[CS164] Final Project

April 23, 2020

Synthetic Control Method Using CVXPY

Convex Optimization Problem

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Contents

1 Problem Description:

Detail your chosen optimization problem here, giving the context and justification for optimizing [#rightproblem]

The fundamental problem of causal inference is that we do not observe both potential outcomes of a single unit across the same timeline. In other words, if we want to disentangle the causal effect of a treatment for a given unit, we need to compare the outcome of the unit in the presence and absence of the treatment (e.g., in a medical trial, a patient cannot be in both the treatment and control group at the same time, thus, we can't observe their outcome in both worlds).

However, we can approximate the counterfactual (the outcome of the treated unit had it not been subjected to treatment) using Synthetic Control Method designed by Abadie, Diamond, and Hainmueller (2010). The plan is to construct a synthetic version of the treated unit based on a convex combination of control group units such the the error between the treated unit and the synthetic unit is minimized during the pre-treatment period.

The aim of this final project is to - Replicate the findings of the original paper of Synthetic Control Method: Using CVXPY, I will formulate the SCM method as a convex minimization problem and compare it with the findings of the original paper. - Provide a basis for Synthetic Control Method in Python: To date, the implementation of SCM in only available in R, MATLAB, and Stata. Thus, the formulation of SCM in Python using CVXPY package can be a start for a Python implementation of the method.

2 Methodology:

The original paper's aim was to disentangle the causal effect of German reunification in 1990 (treatment year) on West Germany's Gross Domestic Product. First, we gather data of the OECD countries from 1960 to 2003, then we split the data into two sections (pre-treatment & post-treatment). We then construct a covariate matrix of the treated unit (West Germany) X_1 and for the control units X_0 (rest of OECD). The benchmark is to have the best fit for West Germany' GDP trend and its constructed synthetic version during the pre-treatment period to have a credible comparison in the post-treatment period. First, I address a simplified version of the problem on which the covariates have the same importance (weights), then I propose an approach to taking specifying weights for the covariates (*Note: The outcome variable (GDP) is excluded from the covariate matrices*)

2.1 Simplified problem formulation:

Assuming that all covariates have the same predictive power with regards to the fit of the outcome variable, The problem can be formalized as:

Objective:

$$\min \sum_{k=1}^{k} (X_1 - W^* X_0)^2$$

Subject to:

$$\sum_{j=1}^{j} W^* = 1$$

$$W \succcurlyeq 0$$

Such as:

- j is the number of control units
- k is the number of covariates
- X_1 is a $(1 \times k)$ vector of observed covariates of the treated unit
- X_0 is a $(j \times k)$ matrix of observed covariates of the control group
- W^* is a $(1 \times j)$ vector of weights

The aim is to minimize the error distance between the outcome of the treated unit Y_1 and the product of the optimal weights W^* by the outcome of the control units Y_0 in the pre-treatment period T_0 such that:

$$W^* \times Y_0 \approx Y_1$$

Where:

- Y_1 is a $(1 \times T_0)$ vector of the outcome variable of the treated unit in the pre-treatment period
- Y_0 is a $(j \times T_0)$ matrix of the outcome variable of the control units in the pre-treatment period

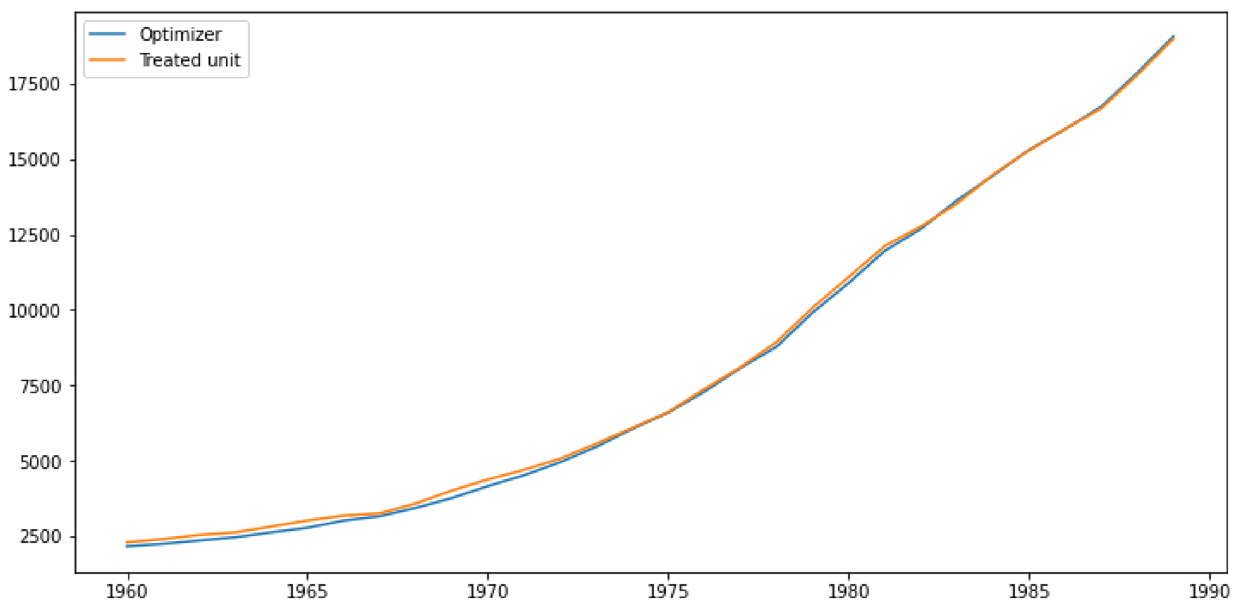


Fig 1. The trend of the outcome variable between the synthetic version based on the weights generated by CVXPY and the actual treated unit

Once we establish a good fit for the synthetic version, we can extrapolate the comparison to the post-treatment period T_1 such that the treatment effect $\hat{\alpha}$ is expressed as:

$$\hat{\alpha} = Y_{1T_1} - \sum_{k=1}^{k} W^* Y_{0T_1}$$

The error between the outcome of the treated unit and its synthetic version is:

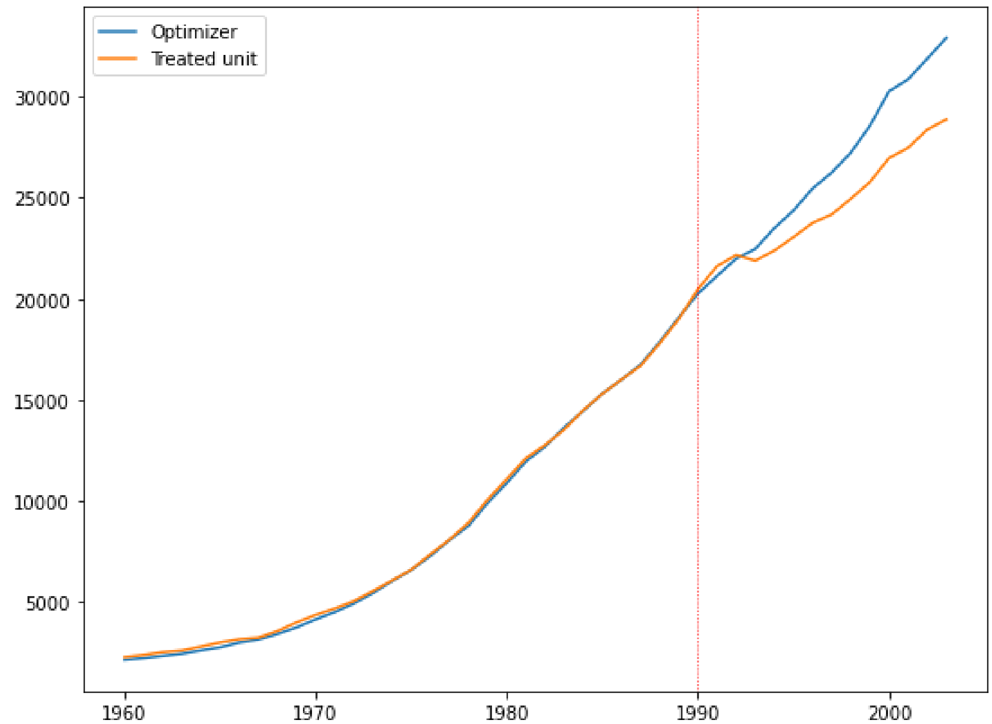


Fig 2. The gap between the synthetic control and the actual treatment unit post treatment period

- Error in the CVXPY optimizer: -7.922435428769463
- Error in the original paper: -9.484920562009224

2.2 Incorporating the importance of covariates :

As mentioned in the methodology, the simplified version attributes the same importance to covariates which can be problematic given their different units and how they contribute differently to the error we're trying to minimize. To illustrate the idea, imagine if we take as covariate

- **trade** in units of millions of dollars
- **unemployment rate** in percentage out of 100%

Both covariates contribute differently to the error rate, an error of 10$ in trade is not equivalent to a 10% gap in unemployment rate. By introducing a diagonal matrix V, each covariate is attributed a weight that reflects its importance in terms of predicting the fit with the treated unit trend. The problem then is formulated as follow:

We pick a diagonal matrix $V \in R^{k \times k}$ to find the optimal weights $W \in R^{j \times 1}$:

$$\min \left(X_1 - X_0 W\right)^T V \left(X_1 - X_0 W\right)$$

We use the matrix V to test how well the it fits the outcome of the treated unit:

$$V^* = \min \left[\left(Y_1 - Y_0 W^* \left(V\right)\right)^T \left(Y_1 - Y_0 W^* \left(V\right)\right)\right]$$

The process of finding V can be seen as an optimization problem on its own as the values of W differ as we change our choice of the importance matrix V. As a result, the problem is rendered non-convex as the following objective function can't contain both W and V as unknown variable subject to optimization:

Objective:

$$\min \left[V \left(X_1 - X_0 W \right)^2 \right]$$

Subject to:

$$\sum_{j=1}^{j} W = 1$$

$$\sum_{1,1}^{k,k} V_{i,i} = 1$$

$$W \succcurlyeq 0 \quad V \succcurlyeq 0$$

A strategy to conserve the DCP rules is to fix one variable to optimize for the other then vice versa, however, the attempt was fruitless as the process didn't converge to a reasonable distribution of weights (all weights were attributed to a single unit).

One approach that I deployed was using Genetic Optimization. The idea was to populated a set of diagonal matrices V then subject them to the fitness function `total_loss_func` to select the matrices with the smallest error. The next generation of matrices is them generated through the process of mutation to tweak the existing optimal set of matrices V. The process is repeated given a tolerance level and defined number of iterations.

The error between the outcome of the treated unit and its synthetic version is:

- Error in the CVXPY optimizer: -8.232488736302061
- Error in the original paper: -9.484920562009224

Covariate	Weight
infrate	0.200
trade	0.177
schooling	0.055
invest60	0.213
invest70	0.188
invest80	0.135
industry	0.033

Table 1. Weights attributed to each covariate

Index	Country	Weight	Index	Country	Weight
0	USA	0.283	8	Norway	0.000
1	UK	0.000	9	Switzerland	0.000
2	Austria	0.630	10	Japan	0.000
3	Belgium	0.000	11	Greece	0.000
4	Denmark	0.000	12	Portugal	0.000
5	France	0.000	13	Spain	0.000
6	Italy	0.000	14	Australia	0.088
7	Netherlands	0.000	15	New Zealand	0.000

Table 2. Weights attributed to each country to construct the synthetic version of West Germany

3 Conclusion

The relaxed version of Synthetic control method is an example of convex problem that strive to minimize the error between the treated unit and its synthetic version. Noticing the weights in table 2, roughly 63% of the weight were attributed to Austria which is a positive sign since it's similar to West Germany in terms of economic and social structure.

Appendix

LO/HC Application:

#audience [HC]: The report breaks down the methodology and its mathematical background in an easy-to-read manner supplied by graphs and tables.

#optimization [HC]: Devise a convex optimization problem for the Synthetic Control Method, then discuss the extended version of the problem.

#gap_analysis [HC]: Bridge the gap between the treated unit and the synthetic control method by minimizing error of the weighted control units.

3.1 Implementation in CVXPY

```
In [ ]: #!pip install scipy -U
```

```
In [1]: import scipy.optimize
```

```
In [2]: import matplotlib.pyplot as plt
        import pandas as pd
        import cvxpy as cvx
        import numpy as np
```

3.2 Processing data

```
In [3]: data = pd.read_csv('german_reunification.csv')
```

```
tr_unit = data[data.country == 'West Germany']
tr_unit = tr_unit[tr_unit.year < 1990]
y_tr = np.array(tr_unit.gdp).reshape(1, 30)

ctr_units = data[data.country != 'West Germany']
ctr_units = ctr_units[ctr_units.year < 1990]
y_ctr = np.array(ctr_units.gdp).reshape(16, 30)

tr_unit_all = data[data.country == 'West Germany']
y_tr_all = np.array(tr_unit_all.gdp).reshape(1, 44)

ctr_units_all = data[data.country != 'West Germany']
y_ctr_all = np.array(ctr_units_all.gdp).reshape(16, 44)
```

```
In [4]: X1 = data[data.country =='West Germany']
        X1 = X1[X1.year < 1990]
        X1 = X1.drop(['code','country','gdp', 'year'], axis=1)
        X1 = X1.set_index(np.arange(len(X1)) // 30).mean(level=0)
        X1 = X1.values

        X0 = data[data.country != 'West Germany']
        X0 = X0[X0.year < 1990]
        X0 = X0.drop(['code','country','gdp', 'year'], axis=1)
        X0 = X0.set_index(np.arange(len(X0)) // 30).mean(level=0)
        X0 = X0.values
        X0.shape, X1.shape
```

```
Out[4]: ((16, 7), (1, 7))
```

3.3 Part I

```
In [5]: # Construct the problem
        w = cvx.Variable((1, 16), nonneg=True)

        objective = cvx.Minimize(cvx.sum_squares(X1 - w @ X0))

        constraints = [cvx.sum(w) == 1]

        prob = cvx.Problem(objective, constraints)

        # The optimal objective value is returned by prob.solve()
        result = prob.solve(verbose=True)

        print('The optimal objective value: ',result,'\n\nWeights: ',w.value)
```

```
-----------------------------------------------------------------
           OSQP v0.6.0  -  Operator Splitting QP Solver
              (c) Bartolomeo Stellato,  Goran Banjac
        University of Oxford  -  Stanford University 2019
```

```
-----------------------------------------------------------------
problem:  variables n = 23, constraints m = 24
          nnz(P) + nnz(A) = 158
settings: linear system solver = qdldl,
          eps_abs = 1.0e-05, eps_rel = 1.0e-05,
          eps_prim_inf = 1.0e-04, eps_dual_inf = 1.0e-04,
          rho = 1.00e-01 (adaptive),
          sigma = 1.00e-06, alpha = 1.60, max_iter = 10000
          check_termination: on (interval 25),
          scaling: on, scaled_termination: off
          warm start: on, polish: on, time_limit: off

iter   objective    pri res    dua res    rho        time
   1   0.0000e+00   5.58e+01   1.71e+04   1.00e-01   1.94e-04s
 200   2.5574e+01   1.11e-02   8.12e-02   3.01e+00   7.71e-04s
 400   3.9777e+01   1.21e-03   9.31e-03   3.01e+00   1.27e-03s
 600   4.1633e+01   1.33e-04   1.02e-03   3.01e+00   1.73e-03s
 800   4.1839e+01   1.45e-05   1.11e-04   3.01e+00   2.99e-03s
 825   4.1845e+01   1.10e-05   8.42e-05   3.01e+00   3.13e-03s
plsh   4.1864e+01   2.83e-13   2.96e-13   --------   3.30e-03s

status:               solved
solution polish:      successful
number of iterations: 825
optimal objective:    41.8644
run time:             3.30e-03s
optimal rho estimate: 6.28e+00

The optimal objective value:  41.86440556366951

Weights:  [[3.07188776e-01 0.00000000e+00 6.61204978e-01 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
  0.00000000e+00 0.00000000e+00 3.16062464e-02 3.77783640e-17]]
```

In [6]:
```python
result = pd.DataFrame({'Country':data[data.country!='West Germany'].country.unique(),
                       'Weight': np.round(w.value[0], decimals=3)})
result
```

Out[6]:

	Country	Weight
0	USA	0.307
1	UK	0.000
2	Austria	0.661
3	Belgium	0.000
4	Denmark	0.000
5	France	0.000
6	Italy	0.000

```
7   Netherlands   0.000
8        Norway   0.000
9   Switzerland   0.000
10        Japan   0.000
11       Greece   0.000
12     Portugal   0.000
13        Spain   0.000
14    Australia   0.032
15  New Zealand   0.000
```

In [7]:
```
# Comparing between the error of the optimizer fit and the Abadie et. al fit
w_opt = np.array([0.22, 0, 0.42, 0, 0, 0, 0, 0.09, 0, 0.11,
                  0.16, 0, 0, 0, 0, 0]).reshape(1, 16)

data_compare = pd.DataFrame({'Optimizer':(w @ X0).value[0],
                             'Opt vs Tr': (w @ X0).value[0] - X1[0],
                             'Paper':(w_opt @ X0)[0],
                             'Paper vs Tr': (w_opt @ X0)[0] - X1[0],
                             'Treated Unit':X1[0]}, index= data.columns[4:])
data_compare
```

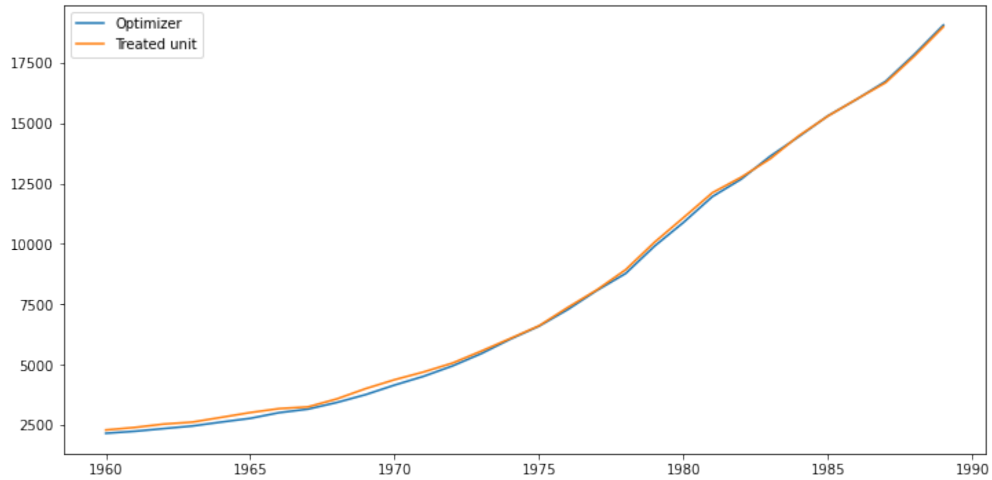
Out[7]:

	Optimizer	Opt vs Tr	Paper	Paper vs Tr	Treated Unit
infrate	4.775253	1.386404	4.793337	1.404488	3.388849
trade	46.588107	0.831420	48.260972	2.504285	45.756687
schooling	50.812894	-4.970439	45.611666	-10.171667	55.783333
invest60	0.257546	-0.079834	0.277653	-0.059727	0.337380
invest70	0.291181	-0.034458	0.316638	-0.009002	0.325640
invest80	25.428058	-1.589940	27.081760	0.063762	27.017998
industry	36.223929	-3.465587	36.472458	-3.217059	39.689516

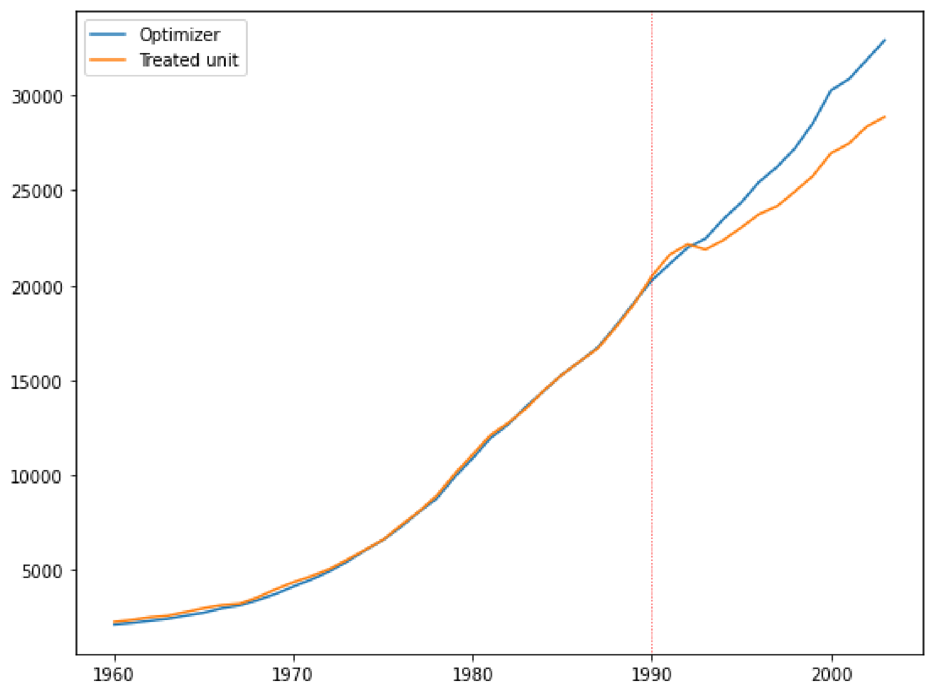
In [8]:
```
sum((w.value @ X0 - X1)[0]), sum((w_opt @ X0 - X1)[0])
```

Out[8]:
```
(-7.922435428769463, -9.484920562009224)
```

In [9]:
```
plt.figure(figsize=(12, 6))
plt.plot(list(range(1960, 1990)), w.value[0] @ y_ctr, label='Optimizer')
#plt.plot(np.array(range(1960, 1990)), (w_opt @ y_ctr)[0], label='Paper')
plt.plot(list(range(1960, 1990)), y_tr[0], label='Treated unit')
plt.legend()
plt.show()
```



In [10]:
```
plt.figure(figsize=(9, 7))
plt.plot(list(data.year.unique()), w.value[0] @ y_ctr_all, label='Optimizer')
#plt.plot(list(data.year.unique()), (w_opt @ y_ctr_all)[0], label='Paper')
plt.plot(list(data.year.unique()), y_tr_all[0], label='Treated unit')
plt.axvline(1990, color='red', linewidth=.7, linestyle=':')
plt.legend()
plt.show()
```



3.4 Part II

```
In [11]: # All covariates

         X0_T = (X0.T)
         X0_T = np.array(X0_T).reshape(7, 16)
         X1_T = (X1.T)
         X1_T = np.array(X1_T).reshape(7, 1)

         y_ctr_T = y_ctr.reshape(30, 16)
         y_tr_T = y_tr.reshape(30, 1)
```

```
In [12]: def total_loss(V_diag, details=False):
             '''
             Finds W that minimizes total loss given V,
             Returns total loss
             '''
             V = np.zeros(shape=(7, 7))
             np.fill_diagonal(V, V_diag)
             # Construct the problem
             w = cvx.Variable((16, 1), nonneg=True)
             objective = cvx.Minimize(cvx.sum(V @ cvx.square(X1_T - X0_T @ w)))
             constraints = [cvx.sum(w) == 1]
             problem = cvx.Problem(objective, constraints)
             result = problem.solve(verbose=details)

             return float((y_tr_T - y_ctr_T @ w.value).T @ (y_tr_T - y_ctr_T @ w.value)), w
```

3.4.1 Generating random V

```
In [13]: # Helpless attempt to avoid the DCP rules
         record = []
         for i in range(1000):
             if (i+1)%100 == 0: print('{}%'.format((i+1)/10))
             new_V = np.random.dirichlet(np.ones(7), size=1)
             record.append([total_loss(new_V)[0], new_V])
```

```
10.0%
20.0%
30.0%
40.0%
50.0%
60.0%
70.0%
80.0%
90.0%
100.0%
```

```
In [81]: results = pd.DataFrame(record, columns=['Error', 'Importance'])\
                        .sort_values(by='Error', ascending=True)
         results = results.reset_index()
         results.head(10)
```

```
Out[81]:    index         Error                                          Importance
         0    801  9.942145e+08  [[0.0005087338248272876, 0.1500022324198057, 0...
         1    242  9.942828e+08  [[0.03737206256530346, 0.2874155630356025, 0.3...
         2    871  9.962772e+08  [[0.02503715369817736, 0.40454433179072724, 0...
         3    685  9.964648e+08  [[0.17318516352589805, 0.19364976271483597, 0...
         4    950  9.969772e+08  [[0.04774285487534092, 0.20207675081546259, 0...
         5    144  9.972595e+08  [[0.036155841489130564, 0.06906128735274225, 0...
         6    430  9.991098e+08  [[0.04555884737334239, 0.19507913975791022, 0...
         7     14  1.000229e+09  [[0.051144883654545385, 0.13424315261686, 0.48...
         8    679  1.000793e+09  [[0.11099965375990348, 0.08697207154132884, 0...
         9    358  1.001793e+09  [[0.08570762477636173, 0.07274630890877992, 0...
```

```
In [83]: W = total_loss(results.iloc[0][1], details=True)[1].value

         pd.DataFrame({'Country':data[data.country!='West Germany'].country.unique(),
                          'Weight': np.round(W.T[0], decimals=3)})
```

```
-----------------------------------------------------------------
           OSQP v0.6.0  -  Operator Splitting QP Solver
              (c) Bartolomeo Stellato,  Goran Banjac
        University of Oxford  -  Stanford University 2019
-----------------------------------------------------------------
problem:  variables n = 23, constraints m = 24
          nnz(P) + nnz(A) = 158
settings: linear system solver = qdldl,
          eps_abs = 1.0e-05, eps_rel = 1.0e-05,
          eps_prim_inf = 1.0e-04, eps_dual_inf = 1.0e-04,
          rho = 1.00e-01 (adaptive),
          sigma = 1.00e-06, alpha = 1.60, max_iter = 10000
          check_termination: on (interval 25),
          scaling: on, scaled_termination: off
          warm start: on, polish: on, time_limit: off

iter   objective    pri res    dua res    rho        time
   1   0.0000e+00   5.58e+01   2.16e+08   1.00e-01   1.33e-04s
 200   6.6906e+06   8.53e-02   1.02e+04   2.28e+01   4.41e-04s
 400   2.0543e+08   7.01e-02   1.79e+05   2.73e+03   7.49e-04s
 600   9.7860e+08   5.93e-02   1.56e+05   2.73e+03   1.05e-03s
 800   2.1213e+09   5.05e-02   1.33e+05   2.73e+03   1.56e-03s
1000   3.2009e+09   4.52e-02   7.43e+04   2.73e+03   1.86e-03s
1200   5.3985e+09   3.76e-02   6.23e+05   1.42e+04   2.17e-03s
1400   8.0430e+09   3.17e-02   5.31e+05   1.42e+04   2.47e-03s
1600   1.1081e+10   2.68e-02   4.49e+05   1.42e+04   2.82e-03s
```

```
1800   1.4470e+10   2.27e-02   1.18e+06   1.42e+04   3.13e-03s
2000   1.8158e+10   1.85e-02   1.00e+06   1.42e+04   3.41e-03s
2200   2.1478e+10   1.50e-02   8.15e+05   1.42e+04   3.69e-03s
2400   2.3537e+10   1.28e-02   1.96e+05   1.42e+04   4.04e-03s
2600   2.4997e+10   1.15e-02   1.94e+05   1.42e+04   4.39e-03s
2800   2.6355e+10   1.03e-02   1.75e+05   1.42e+04   4.78e-03s
3000   2.7634e+10   9.31e-03   1.58e+05   1.42e+04   5.15e-03s
3200   2.8832e+10   8.38e-03   1.42e+05   1.42e+04   5.60e-03s
3400   2.9948e+10   7.55e-03   1.28e+05   1.42e+04   6.08e-03s
3600   3.0983e+10   6.79e-03   1.15e+05   1.42e+04   6.51e-03s
3800   3.1940e+10   6.11e-03   1.04e+05   1.42e+04   7.17e-03s
4000   3.2821e+10   5.50e-03   9.34e+04   1.42e+04   7.47e-03s
4200   3.3631e+10   4.95e-03   8.40e+04   1.42e+04   7.77e-03s
4400   3.4373e+10   4.46e-03   7.56e+04   1.42e+04   8.08e-03s
4600   3.5051e+10   4.01e-03   6.81e+04   1.42e+04   8.35e-03s
4800   3.5671e+10   3.61e-03   6.13e+04   1.42e+04   8.62e-03s
5000   3.6235e+10   3.25e-03   5.52e+04   1.42e+04   8.89e-03s
5200   3.6749e+10   2.93e-03   4.96e+04   1.42e+04   9.17e-03s
5400   3.7216e+10   2.63e-03   4.47e+04   1.42e+04   9.68e-03s
5600   3.7640e+10   2.37e-03   4.02e+04   1.42e+04   9.98e-03s
5800   3.8025e+10   2.13e-03   3.62e+04   1.42e+04   1.03e-02s
6000   3.8374e+10   1.92e-03   3.26e+04   1.42e+04   1.06e-02s
6200   3.8690e+10   1.73e-03   2.93e+04   1.42e+04   1.09e-02s
6400   3.8976e+10   1.56e-03   2.64e+04   1.42e+04   1.11e-02s
6600   3.9235e+10   1.40e-03   2.38e+04   1.42e+04   1.14e-02s
6800   3.9469e+10   1.26e-03   2.14e+04   1.42e+04   1.17e-02s
7000   3.9680e+10   1.13e-03   1.93e+04   1.42e+04   1.20e-02s
7200   3.9871e+10   1.02e-03   1.73e+04   1.42e+04   1.23e-02s
7400   4.0044e+10   9.19e-04   1.56e+04   1.42e+04   1.26e-02s
7600   4.0200e+10   8.28e-04   1.40e+04   1.42e+04   1.29e-02s
7800   4.0340e+10   7.45e-04   1.26e+04   1.42e+04   1.32e-02s
8000   4.0467e+10   6.70e-04   1.14e+04   1.42e+04   1.36e-02s
8200   4.0581e+10   6.03e-04   1.02e+04   1.42e+04   1.40e-02s
8325   4.0647e+10   5.65e-04   9.59e+03   1.42e+04   1.42e-02s

status:               solved
solution polish:      unsuccessful
number of iterations: 8325
optimal objective:    40647136747.7540
run time:             1.43e-02s
optimal rho estimate: 3.51e+04
```

Out[83]:

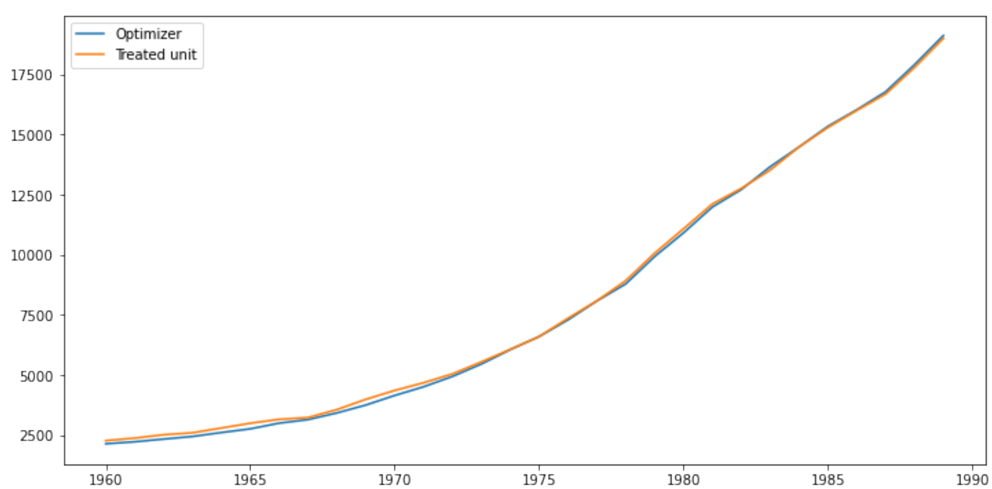
	Country	Weight
0	USA	0.305
1	UK	0.000
2	Austria	0.662

```
3        Belgium   0.000
4        Denmark   0.000
5         France   0.000
6          Italy   0.000
7    Netherlands   0.000
8         Norway   0.000
9    Switzerland   0.000
10         Japan   0.000
11        Greece   0.000
12      Portugal   0.000
13         Spain   0.000
14     Australia   0.036
15   New Zealand   0.000
```

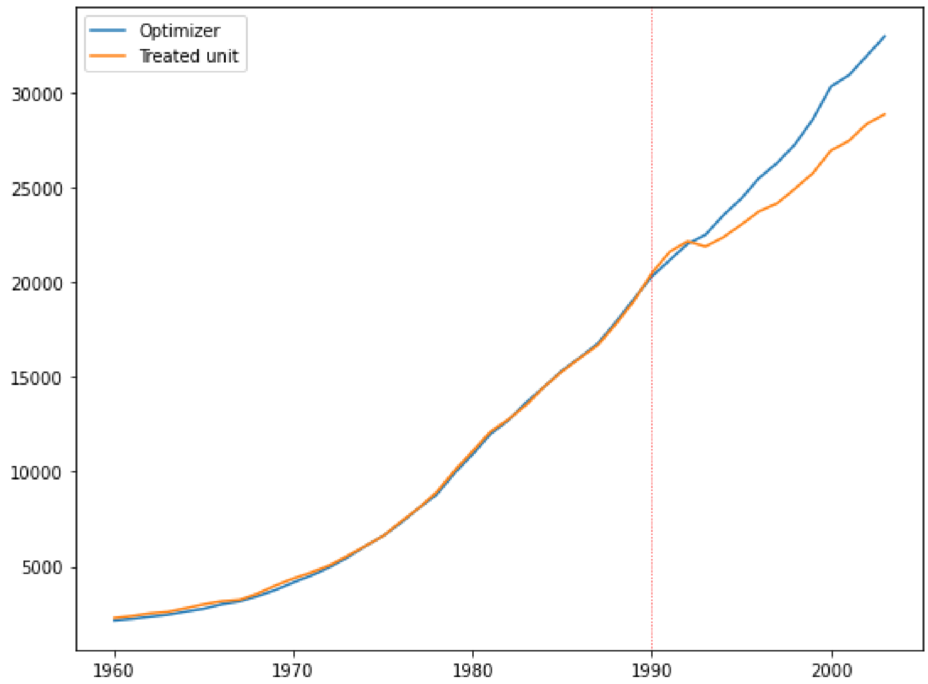
```
In [84]: pd.DataFrame({'Covariate': data.columns[4:],
                       'Weight': np.round(results['Importance'][0][0], 3)})
```

```
Out[84]:    Covariate  Weight
         0    infrate   0.001
         1      trade   0.150
         2  schooling   0.077
         3   invest60   0.559
         4   invest70   0.172
         5   invest80   0.022
         6   industry   0.020
```

```
In [85]: plt.figure(figsize=(12, 6))
         plt.plot(list(range(1960, 1990)), (W.T @ y_ctr)[0], label='Optimizer')
         #plt.plot(np.array(range(1960, 1990)), (w_opt @ y_ctr)[0], label='Paper')
         plt.plot(list(range(1960, 1990)), y_tr[0], label='Treated unit')
         plt.legend()
         plt.show()
```



```
In [86]: plt.figure(figsize=(9, 7))
         plt.plot(list(data.year.unique()), (W.T @ y_ctr_all)[0], label='Optimizer')
         #plt.plot(list(data.year.unique()), (w_opt @ y_ctr_all)[0], label='Paper')
         plt.plot(list(data.year.unique()), y_tr_all[0], label='Treated unit')
         plt.axvline(1990, color='red', linewidth=.7, linestyle=':')
         plt.legend()
         plt.show()
```




```
In [87]: # Comparing between the error of the optimizer fit and the Abadie et. al fit
         data_compare = pd.DataFrame({'Optimizer':(W.T @ X0)[0],
                                      'Opt vs Tr': (W.T @ X0 - X1)[0],
                                      'Paper':(w_opt @ X0)[0],
                                      'Paper vs Tr': (w_opt @ X0 - X1)[0],
                                      'Treated Unit':X1[0]}, index= data.columns[4:])
         data_compare
```

Out[87]:

	Optimizer	Opt vs Tr	Paper	Paper vs Tr	Treated Unit
infrate	4.799544	1.410694	4.793337	1.404488	3.388849
trade	46.725660	0.968973	48.260972	2.504285	45.756687
schooling	50.961441	-4.821892	45.611666	-10.171667	55.783333
invest60	0.258649	-0.078731	0.277653	-0.059727	0.337380
invest70	0.292291	-0.033349	0.316638	-0.009002	0.325640
invest80	25.531910	-1.486088	27.081760	0.063762	27.017998
industry	36.336928	-3.352588	36.472458	-3.217059	39.689516

```
In [88]: sum((W.T @ X0 - X1)[0]), sum((w_opt @ X0 - X1)[0])
```

Out[88]: (-7.392982155039553, -9.484920562009224)

3.4.2 Differential evolution

In [90]:
```
def total_loss_func(V_diag):
    '''
    Finds W that minimizes total loss given V,
    Returns total loss
    '''
    V = np.zeros(shape=(7, 7))
    np.fill_diagonal(V, V_diag)
    # Construct the problem
    w = cvx.Variable((16, 1), nonneg=True)
    objective = cvx.Minimize(cvx.sum(V @ cvx.square(X1_T - X0_T @ w)))
    constraints = [cvx.sum(w) == 10]
    problem = cvx.Problem(objective, constraints)
    result = problem.solve(verbose=False)

    return float((y_tr_T - y_ctr_T @ w.value).T @ (y_tr_T - y_ctr_T @ w.value))
```

In [91]:
```
from scipy.optimize import differential_evolution
bounds = [(0, 1), (0, 1), (0, 1), (0, 1), (0, 1), (0, 1), (0, 1)]

#def constr_f(x): return np.sum(x)
#lc = NonlinearConstraint(constr_f, 0, 1)
#result = scipy.optimize.differential_evolution(total_loss_func, bounds, constraints=(l

result = differential_evolution(total_loss_func, bounds)

'{:.2e}'.format(result.fun)
```

Out[91]: '1.37e+11'

In [92]:
```
gen_V = result.x / np.sum(result.x)
gen_V
```

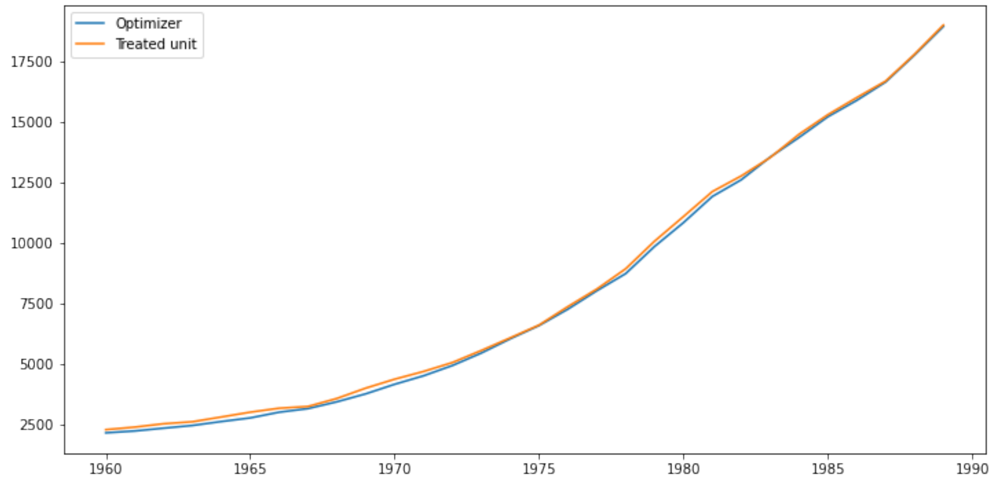
Out[92]:
```
array([0.19968234, 0.17683611, 0.05457288, 0.2129696 , 0.18835117,
       0.13476584, 0.03282207])
```

In [93]:
```
'{:.2e}'.format(total_loss(gen_V)[0])
```

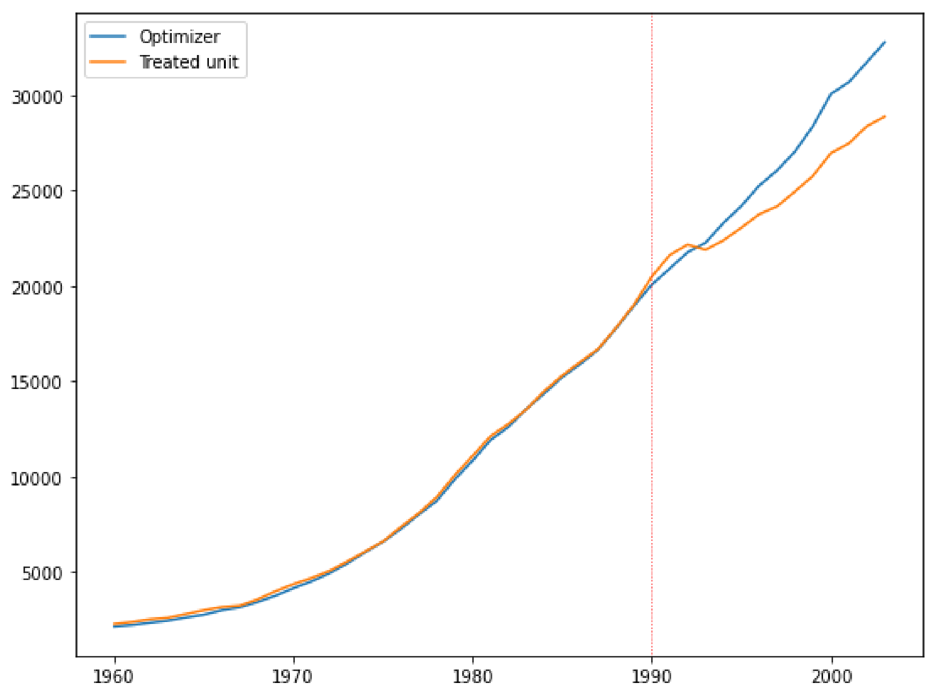
Out[93]: '1.36e+09'

In [94]:
```
gen_W = total_loss(gen_V)[1].value
```

In [95]:
```
plt.figure(figsize=(12, 6))
plt.plot(list(range(1960, 1990)), (gen_W.T @ y_ctr)[0], label='Optimizer')
#plt.plot(np.array(range(1960, 1990)), (w_opt @ y_ctr)[0], label='Paper')
plt.plot(list(range(1960, 1990)), y_tr[0], label='Treated unit')
plt.legend()
plt.show()
```




```
In [97]: plt.figure(figsize=(9, 7))
         plt.plot(list(data.year.unique()), (gen_W.T @ y_ctr_all)[0], label='Optimizer')
         #plt.plot(list(data.year.unique()), (w_opt @ y_ctr_all)[0], label='Paper')
         plt.plot(list(data.year.unique()), y_tr_all[0], label='Treated unit')
         plt.axvline(1990, color='red', linewidth=.7, linestyle=':')
         plt.legend()
         plt.show()
```




```
In [96]: sum((gen_W.T @ X0 - X1)[0]), sum((w_opt @ X0 - X1)[0])

Out[96]: (-8.232488736302061, -9.484920562009224)
```

References

[1] Abadie, A., Diamond, A., Hainmueller, J. (2015). Comparative politics and the synthetic control method: Comparative politics and the synthetic control method. American Journal of Political Science, 59(2), 495-510. doi:10.1111/ajps.12116

[2] Hainmueller, J., Abadie, A., Diamond, A., National Bureau of Economic Research. (2007). Synthetic control methods for comparative case studies : Estimating the effect of California's tobacco control program (Nber working paper series, no. w12831). Cambridge, Mass.: National Bureau of Economic Research. (2007). Retrieved April 21, 2020, from INSERT-MISSING-DATABASE-NAME.