Implementation exercises for the course Heuristic Optimization

Thomas Stützle [1]
stuetzle@ulb.ac.be

IRIDIA, CoDE, ULB

February 25, 2025

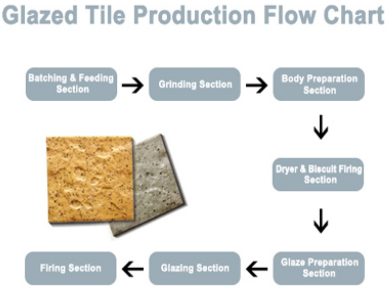
[1] Slides based on 2014 excersises by Dr. Franco Mascia and 2017 by Dr. Federico Pagnozzi.

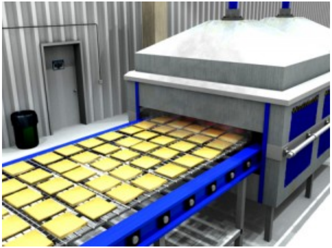
Exercise 1.1: Iterative Improvement for the PFSP

Implement perturbative local search algorithms for the PFSP

1. Permutation Flow Shop Scheduling Problem (PFSP)
2. First-improvement and best-improvement
3. Transpose, exchange and insert neighborhoods
4. Random initialization vs. simplified RZ heuristic
5. Statistical empirical analysis

The Permutation Flow Shop Scheduling Problem (1/4)





Example in ceramic tile production

- Tiles need several processing steps with different machines
- Tiles of different type require specific processing times for each machine
- Goal: find a schedule of the jobs that minimizes an objective function (makespan or total completion time)

The Permutation Flow Shop Scheduling Problem (2/4)

Flow Shop Scheduling

- Several scheduling problems have been proposed with different formulations and constraints.
- In permutation flow shop problems:
  - jobs composed by operations to be executed on several machines
  - all jobs pass through the machines in the same order
  - all jobs available at time zero
  - pre-emption not allowed
  - each operation has to be performed on a specific machine
  - each job at most on one machine at a time
  - each machine at most one job at a time

The Permutation Flow Shop Scheduling Problem (PFSP)

- Jobs pass trough all machines in the same order (FCFS queues)
- No constraints: infinite buffers between machines, no blocking, no no-wait requirements (steel production)

The Permutation Flow Shop Scheduling Problem (2/4)

Flow Shop Scheduling

- Several scheduling problems have been proposed with different formulations and constraints.
- In permutation flow shop problems:
  - jobs composed by operations to be executed on several machines
  - all jobs pass through the machines in the same order
  - all jobs available at time zero
  - pre-emption not allowed
  - each operation has to be performed on a specific machine
  - each job at most on one machine at a time
  - each machine at most one job at a time

The Permutation Flow Shop Scheduling Problem (PFSP)

- Jobs pass trough all machines in the same order (FCFS queues)
- No constraints: infinite buffers between machines, no blocking, no no-wait requirements (steel production)

The Permutation Flow Shop Scheduling Problem (3/4)

Given

A set of n jobs $J_1, \ldots, J_n$ jobs, where each job J_i consists of m operations $o_{i1}, \ldots, o_{im}$ performed on $M_1, \ldots, M_m$ machines in that order, with processing time p_{ij} for operation o_{ij}.

Objective

Find a permutation π that minimizes the sum of the completion times $\sum_{i=1}^{n} C_i$.

The Permutation Flow Shop Scheduling Problem (3/4)

Given

A set of n jobs $J_1, \ldots, J_n$ jobs, where each job J_i consists of m operations $o_{i1}, \ldots, o_{im}$ performed on $M_1, \ldots, M_m$ machines in that order, with processing time p_{ij} for operation o_{ij}.

Objective

Find a permutation π that minimizes the sum of the completion times $\sum_{i=1}^{n} C_i$.

The Permutation Flow Shop Scheduling Problem (3/4)

Given

A set of n jobs $J_1, \ldots, J_n$ jobs, where each job J_i consists of m operations $o_{i1}, \ldots, o_{im}$ performed on $M_1, \ldots, M_m$ machines in that order, with processing time p_{ij} for operation o_{ij}.

Objective

Find a permutation π that minimizes the sum of the completion times $\sum_{i=1}^{n} C_i$.

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots\ m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots\ n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots\ n, j = 2, \ldots\ m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$
\begin{array}{ll}
C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} & j = 1, \ldots\ m \\
C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} & k = 1, \ldots\ n \\
C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} & k = 2, \ldots\ n, j = 2, \ldots\ m
\end{array}
$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \dots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \dots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \dots n, j = 2, \dots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots\ m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots\ n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots\ n, j = 2, \ldots\ m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \dots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \dots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \dots n, j = 2, \dots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$
\begin{aligned}
C_{\pi(1)j} &= \sum_{h=1}^{j} p_{\pi(1)h} && j = 1, \ldots\ m \\
C_{\pi(k)1} &= \sum_{h=1}^{k} p_{\pi(h)1} && k = 1, \ldots\ n \\
C_{\pi(k)j} &= max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} && k = 2, \ldots\ n, j = 2, \ldots\ m
\end{aligned}
$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

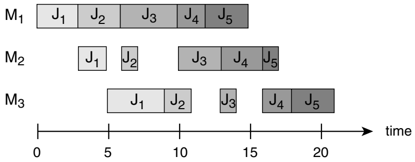
Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3



	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

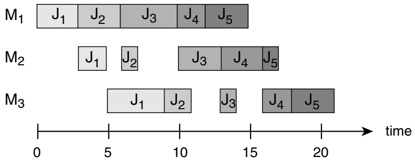
Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3



	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

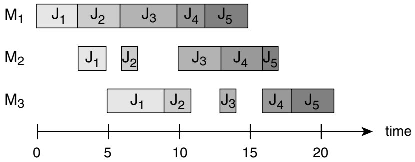
Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3



	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

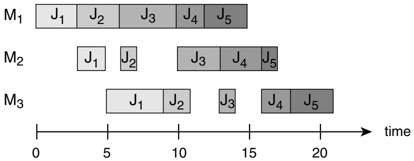
Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3



	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

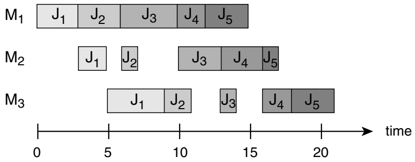
Sum of completion times = 73

The Permutation Flow Shop Scheduling Problem (4/4)

Computing completion times

$$
\begin{aligned}
C_{\pi(1)j} &= \textstyle\sum_{h=1}^{j} p_{\pi(1)h} && j = 1, \ldots\, m\\
C_{\pi(k)1} &= \textstyle\sum_{h=1}^{k} p_{\pi(h)1} && k = 1, \ldots\, n\\
C_{\pi(k)j} &= max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} && k = 2, \ldots\, n, j = 2, \ldots\, m
\end{aligned}
$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3



	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

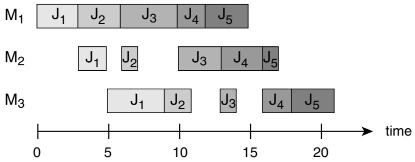
Makespan = 21

Sum of completion times = 73

Computing completion times

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3



	3	6	10	12	15
	5	7	13	16	17
	9	11	14	18	21
C_i	9	11	14	18	21

Makespan = 21

Sum of completion times = 73

Exercise 1.1: Iterative Improvement for the PFSP

Implement 12 iterative improvements algorithms for the PFSP

Exercise 1.1: Iterative Improvement for the PFSP

Implement 12 iterative improvements algorithms for the PFSP

- Pivoting rule:
  1. first-improvement
  2. best-improvement
- Neighborhood:
  1. Transpose
  2. Exchange
  3. Insert
- Initial solution:
  1. Random permutation
  2. Simplified RZ heuristic

Exercise 1.1: Iterative Improvement for the PFSP

Implement 12 iterative improvements algorithms for the PFSP

- Pivoting rule:
  1. first-improvement
  2. best-improvement
- Neighborhood:
  1. Transpose
  2. Exchange
  3. Insert
- Initial solution:
  1. Random permutation
  2. Simplified RZ heuristic

2 pivoting rules $\times$ 3 neighborhoods $\times$ 2 initialization methods $=$
12 combinations

Exercise 1.1: Iterative Improvement for the PFSP

Implement 12 iterative improvements algorithms for the PFSP

Don't implement 12 programs!
Reuse code and use command-line parameters

```
pfsp-ii --first --transpose --srz
pfsp-ii --best --exchange --random-init
...
```

Exercise 1.1: Iterative Improvement for the PFSP

Iterative Improvement

$\pi :=$ `GenerateInitialSolution()`
while π is not a local optimum **do**
  choose a neighbour $\pi' \in \mathcal{N}(\pi)$ such that $F(\pi') < F(\pi)$
  $\pi := \pi'$

Exercise 1.1: Iterative Improvement for the PFSP

Iterative Improvement

$\pi :=$ `GenerateInitialSolution()`
while π is not a local optimum **do**
  choose a neighbour $\pi' \in \mathcal{N}(\pi)$ such that $F(\pi') < F(\pi)$
  $\pi := \pi'$

Which neighbour to choose? Pivoting rule

- **Best Improvement:** choose best from all neighbours of π
  - ✔ Good quality
  - ✘ Requires evaluation of all neighbours in each step
- **First improvement:** evaluate neighbours in fixed order and choose first improving neighbour.
  - ✔ More efficient
  - ✘ Order of evaluation may impact quality / performance

Exercise 1.1: Iterative Improvement for the PFSP

Iterative Improvement

$\pi := \texttt{GenerateInitialSolution()}$
while π is not a local optimum **do**
  choose a neighbour $\pi' \in \mathcal{N}(\pi)$ such that $F(\pi') < F(\pi)$
  $\pi := \pi'$

Which neighbour to choose? Pivoting rule

- **Best Improvement:** choose best from all neighbours of π
  - ✔ Good quality
  - ✘ Requires evaluation of all neighbours in each step
- **First improvement:** evaluate neighbours in fixed order and choose first improving neighbour.
  - ✔ More efficient
  - ✘ Order of evaluation may impact quality / performance

Exercise 1.1: Iterative Improvement for the PFSP

Iterative Improvement

$\pi := \texttt{GenerateInitialSolution()}$

while π is not a local optimum **do**

choose a neighbour $\pi' \in \mathcal{N}(\pi)$ such that $F(\pi') < F(\pi)$

$\pi := \pi'$

Which neighbour to choose? Pivoting rule

- **Best Improvement:** choose best from all neighbours of π
  - ✔ Good quality
  - ✘ Requires evaluation of all neighbours in each step
- **First improvement:** evaluate neighbours in fixed order and choose first improving neighbour.
  - ✔ More efficient
  - ✘ Order of evaluation may impact quality / performance

Exercise 1.1: Iterative Improvement for the PFSP

Iterative Improvement

$\pi := \texttt{GenerateInitialSolution()}$
while π is not a local optimum **do**
  choose a neighbour $\pi' \in \mathcal{N}(\pi)$ such that $F(\pi') < F(\pi)$
  $\pi := \pi'$

Initial solution

- Random permutation
- Simplified RZ heuristic

Exercise 1.1: Iterative Improvement for the PFSP

Iterative Improvement

$\pi :=$ `GenerateInitialSolution()`
while π is not a local optimum **do**
  choose a neighbour $\pi' \in \mathcal{N}(\pi)$ such that $F(\pi') < F(\pi)$
  $\pi := \pi'$

Simplified RZ heuristic

Start by ordering the jobs in ascending order of their sum of processing times.
Construct the solution by inserting **one job at a time** in the position that minimize the CT.

The sum of processing times of job J_i is computed as $\sum_1^m p_{ij}$
Note: the solution is constructed incrementally, and at each iteration C_i corresponds to the makespan of the partial solution.

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$
$$T_i = \sum_1^m p_{ij}$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$	$CT = 16$
$\{J_4\ J_2\}$	$CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$	$CT = 29$
$\{J_2\ J_5\ J_4\}$	$CT = 29$
$\{J_2\ J_4\ J_5\}$	$CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$	$CT = 49$
$\{J_2\ J_3\ J_4\ J_5\}$	$CT = 50$
$\{J_2\ J_4\ J_3\ J_5\}$	$CT = 45$
$\{J_2\ J_4\ J_5\ J_3\}$	$CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$	$CT = 68$
$\{J_2\ J_1\ J_4\ J_3\ J_5\}$	$CT = 67$
$\{J_2\ J_4\ J_1\ J_3\ J_5\}$	$CT = 65$
$\{J_2\ J_4\ J_3\ J_1\ J_5\}$	$CT = 66$
$\{J_2\ J_4\ J_3\ J_5\ J_1\}$	$CT = 66$

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$
$$T_i = \sum_{1}^{m} p_{ij}$$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$ $CT = 16$
$\{J_4\ J_2\}$ $CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$ $CT = 29$
$\{J_2\ J_5\ J_4\}$ $CT = 29$
$\{J_2\ J_4\ J_5\}$ $CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$ $CT = 49$
$\{J_2\ J_3\ J_4\ J_5\}$ $CT = 50$
$\{J_2\ J_4\ J_3\ J_5\}$ $CT = 45$
$\{J_2\ J_4\ J_5\ J_3\}$ $CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$ $CT = 68$
$\{J_2\ J_1\ J_4\ J_3\ J_5\}$ $CT = 67$
$\{J_2\ J_4\ J_1\ J_3\ J_5\}$ $CT = 65$
$\{J_2\ J_4\ J_3\ J_1\ J_5\}$ $CT = 66$
$\{J_2\ J_4\ J_3\ J_5\ J_1\}$ $CT = 66$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Simplified RZ heuristic: an example

$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h}$ $\qquad j = 1, \ldots\ m$

$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1}$ $\qquad k = 1, \ldots\ n$

$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j}$ $\qquad k = 2, \ldots\ n, j = 2, \ldots\ m$

$T_i = \sum_1^m p_{ij}$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$ $\quad CT = 16$

$\{J_4\ J_2\}$ $\quad CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$ $\quad CT = 29$

$\{J_2\ J_5\ J_4\}$ $\quad CT = 29$

$\{J_2\ J_4\ J_5\}$ $\quad CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$ $\quad CT = 49$

$\{J_2\ J_3\ J_4\ J_5\}$ $\quad CT = 50$

$\{J_2\ J_4\ J_3\ J_5\}$ $\quad CT = 45$

$\{J_2\ J_4\ J_5\ J_3\}$ $\quad CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$ $\quad CT = 68$

$\{J_2\ J_1\ J_4\ J_3\ J_5\}$ $\quad CT = 67$

$\{J_2\ J_4\ J_1\ J_3\ J_5\}$ $\quad CT = 65$

$\{J_2\ J_4\ J_3\ J_1\ J_5\}$ $\quad CT = 66$

$\{J_2\ J_4\ J_3\ J_5\ J_1\}$ $\quad CT = 66$

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots\ m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots\ n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots\ n, j = 2, \ldots\ m$$
$$T_i = \sum_{1}^{m} p_{ij}$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3

T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$	$CT = 16$
$\{J_4\ J_2\}$	$CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$	$CT = 29$
$\{J_2\ J_5\ J_4\}$	$CT = 29$
$\{J_2\ J_4\ J_5\}$	$CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$	$CT = 49$
$\{J_2\ J_3\ J_4\ J_5\}$	$CT = 50$
$\{J_2\ J_4\ J_3\ J_5\}$	$CT = 45$
$\{J_2\ J_4\ J_5\ J_3\}$	$CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$	$CT = 68$
$\{J_2\ J_1\ J_4\ J_3\ J_5\}$	$CT = 67$
$\{J_2\ J_4\ J_1\ J_3\ J_5\}$	$CT = 65$
$\{J_2\ J_4\ J_3\ J_1\ J_5\}$	$CT = 66$
$\{J_2\ J_4\ J_3\ J_5\ J_1\}$	$CT = 66$

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$
$$T_i = \sum_{1}^{m} p_{ij}$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$ $CT = 16$

$\{J_4\ J_2\}$ $CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$ $CT = 29$

$\{J_2\ J_5\ J_4\}$ $CT = 29$

$\{J_2\ J_4\ J_5\}$ $CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$ $CT = 49$

$\{J_2\ J_3\ J_4\ J_5\}$ $CT = 50$

$\{J_2\ J_4\ J_3\ J_5\}$ $CT = 45$

$\{J_2\ J_4\ J_5\ J_3\}$ $CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$ $CT = 68$

$\{J_2\ J_1\ J_4\ J_3\ J_5\}$ $CT = 67$

$\{J_2\ J_4\ J_1\ J_3\ J_5\}$ $CT = 65$

$\{J_2\ J_4\ J_3\ J_1\ J_5\}$ $CT = 66$

$\{J_2\ J_4\ J_3\ J_5\ J_1\}$ $CT = 66$

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$
$$T_i = \sum_1^m p_{ij}$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$ $CT = 16$

$\{J_4\ J_2\}$ $CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$ $CT = 29$

$\{J_2\ J_5\ J_4\}$ $CT = 29$

$\{J_2\ J_4\ J_5\}$ $CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$ $CT = 49$

$\{J_2\ J_3\ J_4\ J_5\}$ $CT = 50$

$\{J_2\ J_4\ J_3\ J_5\}$ $CT = 45$

$\{J_2\ J_4\ J_5\ J_3\}$ $CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$ $CT = 68$

$\{J_2\ J_1\ J_4\ J_3\ J_5\}$ $CT = 67$

$\{J_2\ J_4\ J_1\ J_3\ J_5\}$ $CT = 65$

$\{J_2\ J_4\ J_3\ J_1\ J_5\}$ $CT = 66$

$\{J_2\ J_4\ J_3\ J_5\ J_1\}$ $CT = 66$

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$
$$T_i = \sum_{1}^{m} p_{ij}$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$	$CT = 16$
$\{J_4\ J_2\}$	$CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$	$CT = 29$
$\{J_2\ J_5\ J_4\}$	$CT = 29$
$\{J_2\ J_4\ J_5\}$	$CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$	$CT = 49$
$\{J_2\ J_3\ J_4\ J_5\}$	$CT = 50$
$\{J_2\ J_4\ J_3\ J_5\}$	$CT = 45$
$\{J_2\ J_4\ J_5\ J_3\}$	$CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$	$CT = 68$
$\{J_2\ J_1\ J_4\ J_3\ J_5\}$	$CT = 67$
$\{J_2\ J_4\ J_1\ J_3\ J_5\}$	$CT = 65$
$\{J_2\ J_4\ J_3\ J_1\ J_5\}$	$CT = 66$
$\{J_2\ J_4\ J_3\ J_5\ J_1\}$	$CT = 66$

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$
$$T_i = \sum_{1}^{m} p_{ij}$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$ $CT = 16$

$\{J_4\ J_2\}$ $CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$ $CT = 29$

$\{J_2\ J_5\ J_4\}$ $CT = 29$

$\{J_2\ J_4\ J_5\}$ $CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$ $CT = 49$

$\{J_2\ J_3\ J_4\ J_5\}$ $CT = 50$

$\{J_2\ J_4\ J_3\ J_5\}$ $CT = 45$

$\{J_2\ J_4\ J_5\ J_3\}$ $CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$ $CT = 68$

$\{J_2\ J_1\ J_4\ J_3\ J_5\}$ $CT = 67$

$\{J_2\ J_4\ J_1\ J_3\ J_5\}$ $CT = 65$

$\{J_2\ J_4\ J_3\ J_1\ J_5\}$ $CT = 66$

$\{J_2\ J_4\ J_3\ J_5\ J_1\}$ $CT = 66$

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$
$$T_i = \sum_1^m p_{ij}$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$ $CT = 16$

$\{J_4\ J_2\}$ $CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$ $CT = 29$

$\{J_2\ J_5\ J_4\}$ $CT = 29$

$\{J_2\ J_4\ J_5\}$ $CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$ $CT = 49$

$\{J_2\ J_3\ J_4\ J_5\}$ $CT = 50$

$\{J_2\ J_4\ J_3\ J_5\}$ $CT = 45$

$\{J_2\ J_4\ J_5\ J_3\}$ $CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$ $CT = 68$

$\{J_2\ J_1\ J_4\ J_3\ J_5\}$ $CT = 67$

$\{J_2\ J_4\ J_1\ J_3\ J_5\}$ $CT = 65$

$\{J_2\ J_4\ J_3\ J_1\ J_5\}$ $CT = 66$

$\{J_2\ J_4\ J_3\ J_5\ J_1\}$ $CT = 66$

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
$$C_{\pi(k)j} = max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} \qquad k = 2, \ldots n, j = 2, \ldots m$$
$$T_i = \sum_{1}^{m} p_{ij}$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$	$CT = 16$
$\{J_4\ J_2\}$	$CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$	$CT = 29$
$\{J_2\ J_5\ J_4\}$	$CT = 29$
$\{J_2\ J_4\ J_5\}$	$CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$	$CT = 49$
$\{J_2\ J_3\ J_4\ J_5\}$	$CT = 50$
$\{J_2\ J_4\ J_3\ J_5\}$	$CT = 45$
$\{J_2\ J_4\ J_5\ J_3\}$	$CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$	$CT = 68$
$\{J_2\ J_1\ J_4\ J_3\ J_5\}$	$CT = 67$
$\{J_2\ J_4\ J_1\ J_3\ J_5\}$	$CT = 65$
$\{J_2\ J_4\ J_3\ J_1\ J_5\}$	$CT = 66$
$\{J_2\ J_4\ J_3\ J_5\ J_1\}$	$CT = 66$

Simplified RZ heuristic: an example

$$
\begin{aligned}
C_{\pi(1)j} &= \textstyle\sum_{h=1}^{j} p_{\pi(1)h} && j = 1, \ldots\ m \\
C_{\pi(k)1} &= \textstyle\sum_{h=1}^{k} p_{\pi(h)1} && k = 1, \ldots\ n \\
C_{\pi(k)j} &= max\{C_{\pi(k-1)j}, C_{\pi(k)(j-1)}\} + p_{\pi(k)j} && k = 2, \ldots\ n, j = 2, \ldots\ m \\
T_i &= \textstyle\sum_1^m p_{ij}
\end{aligned}
$$

Job	J_1	J_2	J_3	J_4	J_5
p_{i1}	3	3	4	2	3
p_{i2}	2	1	3	3	1
p_{i3}	4	2	1	2	3
T_i	9	6	8	7	7

Starting sequence $= \{J_2\ J_4\ J_5\ J_3\ J_1\}$

Initial Solution $= \{J_2\ J_4\ J_1\ J_3\ J_5\}$

Step 1 $\pi = \{\}$

$\{J_2\ J_4\}$	$CT = 16$
$\{J_4\ J_2\}$	$CT = 16$

Step 2 $\pi = \{J_2\ J_4\}$

$\{J_5\ J_2\ J_4\}$	$CT = 29$
$\{J_2\ J_5\ J_4\}$	$CT = 29$
$\{J_2\ J_4\ J_5\}$	$CT = 29$

Step 3 $\pi = \{J_2\ J_4\ J_5\}$

$\{J_3\ J_2\ J_4\ J_5\}$	$CT = 49$
$\{J_2\ J_3\ J_4\ J_5\}$	$CT = 50$
$\{J_2\ J_4\ J_3\ J_5\}$	$CT = 45$
$\{J_2\ J_4\ J_5\ J_3\}$	$CT = 45$

Step 4 $\pi = \{J_2\ J_4\ J_3\ J_5\}$

$\{J_1\ J_2\ J_4\ J_3\ J_5\}$	$CT = 68$
$\{J_2\ J_1\ J_4\ J_3\ J_5\}$	$CT = 67$
$\{J_2\ J_4\ J_1\ J_3\ J_5\}$	$CT = 65$
$\{J_2\ J_4\ J_3\ J_1\ J_5\}$	$CT = 66$
$\{J_2\ J_4\ J_3\ J_5\ J_1\}$	$CT = 66$

Exercise 1.1: Iterative Improvement for the PFSP

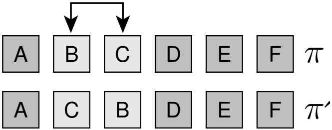
Iterative Improvement

$\pi := \texttt{GenerateInitialSolution()}$
while π is not a local optimum **do**
  choose a neighbour $\pi' \in \mathcal{N}(\pi)$ such that $F(\pi') < F(\pi)$
  $\pi := \pi'$

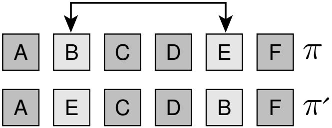
Which neighborhood $\mathcal{N}(\pi)$?

- Transpose
- Exchange
- Insertion

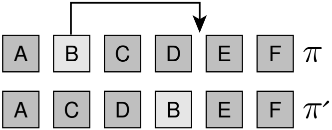
Exercise 1.1: Iterative Improvement for the PFSP



transpose neighbourhood

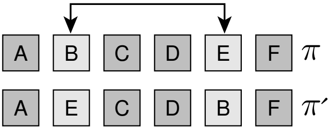


exchange neighbourhood



insert neighbourhood

Exercise 1.1: Iterative Improvement for the PFSP



exchange neighbourhood

Example: Exchange π_i and π_j ($i < j$), $\pi' = \texttt{Exchange}(\pi, i, j)$

Only jobs after i are affected!

Do not recompute the evaluation function from scratch!

Equivalent speed-ups with Transpose and Insertion

Exercise 1.1: Iterative Improvement for the PFSP

Instances

- PFSP instances with 50, 100 and 200 jobs, and 20 machines.
- More info will be available on teams

Experiments

Apply each algorithm k ten times to each instance i and compute:

1. Relative percentage deviation $\Delta_{ki} = 100 \cdot \dfrac{\text{cost}_{ki} - \text{best-known}_i}{\text{best-known}_i}$
2. Computation time (t_{ki})

Report for each algorithm k

- Average relative percentage deviation
- Sum of computation time

Exercise 1.1: Iterative Improvement for the PFSP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Statistical test

- Paired t-test
- Wilcoxon signed-rank test

Exercise 1.1: Iterative Improvement for the PFSP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Background: Statistical hypothesis tests (1)

- *Statistical hypothesis tests* are used to assess the validity of statements about properties of or relations between sets of statistical data.
- The statement to be tested (or its negation) is called the *null hypothesis* (H_0) of the test.
  Example: For the Wilcoxon signed-rank test, the null hypothesis is that 'the median of the differences is zero'.
- The *significance level* (α) determines the maximum allowable probability of incorrectly rejecting the null hypothesis.
  Typical values of α are 0.05 or 0.01.

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Background: Statistical hypothesis tests (2)

- The application of a test to a given data set results in a *p-value*, which represents the probability that the null hypothesis is incorrectly rejected.
- The null hypothesis is rejected iff this p-value is smaller than the previously chosen significance level.
- Most common statistical hypothesis tests are already implemented in statistical software such as the *R software environment* (http://www.r-project.org/).

Exercise 1.1: Iterative Improvement for the PFSP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Example in R

```
best.known <- read.csv ("bestSolutions.txt")
a.cost <- read.table("ii-best-ex-rand.dat")$V1
a.cost <- 100 * (a.cost - best.known) / best.known$BS
b.cost <- read.table("ii-best-ins-rand.dat")$V1
b.cost <- 100 * (b.cost - best.known) / best.known$BS
t.test (a.cost, b.cost, paired=T)$p.value
[1] 0.8819112
wilcox.test (a.cost, b.cost, paired=T)$p.value
[1] 0.0019212
```

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Example in R

```
best.known <- read.csv ("bestSolutions.txt")
a.cost <- read.table("ii-best-ex-rand.dat")$V1
a.cost <- 100 * (a.cost - best.known) / best.known$BS
b.cost <- read.table("ii-best-ins-rand.dat")$V1
b.cost <- 100 * (b.cost - best.known) / best.known$BS
t.test (a.cost, b.cost, paired=T)$p.value
[1] 0.8819112
wilcox.test (a.cost, b.cost, paired=T)$p.value
[1] 0.0019212
```

Exercise 1.1: Iterative Improvement for the PFSP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Example in R

```
best.known <- read.csv ("bestSolutions.txt")
a.cost <- read.table("ii-best-ex-rand.dat")$V1
a.cost <- 100 * (a.cost - best.known) / best.known$BS
b.cost <- read.table("ii-best-ins-rand.dat")$V1
b.cost <- 100 * (b.cost - best.known) / best.known$BS
t.test (a.cost, b.cost, paired=T)$p.value
[1] 0.8819112
wilcox.test (a.cost, b.cost, paired=T)$p.value
[1] 0.0019212
```

Exercise 1.1: Iterative Improvement for the PFSP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Example in R

```
best.known <- read.csv ("bestSolutions.txt")
a.cost <- read.table("ii-best-ex-rand.dat")$V1
a.cost <- 100 * (a.cost - best.known) / best.known$BS
b.cost <- read.table("ii-best-ins-rand.dat")$V1
b.cost <- 100 * (b.cost - best.known) / best.known$BS
t.test (a.cost, b.cost, paired=T)$p.value
[1] 0.8819112
wilcox.test (a.cost, b.cost, paired=T)$p.value
[1] 0.0019212
```

Exercise 1.1: Iterative Improvement for the PFSP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Example in R

```
best.known <- read.csv ("bestSolutions.txt")
a.cost <- read.table("ii-best-ex-rand.dat")$V1
a.cost <- 100 * (a.cost - best.known) / best.known$BS
b.cost <- read.table("ii-best-ins-rand.dat")$V1
b.cost <- 100 * (b.cost - best.known) / best.known$BS
t.test (a.cost, b.cost, paired=T)$p.value
[1] 0.8819112
wilcox.test (a.cost, b.cost, paired=T)$p.value
[1] 0.0019212
```

Exercise 1.1: Iterative Improvement for the PFSP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Example in R

```
best.known <- read.csv ("bestSolutions.txt")
a.cost <- read.table("ii-best-ex-rand.dat")$V1
a.cost <- 100 * (a.cost - best.known) / best.known$BS
b.cost <- read.table("ii-best-ins-rand.dat")$V1
b.cost <- 100 * (b.cost - best.known) / best.known$BS
t.test (a.cost, b.cost, paired=T)$p.value
[1] 0.8819112
wilcox.test (a.cost, b.cost, paired=T)$p.value
[1] 0.0019212
```

Exercise 1.2 VND algorithms for the PFSP

Implement 2 VND algorithms for the PFSP

- Pivoting rule: first-improvement
- Neighborhood order:
  1. transpose $\rightarrow$ exchange $\rightarrow$ insert
  2. transpose $\rightarrow$ insert $\rightarrow$ exchange
- Initial solution:
  1. Simplified RZ heuristic

Exercise 1.2 VND algorithms for the PFSP

Variable Neighbourhood Descent (VND)

k neighborhoods $\mathcal{N}_1, \ldots, \mathcal{N}_k$
$\pi := \texttt{GenerateInitialSolution()}$
$i := 1$
repeat
  choose the first improving neighbor $\pi' \in \mathcal{N}_i(\pi)$
  if $\nexists \pi'$ **then**
    $i := i + 1$
  else
    $\pi := \pi'$
    $i := 1$
until $i > k$

Implement 2 VND algorithms for the PFSP

- Instances: Same as 1.1
- Experiments: ten runs of each algorithm per instance
- Report: Same as 1.1
- Statistical tests: Same as 1.1

- Instances and barebone code will be soon available on Teams
- Deadline is April 8 (23:59)
- Questions in the meantime?
  `stuetzle@ulb.ac.be`