Lectures by Claudio Castelnovo

1 Lecture 1

1.1 Introduction

Aims: Brief introduction to fractionalised quasiparticles that can appear in frustrated magnetism.

Frustrated magnetism: The inability to minimise all terms of Hamiltonian simultanously, cannot be described by Landau-Ginzburg theory.

Has many interesting properties like

- Degeneracy
- Emergent Symmetries
- Topological Order
- Fractional Excitations

Lectures will be structured as series of examples.

1.2 Conventional Magnetism

We have crystal structure, lattice Λ and each site has magnetic moments μ_i. We shall take them to be along 1 axis, and so assume they are of Ising type with only nearest neigbour interactions.

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

where $J > 0$ for ferromagnetic interactions, which favour alignment. This is combatted by entropy, which favours random $\{\sigma_i\}$. The interactions come about from exchange couplings of neighbouring electrons[1]

low sym 1 High symm phase ———*———¿ T/J

Has $\mathbb{Z}_2$ symmetry in high temperature phase and this symmetry is *broken*.

Remark. Each individual energy term is minimised

Example (Spin correlations)**.** How do we compute correlations? Define partition function

$$Z = \sum_{\{\sigma_i\}} \mathrm{e}^{-\beta H}$$

which then allows us to compute correlation

$$\langle \sigma_l \sigma_m \rangle = \frac{1}{Z} \sum_{\{\sigma_i\}} \sigma_l \sigma_k \mathrm{e}^{\beta J \sum_{\langle ij \rangle} \sigma_i \sigma_j}$$

At high temperature $\beta J \ll 1$, so that the lowest terms in the taylor expansion of the probality density dominate, and we obtain

$$\langle \sigma_l \sigma_k \rangle \simeq \frac{2\beta J_{lm}}{Z}$$

[1] Limit of Hubbard model? Must investigate

where $J_{lm} = \begin{cases} J & l, m \text{ are n.neighbours} \\ 0 & \text{otherwise} \end{cases}$ These correlations are *trivial* correlation as the spins only reflect the hamiltonian.

For conventional magnetism, the behaviour is therefore trival everywhere except near critical point. At low temperature the order is trival; If $\sigma = 1$ then most likely all other σ will also be 1. At high temperature the correlations are trivial $\langle \sigma\sigma \rangle \sim \beta H$, where the part of the hamiltonian refered to is the interaction of the two appropriate spins. The kind of interesting behaviour seen near critical point

- Spin correlations decay with power law
- Scale invariance
- Critical scaling of physical properties with distance from C.P. in parameter space. ($C_V \sim |T - T_C|^\alpha$, $\langle \sigma \rangle \sim B^{1/\delta}$)
- Universality

Example (Quantum phase transitions)**.** Now assume quantum spins and add a transverse field Γ (Quantum phase transition arises due to non-commutativity of terms)

$$H = -J \sum_{\langle ij \rangle} \sigma_i^Z \sigma_j^Z + \Gamma \sum_i \sigma_i^X.$$

Now 2D parameter space of T, Γ. phase boundary of ordered and disordered phases is now a line, and at $T = 0$, we have a Quantum critical point. Fluctations near the quantum critical point take much different form from usual, as it will be of $d + 1$ dimensions in contrast to the classical which is at d dimensions. These fluctations spreaed further into the phase diagram into a cone in the quantum critical regime.

Conventional magnetism has trivial correlations apart from critical behaviour. Relies on being minimise all energy terms at the same time. Instead can have competition between lattice geometry and interaction terms.

1.3 Frustrated Magnetism

$$H = \sum_{ij} H_{ij}$$

such that not all H_{ij} can be minimised at the same time. This supresses the critcal temperature significantly $T_c/J \ll 1$, where J is characteristic scale of H_{ij}, and in the region $T_c < T \lesssim J$ non trivial region. One can pseudo quantify the degree of frustration by the ratio $f = \frac{T_c}{|\Theta_{CW}|}$, of the critical temperature and the Curie-Weiss temperature. This region is a new phase called a *spin liquid.*

Remark (Properties of Spin liquids)**.** Spin liquids can both be quantum and classical show

- No long range order
- Strong correlations, as $T < J$

- Have large degeneracy/entanglement
- Emergent Symmetries
- Topological order
- Fractionalised Quantum particles.

and examples of systems exhibiting it include Triangular Ising AFM4, 8-vertex model, and the Quantum Toric Code. Note that the frustration does not always lead to these properties, but it does allow them. Note that it is only bounded by a critical point on the lower bound in classical case, hence it is technically a conventionally defined states. This may not be the case for quantum systems, where the system may be gapped[2]. Note theoretical models can be *fully frustrated* with $f = 0$, though these are not physical, as they have finite 0 temperature entropy.

Example (Triangular Ising AFM)**.** A completely classical anti-ferromagnetic model

$$H = H \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

with $J > 0$. On triangular lattice, cannot minimise all terms. Consider a triangle; We can at most have 2 anti-ferromagnetic bonds for every 3, and once two spins have been chosen, have degeneracy for the choice of the third. These are *2-1 triangles* and can be used to tile the whole lattice. In particular, can label the ferromagnetic bond of each triangle by a dimer going between the centers of the two triangles which the bond touches. This gives a $2:1$ mapping of spins to dimers,called *the dimer model*.

Remark. For square lattice, can easily minimise all energy terms by making them anti-ferromagnetic.

[2] expand on this point?