Map Based Localisation

A non-holonomic robot navigates in a partial unknown environment.

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$\boldsymbol{\mu}_0 = \begin{bmatrix} s_{x,0} \\ s_{y,0} \\ s_{\theta,0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ robot initial position $\boldsymbol{\Sigma}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ initial covariance matrix $\boldsymbol{m} = \begin{bmatrix} m_x \\ m_y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ landmark position Assume the following conditions remain constant $\forall\ k$ $\boldsymbol{Q_k} = \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$ motion model covariance matrix $\boldsymbol{R_k} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.02 \end{bmatrix}$ observation model covariance matrix $V_k = 1m/s$ mobile robot linear velocity $\omega_k = 1rad/s$ mobile robot angular velocity $\Delta t = 0.1s$ sampling time Assumed measurements at each step k using the LiDAR $\boldsymbol{z}_{1,1} = [4.87\ \ 0.8]^T$ $\boldsymbol{z}_{1,2} = [4.72\ \ 0.72]^T$ $\boldsymbol{z}_{1,3} = [4.69\ \ 0.65]^T$	Using Kalman filter estimate the position of the robot for three time steps, i.e., μ_1, μ_2, μ_3 and Σ_1, Σ_2, Σ_3.

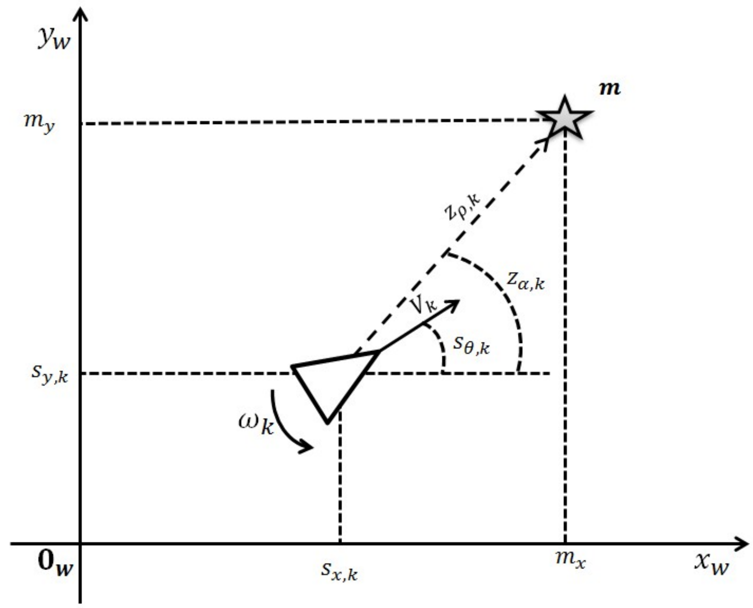

Figure 1 A non-holonomic robot moving in a 2D environment

First Iteration

(1.1) Calculate the estimated position of the robot

$$\widehat{\boldsymbol{\mu}}_{\mathbf{1}} = \mathbf{h}(\boldsymbol{\mu}_{\mathbf{0}}, \mathbf{u}_{\mathbf{1}})$$

$$\widehat{\boldsymbol{\mu}}_{\mathbf{1}} = \begin{bmatrix} \hat{s}_{x,1} \\ \hat{s}_{y,1} \\ \hat{s}_{\theta,1} \end{bmatrix} = \begin{bmatrix} s_{x,0} + \Delta t \cdot V_1 \cdot \cos(s_{\theta,0}) \\ s_{y,0} + \Delta t \cdot V_1 \cdot \sin(s_{\theta,0}) \\ s_{\theta,0} + \Delta t \cdot \omega_1 \end{bmatrix} = \begin{bmatrix} 0 + 0.1 \cdot 1 \cdot \cos(0) \\ 0 + 0.1 \cdot 1 \cdot \sin(0) \\ 0 + 0.1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

(1.2) Calculate the linearized model to be used in the uncertainty propagation

$$\boldsymbol{H}_{\mathbf{1}} = \begin{bmatrix} 1 & 0 & -\Delta t \cdot V_1 \cdot \sin(s_{\theta,0}) \\ 0 & 1 & \Delta t \cdot V_1 \cdot \cos(s_{\theta,0}) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$$

(1.3) Calculate the propagation of the uncertainty

$$\widehat{\boldsymbol{\Sigma}}_{\mathbf{1}} = \mathbf{H}_{\mathbf{1}} \cdot \boldsymbol{\Sigma}_0 \cdot \mathbf{H}_1^{\boldsymbol{T}} + \mathbf{Q}_1$$

$$\widehat{\boldsymbol{\Sigma}}_{\mathbf{1}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.1 & 1 \end{bmatrix} + \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$

$$\widehat{\boldsymbol{\Sigma}}_{\mathbf{1}} = \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$

(1.4) Define

$$\Delta x = m_x - \hat{s}_{x,1} = 2.9$$

$$\Delta y = m_y - \hat{s}_{y,1} = 4$$

$$p = \Delta x^2 + \Delta y^2 = 24.41$$

(1.5) Calculate the observation model

$$\hat{\boldsymbol{z}}_1 = \mathbf{g}(\boldsymbol{m}, \widehat{\boldsymbol{\mu}}_1)$$

$$\hat{\boldsymbol{z}}_1 = \begin{bmatrix} \hat{z}_{\rho,1} \\ \hat{z}_{\theta,1} \end{bmatrix} = \begin{bmatrix} \sqrt{\Delta x^2 + \Delta y^2} \\ \operatorname{atan2}(\Delta y, \Delta x) - \hat{s}_{\theta,1} \end{bmatrix} = \begin{bmatrix} 4.9406 \\ 0.8435 \end{bmatrix}$$

(1.6) Linearise the observation model

$$\mathbf{G_1} = \nabla_{\mathbf{s_1}} \mathbf{g}(\boldsymbol{m}, \boldsymbol{s_1})|_{\mathbf{s_1} = \widehat{\boldsymbol{\mu}}_1}$$

$$\mathbf{G_1} = \begin{bmatrix} \dfrac{\partial g_1}{\partial s_{x,k}} & \dfrac{\partial g_1}{\partial s_{y,k}} & \dfrac{\partial g_1}{\partial s_{\theta,k}} \\ \dfrac{\partial g_2}{\partial s_{x,k}} & \dfrac{\partial g_2}{\partial s_{y,k}} & \dfrac{\partial g_2}{\partial s_{\theta,k}} \end{bmatrix} = \begin{bmatrix} -\dfrac{\Delta x}{\sqrt{p}} & -\dfrac{\Delta y}{\sqrt{p}} & 0 \\ \dfrac{\Delta y}{p} & -\dfrac{\Delta x}{p} & -1 \end{bmatrix} = \begin{bmatrix} -0.5870 & -0.8096 & 0 \\ 0.1639 & -0.1188 & -1 \end{bmatrix}$$

(1.7) Using the linearised model compute the measurement uncertainty propagation

$$\mathbf{Z_1} = \mathbf{G_1} \cdot \widehat{\mathbf{\Sigma}}_\mathbf{1} \cdot \mathbf{G}_\mathbf{1}^{\boldsymbol{T}} + \boldsymbol{R_1}$$

$$\mathbf{Z_1} = \begin{bmatrix} -0.5870 & -0.8096 & 0 \\ 0.1639 & -0.1188 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix} \cdot \begin{bmatrix} -0.5870 & 0.1639 \\ -0.8096 & -0.1188 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.02 \end{bmatrix}$$

$$\mathbf{Z_1} = \begin{bmatrix} 0.6095 & 0.0133 \\ 0.0133 & 0.2392 \end{bmatrix}$$

(1.8) Calculate Kalaman Gain

$$\mathbf{K_1} = \widehat{\mathbf{\Sigma}}_\mathbf{1} \cdot \mathbf{G}_\mathbf{1}^{\boldsymbol{T}} \cdot \mathbf{Z}_\mathbf{1}^{-\mathbf{1}}$$

$$\mathbf{K_1} = \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix} \cdot \begin{bmatrix} -0.5870 & 0.1639 \\ -0.8096 & -0.1188 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1.6427 & -0.0916 \\ -0.0916 & 4.1858 \end{bmatrix}$$

$$\mathbf{K_1} = \begin{bmatrix} -0.5019 & 0.3238 \\ -0.6684 & -0.2460 \\ -0.0047 & -0.8340 \end{bmatrix}$$

(1.9) Calculate position of the robot

$$\boldsymbol{\mu_1} = \widehat{\boldsymbol{\mu}}_\mathbf{1} + \mathbf{K_1}(\mathbf{z}_1 - \hat{\mathbf{z}}_1)$$

$$\boldsymbol{\mu_1} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix} + \begin{bmatrix} -0.5019 & 0.3238 \\ -0.6684 & -0.2460 \\ -0.0047 & -0.8340 \end{bmatrix} \cdot \left(\begin{bmatrix} 4.87 \\ 0.8 \end{bmatrix} - \begin{bmatrix} 4.9406 \\ 0.8435 \end{bmatrix} \right)$$

$$\boldsymbol{\mu_1} = \begin{bmatrix} 0.1214 \\ 0.0579 \\ 0.1366 \end{bmatrix}$$

(1.10) Calculate covariance

$$\boldsymbol{\Sigma_1} = (\mathbf{I} - \mathbf{K_1} \cdot \mathbf{G_1}) \cdot \widehat{\boldsymbol{\Sigma}}_1$$

$$\boldsymbol{\Sigma_1} = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -0.5019 & 0.3238 \\ -0.6684 & -0.2460 \\ -0.0047 & -0.8340 \end{bmatrix} \cdot \begin{bmatrix} -0.5870 & -0.8096 & 0 \\ 0.1639 & -0.1188 & -1 \end{bmatrix} \right) \cdot \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$

$$\boldsymbol{\Sigma_1} = \begin{bmatrix} 0.3257 & -0.1742 & 0.0676 \\ -0.1742 & 0.2088 & -0.0484 \\ 0.0676 & -0.0484 & 0.0335 \end{bmatrix}$$

Second Iteration

(2.1) Calculate the estimated position of the robot

$$\widehat{\boldsymbol{\mu}}_{\mathbf{2}} = \mathbf{h}(\boldsymbol{\mu}_{\mathbf{1}}, \mathbf{u}_{\mathbf{2}})$$

$$\widehat{\boldsymbol{\mu}}_{\mathbf{2}} = \begin{bmatrix} \hat{s}_{x,2} \\ \hat{s}_{y,2} \\ \hat{s}_{\theta,2} \end{bmatrix} = \begin{bmatrix} s_{x,1} + \Delta t \cdot V_2 \cdot \cos(s_{\theta,1}) \\ s_{y,1} + \Delta t \cdot V_2 \cdot \sin(s_{\theta,1}) \\ s_{\theta,1} + \Delta t \cdot \omega_2 \end{bmatrix} = \begin{bmatrix} 0.2204 \\ 0.0715 \\ 0.2366 \end{bmatrix}$$

(2.2) Calculate the linearized model to be used in the uncertainty propagation

$$\boldsymbol{H}_{\mathbf{2}} = \begin{bmatrix} 1 & 0 & -\Delta t \cdot V_2 \cdot \sin(s_{\theta,1}) \cdot \\ 0 & 1 & \Delta t \cdot V_2 \cdot \cos(s_{\theta,1}) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.0136 \\ 0 & 1 & 0.0991 \\ 0 & 0 & 1 \end{bmatrix}$$

(2.3) Calculate the propagation of the uncertainty)

$$\widehat{\boldsymbol{\Sigma}}_{\mathbf{2}} = \mathbf{H}_{\mathbf{2}} \cdot \boldsymbol{\Sigma}_1 \cdot \mathbf{H}_2^{\boldsymbol{T}} + \mathbf{Q}_2$$

$$\widehat{\boldsymbol{\Sigma}}_{\mathbf{2}} = \begin{bmatrix} 1 & 0 & -0.0136 \\ 0 & 1 & 0.0991 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.3257 & -0.1742 & 0.0676 \\ -0.1742 & 0.2088 & -0.0484 \\ 0.0676 & -0.0484 & 0.0335 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -0.0136 & 0.0991 & 1 \end{bmatrix}$$

$$+ \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$

$$\widehat{\boldsymbol{\Sigma}}_{\mathbf{2}} = \begin{bmatrix} 0.8239 & -0.1569 & 0.0771 \\ -0.1569 & 0.6996 & -0.0351 \\ 0.0771 & -0.0351 & 0.2335 \end{bmatrix}$$

(2.4) Define

$$\Delta x = m_x - \hat{s}_{x,2} = 2.7796$$

$$\Delta y = m_y - \hat{s}_{y,2} = 3.9285$$

$$p = \Delta x^2 + \Delta y^2 = 23.1587$$

(2.5) Calculate the observation model

$$\hat{\boldsymbol{z}}_2 = \mathbf{g}(\boldsymbol{m}, \widehat{\boldsymbol{\mu}}_{\mathbf{2}})$$

$$\hat{\boldsymbol{z}}_2 = \begin{bmatrix} \hat{z}_{\rho,2} \\ \hat{z}_{\theta,2} \end{bmatrix} = \begin{bmatrix} \sqrt{\Delta x^2 + \Delta y^2} \\ \operatorname{atan2}(\Delta y, \Delta x) - \hat{s}_{\theta,2} \end{bmatrix} = \begin{bmatrix} 4.8124 \\ 0.7184 \end{bmatrix}$$

(2.6) Linearise the observation model

$$\mathbf{G_2} = \nabla_{\mathbf{s_2}}\, \mathbf{g}(\boldsymbol{m}, \boldsymbol{s_2})|_{\mathbf{s_2}=\widehat{\boldsymbol{\mu}}_2}$$

$$\mathbf{G_2} = \begin{bmatrix} \frac{\partial g_1}{\partial s_{x,k}} & \frac{\partial g_1}{\partial s_{y,k}} & \frac{\partial g_1}{\partial s_{\theta,k}} \\ \frac{\partial g_2}{\partial s_{x,k}} & \frac{\partial g_2}{\partial s_{y,k}} & \frac{\partial g_2}{\partial s_{\theta,k}} \end{bmatrix} = \begin{bmatrix} -\frac{\Delta x}{\sqrt{p}} & -\frac{\Delta y}{\sqrt{p}} & 0 \\ \frac{\Delta y}{p} & -\frac{\Delta x}{p} & -1 \end{bmatrix} = \begin{bmatrix} -0.5776 & -0.8163 & 0 \\ 0.1696 & -0.1200 & -1 \end{bmatrix}$$

(2.7) Using the linearised model compute the measurement uncertainty propagation

$$\mathbf{Z_2} = \mathbf{G_2} \cdot \widehat{\mathbf{\Sigma}}_2 \cdot \mathbf{G}_2^{\boldsymbol{T}} + \boldsymbol{R_2}$$

$$\mathbf{Z_2} = \begin{bmatrix} -0.5776 & -0.8163 & 0 \\ 0.1696 & -0.1200 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0.8239 & -0.1569 & 0.0771 \\ -0.1569 & 0.6996 & -0.0351 \\ 0.0771 & -0.0351 & 0.2335 \end{bmatrix} \cdot \begin{bmatrix} -0.5776 & 0.1696 \\ -0.8163 & -0.1200 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.02 \end{bmatrix}$$

$$\mathbf{Z_2} = \begin{bmatrix} 0.6931 & 0.0146 \\ 0.0146 & 0.2591 \end{bmatrix}$$

(2.8) Calculate Kalaman Gain

$$\mathbf{K_2} = \widehat{\mathbf{\Sigma}}_2 \cdot \mathbf{G}_2^{\boldsymbol{T}} \cdot \mathbf{Z}_2^{-1}$$

$$\mathbf{K_2} = \begin{bmatrix} 0.8239 & -0.1569 & 0.0771 \\ -0.1569 & 0.6996 & -0.0351 \\ 0.0771 & -0.0351 & 0.2335 \end{bmatrix} \cdot \begin{bmatrix} -0.5776 & 0.1696 \\ -0.8163 & -0.1200 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1.4444 & -0.0812 \\ -0.0812 & 3.8643 \end{bmatrix}$$

$$\mathbf{K_2} = \begin{bmatrix} -0.5090 & 0.3430 \\ -0.6879 & -0.2526 \\ -0.0054 & -0.8342 \end{bmatrix}$$

(2.9) Calculate position of the robot

$$\boldsymbol{\mu_2} = \widehat{\boldsymbol{\mu}}_2 + \mathbf{K_2}(\mathbf{z}_2 - \hat{\mathbf{z}}_2)$$

$$\boldsymbol{\mu_2} = \begin{bmatrix} 0.2204 \\ 0.0715 \\ 0.2366 \end{bmatrix} + \begin{bmatrix} -0.5090 & 0.3430 \\ -0.6879 & -0.2526 \\ -0.0054 & -0.8342 \end{bmatrix} \cdot \left(\begin{bmatrix} 4.72 \\ 0.72 \end{bmatrix} - \begin{bmatrix} 4.8124 \\ 0.7184 \end{bmatrix} \right)$$

$$\boldsymbol{\mu_2} = \begin{bmatrix} 0.2782 \\ 0.1484 \\ 0.2359 \end{bmatrix}$$

(2.10) Calculate covariance

$$\mathbf{\Sigma_2} = (\mathbf{I} - \mathbf{K_2} \cdot \mathbf{G_2}) \cdot \widehat{\mathbf{\Sigma}}_2$$

$$\mathbf{\Sigma_2} = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -0.5090 & 0.3430 \\ -0.6879 & -0.2526 \\ -0.0054 & -0.8342 \end{bmatrix} \cdot \begin{bmatrix} -0.5776 & -0.8163 & 0 \\ 0.1696 & -0.1200 & -1 \end{bmatrix} \right)$$

$$\cdot \begin{bmatrix} 0.8239 & -0.1569 & 0.0771 \\ -0.1569 & 0.6996 & -0.0351 \\ 0.0771 & -0.0351 & 0.2335 \end{bmatrix}$$

$$\mathbf{\Sigma_2} = \begin{bmatrix} 0.6189 & -0.3756 & 0.1432 \\ -0.3756 & 0.3500 & -0.1007 \\ 0.1432 & -0.1007 & 0.0531 \end{bmatrix}$$

Third Iteration

(3.1) Calculate the estimated position of the robot

$$\hat{\boldsymbol{\mu}}_3 = \mathbf{h}(\boldsymbol{\mu}_2, \mathbf{u}_3)$$

$$\hat{\boldsymbol{\mu}}_3 = \begin{bmatrix} \hat{s}_{x,3} \\ \hat{s}_{y,3} \\ \hat{s}_{\theta,3} \end{bmatrix} = \begin{bmatrix} s_{x,2} + \Delta t \cdot V_3 \cdot \cos(s_{\theta,2}) \\ s_{y,2} + \Delta t \cdot V_3 \cdot \sin(s_{\theta,2}) \\ s_{\theta,2} + \Delta t \cdot \omega_3 \end{bmatrix} = \begin{bmatrix} 0.3754 \\ 0.1718 \\ 0.3359 \end{bmatrix}$$

(3.2) Calculate the linearized model to be used in the uncertainty propagation

$$\boldsymbol{H}_3 = \begin{bmatrix} 1 & 0 & -\Delta t \cdot V_3 \cdot \sin(s_{\theta,2}) \cdot \\ 0 & 1 & \Delta t \cdot V_3 \cdot \cos(s_{\theta,2}) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.0234 \\ 0 & 1 & 0.0972 \\ 0 & 0 & 1 \end{bmatrix}$$

(3.3) Calculate the propagation of the uncertainty)

$$\hat{\boldsymbol{\Sigma}}_3 = \mathbf{H}_3 \cdot \boldsymbol{\Sigma}_2 \cdot \mathbf{H}_3^{\boldsymbol{T}} + \mathbf{Q}_3$$

$$\hat{\boldsymbol{\Sigma}}_3 = \begin{bmatrix} 1 & 0 & -0.0234 \\ 0 & 1 & 0.0972 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.6189 & -0.3756 & 0.1432 \\ -0.3756 & 0.3500 & -0.1007 \\ 0.1432 & -0.1007 & 0.0531 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -0.0234 & 0.0972 & 1 \end{bmatrix} + \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$

$$\hat{\boldsymbol{\Sigma}}_3 = \begin{bmatrix} 1.1123 & -0.3494 & 0.1520 \\ -0.3494 & 0.8309 & -0.0855 \\ 0.1520 & -0.0855 & 0.2531 \end{bmatrix}$$

(3.4) Define

$$\Delta x = m_x - \hat{s}_{x,3} = 2.6246$$

$$\Delta y = m_y - \hat{s}_{y,3} = 3.8282$$

$$p = \Delta x^2 + \Delta y^2 = 21.5436$$

(3.5) Calculate the observation model

$$\hat{\boldsymbol{z}}_3 = \mathbf{g}(\boldsymbol{m}, \hat{\boldsymbol{\mu}}_3)$$

$$\hat{\boldsymbol{z}}_3 = \begin{bmatrix} \hat{z}_{\rho,3} \\ \hat{z}_{\theta,3} \end{bmatrix} = \begin{bmatrix} \sqrt{\Delta x^2 + \Delta y^2} \\ \operatorname{atan2}(\Delta y, \Delta x) - \hat{s}_{\theta,3} \end{bmatrix} = \begin{bmatrix} 4.6415 \\ 0.6339 \end{bmatrix}$$

(3.6) Linearise the observation model

$$\mathbf{G_3} = \nabla_{s_3} \mathbf{g}(\boldsymbol{m}, \boldsymbol{s_3})|_{s_3=\hat{\boldsymbol{\mu}}_3}$$

$$\mathbf{G_3} = \begin{bmatrix} \frac{\partial g_1}{\partial s_{x,k}} & \frac{\partial g_1}{\partial s_{y,k}} & \frac{\partial g_1}{\partial s_{\theta,k}} \\ \frac{\partial g_2}{\partial s_{x,k}} & \frac{\partial g_2}{\partial s_{y,k}} & \frac{\partial g_2}{\partial s_{\theta,k}} \end{bmatrix} = \begin{bmatrix} -\frac{\Delta x}{\sqrt{p}} & -\frac{\Delta y}{\sqrt{p}} & 0 \\ \frac{\Delta y}{p} & -\frac{\Delta x}{p} & -1 \end{bmatrix} = \begin{bmatrix} -0.5655 & -0.8248 & 0 \\ 0.1777 & -0.1218 & -1 \end{bmatrix}$$

(3.7) Using the linearised model compute the measurement uncertainty propagation

$$\mathbf{Z_3} = \mathbf{G_3} \cdot \hat{\boldsymbol{\Sigma}}_3 \cdot \mathbf{G}_3^T + \boldsymbol{R_3}$$

$$\mathbf{Z_3} = \begin{bmatrix} -0.5655 & -0.8248 & 0 \\ 0.1777 & -0.1218 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1.1123 & -0.3494 & 0.1520 \\ -0.3494 & 0.8309 & -0.0855 \\ 0.1520 & -0.0855 & 0.2531 \end{bmatrix} \cdot \begin{bmatrix} -0.5655 & 0.1777 \\ -0.8248 & -0.1218 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.02 \end{bmatrix}$$

$$\mathbf{Z_3} = \begin{bmatrix} 0.6950 & 0.0143 \\ 0.0143 & 0.2608 \end{bmatrix}$$

(3.8) Calculate Kalaman Gain

$$\mathbf{K_3} = \hat{\boldsymbol{\Sigma}}_3 \cdot \mathbf{G}_3^T \cdot \mathbf{Z}_3^{-1}$$

$$\mathbf{K_3} = \begin{bmatrix} 1.1123 & -0.3494 & 0.1520 \\ -0.3494 & 0.8309 & -0.0855 \\ 0.1520 & -0.0855 & 0.2531 \end{bmatrix} \cdot \begin{bmatrix} -0.5655 & 0.1777 \\ -0.8248 & -0.1218 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1.4405 & -0.0789 \\ -0.0789 & 3.8387 \end{bmatrix}$$

$$\mathbf{K_3} = \begin{bmatrix} -0.4978 & 0.3656 \\ -0.6965 & -0.2602 \\ -0.0052 & -0.8266 \end{bmatrix}$$

(3.9) Calculate position of the robot

$$\boldsymbol{\mu_3} = \hat{\boldsymbol{\mu}}_3 + \mathbf{K_3}(\mathbf{z}_3 - \hat{\mathbf{z}}_3)$$

$$\boldsymbol{\mu_3} = \begin{bmatrix} 0.3754 \\ 0.1718 \\ 0.3359 \end{bmatrix} + \begin{bmatrix} -0.4978 & 0.3656 \\ -0.6965 & -0.2602 \\ -0.0052 & -0.8266 \end{bmatrix} \cdot \left(\begin{bmatrix} 4.69 \\ 0.65 \end{bmatrix} - \begin{bmatrix} 4.6415 \\ 0.6339 \end{bmatrix} \right)$$

$$\boldsymbol{\mu_3} = \begin{bmatrix} 0.3571 \\ 0.1338 \\ 0.3223 \end{bmatrix}$$

(3.10) Calculate covariance

$$\boldsymbol{\Sigma_3} = (\mathbf{I} - \mathbf{K_3} \cdot \mathbf{G_3}) \cdot \widehat{\boldsymbol{\Sigma}}_3$$

$$\boldsymbol{\Sigma_3} = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -0.4978 & 0.3656 \\ -0.6965 & -0.2602 \\ -0.0052 & -0.8266 \end{bmatrix} \cdot \begin{bmatrix} -0.5655 & -0.8248 & 0 \\ 0.1777 & -0.1218 & -1 \end{bmatrix} \right)$$

$$\cdot \begin{bmatrix} 1.1123 & -0.3494 & 0.1520 \\ -0.3494 & 0.8309 & -0.0855 \\ 0.1520 & -0.0855 & 0.2531 \end{bmatrix}$$

$$\boldsymbol{\Sigma_3} = \begin{bmatrix} 0.9103 & -0.5638 & 0.2231 \\ -0.5638 & 0.4710 & -0.1523 \\ 0.2231 & -0.1523 & 0.0747 \end{bmatrix}$$