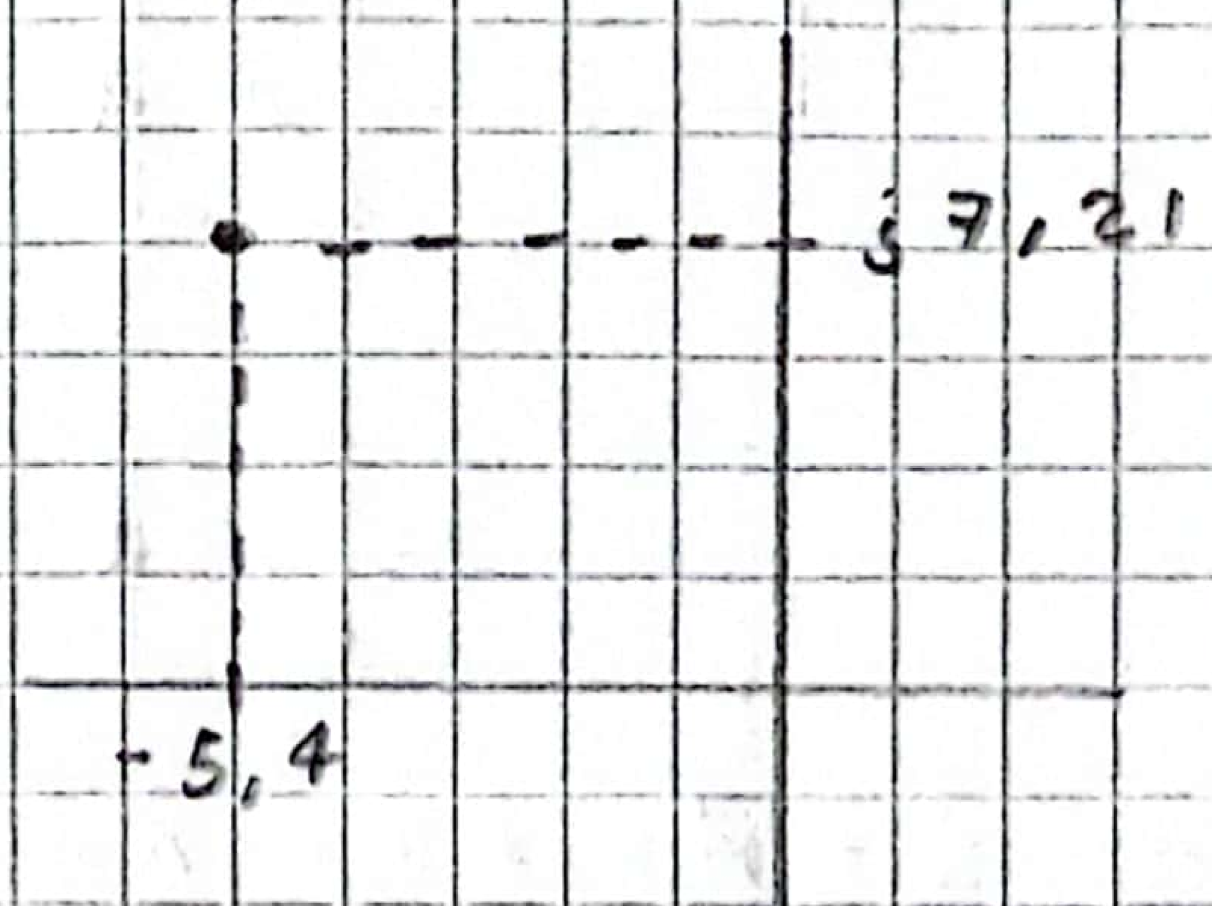
$\det(sI - A(BK)) = s^3 + (5 + K_3)s^2 + (4 + K_2)s + K_1 = 0$

Ecuación característica del sistema

$$(s + 5,4 - j7,2)(s + 5,4 + j7,2)(s + 5,1) = 0$$

$$T \rightarrow s^3 + 15,9 s^2 + 136,22 s + 413,83 = 0$$





$$s^3 + (5 + k_3)s^2 + (4 + k_2)s + k_1 = s^3 + 15,9 s^2 + 136,22 s + 413,83$$

$$(5 + k_3)s^2 = 15,93 s^2 \qquad (4 + k_2)s = 136,27 s$$

$$5 + k_3 = 15,9 \qquad 4 + k_2 = 136,27$$

$$k_3 = 10,9 \quad ; \quad k_2 = 132,22 \quad ; \quad k_1 = 413,83$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -413,8 & -136,22 & -15,9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$