Open quantum dynamics of quantum optical many body systems

1 Lecture 1

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No microscopic derivation, but more useful persepctive: **collision model.**

Model your environment as consisting of several subsystem think of them as particles.

Your system collides with each of the subsystem of the environment sequentially, pairwise collisions.

Example (Micromason)**.** Cavity inject stream of atom such that only 1 atom is inside at a time. Beam of atoms acts as environment, and by construction is collision model.

Collision model allow us to follow both environemnt and system, in contrast to more standard master equation where you only follow reduced dynamics.

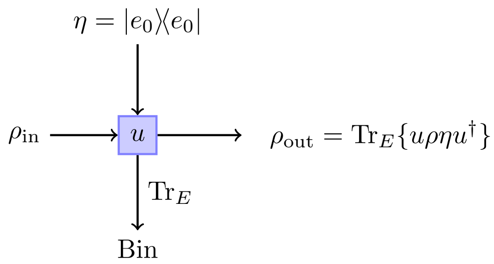
Can follow quantum trajectory :)

Useful: - Want to follow trajectory or environment.

1.1 Technical details

Interaction picture

System density $\rho = r$, Environment $\eta = e$.



$$\begin{aligned}
\rho_{\text{out}} &= \sum_k \langle e_k| \, u\rho_{in} \, |e_0\rangle\langle e_0| \, u^\dagger \, |e_k\rangle \\
&= \sum_k A_k \rho_{\text{in}} A_k^\dagger \\
&= \Phi(\rho_{\text{in}})
\end{aligned}$$

$A_k = \langle e_k|u|e_0\rangle$ is operator acting on system

Number of ancilla's N each in state $\eta = |e_0\rangle\langle e_0|$

e e e r0-¿[ ]-r1-¿ [ ]-¿...-¿ [ ]-¿rn

Assumptions about interaction: Due to interaction hamiltionian V, so $U = e^{-iVg\tau}$, lasting time τ, which is *short* compared to $1/g$. The environment is so large that the system never collide

$$U = e^{-igV\tau} = 1 - igV\tau - \frac{1}{2}V^2 g^2 \tau^2 + ..$$

$$\simeq \mathrm{Tr}\{(1 - iVg\tau - \tfrac{1}{2}V^2g^2\tau^2)\rho\eta(1 + iVg\tau - \tfrac{1}{2}V^2g^2\tau^2)\}$$
$$= \rho_{n-1} - ig\tau\,\mathrm{Tr}_E\{[V, \rho_{n-1}\eta]\} + g^2\tau^2\,\mathrm{Tr}\{V\rho\eta V - \tfrac{1}{2}V^2\rho\eta - \rho\eta V^2\}$$

where $V_{\text{eff}} = \mathrm{Tr}\{V\eta\}$, can be assumed to be reabsorbed into the definition of free hamiltonian.

Coarse grained time derivative; only look between collision $*\rho t = \frac{\rho_n - \rho_{n-1}}{\tau}$ take limit $\tau \to 0$. Time can be found as $t = n\tau$. With this information we can derive the Linblad master equation for markovian open quantum systems:

$$\rho t = \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\right.$$

where $\sqrt{gamma_k} L_k = g\sqrt{\tau}\,\langle e_k|V|e_0\rangle$.

L_k jump operators are transition rates between different states of the environment.

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$$\rho_{\text{out}} = \sum_k A_k \rho_{\text{in}} A_k^\dagger = \Phi(\rho_{\text{in}})$$

$A_k = \langle e_k|u|e_0\rangle$

Completely positive: A map that even when only acting on a subsystem of the full density matrix produces a valid density operator

Lecture 2:

Convention $|{\downarrow}\rangle = |1\rangle$ and $|{\uparrow}\rangle = |0\rangle$ The interaction is assumed to take the form

$$V = \Omega\sigma_z^{(S)}\sigma_x^{(E)}; U(\tau) = \mathrm{e}^{-i\Omega\tau\sigma_Z\sigma_x}$$

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$$\rho t = \gamma(\sigma_z\rho\sigma_z - \rho)$$

First wave function investgation yields behaviour $(c_0\,|0\rangle + c_1\,|1\rangle)\,|0\rangle \to c_0\,|0\rangle\,|\phi_0\rangle + c_1\,|1\rangle\,|\phi_1\rangle$

Now collide n times becomes $c_0\,|0\rangle\,|\phi_0\rangle\,|\phi_0\rangle \ldots$

——- Quantum Darwinism:

System+environment interaction has 'system eigenstates' $|0\rangle$ and $|1\rangle$

Pointer states: The system part of the interaction eigenvectors.

Quantum superposition of pointer states -¿ Classical superposition of pointer states; no coherence

Why do people agree on reality? Because environment is made of many small part, and they must contain essentially the same information; the classical information. ——-

$|e_0\rangle = |\Phi_{\text{out}}\rangle$

So we have either stays in same state, or jumps. Probability of jump is $P_k = \tau\gamma_k \langle\psi|L_k^\dagger L_k\psi\rangle$, and state changes $|psi\rangle \rightarrow \frac{L_k|\psi\rangle}{\sqrt{p_k/\tau}}$. Probability of no jump is $P_0 = 1 - \sum P_k$, and state changes $|\psi\rangle = \frac{1-iH_{\text{eff}}}{\sqrt{p_0}}|\psi\rangle$, where the effective hamiltonian is non-hermitian $H_{\text{eff}} = -\frac{i}{2}\sum_k L_k^\dagger L_k$.

This can be thought for two level system: 1. If we at any time observe a photon, the system is definitely in the ground state 2. If we do not observe a photon, over time, we get more confident that the system is in ground state. Hence we get exponential decay until we get

Note we renormalise the wavefunction after each step.