since one of the four tuples in PAY satisfy m_2. We denote the selectivity of a minterm m_i as $sel(m_i)$.

2. *Access frequency*: frequency with which user applications access data. If $Q = \{q_1, q_2, \ldots, q_q\}$ is a set of user queries, $acc(q_i)$ indicates the access frequency of query q_i in a given period.

Note that minterm access frequencies can be determined from the query frequencies. We refer to the access frequency of a minterm m_i as $acc(m_i)$.

3.3.1.2 Primary Horizontal Fragmentation

Before we present a formal algorithm for horizontal fragmentation, we intuitively discuss the process for primary (and derived) horizontal fragmentation. A *primary horizontal fragmentation* is defined by a selection operation on the owner relations of a database schema. Therefore, given relation R, its horizontal fragments are given by

$$R_i = \sigma_{F_i}(R),\ 1 \le i \le w$$

where F_i is the selection formula used to obtain fragment R_i (also called the *fragmentation predicate*). Note that if F_i is in conjunctive normal form, it is a minterm predicate (m_i). The algorithm we discuss will, in fact, insist that F_i be a minterm predicate.

Example 3.7. The decomposition of relation PROJ into horizontal fragments PROJ_1 and PROJ_2 in Example 3.1 is defined as follows[1]:

$$\text{PROJ}_1 = \sigma_{\text{BUDGET} \le 200000}\ (\text{PROJ})$$
$$\text{PROJ}_2 = \sigma_{\text{BUDGET} > 200000}\ (\text{PROJ})$$

♦

Example 3.7 demonstrates one of the problems of horizontal partitioning. If the domain of the attributes participating in the selection formulas are continuous and infinite, as in Example 3.7, it is quite difficult to define the set of formulas $F = \{F_1, F_2, \ldots, F_n\}$ that would fragment the relation properly. One possible course of action is to define ranges as we have done in Example 3.7. However, there is always the problem of handling the two endpoints. For example, if a new tuple with a BUDGET value of, say, \$600,000 were to be inserted into PROJ, one would have had to review the fragmentation to decide if the new tuple is to go into PROJ_2 or if the fragments need to be revised and a new fragment needs to be defined as

[1] We assume that the non-negativity of the BUDGET values is a feature of the relation that is enforced by an integrity constraint. Otherwise, a simple predicate of the form $0 \le$ BUDGET also needs to be included in Pr. We assume this to be true in all our examples and discussions in this chapter.

$$\begin{aligned}
PROJ_2 &= \sigma_{200000 < \text{BUDGET} \leq 400000}\ (\text{PROJ})\\
PROJ_3 &= \sigma_{\text{BUDGET} > 400000}\ (\text{PROJ})
\end{aligned}$$

Example 3.8. Consider relation PROJ of Figure 3.3. We can define the following horizontal fragments based on the project location. The resulting fragments are shown in Figure 3.8.

$$\begin{aligned}
PROJ_1 &= \sigma_{\text{LOC=``Montreal''}}\ (\text{PROJ})\\
PROJ_2 &= \sigma_{\text{LOC=``New York''}}\ (\text{PROJ})\\
PROJ_3 &= \sigma_{\text{LOC=``Paris''}}\ (\text{PROJ})
\end{aligned}$$

♦

$PROJ_1$

PNO	PNAME	BUDGET	LOC
P1	Instrumentation	150000	Montreal

$PROJ_2$

PNO	PNAME	BUDGET	LOC
P2	Database Develop.	135000	New York
P3	CAD/CAM	250000	New York

$PROJ_3$

PNO	PNAME	BUDGET	LOC
P4	Maintenance	310000	Paris

Fig. 3.8 Primary Horizontal Fragmentation of Relation PROJ

Now we can define a horizontal fragment more carefully. A horizontal fragment R_i of relation R consists of all the tuples of R that satisfy a minterm predicate m_i. Hence, given a set of minterm predicates M, there are as many horizontal fragments of relation R as there are minterm predicates. This set of horizontal fragments is also commonly referred to as the set of *minterm fragments*.

From the foregoing discussion it is obvious that the definition of the horizontal fragments depends on minterm predicates. Therefore, the first step of any fragmentation algorithm is to determine a set of simple predicates that will form the minterm predicates.

An important aspect of simple predicates is their *completeness*; another is their *minimality*. A set of simple predicates Pr is said to be *complete* if and only if there is an equal probability of access by every application to any tuple belonging to any minterm fragment that is defined according to Pr[2].

Example 3.9. Consider the fragmentation of relation PROJ given in Example 3.8. If the only application that accesses PROJ wants to access the tuples according to the location, the set is complete since each tuple of each fragment PROJ$_i$ (Example 3.8) has the same probability of being accessed. If, however, there is a second application which accesses only those project tuples where the budget is less than or equal to \$200,000, then Pr is not complete. Some of the tuples within each PROJ$_i$ have a higher probability of being accessed due to this second application. To make the set of predicates complete, we need to add (BUDGET $\leq$ 200000, BUDGET $>$ 200000) to Pr:

$$\begin{aligned} Pr = \{&\text{LOC=“Montreal”, LOC=“New York”, LOC=“Paris”,} \\ &\text{BUDGET} \leq 200000, \text{BUDGET} > 200000\} \end{aligned}$$

♦

The reason completeness is a desirable property is because fragments obtained according to a complete set of predicates are logically uniform since they all satisfy the minterm predicate. They are also statistically homogeneous in the way applications access them. These characteristics ensure that the resulting fragmentation results in a balanced load (with respect to the given workload) across all the fragments. Therefore, we will use a complete set of predicates as the basis of primary horizontal fragmentation.

It is possible to define completeness more formally so that a complete set of predicates can be obtained automatically. However, this would require the designer to specify the access probabilities for *each* tuple of a relation for *each* application under consideration. This is considerably more work than appealing to the common sense and experience of the designer to come up with a complete set. Shortly, we will present an algorithmic way of obtaining this set.

The second desirable property of the set of predicates, according to which minterm predicates and, in turn, fragments are to be defined, is minimality, which is very intuitive. It simply states that if a predicate influences how fragmentation is performed (i.e., causes a fragment f to be further fragmented into, say, f_i and f_j), there should be at least one application that accesses f_i and f_j differently. In other words, the simple predicate should be *relevant* in determining a fragmentation. If all the predicates of a set Pr are relevant, Pr is *minimal*.

A formal definition of relevance can be given as follows [Ceri et al., 1982b]. Let m_i and m_j be two minterm predicates that are identical in their definition, except that m_i contains the simple predicate p_i in its natural form while m_j contains $\neg p_i$. Also, let f_i and f_j be two fragments defined according to m_i and m_j, respectively. Then p_i is *relevant* if and only if

[2] It is clear that the definition of completeness of a set of simple predicates is different from the completeness rule of fragmentation given in Section 3.2.4.

$$\frac{acc(m_i)}{card(f_i)} \neq \frac{acc(m_j)}{card(f_j)}$$

Example 3.10. The set *Pr* defined in Example 3.9 is complete and minimal. If, however, we were to add the predicate

PNAME = "Instrumentation"

to *Pr*, the resulting set would not be minimal since the new predicate is not relevant with respect to *Pr* – there is no application that would access the resulting fragments any differently. ♦

We can now present an iterative algorithm that would generate a complete and minimal set of predicates Pr' given a set of simple predicates *Pr*. This algorithm, called COM_MIN, is given in Algorithm 3.1. To avoid lengthy wording, we have adopted the following notation:

Rule 1: each fragment is accessed differently by at least one application.'

f_i *of* Pr': fragment f_i defined according to a minterm predicate defined over the predicates of Pr'.

Algorithm 3.1: COM_MIN Algorithm

```
Input: $R$: relation; $Pr$: set of simple predicates
Output: $Pr'$: set of simple predicates
Declare: $F$: set of minterm fragments
begin
    find $p_i \in Pr$ such that $p_i$ partitions $R$ according to Rule 1 ;
    $Pr' \leftarrow p_i$ ;
    $Pr \leftarrow Pr - p_i$ ;
    $F \leftarrow f_i$                {$f_i$ is the minterm fragment according to $p_i$} ;
    repeat
        find a $p_j \in Pr$ such that $p_j$ partitions some $f_k$ of $Pr'$ according to Rule 1
        ;
        $Pr' \leftarrow Pr' \cup p_j$ ;
        $Pr \leftarrow Pr - p_j$ ;
        $F \leftarrow F \cup f_j$ ;
        if $\exists p_k \in Pr'$ which is not relevant then
            $Pr' \leftarrow Pr' - p_k$ ;
            $F \leftarrow F - f_k$ ;
    until $Pr'$ is complete ;
end
```

The algorithm begins by finding a predicate that is relevant and that partitions the input relation. The **repeat-until** loop iteratively adds predicates to this set, ensuring minimality at each step. Therefore, at the end the set Pr' is both minimal and complete.

The second step in the primary horizontal design process is to derive the set of minterm predicates that can be defined on the predicates in set Pr'. These minterm predicates determine the fragments that are used as candidates in the allocation step. Determination of individual minterm predicates is trivial; the difficulty is that the set of minterm predicates may be quite large (in fact, exponential on the number of simple predicates). We look at ways of reducing the number of minterm predicates that need to be considered in fragmentation.

This reduction can be achieved by eliminating some of the minterm fragments that may be meaningless. This elimination is performed by identifying those minterms that might be contradictory to a set of implications I. For example, if $Pr' = \{p_1, p_2\}$, where

$p_1 : att = value\_1$
$p_2 : att = value\_2$

and the domain of att is $\{value\_1, value\_2\}$, it is obvious that I contains two implications:

$i_1 : (att = value\_1) \Rightarrow \neg(att = value\_2)$
$i_2 : \neg(att = value_1) \Rightarrow (att = value\_2)$

The following four minterm predicates are defined according to Pr':

$m_1 : (att = value\_1) \wedge (att = value\_2)$
$m_2 : (att = value\_1) \wedge \neg(att = value\_2)$
$m_3 : \neg(att = value\_1) \wedge (att = value\_2)$
$m_4 : \neg(att = value\_1) \wedge \neg(att = value\_2)$

In this case the minterm predicates m_1 and m_4 are contradictory to the implications I and can therefore be eliminated from M.

The algorithm for primary horizontal fragmentation is given in Algorithm 3.2. The input to the algorithm PHORIZONTAL is a relation R that is subject to primary horizontal fragmentation, and Pr, which is the set of simple predicates that have been determined according to applications defined on relation R.

Example 3.11. We now consider the design of the database scheme given in Figure 3.7. The first thing to note is that there are two relations that are the subject of primary horizontal fragmentation: PAY and PROJ.

Suppose that there is only one application that accesses PAY, which checks the salary information and determines a raise accordingly. Assume that employee records are managed in two places, one handling the records of those with salaries less than

Algorithm 3.2: PHORIZONTAL Algorithm

```
Input: $R$: relation; $Pr$: set of simple predicates
Output: $M$: set of minterm fragments
begin
    $Pr' \leftarrow$COM_MIN($R, Pr$) ;
    determine the set $M$ of minterm predicates ;
    determine the set $I$ of implications among $p_i \in Pr'$ ;
    foreach $m_i \in M$ do
        if $m_i$ is contradictory according to $I$ then
            $M \leftarrow M - m_i$
end
```

or equal to \$30,000, and the other handling the records of those who earn more than \$30,000. Therefore, the query is issued at two sites.

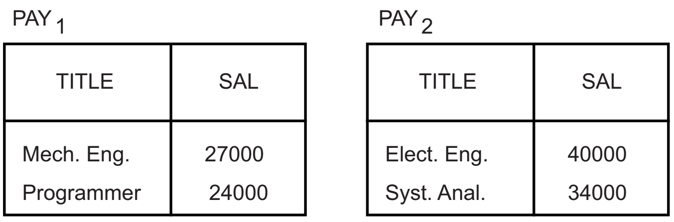
The simple predicates that would be used to partition relation PAY are

p_1: SAL $\leq$ 30000
p_2: SAL $>$ 30000

thus giving the initial set of simple predicates $Pr = \{p_1, p_2\}$. Applying the COM_MIN algorithm with $i = 1$ as initial value results in $Pr' = \{p_1\}$. This is complete and minimal since p_2 would not partition f_1 (which is the minterm fragment formed with respect to p_1) according to Rule 1. We can form the following minterm predicates as members of M:

m_1: (SAL $<$ 30000)
m_2: $\neg$(SAL $\leq$ 30000) $=$ SAL $>$ 30000

Therefore, we define two fragments $F_s = \{S_1, S_2\}$ according to M (Figure 3.9).



PAY 1

TITLE	SAL
Mech. Eng.	27000
Programmer	24000

PAY 2

TITLE	SAL
Elect. Eng.	40000
Syst. Anal.	34000

Fig. 3.9 Horizontal Fragmentation of Relation PAY

Let us next consider relation PROJ. Assume that there are two applications. The first is issued at three sites and finds the names and budgets of projects given their location. In SQL notation, the query is

```
SELECT PNAME, BUDGET
FROM   PROJ
WHERE  LOC=Value
```

For this application, the simple predicates that would be used are the following:

p_1: LOC = “Montreal”
p_2: LOC = “New York”
p_3: LOC = “Paris”

The second application is issued at two sites and has to do with the management of the projects. Those projects that have a budget of less than or equal to \$200,000 are managed at one site, whereas those with larger budgets are managed at a second site. Thus, the simple predicates that should be used to fragment according to the second application are

p_4: BUDGET $\leq$ 200000
p_5: BUDGET $>$ 200000

If the COM_MIN algorithm is followed, the set $Pr' = \{p_1, p_2, p_4\}$ is obviously complete and minimal. Actually COM_MIN would add any two of p_1, p_2, p_3 to Pr'; in this example we have selected to include p_1, p_2.

Based on Pr', the following six minterm predicates that form M can be defined:

m_1: (LOC = “Montreal”) $\wedge$ (BUDGET $\leq$ 200000)
m_2: (LOC = “Montreal”) $\wedge$ (BUDGET $>$ 200000)
m_3: (LOC = “New York”) $\wedge$ (BUDGET $\leq$ 200000)
m_4: (LOC = “New York”) $\wedge$ (BUDGET $>$ 200000)
m_5: (LOC = “Paris”) $\wedge$ (BUDGET $\leq$ 200000)
m_6: (LOC = “Paris”) $\wedge$ (BUDGET $>$ 200000)

As noted in Example 3.6, these are not the only minterm predicates that can be generated. It is, for example, possible to specify predicates of the form

$$p_1 \wedge p_2 \wedge p_3 \wedge p_4 \wedge p_5$$

However, the obvious implications

$i_1 : p_1 \Rightarrow \neg p_2 \wedge \neg p_3$

$i_2 : p_2 \Rightarrow \neg p_1 \wedge \neg p_3$

$i_3 : p_3 \Rightarrow \neg p_1 \wedge \neg p_2$

$i_4 : p_4 \Rightarrow \neg p_5$

$i_5 : p_5 \Rightarrow \neg p_4$

$i_6 : \neg p_4 \Rightarrow p_5$

$i_7 : \neg p_5 \Rightarrow p_4$

eliminate these minterm predicates and we are left with m_1 to m_6.

Looking at the database instance in Figure 3.3, one may be tempted to claim that the following implications hold:

i_8: LOC = “Montreal” $\Rightarrow \neg$ (BUDGET $>$ 200000)
i_9: LOC = “Paris” $\Rightarrow \neg$ (BUDGET $\leq$ 200000)
i_{10}: $\neg$ (LOC = “Montreal”) $\Rightarrow$ BUDGET $\leq$ 200000
i_{11}: $\neg$ (LOC = “Paris”) $\Rightarrow$ BUDGET $>$ 200000

However, remember that implications should be defined according to the semantics of the database, not according to the current values. There is nothing in the database semantics that suggest that the implications i_8 through i_{11} hold. Some of the fragments defined according to $M = \{m_1, \ldots, m_6\}$ may be empty, but they are, nevertheless, fragments.

The result of the primary horizontal fragmentation of PROJ is to form six fragments $F_{PROJ} = \{$PROJ$_1$, PROJ$_2$, PROJ$_3$, PROJ$_4$, PROJ$_5$, PROJ$_6\}$ of relation PROJ according to the minterm predicates M (Figure 3.10). Since fragments PROJ$_2$, and PROJ$_5$ are empty, they are not depicted in Figure 3.10. ♦

PROJ$_1$

PNO	PNAME	BUDGET	LOC
P1	Instrumentation	150000	Montreal

PROJ$_3$

PNO	PNAME	BUDGET	LOC
P2	Database Develop.	135000	New York

PROJ$_4$

PNO	PNAME	BUDGET	LOC
P3	CAD/CAM	250000	New York

PROJ$_6$

PNO	PNAME	BUDGET	LOC
P4	Maintenance	310000	Paris

Fig. 3.10 Horizontal Partitioning of Relation PROJ

3.3.1.3 Derived Horizontal Fragmentation

A derived horizontal fragmentation is defined on a member relation of a link according to a selection operation specified on its owner. It is important to remember two points. First, the link between the owner and the member relations is defined as an equi-join. Second, an equi-join can be implemented by means of semijoins. This second point is especially important for our purposes, since we want to partition a