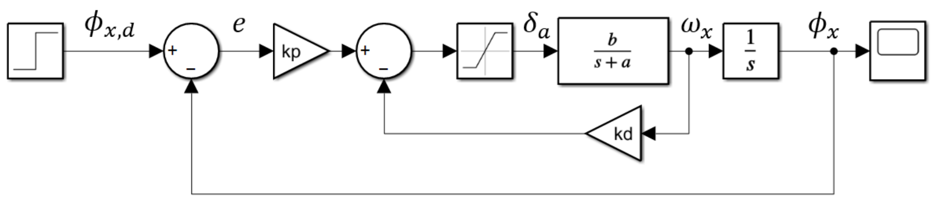
Successive Loop Closure for Yaw Control

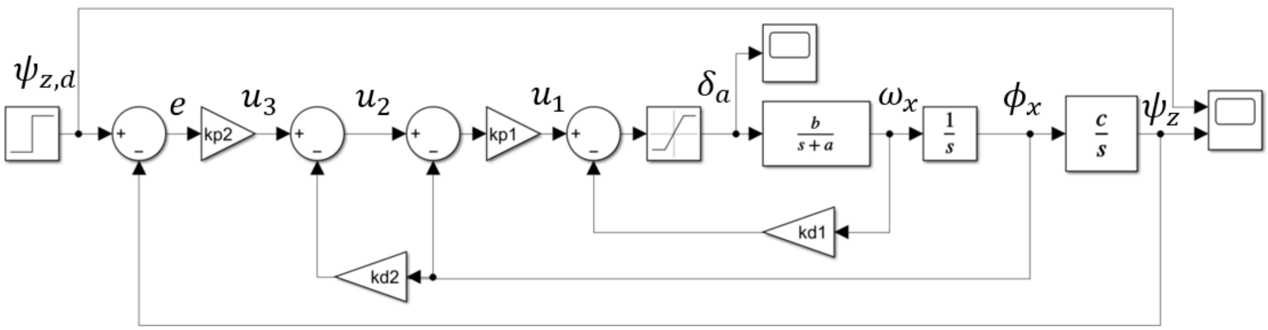
Due May 10 by 11:59pm **Points** 100 **Submitting** a file upload **File Types** pdf

The objective of this assignment is to use successive loop closure to design a yaw control strategy for the airplane.

In a previous assignment, you should have designed the following PD (Proportional-Derivative) controller for the roll dynamics of the airplane:



This PD roll controller has an inner loop: the derivative feedback loop. It has an outer loop: the proportional feedback loop. In this assignment, another outer loop will be added for yaw control. The complete block diagram will appear as follows:



This block diagram consists of an innermost loop from u_1 to ω_x, a second inner loop from u_2 to ϕ_x, a third inner loop from u_3 to ϕ_x, and an outer loop from $\psi_{z,d}$ to ψ_z. In this assignment, we will assume constant values for a, b, and c as follows:

$a = 9$

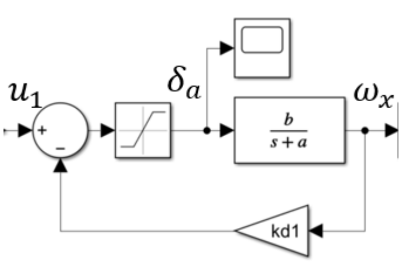
$b = 33$

$c = 0.5$

In reality, a, b, and c are not constant, but depend on airspeed, elevator angle, and rudder angle.

Successive Loop Closure

Successive loop closure is a control design technique. It begins by choosing controller gains for the innermost loop. The chosen gains result in a time-constant τ_1 for the innermost loop. If the time constant for the innermost loop is at least 5 to 10 times faster than the successive outer loops, $\tau_2 \geq 5\tau_1$, the inner loop dynamics can be approximated by a constant static gain $\mathcal{G}_1$. This approximation simplifies the block diagram. For example, in the yaw controller, the innermost loop is



The controller gain to be determined is k_{d1}. The closed loop transfer function (assuming no saturation) for the loop is

$$\frac{\omega_x}{u_1} = \frac{b}{s+a+k_{d1}b}$$

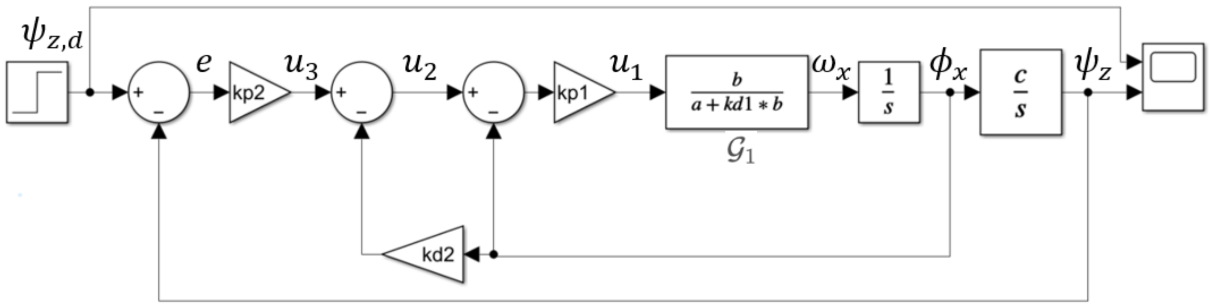
The time-constant for this transfer function is

$$\tau_1 = \frac{1}{a+k_{d1}b}$$

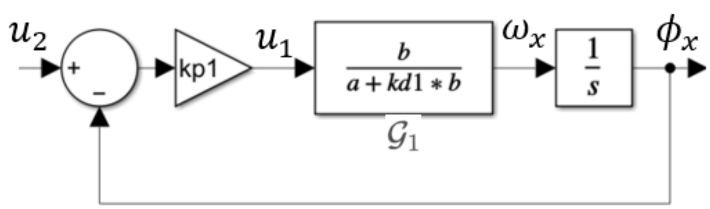
The steady-state gain is found by setting $s=0$ to be

$$\mathcal{G}_1 = \frac{b}{a+k_{d1}b}$$

If the time-constant for the second loop is at least 5 to 10 times slower, *i.e.*, $\tau_2 \geq 5\tau_1$, then the innermost loop can be replaced by the gain $\mathcal{G}_1$, and the block diagram becomes



Now the innermost loop is from u_2 to ϕ_x:



The transfer function is

$$\frac{\phi_x}{u_2} = \frac{k_{p1}b}{s(a+k_{d1}b)+k_{p1}b}$$

Its time constant is

$$\tau_2 = \frac{a+k_{d1}b}{k_{p1}b}$$

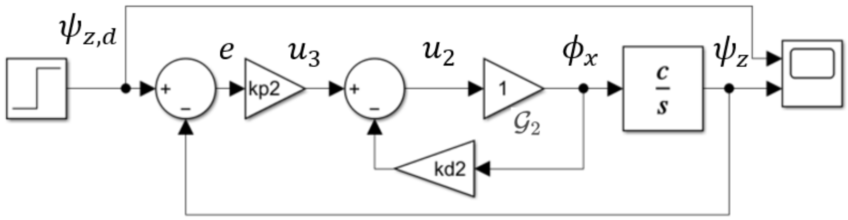
Its steady-state gain is

$$\mathcal{G}_2 = 1$$

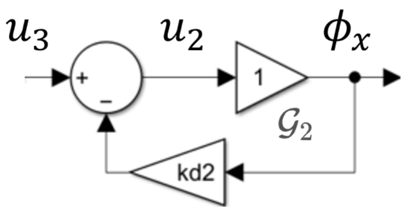
To guarantee that $\tau_2 \geq 5\tau_1$ requires that

$$k_{p1} < \frac{(a+k_{d1}b)^2}{5b}$$

If τ_2 is fast enough, *i.e.*, $\tau_3 \geq 5\tau_2$, then the transfer function $\frac{\phi_x}{u_2}$ can be replaced by its gain $\mathcal{G}_2 = 1$, and the block diagram becomes



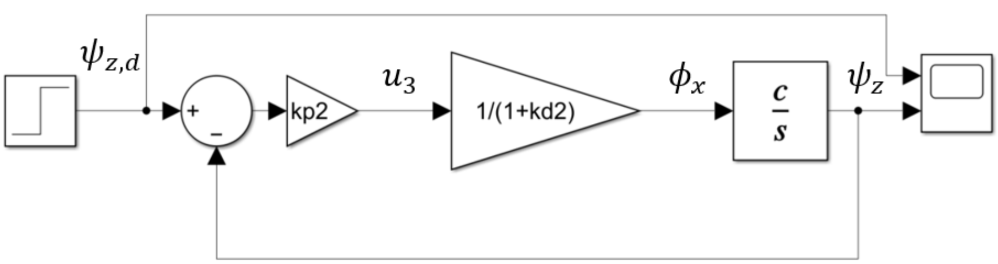
Now the innermost loop is



This loop is actually just a gain:

$$\frac{\phi_x}{u_3} = \frac{1}{1+k_{d2}} = \mathcal{G}_3$$

It does not have a time-constant, so the requirement becomes $\tau_4 \geq 5\tau_2$ instead of $\tau_3 \geq 5\tau_2$. Now the yaw control block diagram is



The transfer function is

$$\frac{\psi_z}{\psi_{z,d}} = \frac{k_{p2}c}{s(1+k_{d2})+k_{p2}c}$$

The time-constant is

$$\tau_4 = \frac{1+k_{d2}}{k_{p2}c}$$

To satisfy the requirement $\tau_4 \geq 5\tau_2$, the requirement is that

$$\frac{k_{p2}c}{1+k_{d2}} < \frac{k_{p1}b}{5(a+k_{d1}b)}$$

Summary

The above analysis resulted in three different time constants, each corresponding to one of the four control loops:

$$\tau_1 = \frac{1}{a+k_{d1}b}$$

$$\tau_2 = \frac{a+k_{d1}b}{k_{p1}b}$$

$$\tau_4 = \frac{1+k_{d2}}{k_{p2}c}$$

Successive loop closure requires that

$$\tau_4 \geq 5\tau_2 \text{ and } \tau_2 \geq 5\tau_1$$

These time constants help determine the user-selected control gains k_{p1}, k_{d1}, k_{p2}, and k_{d2}. For example, if we want the time-constant for the airplane yaw dynamics to be $\tau_4 = 1$s, then $\tau_4 = 5\tau_2 = 25\tau_1$ means that $\tau_2 = 0.2$s and $\tau_1 = 0.04$s. Using the equations for the time-constants above, the controller gains are

$$k_{d1} = \frac{\frac{1}{\tau_1} - a}{b} = \frac{\frac{1}{0.04} - 9}{33} = 0.4848$$

$$k_{p1} = \frac{a + k_{d1} b}{\tau_2 b} = \frac{9 + 0.4848(33)}{(0.2)(33)} = 3.7879$$

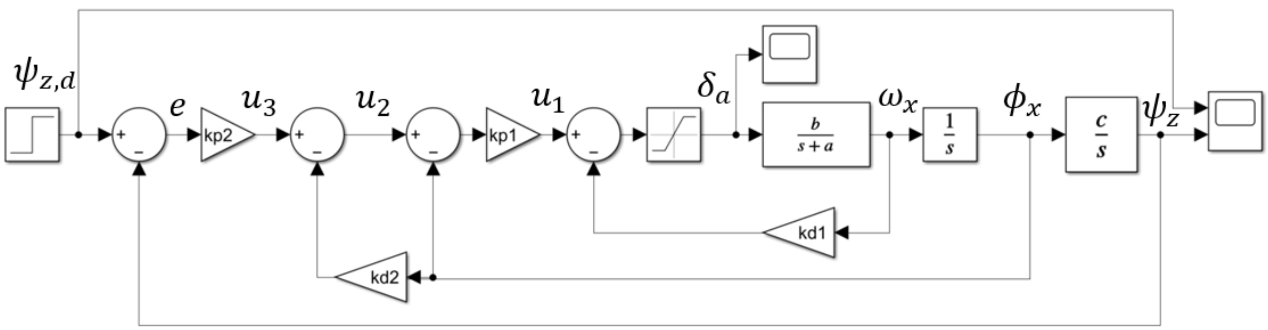
$$k_{d2} = 1$$

$$k_{p2} = \frac{1 + k_{d2}}{\tau_4 c} = \frac{1+1}{(1)(0.5)} = 4$$

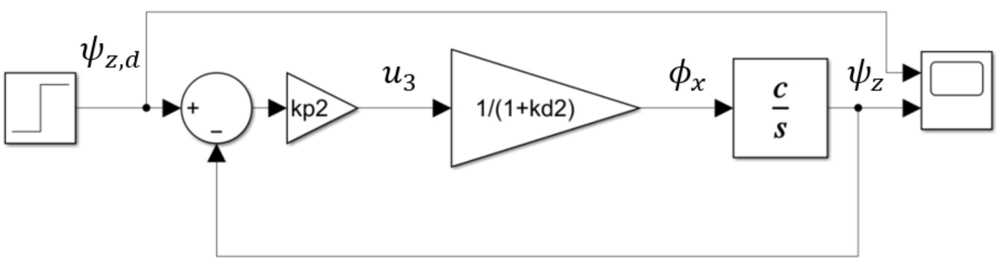
Because τ_4 depended on two control gains k_{p2} and k_{d2} we could select one of them somewhat randomly as long as the other is used to satisfy the time-constant equation for τ_4. Because of that, we chose $k_{d2} = 1$, but k_{p2} depended on the value of k_{d2}.

Your Assignment

Set $\tau_4 = 2$s and select the controller gains to require that $\tau_4 = 5\tau_2 = 25\tau_1$. Show using Simulink that the successive loop closure technique results in a similar response as the complete block diagram. In other words, show that the block diagrams



and



produce very similar results when the values of k_{p1}, k_{d1}, k_{p2}, and k_{d2} are selected using the successive loop closure technique. Set the limits of the saturation block to 1 and -1. Plot the results on a single scope.

What to Submit

Copy and paste a screenshot of the Simulink model into a word document. Copy and paste a screenshot of the scope plot showing the resulting signals on a single graph. Save the document as a pdf and submit it.