Analyzing email spam data with a Bernoulli model

The SpamBase dataset from the UCI repository consists of $n = 4601$ emails that have been manually classified as *spam* (junk email) or *ham* (non-junk email).
The dataset also contains a vector of covariates/features for each email, such as the number of capital letters or $-signs; this information can be used to build a spam filter that automatically separates spam from ham.
This notebook analyzes only the proportion of spam emails without using the covariates.

First, some housekeeping: loading libraries and setting up colors.

```
options(repr.plot.width=16, repr.plot.height=5, lwd = 4)
library("RColorBrewer") # for pretty colors
library("tidyverse")    # for string interpolation to print variables in plots.
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.0 --

## v ggplot2 3.3.2     v purrr   0.3.4
## v tibble  3.0.4     v dplyr   1.0.5
## v tidyr   1.1.3     v stringr 1.4.0
## v readr   1.4.0     v forcats 0.5.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
library("latex2exp")    # the TeX() function makes it possible to print latex math
colors = brewer.pal(12, "Paired")[c(1,2,7,8,3,4,5,6,9,10)];
```

```
data = read.csv("https://archive.ics.uci.edu/ml/machine-learning-databases/spambase/spambase.data", sep
spam = data$X1 # This is the binary data where spam = 1, ham = 0.
n = length(spam)
spam = sample(spam, size = n) # Randomly shuffle the data.
```

Let us define a function that computes the posterior and plots it.

```
BernPost <- function(x, alphaPrior, betaPrior, legend = TRUE){
    thetaGrid = seq(0,1, length = 1000)
    n = length(x)
    s = sum(x)
    f = n - s
    alphaPost = alphaPrior + s
    betaPost = betaPrior + f
    priorPDF = dbeta(thetaGrid, alphaPrior, betaPrior)
    normLikePDF = dbeta(thetaGrid, s + 1, f + 1) # Trick to get the normalized likelihood
    postPDF = dbeta(thetaGrid, alphaPost, betaPost)

    plot(1, type="n", axes=FALSE, xlab = expression(theta), ylab = "",
         xlim=c(min(thetaGrid),max(thetaGrid)),
         ylim = c(0,max(priorPDF,postPDF,normLikePDF)),
         main = TeX(sprintf("Prior: $\\mathrm{Beta}(\\alpha = %0.0f, \\beta = %0.0f)",
                            alphaPrior, betaPrior)))
    axis(side = 1)
```

```
    lines(thetaGrid, priorPDF, type = "l", lwd = 4, col = colors[6])
    lines(thetaGrid, normLikePDF, lwd = 4, col = colors[2])
    lines(thetaGrid, postPDF, lwd = 4, col = colors[4])
    if (legend){
        legend(x = "topleft", inset=.05,
           legend = c("Prior", "Likelihood (normalized)", "Posterior"),
           lty = c(1, 1, 1), pt.lwd = c(3, 3, 3),
           col = c(colors[6], colors[2], colors[4]))
    }
    cat("Posterior mean is ", round(alphaPost/(alphaPost + betaPost),3), "\n")
    cat("Posterior standard deviation is ",
        round(sqrt(alphaPost*betaPost/((alphaPost+betaPost)^2*(alphaPost+betaPost+1))),3),"\n")
    return(list("alphaPost" = alphaPrior + s, "betaPost" = betaPrior + f))
}
```

Let start by analyzing only the first 10 data points.

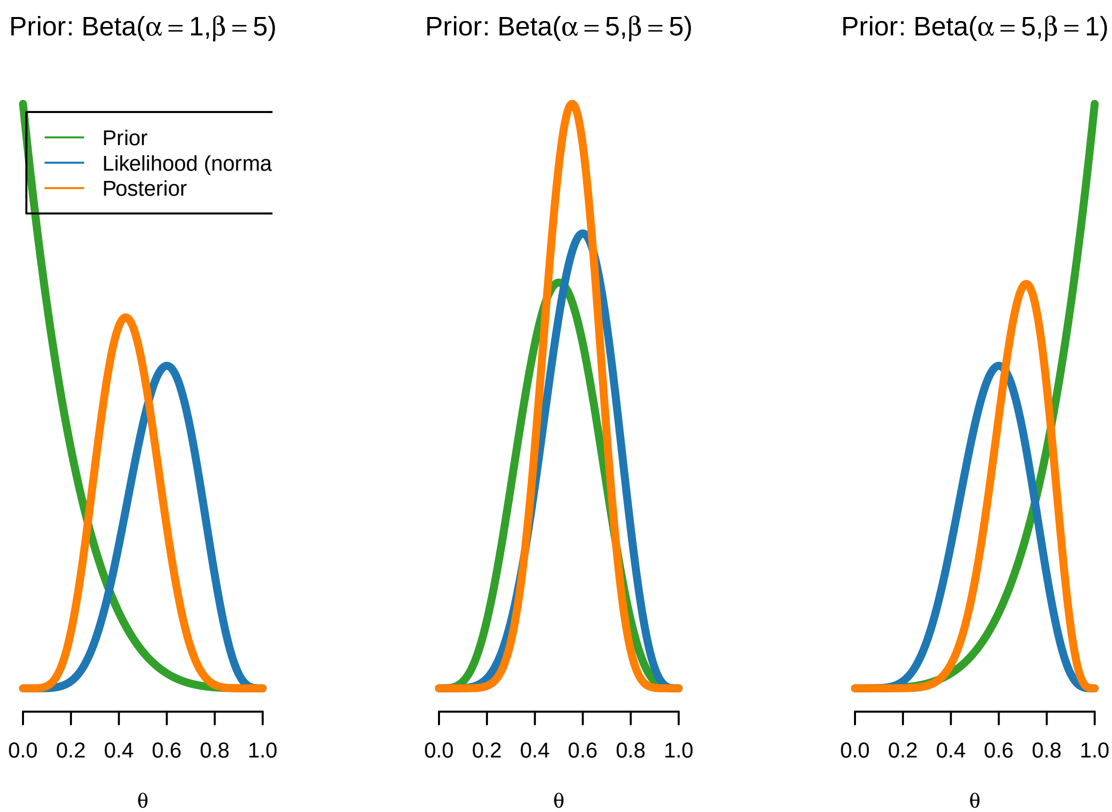
```
n = 10
x = spam[1:n]
par(mfrow = c(1,3))
post = BernPost(x, alphaPrior = 1, betaPrior = 5, legend = TRUE)
```

```
## Posterior mean is  0.438
## Posterior standard deviation is  0.12
```

```
post = BernPost(x, alphaPrior = 5, betaPrior = 5, legend = FALSE)
```

```
## Posterior mean is  0.55
## Posterior standard deviation is  0.109
```

```
post = BernPost(x, alphaPrior = 5, betaPrior = 1, legend = FALSE)
```




```
## Posterior mean is  0.688
## Posterior standard deviation is  0.112
```

Since we only have $n = 10$ data points, the posteriors for the three different priors differ a lot. Priors matter when the data are weak. Let's try with the $n = 100$ first observations.

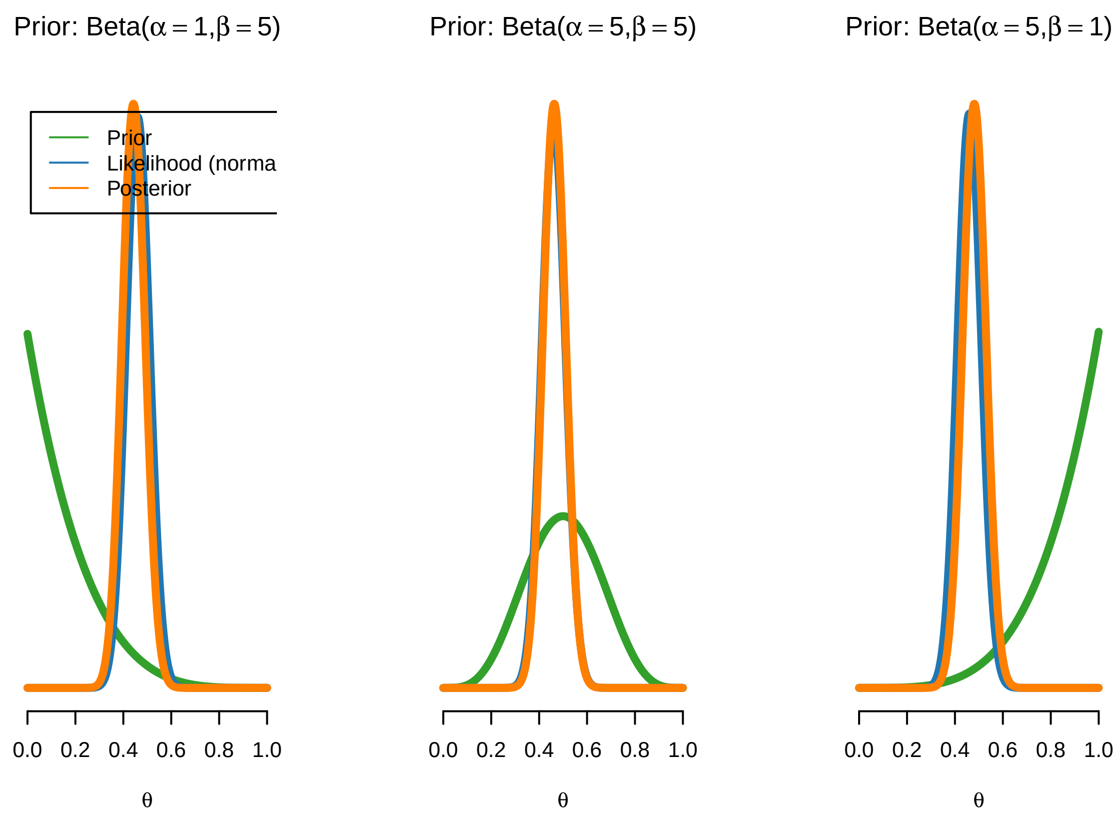
```
n = 100
x = spam[1:n]
par(mfrow = c(1,3))
post = BernPost(x, alphaPrior = 1, betaPrior = 5, legend = TRUE)
```

```
## Posterior mean is  0.443
## Posterior standard deviation is  0.048
```

```
post = BernPost(x, alphaPrior = 5, betaPrior = 5, legend = FALSE)
```

```
## Posterior mean is  0.464
## Posterior standard deviation is  0.047
```

```
post = BernPost(x, alphaPrior = 5, betaPrior = 1, legend = FALSE)
```




```
## Posterior mean is  0.481
## Posterior standard deviation is  0.048
```

The effect of the prior is now almost gone. Finally let's use all $n = 4601$ observations in the dataset:

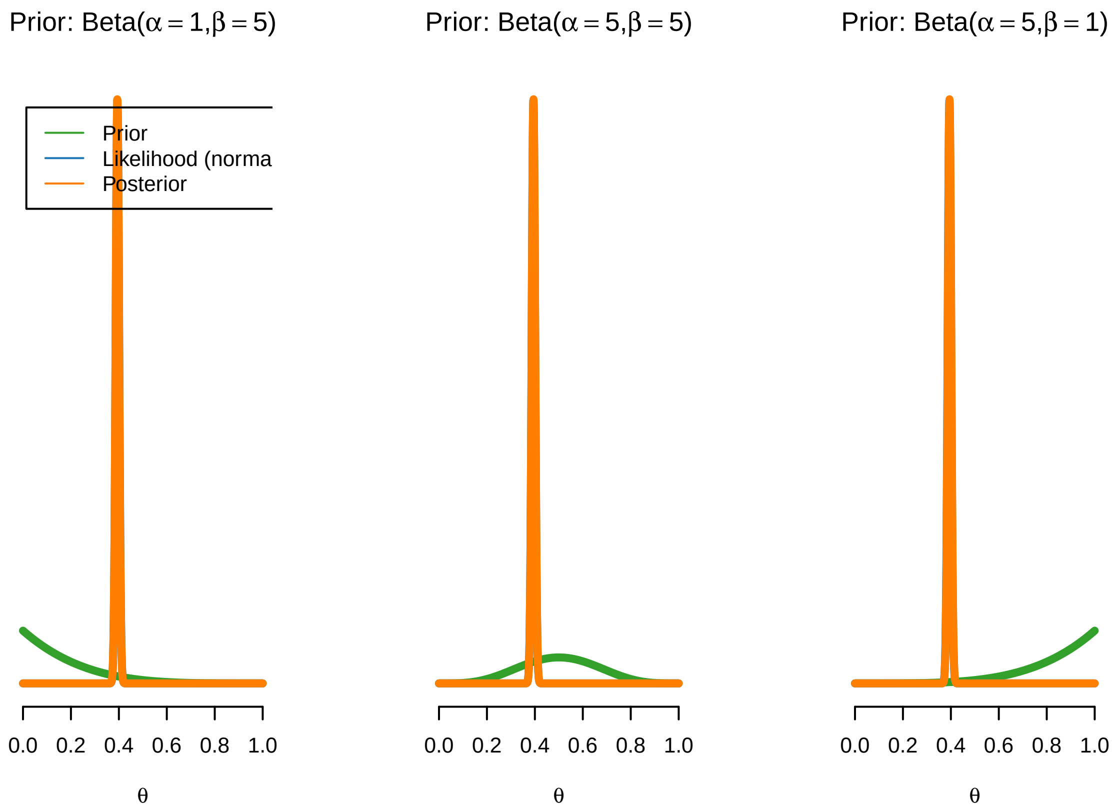
```
x = spam
par(mfrow = c(1,3))
post = BernPost(x, alphaPrior = 1, betaPrior = 5, legend = TRUE)
```

```
## Posterior mean is  0.394
## Posterior standard deviation is  0.007
```

```
post = BernPost(x, alphaPrior = 5, betaPrior = 5, legend = FALSE)
```

```
## Posterior mean is  0.394
## Posterior standard deviation is  0.007
```

```
post = BernPost(x, alphaPrior = 5, betaPrior = 1, legend = FALSE)
```




```
## Posterior mean is  0.394
## Posterior standard deviation is  0.007
```

We see two things: * The effect of the prior is completely gone. All three prior give identical posteriors. We have reached a subjective consensus among the three persons. * We are quite sure now that the spam probability θ is around 0.4.

A later notebook will re-analyze this data using for example logistic regression.