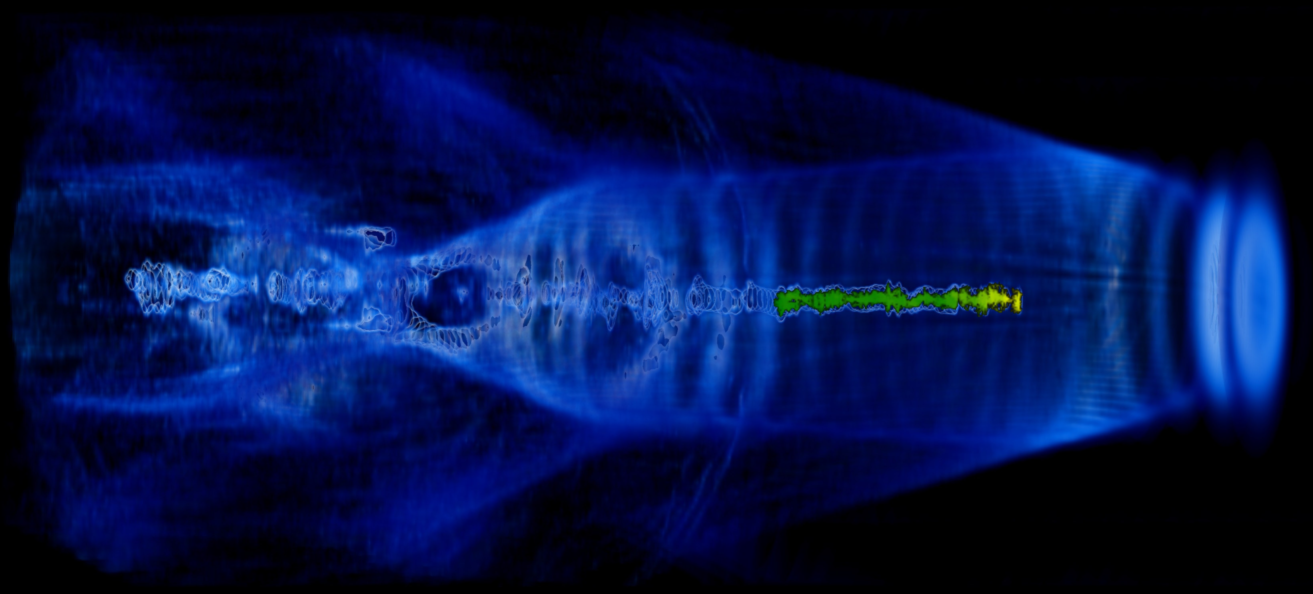
Master's thesis 2019

A compact plasma beam dump for next generation particle accelerators

OSCAR JAKOBSSON



School of Physics and Astronomy
The University of Manchester

A compact plasma beam dump for next generation particle accelerators

OSCAR JAKOBSSON

School of Physics and Astronomy
Cockcroft Accelerator Group
The University of Manchester
Manchester, United Kingdom 2019

A compact plasma beam dump for next generation particle accelerators
OSCAR JAKOBSSON

Supervisor: Guoxing Xia, Cockcroft Accelerator Group

Master's Thesis 2019
School of Physics and Astronomy
Cockcroft Accelerator Group
The University of Manchester

Cover: Wind visualization constructed in Matlab showing a surface of constant wind speed along with streamlines of the flow.

Typeset in LaTeX

A compact plasma beam dump for next generation particle accelerators
OSCAR JAKOBSSON
School of Physics and Astronomy
The University of Manchester

Abstract

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

Keywords: Plasma wakefield acceleration, deceleration, beam dump, ILC, EuPRAXIA

Acknowledgements

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

Oscar Jakobsson, Manchester, January 2019

Contents

List of Figures

List of Tables

1

Introduction

This chapter presents the section levels that can be used in the template.

1.1 Motivation

1.2 Thesis Outline

The following table presents an overview of the section levels that are used in this document. The number of levels that are numbered and included in the table of contents is set in the settings file `Settings.tex`. The levels are shown in Section 1.3.

Name	Command
Chapter	`\chapter{`*`Chapter name`*`}`
Section	`\section{`*`Section name`*`}`
Subsection	`\subsection{`*`Subsection name`*`}`
Subsubsection	`\subsubsection{`*`Subsubsection name`*`}`
Paragraph	`\paragraph{`*`Paragraph name`*`}`
Subparagraph	`\paragraph{`*`Subparagraph name`*`}`

1.3 Section

1.3.1 Subsection

1.3.1.1 Subsubsection

1.3.1.1.1 Paragraph

1.3.1.1.1.1 Subparagraph

2

Theory

2.1 PWFA - Linear-Fluid Wakefield Theory

2.1.1 Density perturbations

Perturbation due to beam $n(r,\xi) \rightarrow n(r,\xi) + n_1(r,\xi)$, use Maxwell's equations and continuity equation.

$$-\frac{1}{k_p^2}\left(\frac{\partial^2}{\partial \xi^2} + k_p^2\right) n_1(r,\xi) = n_b(r,\xi) \quad , \quad n_1(r,\xi<0) = 0 \tag{2.1}$$

$$\mathcal{L}_\xi n_1(r,\xi) = n_b(r,\xi) \quad \Rightarrow \quad \mathcal{L}_\xi G(\xi,\xi') = \delta(\xi-\xi') \tag{2.2}$$

$$G(\xi,\xi') = \begin{cases} 0 & , \ -\infty < \xi < \xi' \\ A\sin(k_p\xi) + B\cos(k_p\xi) & , \ \xi' < \xi < \infty \end{cases} \tag{2.3}$$

where the Green's function obeys the same b.c as the density perturbation, i.e it is continuous across the boundary with a discontinuous derivative across the boundary. Integrate across discontinuity at $\xi = 0$

$$\lim_{\epsilon\to 0}\int_{\xi'-\epsilon}^{\xi'+\epsilon} \mathcal{L}_\xi G(\xi,\xi')\,\mathrm{d}\xi = \lim_{\epsilon\to 0}\int_{\xi'-\epsilon}^{\xi'+\epsilon} \delta(\xi)\,\mathrm{d}\xi = 1 \quad \Rightarrow \quad \lim_{\epsilon\to 0}\left[-\frac{1}{k_p^2}\frac{\partial G}{\partial \xi}\right]_{\xi'-\epsilon}^{\xi'+\epsilon} = 1 \tag{2.4}$$

For simplicity set arrival of beam at $t = 0$, such that $\xi' = 0$.

$$G(\xi,\xi') = -k_p\sin(k_p\xi)\,\Theta(\xi) \quad \Rightarrow \quad n_1(r,\xi) = \int_{-\infty}^{\infty} G(\xi,\xi')\, n_b(r,\xi')\,\mathrm{d}\xi' \tag{2.5}$$

2.1.2 Longitudinal Accelerating Field

$$\boldsymbol{\nabla}\times\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{2.6}$$

$$\boldsymbol{\nabla}\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{2.7}$$

gives

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial \mathbf{J}}{\partial t} + 4\pi\boldsymbol{\nabla}\rho \tag{2.8}$$

Lorentz for law ($\mathbf{v}$ is the velocity of the plasma):

$$m\frac{\partial n\mathbf{v}}{\partial t} = en\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right) \approx en\mathbf{E} \quad \Rightarrow \quad \frac{\partial \mathbf{J}_p}{\partial t} = \frac{e^2 n}{m}\mathbf{E} \tag{2.9}$$

Letting $\rho = \rho_b + \rho_p$ and $\mathbf{J} = \mathbf{J}_b + \mathbf{J}_p$ for the beam and plasma respectively, and $\mathbf{J}_b = c\rho_b\hat{\mathbf{z}}$, gives

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - k_p^2\right)\mathbf{E} = \frac{4\pi}{c}\frac{\partial \rho_b}{\partial t}\hat{\mathbf{z}} + 4\pi\boldsymbol{\nabla}\left(\rho_b + \rho_p\right) \tag{2.10}$$

where $k_p = \omega_p/c$ is the plasma wave number. To find the electric field along the beam, z-direction, we proceed by solving

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - k_p^2\right)E_z = \frac{4\pi}{c}\frac{\partial \rho_b}{\partial t} + 4\pi\frac{\partial}{\partial z}\left(\rho_b + \rho_p\right) \tag{2.11}$$

using $\nabla^2 = \nabla_\perp^2 + \partial_z^2$, in Fourier transform space, where

$$\widetilde{E}_z\left(k\right) = \int_{-\infty}^{\infty} E_z(\xi)e^{-ik\xi}\mathrm{d}k \quad , \quad \widetilde{\rho}_b\left(\xi\right) + \widetilde{\rho}_p\left(\xi\right) = \int_{-\infty}^{\infty}\left(\rho_b(k) + \rho_b(k)\right)e^{ik\xi}\mathrm{d}k \tag{2.12}$$

such that

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\widetilde{E}_z\left(\xi\right) = 0 \tag{2.13}$$

which gives

$$\left(\nabla_\perp^2 - k_p^2\right)\widetilde{E}_z\left(\xi\right) = \frac{4\pi}{c}\frac{\partial \widetilde{\rho}_b}{\partial t} + 4\pi\frac{\partial}{\partial z}\left(\widetilde{\rho}_b + \widetilde{\rho}_p\right) \tag{2.14}$$

From eq. XXX we have

$$\frac{\partial^2 \rho_p}{\partial t^2} + \omega_p^2\rho_p = -\omega_p^2\rho_b \quad \Rightarrow \quad -k^2\widetilde{\rho}_p + k_p^2\widetilde{\rho}_p = -k_p^2\widetilde{\rho}_b \quad \Rightarrow \quad \widetilde{\rho}_p = \frac{k_p^2}{k^2 - k_p^2}\widetilde{\rho}_b \tag{2.15}$$

$$\nabla_\perp^2 = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} \tag{2.16}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - k_p^2\right)\widetilde{E}_z = \left[\frac{4\pi}{c}\frac{\partial}{\partial t} + 4\pi\left(1 + \frac{k_p^2}{k^2 - k_p^2}\right)\frac{\partial}{\partial z}\right]\widetilde{\rho}_b \tag{2.17}$$

We now rewrite this equation as

$$\mathscr{L}\widetilde{E}_z = \widetilde{f}(r) \tag{2.18}$$

We proceed as before and solve this PDE by finding the Green's function. We assume that the source is radially symmetric such that $\mathscr{L}G(r, r') = \delta(r - r')$. The LHS of eq. XXX is the modified Bessel function of order zero and the RHS represents our source term. Consequently the Green's function is formed by linear combinations of modified Bessels functions of order zero.

$$G\left(r, r'\right) = \begin{cases} A(r_0)(A_1 I_0(r) + B_1 K_0(r)) & , \ 0 < r < r' \\ B(r_0)(A_2 I_0(r) + B_2 K_0(r)) & , \ r' < r < \infty \end{cases} \tag{2.19}$$

Now lets solve this for the a beam charge distribution that is a delta function in z, yet with radial symmetry

$$\rho_b = \frac{e}{2\pi r}\delta(r - r_0)\delta(\xi) \quad \Rightarrow \quad \left(r^2\frac{\partial^2}{\partial r^2} + r\frac{\partial}{\partial r} - r^2 k_p^2\right)\widetilde{E}_z = 2ei\frac{kk_p^2}{k^2 - k_p^2} r\delta(r - r_0) \tag{2.20}$$

We may compute the electric field from the Green's function directly, or by first computing the point-particle and then convolving it with the point-field, which ever is more computationally demanding.

3
Methods

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

4
Results

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

5

Conclusion

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

Bibliography

[1] John David Jackson. Jackson - Classical Electrodynamics (3rd Ed.).pdf, 1962.

A
Appendix 1

Lorem ipsum dolor sit amet, consectetur adipisicing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.