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Keyframe-based Visual-Inertial Odometry for Small Workspace

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Saarbrücken, June 2016

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Acknowledgments

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by

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Abstract

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Contents

Chapter 1

Introduction

In past few years, the development of *Robotics* has surpassed people's expectation. The word *Robotics* has been first appeared in science fiction "Liar!" by Issac Asimov [38], it referred to science and technology of robots. By definition, *Robotics* is a research branch that related to design, control and application of robots, as well as processing feedback from robots.

Modern robots have been classified into several categories(e.g., Mobile robot, industrial robot, service robot, education robot etc.) with their usages. Among those categories, full-autonomous or semi-autonomous mobile robots attracts more and more researches. Such robots have abilities to move around in their moving space, with or without humans' control. The aim of research in mobile robots is to help us accomplish various hard tasks, whether domestically, commercially or militarily. These tasks, such as assisting disabled people, defusing bombs, or repair equipment in dangerous place is either risky or high expense for human beings.

For mobile robot, finding physical location of itself in unknown environment is normally crucial, such an ability(e.g., *Robot Navigation*) allows mobile robots avoid risky obstacles and finally arrive the goal position. Roughly speaking, *Robot Navigation* is a computing system which processes the information from external sources(e.g., sensors) and apply an algorithm to navigate robot, and sometimes build a map of environment. In *Robot Navigation*, robots sense environmental information by their sensors. These sensors, either locally (e.g., camera, inertial measurement unit (IMU)), or globally (e.g., Global Positioning System) detect events or changes in environment, and transfer their data to robots. Generally, sensors equipped in mobile robot (Local Sensor) are designed light, small, and inexpensive considering convenient movement and low expense. Camera and IMU sensor are considered most common local sensor in small mobile robot system.

A camera is a optical instrument for capturing images. Modern camera has several advantages for *Robot Navigation*. First, the core of camera chip set is cheap and easily installed in any mobile robot system; Second, camera often brings very rich information as it simulates the functioning of human eye. By recognizing key-points [29, 20, 28] in certain images, system observes the *landmarks* in environment, and those *landmarks* will localize robots by *Triangulation* [4, 15, 6].

A IMU sensor (Figure 1-1) often combines *gyroscope* and *accelerometer*, sometimes

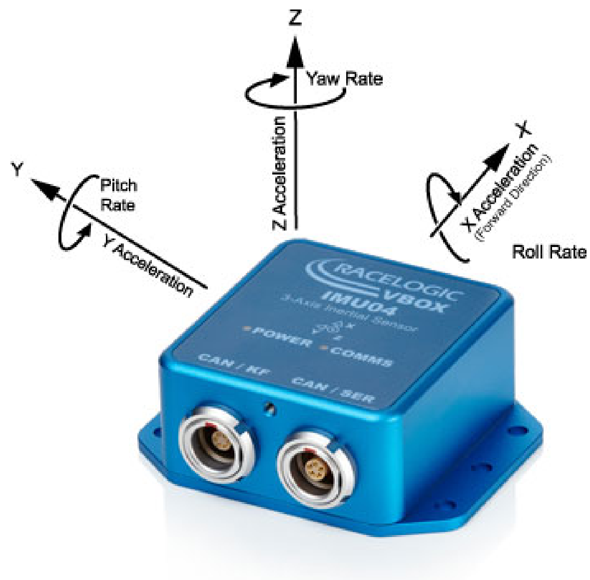


Figure 1-1: IMU sensor with gyroscope and accelerometer measures rotational rate and acceleration of X, Y, Z axis regarding its local frame. Note that IMU sensor normally has bias and irreducible noises, it is necessary to apply a calibration like cameras. The frequency of IMU output is normally larger than 100 Hz. Source: [1]

magnetometer, to measure specific force, angular rate and magnetic field regarding to its local frame. IMU is one of main component in *Inertial Navigation System*, which firstly used in air plane, spacecraft, guided missiles, and now also in mobile robot [2, 16, 24, 9]. IMU sensor utilize *Dead Reckoning* to track device's position, such technology tries to integrate IMU data over time by assuming the movement model of devices fixed(i.e., acceleration and rotation rate is constant over small period of time).

1.1 Motivation and Contribution

The main motivation of this thesis is to improve the navigation accuracy by fusing camera data and IMU sensor data.

Single sensor-based navigation system may not satisfy the requirements of high-quality localization by mobile robot. GPS-based navigation system has been used for outdoor devices for long time. However, such a system suffered from localizing in in-door environment, and also the accuracy of general GPS is not high, i.e., 3 to 5 meters error [37]. For mobile robot, which may move from in-door to out-door, and requires high-accuracy navigation, GPS is mostly not used, or as baseline of navigation [13].

Vision-based navigation system [4, 15], or simultaneous localization and mapping (SLAM) [5, 7, 25] gives an acceptable navigation result. [4] recognizes corner feature by single camera, and it uses a extended kalman filter framework to track the uncertainty and propagates the system state, however it can not handle large-scale scene. [15] utilizes key-frame based bundle adjustment to update the map, improves both accuracy and efficiency, it still met some problem in large-scale scene, becasue vision-based method often is a trade-off between computational complexity and localization accuracy due to the rich information and low output frequency by camera.

Single *Inertial Navigation System* recognize its pose by *Dead Reckoning* [22, 17]. However, the problem of *Dead Reckoning* error accumulation; Only few directions are observable [13] by IMU during whole navigation process. When object corrupts movement assumption, a correction step by external data will be needed.

Fusing camera and IMU data has many advantages. On the one hand, it can decrease computational time by making good use of high-frequency IMU data; On the other hand, it can reduce the error accumulation of IMU integration by the correction of vision-based navigation result within several turn. Fuse the camera and IMU data for robot navigation is not novel. [24] applies multi-state kalman filter (MSCKF) on state update, decrease the computational complexity by only keeping few last information of keyframe. [13] analysis the consistency of MSCKF, correct the ways of IMU integration to obtain consistency of sytem, therefore increase the accuracy. [9] exploits a way to optimize manifold information, therefore solving data fusing problem in a non-linear optimization scheme. Unfortunately, codes and data of above methods are not accessible, that motivates to explore possible methods to increase accuracy of *vision-inertial navigation system* both theoretically and experimentally.

The contributions of this thesis are as follows,

- We exploit a highly flexible, realistic software to generate synthetic IMU sensor data and corresponding vision data, that is the main source to provide experimental data in this thesis.
- We present error-state kalman filter system kinematic based on quaternion, explore the multiple ways to integrate IMU data due to different movement models.
- We propose a novel method to update fusing result based on key-framed bundle adjustment.
- We propose a real-time visual-inertial framework implemented in C++, which is stable, scalable, high-accuracy and low-latency.

1.2 Outline of the Thesis

In Chapter 2 we first overview our visual-inertial odometry, including world representation, important notations, we also compare filter method and key-frame based method in SLAM problem, and in the end we explain our choice in this thesis.

Then we enter the Chapter 3, which introduces *Quaternion Algebra*. In this chapter, we introduces the basic operations on quaternion, the relationship among quaternion, rotation matrix and rotation vector. In this chapter, we also explain how to integrate or derivative quaternion over time.

Chapter 4 is main part of this thesis. In Section 4.1, we study the Error-State Kalman Filter (ESKF), and apply it to IMU integration; Section 4.2 we summarize how to fuse camera into ESKF, and how to optimize it by key-frame based bundle adjustment. In Section 4.3, we overview the pipeline of our visual-inertial odometry system.

We show our experiment results in Chapter 5. First, the process of generating synthetic IMU and camera data is presented. Then we run several experiments to show our proposed visual-inertial odometry has higher accuracy than single IMU integration, visual SLAM, as our system is still running in real-time.

In the end, we summarize and discuss our work and analysis the potential future work in Chapter 6.

Chapter 2

Overview of Visual-inertial Odometry

In this chapter, we will overview visual-inertial odometry system. In section 2.1, we will introduce world representation(e.g., world frame, camera frame and IMU frame) together with basic notations in our odometry system. Then in section 2.2, we will discuss two important scheme, filter method and keyframe Bundle Adjustment(keyframe BA) in SLAM algorithm, and explain why we finally choose keyframe-based method.

2.1 World Representations and Notations

Visual-inertial odometry [18], literally, is an odometry system, which received environment information by visual (camera) and inertial (IMU) sensor. VIO is similar with well-known visual odometry (VO) problem [27], with an additional IMU sensor, it tries to estimate agent's pose as agent keep moving in the environment. One big difference between VIO and SLAM algorithm is that VIO does not or only build a simple map, whereas SLAM normally maintains and continuously updates a map.

To setup a VIO system, we need to first define the ways to represent the world. Globally, we have a world frame $\mathcal{W}$; World frame $\mathcal{W}$ is set to a right-handed Cartesian coordinate system that every objects has an absolute pose (translation and rotation) in it. Then we have local frame for each sensor, i.e., IMU frame $\mathcal{I}$ and camera frame $\mathcal{C}$; Every time camera and IMU sensor obtain observations within their own local frame, we need to integrate those data and estimate the pose of those sensors in world frame $\mathcal{W}$. Figure 2-1 shows the overall world representations. Both IMU frame and Camera frame are right-handed Cartesian coordinate system.

In this master thesis, we use following notations,

- We denote scalars as a, b, c, vectors as $\mathbf{a}, \mathbf{b}, \mathbf{c}$, matrices as $\mathbf{A}, \mathbf{B}, \mathbf{C}$, frames as $\mathcal{A}, \mathcal{B}, \mathcal{C}$.

- We denote measurement $\mathbf{m}$ in particular frame $\mathcal{F}$ as $\mathbf{m}_{\mathcal{F}}$. To further simplify, any parameter that is **not** in world frame shall be denoted particularly. For example, the translation $\mathbf{p}$ in camera frame will be denoted as $\mathbf{p}_{\mathcal{C}}$, and the translation $\mathbf{p}$ in world frame will be denoted as $\mathbf{p}$.

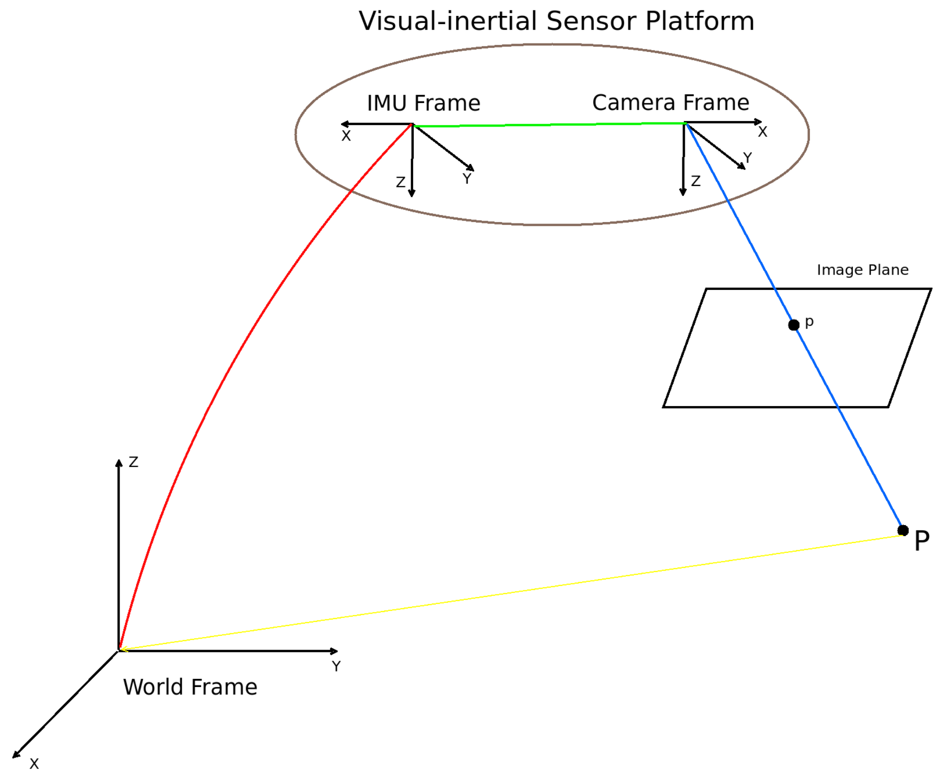


Figure 2-1: This figure shows the connection among world frame $\mathcal{W}$, IMU frame $\mathcal{I}$ and camera frame $\mathcal{C}$. Green line shows the transformation between camera and IMU, which can be pre-calibrated. Red line is the pose of IMU in world frame. Camera frame observes point $\mathbf{p}$ of object P in image plane, and connects a object P by blue line, and the coordinate of object P in world frame is presented as yellow line.

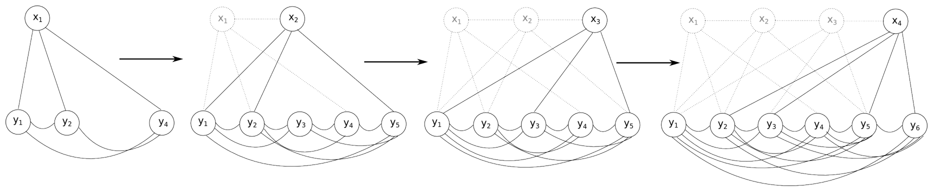
- A general translation $\mathbf{t}$ should express a translation from point A to point B in frame $\mathcal{C}$, which is denoted as $\mathbf{t}_{\mathcal{C}}^{AB}$. We simplify a point $\mathbf{p}$ in frame $\mathcal{A}$ as $\mathbf{p}_{\mathcal{A}}$, when this point is the translation $\mathbf{t}_{\mathcal{A}}^{OP}$, O is origin of frame $\mathcal{A}$, and $\mathbf{p} = P$, this holds same for vector.

- A general rotation is either expressed in quaternion $\mathbf{q}$ or rotation matrix $\mathbf{R}$. We use quaternion $\mathbf{q}$ as example. A quaternion is a orientation operation from frame $\mathcal{B}$ to frame $\mathcal{A}$, and it is denoted as $\mathbf{q}_{\mathcal{AB}}$ in this thesis. Noted that if such a operation is from world frame $\mathcal{W}$ to some frame $\mathcal{B}$, we omit both frame for simplification, i.e., $\mathbf{q}_{\mathcal{WB}} \triangleq \mathbf{q}$.

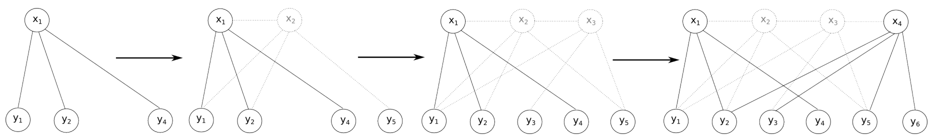
- We use hat operator $\hat{\boldsymbol{x}}$ to represent the estimation of state $\boldsymbol{x}$.

2.2 Filter Versus Keyframe

It is important to note that though this thesis focus on visual-inertial odometry for small workspace, we still tend to keep the possibility to extend our system to a general, scalable and efficient SLAM system. SLAM system usually have two parallel process,



(a) Filter



(b) Keyframe-based BA

Figure 2-2: (a) Filter method for SLAM. (b) Keyframe-based Bundle Adjustment (BA) for SLAM. We denote the i^{th} camera position as $\mathbf{x}_i$, i^{th} image feature as $\mathbf{y}_i$. We connect the line between camera and image feature if this feature is observed by this camera, the vanished observations is presented as dotted line, and the vanished camera is expressed as grey font. Both graph changes as time goes on from left to right. One can see from (a) that though only the latest camera pose is reserved, the edges between features are increasing exponentially. (b) stores some of former camera poses (keyframe) (i.e., $\mathbf{x}_1$ and $\mathbf{x}_4$) by keeping graph stay sparsity.

one is for localization and the other is for mapping, the crucial point of building such a system is to keep both processes efficient. There are two general framework (e.g., filter-based method and keyframe-based method) in SLAM. In this section, we want to discuss whether filter-based method or keyframe-based method are more suitable for our case.

Filter-based SLAM [5, 6, 4] uses *Extended Kalman Filter* (EKF) to propagate state and update the covariance of the state. In each step, system obtain the current pose estimation and map update by marginalising all former information. This marginalising step usually eliminates the former pose and adds connections to image features. As showed in Figure 2-2a, the graph will not grow fast with time since the former pose has been eliminated and features in environment is limited. However, once the system moves to large scale scene, the problem of limiting the number of features become severe as the graph tends to be fully-connected.

Keyframe-based SLAM [15, 24, 10, 7, 25] applies *bundle adjustment* (BA) for keyframes to update the map in each step. In keyframe-based SLAM, it stores some historical poses (keyframes), and combines with image feature points to do a BA step. The chosen of keyframes varies from implementations, the idea is to pick up the poses that is not very close to last keyframe, otherwise the information might be redundant, which might lead to increase the computational cost. In Figure 2-2b, the graph still stays sparsity as the number of poses and features increases. The drawback might be the behaviour of pose estimation is inadequate as it ignores some of former information.

In [30], they have shown that the computational cost for keyframe BA in $O(m^2 \cdot n)$, and for filter method is $O(n^3)$ where m is the number of key frames, and n is the number of landmarks. Therefore they conclude that keyframe-based SLAM is slightly better than filter-based SLAM in their experiment settings, especially when scale of scene becomes larger considering the number of landmarks will be much larger than key frames.

In this master thesis, we choose keyframe-based method for visual part and filter method for IMU integration part. We choose filter method for IMU integration part is that we do not keep former information (e.g., image features or landmarks) in integration step, hence each filter step can be regarded as an optimization step. The reason why we use keyframe-based method for visual part is that we want keep the scalability of our system, besides the results from IMU integration can be a good compensation in case of the lack of pose estimation in keyframe-based BA.

Chapter 3

Background on Quaternion Algebra

One important task for this master thesis is to integrate IMU data over time to estimate camera pose (e.g., position and orientation). By assuming movement model, it is straightforward to integrate position in Cartesian space, however integrating orientation in manifold space is often not a easy task. In this chapter, we introduce the quaternion algebra, and explore the way to operate quaternion over time.

3.1 Definition of Quaternion

A quaternion Q is defined as,

$$Q = q_w + q_x i + q_y j + q_z k \tag{3.1}$$

where $\{q_w, q_x, q_y, q_z\} \in \mathbb{R}$, and $\{i, j, k\}$ are three imaginary unit length, e.g., $i^2 = j^2 = k^2 = ijk = -1$.

In most cases, we represent quaternion Q as a four-element vector $\mathbf{q}$ composed by above four real number $\{q_w, q_x, q_y, q_z\}$, i.e.,

$$\mathbf{q} \triangleq [q_w \ \mathbf{q}_v]^T = [q_w \ q_x \ q_y \ q_z]^T \tag{3.2}$$

where q_w is the real part of $\mathbf{q}$, and q_v is a 3-vector to represent imaginary part of $\mathbf{q}$.

It is worth to be noted that there are two different conventions of quaternion $\mathbf{q}$, *Hamilton way* [12] and *JPL way* [3]. In Hamilton convention, the real part q_w is the first component of $\mathbf{q}$, i.e., $[q_w \ \mathbf{q_v}]$, whereas in JPL way, the real part is the fourth component, i.e., $[\mathbf{q_v} \ q_w]$. To avoid confusions, and considering Hamilton way is more common to use, especially for implementation [11, 14], we hereby claim that we use ***Hamilton way*** to represent quaternion $\mathbf{q}$ throughout this master thesis.

3.2 Properties of Quaternion

In this section, we will introduce properties of quaternion.

Summation We start with summation of two quaternions $\mathbf{q}$ and $\mathbf{p}$,

$$\boldsymbol{q}+\boldsymbol{p}=[q_w+p_w \ \mathbf{q}_v+\mathbf{p}_v]^T=[q_w+p_w \ q_x+p_x \ q_y+p_y \ q_z+p_z \ ]^T \tag{3.3}$$

Product We use $\otimes$ to denote the product operation on quaternions, which gives,

$$\boldsymbol{q}\otimes\boldsymbol{p}=\begin{bmatrix}p_wq_w-p_xq_x-p_yq_y-p_zq_z\\ p_wq_x+p_xq_w+p_yq_z-p_zq_y\\ p_wq_y-p_xq_q+p_yq_w+p_zq_x\\ p_wq_z-p_xq_y-p_yq_z+p_zq_w\end{bmatrix} \tag{3.4}$$

The product of two quaternions can be expressed as two equivalent matrix products,

$$\boldsymbol{q_1}\otimes\boldsymbol{q_2}=Q_1^+\boldsymbol{q_2} \tag{3.5}$$

with

$$Q_\mathbf{1}^+=q_w\mathbb{I}+\begin{bmatrix}0 & -\boldsymbol{q_v}^T\\ \boldsymbol{q_v} & [\boldsymbol{q_v}]_\times\end{bmatrix} \tag{3.6}$$

where $[\boldsymbol{q}_v]_\times$ represents the cross-product matrix of $\boldsymbol{q}_v$, which the cross-product matrix of a vector $\boldsymbol{v}$ is defined by

$$[\boldsymbol{v}]_\times=\begin{bmatrix}0 & -v_z & v_y\\ v_z & 0 & -v_x\\ -v_y & v_x & 0\end{bmatrix} \tag{3.7}$$

note that the quaternion product is not commutative, i.e.,

$$\boldsymbol{p}\otimes\boldsymbol{q}\neq\boldsymbol{q}\otimes\boldsymbol{p} \tag{3.8}$$

however it is associative, and distributive over sum, i.e.,

$$\boldsymbol{p}\otimes(\boldsymbol{q}\otimes\boldsymbol{k})=(\boldsymbol{p}\otimes\boldsymbol{q})\otimes\boldsymbol{k} \tag{3.9}$$

$$\boldsymbol{p}\otimes(\boldsymbol{q}+\boldsymbol{k})=\boldsymbol{p}\otimes\boldsymbol{q}+\boldsymbol{p}\otimes\boldsymbol{k} \tag{3.10}$$

$$(\boldsymbol{q}+\boldsymbol{k})\otimes\boldsymbol{p}=\boldsymbol{q}\otimes\boldsymbol{p}+\boldsymbol{k}\otimes\boldsymbol{p} \tag{3.11}$$

Conjugate The conjugate $\boldsymbol{q}^*$of a quaternion is defined by

$$\boldsymbol{q}^*\triangleq q_w-\boldsymbol{q_v}=[q_w \ \ -\boldsymbol{q_v}]^T \tag{3.12}$$

Identity We call a quaternion $\mathbf{q}$ identical if, for any given quaternion $\mathbf{p}$, $\boldsymbol{q}\otimes\boldsymbol{p}=\boldsymbol{p}\otimes\boldsymbol{q}=\boldsymbol{p}$. An identical quaternion $\mathbf{q}$ satisfies that,

$$\boldsymbol{q}=[1 \ 0 \ 0 \ 0]^T \tag{3.13}$$

In this master thesis, we denote identical quaternion as $\boldsymbol{q}_\mathbb{I}$.

Norm The norm of a quaternion $\|\boldsymbol{q}\|$ is defined similar to the norm of a vector, which is,

$$\|\boldsymbol{q}\| = \sqrt{q_w^2 + q_x^2 + q_y^2 + q_z^2} \tag{3.14}$$

Inverse The inverse of a quaternion $\boldsymbol{q}^{-1}$ is defined as,

$$\boldsymbol{q}^{-1} = \boldsymbol{q}^* / \|\boldsymbol{q}\| \tag{3.15}$$

which leads to,

$$\boldsymbol{q}^* \otimes \boldsymbol{q}^{-1} = \boldsymbol{q}^{-1} \otimes \boldsymbol{q}^* = \boldsymbol{q}_{\mathbb{I}} \tag{3.16}$$

Unit quaternion The norm of a unit quaternion $\|\boldsymbol{q}\|$ is 1, and the inverse of such a unit quaternion is equal to the conjugate of this quaternion,

$$\boldsymbol{q}^{-1} = \boldsymbol{q}^* \tag{3.17}$$

Pure quaternion A pure quaternion $\boldsymbol{q}$ is defined as,

$$\boldsymbol{q} = [0 \ \boldsymbol{q}_v]^T = [0 \ q_x \ q_y \ q_z]^T \tag{3.18}$$

Let pure quaternion $\boldsymbol{q} = \theta \boldsymbol{u}$, where $\theta = \|\boldsymbol{q}\|$, we can compute the exponential of $\boldsymbol{q}$ with the help of Euler formula,

$$\mathrm{e}^{\boldsymbol{q}} = \mathrm{e}^{\theta \boldsymbol{u}} = \cos\theta + \boldsymbol{u}\sin\theta = [\cos\theta \ \boldsymbol{u}\sin\theta]^T \tag{3.19}$$

which is still a quaternion, and moreover $\mathrm{e}^{\boldsymbol{q}}$ is a unit quaternion because its norm $\|\mathrm{e}^{\boldsymbol{q}}\|^2 = \cos\theta^2 + \sin\theta^2 = 1$.

3.3 Quaternions and Rotation operations

We discuss the relationship between quaternions and rotation operations by first introducing rotation vector $\boldsymbol{v}$.

Given a rotation vector $\boldsymbol{v} = \phi \boldsymbol{u}$, where ϕ is the norm of $\boldsymbol{v}$ and $\boldsymbol{u}$ is a unit vector, we can rotate a vector $\boldsymbol{x}$ by an angle ϕ around the axis $\boldsymbol{u}$ following right-handed rule, and obtain a new vector $\boldsymbol{x'}$,

$$\boldsymbol{x'} = \boldsymbol{x}_{||} + \boldsymbol{x}_{\perp}\cos\phi + (\boldsymbol{u} \times \boldsymbol{x})\sin\phi \tag{3.20}$$

where $\boldsymbol{x}_{||} = (\boldsymbol{x} \cdot \boldsymbol{u})\boldsymbol{u}$ is the component parallel to $\boldsymbol{x}$, and $\boldsymbol{x}_{\perp} = -\boldsymbol{u} \times (\boldsymbol{u} \times \boldsymbol{x})$ is the component perpendicilar to $\boldsymbol{x}$, therefore $\boldsymbol{x} = \boldsymbol{x}_{||} + \boldsymbol{x}_{\perp}$. This formula is known as *vector rotation formular* or *Rodrigues formula.*

We then can define a rotation matrix $\mathbf{R}$ by rotation vector $\boldsymbol{v} = \phi \boldsymbol{u}$ by the help of Equation (3.7) as,

$$\mathbf{R} = \mathrm{e}^{[\boldsymbol{v}]_\times} \tag{3.21}$$

we can rotate a vector $\boldsymbol{x}$ by an angle ϕ around the axis $\boldsymbol{u}$ using $\mathbf{R}$ in a clean way,

$$\boldsymbol{x'} = \mathbf{R}\boldsymbol{x} \tag{3.22}$$

one can show that result in Equation (3.22) is equivalent with the result in Equation (3.20) [35].

Constructing a unit quaternion $\boldsymbol{q}$ by Equation (3.19) with a rotation vector $\boldsymbol{v} = \phi\boldsymbol{u}$,

$$\boldsymbol{q} = \mathrm{e}^{\boldsymbol{v}/2} = \begin{bmatrix} \cos\phi/2 \\ \boldsymbol{v}\sin\phi/2 \end{bmatrix} \tag{3.23}$$

we can rotate a vector $\boldsymbol{x}$ by an angle ϕ around the axis $\boldsymbol{u}$ by,

$$\boldsymbol{x'} = \boldsymbol{q} \otimes \boldsymbol{x} \otimes \boldsymbol{q}^* \tag{3.24}$$

we then show $\boldsymbol{x'}$ in Equation (3.24) is equivalent with $\boldsymbol{x'}$ in Equation (3.20).

We transferred the vector $\boldsymbol{x}$ into pure quaternion form as,

$$\boldsymbol{x}_q = [0\ \boldsymbol{x}]^T \tag{3.25}$$

then we rewrite formula (3.24) as,

$$\begin{bmatrix} 0 \\ \boldsymbol{x'} \end{bmatrix} = \begin{bmatrix} \cos\phi/2 \\ \boldsymbol{v}\sin\phi/2 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ \boldsymbol{x} \end{bmatrix} \otimes \begin{bmatrix} \cos\phi/2 \\ -\boldsymbol{v}\sin\phi/2 \end{bmatrix} \tag{3.26}$$

expanding it by Equation (3.4), it is easily to show that,

$$\boldsymbol{x'} = \boldsymbol{x}_{||} + \boldsymbol{x}_{\perp}\cos\phi + (\boldsymbol{u} \times \boldsymbol{x})\sin\phi \tag{3.27}$$

which is exactly Equation (3.20).

To summarize here, we can construct a quaternion $\boldsymbol{q}$ or a rotation matrix $\mathbf{R}$ by any rotation vector $\boldsymbol{v} = \phi\boldsymbol{u}$, we denote such a quaternion as $\boldsymbol{q}\{\boldsymbol{v}\}$ and rotation matrix as $\mathbf{R}\{\boldsymbol{v}\}$ respectively. And a rotation operation of a vector $\boldsymbol{x}$ related to $\boldsymbol{v}$ can either be expressed as quaternion $\boldsymbol{q}\{\boldsymbol{v}\} \otimes \boldsymbol{x} \otimes \boldsymbol{q}\{\boldsymbol{v}\}^*$, or a rotation matrix $\mathbf{R}\{\boldsymbol{v}\}\boldsymbol{x}$. Note that we sometimes simplify $\mathbf{R}\{\boldsymbol{v}\}$ to $\mathbf{R}$, and/or $\boldsymbol{q}\{\boldsymbol{v}\}$ to $\boldsymbol{q}$ in this master thesis.

We also give conversion from rotation matrix $\mathbf{R}$ to quaternion $\boldsymbol{q}$. Knowing that,

$$\boldsymbol{q} \otimes \boldsymbol{x} \otimes \boldsymbol{q}^* = \mathbf{R}\boldsymbol{x} \tag{3.28}$$

we can construct $\mathbf{R} = \mathbf{R}\{\boldsymbol{q}\}$ by,

$$\mathbf{R} = \begin{bmatrix} q_w^2 + q_x^2 - q_y^2 - q_z^2 & 2(q_xq_y - q_wq_z) & 2(q_xq_z + q_wq_y) \\ 2(q_xq_y + q_wq_z) & q_w^2 - q_x^2 + q_y^2 - q_z^2 & 2(q_yq_z - q_wq_x) \\ 2(q_xq_z - q_wq_y) & 2(q_yq_z + q_wq_x) & q_w^2 - q_x^2 - q_y^2 + q_z^2 \end{bmatrix} \tag{3.29}$$

and a conversion from quaternion to rotation matrix can be found in [33].

3.4 Time-derivatives on Quaternion

We introduce the time-derivative on quaternion by first define,

$$\boldsymbol{q}(t+\Delta t) \triangleq \boldsymbol{q}(t) \otimes \Delta \boldsymbol{q} \tag{3.30}$$

where $\boldsymbol{q}(t)$ is the quaternion at time t and $\Delta \boldsymbol{q}$ is quaternion transformation within a small period time Δt.

One can expand $\Delta \boldsymbol{q}$ by Taylor expansions with Equation (3.23) to,

$$\Delta \boldsymbol{q} = \begin{bmatrix} 1 \\ \frac{1}{2}\Delta \boldsymbol{\theta} \end{bmatrix} + O(\|\Delta \boldsymbol{\theta}\|^2) \tag{3.31}$$

where $\Delta \boldsymbol{\theta}$ is a angular vector corresponding to $\Delta \boldsymbol{q}$. In fact, the angular rate $\boldsymbol{\omega}$ at time t is defined as,

$$\boldsymbol{\omega}(t) \triangleq \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{\theta}}{\Delta t} \tag{3.32}$$

which is one of measurements we can obtain from IMU sensor.

By definition of the derivative, we can obtain the time-derivative $\dot{\boldsymbol{q}}$ of quaternion $\boldsymbol{q}$ as,

$$\dot{\boldsymbol{q}} = \frac{d\boldsymbol{q}(t)}{dt} \triangleq \lim_{\Delta t \to 0} \frac{\boldsymbol{q}(t+\Delta t) - \boldsymbol{q}(t)}{\Delta t} \tag{3.33}$$

which follows,

$$\begin{aligned} \dot{\boldsymbol{q}} &\triangleq \lim_{\Delta t \to 0} \frac{\boldsymbol{q}(t+\Delta t) - \boldsymbol{q}(t)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{\boldsymbol{q} \otimes \Delta \boldsymbol{q} - \boldsymbol{q}}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{\boldsymbol{q} \otimes (\begin{bmatrix} 1 \\ \frac{1}{2}\Delta \boldsymbol{\theta} \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix})}{\Delta t} \\ &= \frac{1}{2}\boldsymbol{q} \otimes \begin{bmatrix} 0 \\ \boldsymbol{\omega} \end{bmatrix} \end{aligned} \tag{3.34}$$

here we simplify $\boldsymbol{q}(t)$ to $\boldsymbol{q}$. Then we can obtain the time-derivative on quaternion by writing angular rate into pure quaternion form (3.18), which is,

$$\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{q} \otimes \boldsymbol{\omega} \tag{3.35}$$

3.5 Time-integration on Quaternion

To integrate quaternion over time, we explore the relationship between $\boldsymbol{q}(t_n)$ and $\boldsymbol{q}(t_{n+1})$ where $t_n = n\Delta t$. Expanding $\boldsymbol{q}(t_{n+1})$ using Taylor series, we have

$$\boldsymbol{q}(t_{n+1}) = \boldsymbol{q}(t_n) + \dot{\boldsymbol{q}}(t_n)\Delta t + \frac{1}{2!}\ddot{\boldsymbol{q}}(t_n)\Delta t^2 + \frac{1}{3!}\dddot{\boldsymbol{q}}(t_n)\Delta t^3 + \dots \tag{3.36}$$

Assume that the second order derivative of rotational rate is zero, which is $\ddot{\boldsymbol{\omega}} = 0$, we have

$$\dot{\boldsymbol{q}}(t_{n+1}) = \frac{1}{2}\boldsymbol{q}(t_n) \otimes \boldsymbol{\omega}()(t_n) \tag{3.37}$$

$$\ddot{\boldsymbol{q}}(t_{n+1}) = \frac{1}{2^2}\boldsymbol{q}(t_n) \otimes \boldsymbol{\omega}^2(t_n) + \frac{1}{2}\boldsymbol{q}(t_n) \otimes \dot{\boldsymbol{\omega}} \tag{3.38}$$

$$\dddot{\boldsymbol{q}}(t_{n+1}) = \frac{1}{2^3}\boldsymbol{q}(t_n) \otimes \boldsymbol{\omega}^3(t_n) + \frac{1}{4}\boldsymbol{q}(t_n) \otimes \dot{\boldsymbol{\omega}}\boldsymbol{\omega}(t_n) + \frac{1}{2}\boldsymbol{q}(t_n) \otimes \boldsymbol{\omega}(t_n)\dot{\boldsymbol{\omega}} \tag{3.39}$$

$$\vdots$$

and so forth and so on. We then get the result of time integration by taking Equation (3.37, 3.38, 3.39) back into Equation (3.36).

We hereby gives a stronger assumption that angular rate $\boldsymbol{\omega}(t_n)$ remains constant during a small time period Δt, which is $\dot{\boldsymbol{\omega}} = 0$. However, considering the sampling rate of IMU sensor is usually high ($>$ 100 Hz), this assumption actually is general and also gives us a cleaner expression of time integration.

Given $\dot{\boldsymbol{\omega}} = 0$, we can get

$$\boldsymbol{q}_{n+1} = \boldsymbol{q}_n \otimes (1 + \frac{1}{2}\boldsymbol{\omega}_n\Delta t + \frac{1}{2!}(\frac{1}{2}\boldsymbol{\omega}_n\Delta t)^2 + \frac{1}{3!}(\frac{1}{2}\boldsymbol{\omega}_n\Delta t)^3 + \frac{1}{4!}(\frac{1}{2}\boldsymbol{\omega}_n\Delta t)^4 + \dots) \tag{3.40}$$

here we regard $\boldsymbol{q}$ and $\boldsymbol{\omega}$ as series, which is exactly

$$\boldsymbol{q}_{n+1} = \boldsymbol{q}_n \otimes \mathrm{e}^{\boldsymbol{\omega}\Delta t/2} \tag{3.41}$$

we can rewrite it by Equation (3.23) as,

$$\boldsymbol{q}_{n+1} = \boldsymbol{q}_n \otimes \boldsymbol{q}\{\boldsymbol{\omega}_n\Delta t\} \tag{3.42}$$

which is called ***Zeroth order forward integration*** of quaternion over time.

We can obtain ***Zeroth order backward integration*** by assuming the angular rate remains $\boldsymbol{\omega}_{n+1}$ within Δt, then we have

$$\boldsymbol{q}_{n+1} = \boldsymbol{q}_n \otimes \boldsymbol{q}\{\boldsymbol{\omega}_{n+1}\Delta t\} \tag{3.43}$$

and ***Zeroth order midward integration*** by assuming the angular rate holds $\bar{\boldsymbol{\omega}}_{n+1} = (\boldsymbol{\omega}_n + \boldsymbol{\omega}_{n+1})/2$ within Δt,

$$\boldsymbol{q}_{n+1} = \boldsymbol{q}_n \otimes \boldsymbol{q}\{\bar{\boldsymbol{\omega}}_n\Delta t\} \tag{3.44}$$

Though not used in this master thesis, we notice that [31] gives ***First order integration*** by assuming angular rate is linear with time, i.e., $\dot{\boldsymbol{\omega}} = \frac{\boldsymbol{\omega}_{n+1}-\boldsymbol{\omega}_n}{\Delta t}$, which is

$$\boldsymbol{q}_{n+1} = \boldsymbol{q}_n \otimes \boldsymbol{q}\{\bar{\boldsymbol{\omega}}_n \Delta t\} + \frac{\Delta t^2}{48} \boldsymbol{q}_n \otimes (\boldsymbol{\omega}_n \otimes \boldsymbol{\omega}_{n+1} - \boldsymbol{\omega}_{n+1} \otimes \boldsymbol{\omega}_n) + \dots \tag{3.45}$$

in our quaternion convention.

Chapter 4

Modular Sensor Fusing

In previous chapter, we learned how to represent each frame and analysis the difference between filter method and keyframe BA method for our odometry system. We also learned quaternion algebra and basic approaches for quaternion integration or derivative overtime under some general assumptions. In this chapter, we will explore the details of our sensor fusing approach, which roughly uses so called *IMU loose integration framework* [34]. In such a framework, system propagates states via Kalman filter (KF) based on IMU measurements, extrasensory (e.g., camera, GPS etc.) data are used in correction step. Computational cost for KF-styled approach is usually linear, hence *IMU loose integration framework* provides a good trade-off between computational complexity and accuracy in a real-time robotic navigation system.

4.1 Error-state Kalman Filter for IMU Integration

The error-state Kalman Filter (ESKF) followed the paradigm of Kalman filter, it also has prediction and correction step. However, ESKF separates system into three different states: true state nominal state and error state. Nominal state processes large signal, which is integrable in non-linear fashion, whereas error state keeps track of error and noise term, which can be integrated in linear way. The composition of nominal state and error state, we call it true state, which is the final guess of system.

The ESKF has some nice properties when building a visual-inertial odometry:

1. The computation of Jacobian may be very fast, because the error state is small and all second order products are negligible. This is very important since we want the system run in real-time.

2. Integration of vision data with IMU data is straightforward in KF correction step. One can utilize the result of tracking to correct the IMU integration state.

3. Large signal has been integrated in nominal state, so that we can apply the correction step in a lower rate than prediction step.

The procedure of ESKF in this system can be explained as follows. IMU data first is integrated into nominal state via numerical integration methods, note that nominal state does not take noise term or error term into account, hence nominal state will accumulate errors. The error state then predict the error and noise using normal extended KF paradigm, meaning it will predict the mean and covariance of system's error. In parallel a correction step is performed at a lower rate, the results of visual tracking are used to correct error state, the error state then is injected into nominal state, which nominal state become the final guess of system at that time point. The system goes on until the criterion condition has been met.

We explain the ESKF for IMU integration in this section, and visual sensor as correction data in Section 4.2.

4.1.1 System Kinematics

We denote our true state $\boldsymbol{x}_t$ as,

$$\boldsymbol{x}_t = \boldsymbol{x}_n \oplus \boldsymbol{x}_e \tag{4.1}$$

where $\boldsymbol{x}_n$ is the nominal state for large signals, and $\boldsymbol{x}_e$ is error state for small error/noise signal, we use $\oplus$ to denote a general composition step.

We then introduce position $\boldsymbol{p}$, velocity $\boldsymbol{v}$, quaternion $\boldsymbol{q}$, accelerometer bias $\boldsymbol{a}_b$, gyroscope bias $\boldsymbol{\omega}_b$ and gravity vector $\boldsymbol{g}$ into true state, nominal state and error state respectively. The general composition step can be shown as,

$$\begin{align}
\boldsymbol{p}_t =& \boldsymbol{p}_n + \boldsymbol{p}_e \tag{4.2}\\
\boldsymbol{v}_t =& \boldsymbol{v}_n + \boldsymbol{v}_e \tag{4.3}\\
\boldsymbol{q}_t \approx& \boldsymbol{q}_n \otimes \begin{bmatrix} 1 \\ \boldsymbol{\theta}_e/2 \end{bmatrix} \tag{4.4}\\
\boldsymbol{a}_{bt} =& \boldsymbol{a}_{bn} + \boldsymbol{a}_{be} \tag{4.5}\\
\boldsymbol{\omega}_{bt} =& \boldsymbol{\omega}_{bn} + \boldsymbol{\omega}_{be} \tag{4.6}\\
\boldsymbol{g}_t =& \boldsymbol{g}_n + \boldsymbol{g}_e \tag{4.7}
\end{align}$$

note that we derive Equation (4.4) by small angle approximation (Equation (3.31)), we apply angular error $\boldsymbol{\theta}_e$ instead of quaternion error in error state following classical approaches.

We then construct kinematic equations for true state, which are

$$\dot{\boldsymbol{p}}_t = \boldsymbol{v}_t \tag{4.8}$$
$$\dot{\boldsymbol{v}}_t = \mathbf{R}_t(\boldsymbol{a}_m - \boldsymbol{a}_{bt} - \boldsymbol{a}_n) + \boldsymbol{g}_t \tag{4.9}$$
$$\dot{\boldsymbol{q}}_t = \frac{1}{2}\boldsymbol{q}_t \otimes (\boldsymbol{\omega}_m - \boldsymbol{\omega}_{bt} - \boldsymbol{\omega}_n) \tag{4.10}$$
$$\dot{\boldsymbol{a}_{bt}} = \boldsymbol{a}_w \tag{4.11}$$
$$\dot{\boldsymbol{\omega}_{bt}} = \boldsymbol{\omega}_w \tag{4.12}$$
$$\dot{\boldsymbol{g}}_t = 0 \tag{4.13}$$

where $\boldsymbol{a}_m$, $\boldsymbol{\omega}_m$ are the measurements from accelerometer and gyroscope respectively within **local frame**, $\boldsymbol{a}_n$, $\boldsymbol{\omega}_n$ are noises with those measurements, $\boldsymbol{a}_w$, $\boldsymbol{\omega}_w$ are white Gaussian noise together with accelerometer and gyroscope bias, and $\mathbf{R}_t$ is the rotation matrix corresponding to true state quaternion, i.e., $\mathbf{R}_t \triangleq \mathbf{R}_t\{\boldsymbol{q}\}$ regarding to Equation (3.29). We use similar notations in nominal state and error state.

We obtain kinematic equations for nominal state by cutting off all small signals, which leads to

$$\dot{\boldsymbol{p}}_n = \boldsymbol{v}_n \tag{4.14}$$
$$\dot{\boldsymbol{v}}_n = \mathbf{R}_n(\boldsymbol{a}_m - \boldsymbol{a}_{bt}) + \boldsymbol{g}_n \tag{4.15}$$
$$\dot{\boldsymbol{q}}_n = \frac{1}{2}\boldsymbol{q}_n \otimes (\boldsymbol{\omega}_m - \boldsymbol{\omega}_{bn}) \tag{4.16}$$
$$\dot{\boldsymbol{a}_{bn}} = 0 \tag{4.17}$$
$$\dot{\boldsymbol{\omega}_{bn}} = 0 \tag{4.18}$$
$$\dot{\boldsymbol{g}}_n = 0 \tag{4.19}$$

and error state with small signals,

$$\dot{\boldsymbol{p}}_e = \boldsymbol{v}_e \tag{4.20}$$
$$\dot{\boldsymbol{v}}_e = \mathbf{R}_n\left[\boldsymbol{a}_m - \boldsymbol{a}_{bn}\right]_\times - \mathbf{R}_n\boldsymbol{a}_{be} + \boldsymbol{g}_e - \mathbf{R}_n\boldsymbol{a}_n \tag{4.21}$$
$$\dot{\boldsymbol{\theta}}_e = \left[\boldsymbol{\omega}_m - \boldsymbol{\omega}_{bn}\right]_\times - \boldsymbol{\omega}_{be} - \boldsymbol{\omega}_n \tag{4.22}$$
$$\dot{\boldsymbol{a}_{be}} = \boldsymbol{a}_w \tag{4.23}$$
$$\dot{\boldsymbol{\omega}_{be}} = \boldsymbol{\omega}_w \tag{4.24}$$
$$\dot{\boldsymbol{g}}_e = 0 \tag{4.25}$$

it is trivial to derive Equation (4.20, 4.23, 4.24, 4.25), see Appendix A for derivation of Equation (4.21 and 4.22).

4.1.2 State Time-integration and Error-state Jacobian

We then gives time-integration equations between any two time stamp t_n and t_{n+1} where we measure the time difference Δt as $\Delta t = t_{n+1} - t_n$. In order to simplify our notations, we denote last state parameters as $\boldsymbol{x}$, and denote current state parameters as $\boldsymbol{x}'$, where current state is measured at time stamp t_n, and last state is measured at t_{n-1}. Same notations for error state. Therefore, time-integration equations for nominal state for one updating are

$$\begin{aligned}
\boldsymbol{p}_n' =& \boldsymbol{p}_n + \boldsymbol{v}_n \Delta t + \frac{1}{2}(\mathbf{R}_n(\boldsymbol{a}_m - \boldsymbol{a}_{bt}) + \boldsymbol{g}_n)\Delta t^2 & (4.26)\\
\boldsymbol{v}_n' =& \boldsymbol{v}_n + (\mathbf{R}_n(\boldsymbol{a}_m - \boldsymbol{a}_{bt}) + \boldsymbol{g}_n)\Delta t & (4.27)\\
\boldsymbol{q}_n' =& \boldsymbol{q}_n \otimes \boldsymbol{q}\{(\boldsymbol{\omega}_m - \boldsymbol{\omega}_{bn})\Delta t\} & (4.28)\\
\boldsymbol{a}_{bn}' =& \boldsymbol{a}_{bn} & (4.29)\\
\boldsymbol{\omega}_{bn}' =& \boldsymbol{\omega}_{bn} & (4.30)\\
\boldsymbol{g}_n' =& \boldsymbol{g}_n & (4.31)
\end{aligned}$$

we use ***Zeroth order forward integration*** explained in Section 3.5 to integrate our state over time, this is also called Euler method in Runge-Kutta numerical integration methods (see Appendix B.1).

We integrate our error state in same manner, except truncating second-order signal out. Hence we obtain the integration equations for error state

$$\begin{aligned}
\boldsymbol{p}_e' =& \boldsymbol{p}_e + \boldsymbol{v}_e \Delta t & (4.32)\\
\boldsymbol{v}_e' =& \boldsymbol{v}_e + (\mathbf{R}_n\left[\boldsymbol{a}_m - \boldsymbol{a}_{bn}\right]_\times - \mathbf{R}_n \boldsymbol{a}_{be} + \boldsymbol{g}_e)\Delta t + \boldsymbol{v}_i & (4.33)\\
\boldsymbol{\theta}_e' =& (\mathbf{R}_n^T\{\boldsymbol{\omega}_m - \boldsymbol{\omega}_{bn}\}\boldsymbol{\theta}_e - \boldsymbol{\omega}_{be})\Delta t + \boldsymbol{\theta}_i & (4.34)\\
\boldsymbol{a}_{be}' =& \boldsymbol{a}_{be} + \boldsymbol{a}_i & (4.35)\\
\boldsymbol{\omega}_{be}' =& \boldsymbol{\omega}_{be} + \boldsymbol{\omega}_i & (4.36)\\
\boldsymbol{g}_e' =& \boldsymbol{g}_e & (4.37)
\end{aligned}$$

where $\boldsymbol{v}_i$, $\boldsymbol{\theta}_i$, $\boldsymbol{a}_i$ and $\boldsymbol{\omega}_i$ are random impulses for velocity, angular error, accelerometer bias and gyroscope. Those impulses can be modelled by Gaussian process. We derive Equation (4.34) by close-formed integration methods described in Appendix B.2.

We then give the Jacobian of error state $\mathbf{J}_{x_e}$ for ESKF prediction step usage,

$$\mathbf{J}_{e'e} = \frac{\partial \boldsymbol{x}_e'}{\partial \boldsymbol{x}_e} = \begin{bmatrix} \mathbf{1} & \mathbf{1}\Delta t & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & \mathbf{R}_n\left[\boldsymbol{a}_m - \boldsymbol{a}_{bn}\right]_\times \Delta t & \mathbf{R}_n \Delta t & 0 & \mathbf{1}\Delta t \\ 0 & 0 & \mathbf{R}_n^T\{\boldsymbol{\omega}_m - \boldsymbol{\omega}_{bn}\}\Delta t & 0 & -\mathbf{1}\Delta t & 0 \\ 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1} \end{bmatrix} \quad (4.38)$$

Noted that partial derivative between true state $\boldsymbol{x}_t$ and $\boldsymbol{x}_e$ is not identity because we use different parameters to represent orientations, e.g.quaternion in true state, angular error in error state. We then give the Jacobian of true state with respect to error state by

$$\mathbf{J}_{te} = \frac{\partial \boldsymbol{x}_t}{\partial \boldsymbol{x}_e} = \begin{bmatrix} \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial \boldsymbol{q}_t}{\partial \boldsymbol{\theta}_e} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1} \end{bmatrix} \tag{4.39}$$

and by Equation (3.6), we have

$$\frac{\partial \boldsymbol{q}_t}{\partial \boldsymbol{\theta}_e} = \frac{1}{2} Q^{+}(\boldsymbol{q}) \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.40}$$

$$= \frac{1}{2} \begin{bmatrix} -q_x & -q_y & -q_z \\ q_w & -q_x & q_y \\ q_z & q_w & -q_x \\ -q_y & q_x & q_w \end{bmatrix} \tag{4.41}$$

$$\tag{4.42}$$

4.1.3 State Propagation

Initially, nominal state $\boldsymbol{x}_n$ has been set to a initial guess based on some prior knowledge, and there is no error at start, i.e., error state is set to zero. We assume error state $\boldsymbol{x}_e$ as a normal distribution, i.e., $\boldsymbol{x}_e = \mathcal{N}(\hat{\boldsymbol{x}}_e, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma}$ denotes the covariance matrix for error state, which helps us to track the uncertainty of error state. Note that $\boldsymbol{\Sigma}$ is initialized to a very small diagonal matrix.

At certain round, we first obtain the measurements from accelerator and gyroscope, and compute a new nominal state estimation $\hat{\boldsymbol{x}'_n}$ by Equation (4.14) to (4.19). We then compute error state Jacobian $\mathbf{J}'_{e'e}$ by Equation(4.38), then update the error state and covariance matrix of current error state by,

$$\hat{\boldsymbol{x}'_e} = \mathbf{J}'_{e'e} \hat{\boldsymbol{x}}_e \tag{4.43}$$

$$\boldsymbol{\Sigma}' = \mathbf{J}'_{e'e} \boldsymbol{\Sigma} (\mathbf{J}'_{e'e})^T \tag{4.44}$$

which is called *prediction step* in ESKF. We omit prime symbol in next step, i.e.current state $\boldsymbol{x}$ is replaced by $\boldsymbol{x}'$.

We then assume correction measurement $\boldsymbol{y}$ from extrasensory data is a non-linear function with additional white Gaussian noise $w = \mathcal{N}(0, \mathbf{W})$ of our true state, i.e.,

$$\boldsymbol{y} = h(\boldsymbol{x}_t) + w \tag{4.45}$$

and the *correction step* of ESKF are as follows,

$$\mathbf{K} = \mathbf{\Sigma}\mathbf{H}^T(\mathbf{H}\mathbf{\Sigma}\mathbf{H}^T + \mathbf{W})^{-1} \tag{4.46}$$
$$\hat{\boldsymbol{x}}'_e = \mathbf{K}(\boldsymbol{y} - h(\hat{\boldsymbol{x}}_t)) \tag{4.47}$$
$$\mathbf{\Sigma}' = (\mathbf{1} - \mathbf{K}\mathbf{H})\mathbf{\Sigma} \tag{4.48}$$

where $\mathbf{H}$ is the Jacobian matrix of measurement function $h()$ with respect to error state $\boldsymbol{x}_e$ (see Section 4.2.2). Noted the estimation of true state here is the nominal state since we have not observed the mean of error state. The true state is estimated by Equation (4.1) and Equation (4.8) to (4.13). Depending on the output frequency of extra sensor, *correction step* often happens on a lower rate than *prediction step*. As always, we omit prime symbol in next few steps as state has been refreshed.

Before system enters into next round, we reset error state to initial state, i.e., $\hat{\boldsymbol{x}}_e = 0$ in our case. We update covariance matrix $\mathbf{\Sigma}$ by

$$\mathbf{\Sigma}' = \mathbf{J}_{ge}\mathbf{\Sigma}(\mathbf{J}_{ge})^T \tag{4.49}$$

where $\mathbf{J}_{ge}$ is Jacobian matrix of updated error state with respect to old error state, and it is given by

$$\mathbf{J}_{ge} = \begin{bmatrix} \mathbf{1}_6 & 0 & 0 \\ 0 & \mathbf{1} - \left[\frac{1}{2}\hat{\boldsymbol{\theta}}_e\right]_\times & 0 \\ 0 & 0 & \mathbf{1}_9 \end{bmatrix} \tag{4.50}$$

we derive it as similar way with the one shows in Appendix A.

4.2 Camera as Complementary Sensory Data

As we discussed in Section 4.1, we somehow need extrasensory data in ESKF *correction step*, we modelled this sensor as a non-linear measurement of true state $\boldsymbol{x}_t$ plus a white Gaussian noise as showed in Equation (4.45). This sensor in our case should satisfy following conditions,

- This sensor should carry rich information as we need to obtain accurate camera pose estimation from it.
- This sensor, unlike GPS, should work both inside and outside environment as our system is designed for mobile robots.
- This sensor should be light-weight, low-expense and better easy to handle as this is the general requirements for mobile robots.

which lead to our choice — camera. It is worthy to note that though we treat data in correction step as black box, it is also possible to choose a multiple sensory platform. However on the one hand, we choose camera not only it satisfies all above conditions, also the camera as most commonly-used sensor has been well-investigated and well-understanding, this could help us to find multiple possible approaches to solve our problem, e.g., camera pose estimation in our case; on the other hand, multiple extra sensors (i.e., camera+GPS) probably meet the some problems such as synchronizing between multiple sensory platforms. Therefore we choose camera as complementary data in this work.

4.2.1 Introduction to Monocular Visual Odometry

Visual odometry estimates the camera pose , and it can provide the measurements (e.g., global translation and rotation) we need in ESKF prediction step as the transformation between camera and IMU has been pre-calibrated. We here briefly introduce different types of visual odometry system. There are stereo-based odometry [23], depth-camera odometry [26], however considering we only use one camera in this work, we here mainly introduce monocular visual odometry.

A monocular odometry usually works as follows. Initially, system obtain the first guess of camera's pose by Homography [8] between two images. After a new image is acquired, system then tries to find the correspondences among common *landmarks*, where a landmark is defined as the most distinctive object in real world. Depending on the types of correspondences, we have

- **Feature-based methods** which is proposed in [5, 6, 15, 25], feature-based methods use image features to denote these correspondences, e.g., points are same if the features corresponds to it are same. System then reproject similar points from last image to current one using estimated camera pose transformation, error between points in current visual frame is called reprojection error.
- **Direct methods** which is proposed in [7], it uses image intensity (e.g., photometric error) to find such correspondences.
- **Semi-direct methods** which is showed in [10], it uses half image feature and half image intensity to find correspondences.

After the points connection between two images has been established, camera pose is estimated by minimizing the re-projection error or image intensity error.

Building a map based on such odometry is straightforward. A point is recognized as a part of landmark if it has been frequently tracked, and then will be inserted into the map as a recognized "map point"; A map is then constructed by such map points, normally a batch processing (e.g., bundle adjustment) is used for optimizing the map during mapping process. However such a technique can also be used for improving the pose estimation quality as we introduced in Section 4.2.3.

4.2.2 Self-adapt Map Scale

Estimation of camera pose from monocular visual odometry (VO) usually is a good compensation to ESKF IMU integration since global translation is unobservable to IMU integration. However since camera is a angle-sensor, it is impossible to obtain the scale of map [10], i.e., global translation from monocular VO has been scaled up/down with real world scale. People used to estimate map scale factor by aligning first few frames with ground-truth data [10], or initialize map scale by a standard object (e.g., a A4 paer) [5], it is still likely to accumulate scale drift with time goes on. In our framework, since IMU measures under a real world scale, we can propagate the map scale in our ESKF framework by introducing a scale factor in our state.

We first add scale factor $\boldsymbol{\lambda}$ which 3-vector to represent the scalability with x-axis, y-axis and z-axis respectively, now our true state, nominal state and error state has been changed into,

$$\boldsymbol{x}_t = [\boldsymbol{p}_t \; \boldsymbol{v}_t \; \boldsymbol{q}_t \; \boldsymbol{a}_{bt} \; \boldsymbol{\omega}_{bt} \; \boldsymbol{g}_t \; \boldsymbol{\lambda}_t]^T \tag{4.51}$$

$$\boldsymbol{x}_n = [\boldsymbol{p}_n \; \boldsymbol{v}_n \; \boldsymbol{q}_n \; \boldsymbol{a}_{bn} \; \boldsymbol{\omega}_{bn} \; \boldsymbol{g}_n \; \boldsymbol{\lambda}_n]^T \tag{4.52}$$

$$\boldsymbol{x}_e = [\boldsymbol{p}_e \; \boldsymbol{v}_e \; \boldsymbol{\theta}_e \; \boldsymbol{a}_{be} \; \boldsymbol{\omega}_{be} \; \boldsymbol{g}_e \; \boldsymbol{\lambda}_e]^T \tag{4.53}$$

the derivative with $\boldsymbol{\lambda}$ with respect to time is easily derived since we assume scale factor is independent with time, therefore we have

$$\dot{\boldsymbol{\lambda}}_t = 0 \tag{4.54}$$

$$\dot{\boldsymbol{\lambda}}_n = 0 \tag{4.55}$$

$$\dot{\boldsymbol{\lambda}}_e = 0 \tag{4.56}$$

it is then trivial to make corresponding changes to system kinematic equations (e.g., Equation 4.14 to 4.8) and error state Jacobian (e.g., Equation 4.39 and 4.40). The measurement function $h()$ of estimated true state $\dot{\boldsymbol{x}}_t$ can then be derived as

$$h(\boldsymbol{p}_t) = \boldsymbol{\lambda}_t \odot (\boldsymbol{p}_t - \boldsymbol{p}_{t0}) + \boldsymbol{p}_{t0} \tag{4.57}$$

$$h(\boldsymbol{v}_t) = \boldsymbol{v}_t \tag{4.58}$$

$$h(\boldsymbol{q}_t) = \boldsymbol{q}_t \tag{4.59}$$

$$h(\boldsymbol{a}_{bt}) = \boldsymbol{a}_{bt} \tag{4.60}$$

$$h(\boldsymbol{\omega}_{bt}) = \boldsymbol{\omega}_{bt} \tag{4.61}$$

$$h(\boldsymbol{g}_t) = \boldsymbol{g}_t \tag{4.62}$$

$$h(\boldsymbol{\lambda}_t) = \boldsymbol{\lambda}_t \tag{4.63}$$

where $\odot$ is point-wise vector multiplication and $\dot{\boldsymbol{p}}_{t0}$ is the camera/IMU position in reference frame, i.e., the position obtained from Homography of first two keyframes. Noted here, we assume the camera sensor and IMU sensor has identical position for simplification, i.e., $\dot{\boldsymbol{p}}_t$ is the estimated camera position in world frame. A more complicated case can be found in [21].

In Section 4.1, we did not give the explicit expression of $\mathbf{H}$ in Equation (4.46). $\mathbf{H}$ is defined as the Jacobian matrix of extrasensory measurement function $h()$ with respect to error state at nominal state $\boldsymbol{x}_n$, as $\boldsymbol{x}_n$ is the estimation of true state $\boldsymbol{x}_t$

here. By chain rule, $\mathbf{H}$ can be written as

$$\mathbf{H} \triangleq \frac{\partial h}{\partial \boldsymbol{x}_e}\Big|_{\boldsymbol{x}_n} = \frac{\partial h}{\partial \boldsymbol{x}_t}\Big|_{\boldsymbol{x}_n} \frac{\partial \boldsymbol{x}_t}{\partial \boldsymbol{x}_e}\Big|_{\boldsymbol{x}_n} = \mathbf{J}_H \mathbf{J}_{te} \tag{4.64}$$

we already give $\mathbf{J}_{te}$ in Equation (4.39), we then derive $\mathbf{J}_H$, which leads to

$$\mathbf{J}_H = \begin{bmatrix} \mathbf{J}_p & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1}_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{1}_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1}_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{1}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{1}_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1}_3 \end{bmatrix} \tag{4.65}$$

where $\mathbf{J}_p$ is a 3×3 matrix where diagonal is three elements of $\boldsymbol{\lambda}_n$ respectively.

4.2.3 Keyframe-based Bundle Adjustment

At last, a keyframe-based bundle adjustment is used to further increase pose estimation accuracy.

A general bundle adjustment tries to optimize bunch of camera poses and 3D points together by minimizing the reprojection error between reprojected 3D points and predicted image points. Mathematically, we obtain our optimized camera poses vector $\boldsymbol{c}$ and 3D points vector $\boldsymbol{p}$ by

$$\{\boldsymbol{c}, \boldsymbol{p}\} = \arg\min_{\boldsymbol{c}_i, \boldsymbol{p}_j} \sum_{i=1}^{n} \sum_{j=1}^{m} Obj(CamProj(\boldsymbol{c}_i, \boldsymbol{p}_j), \mathbf{I}_{ij})^2 \tag{4.66}$$

where n, m is the number of camera poses and 3D points respectively, $\mathbf{I}_{ij}$ is the predicted 2D point position corresponding to j^{th} 3D point in i^{th} image frame, function $CamProj()$ reprojects the 3D point from global frame into camera frame and $Obj()$ is a general function (e.g., Euclidean distance) that measures the error between two 2D points.

As long as we assume our camera is pin-hole model, we then can give function $CamProj()$ as

$$CamProj(\boldsymbol{c}_i, \boldsymbol{p}_j) = K(\mathbf{R}\{\boldsymbol{c}_i\}\boldsymbol{p}_j) \tag{4.67}$$

$$K(\boldsymbol{P}_{\mathcal{C}}) = [u_0 \; v_0]^T + \begin{bmatrix} f_u & 0 \\ 0 & f_v \end{bmatrix} r \begin{bmatrix} \frac{P_x}{P_z} & \frac{P_y}{P_z} \end{bmatrix}^T \tag{4.68}$$

where $\mathbf{R}\{\boldsymbol{c}_i\}$ is the rotation matrix corresponding to camera pose, function K transforms a point P in camera frame $\mathcal{C}$ into 2D point in image plane, parameters u_0, v_0 are principle point, f_u, f_u are focal length, and r is the distortion factor, which those parameters are obtained by camera calibration.

At each key frame turn after correction step, we employ such bundle adjustment

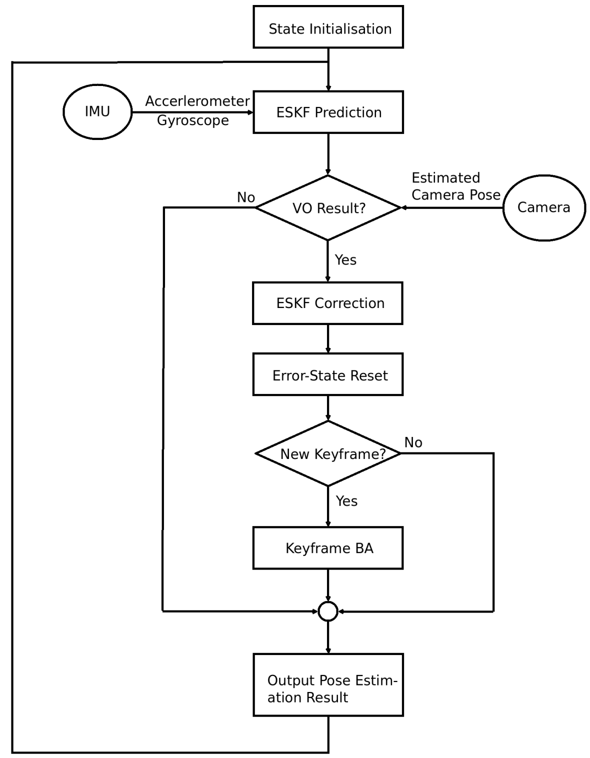


Figure 4-1: Pipeline of our visual-inertial odometry. Noted that measurement frequency from camera is 4 times lower than measurement frequency of IMU sensor in our experiment setting, therefore we do not have visual odometry (VO) result in some certain turn. Also whether this frame is keyframe is provided by VO.

step by inputting the camera pose in nominal state and 3D points obtained by VO; In order to ensure the efficiency, we throw out the oldest key frame after key frame queue has been reached maximum number, we set this number to 20 in all our experiments. We observe this number is suitable and keep our visual-inertial odometry runs in real-time.

Solutions to general bundle adjustment varies [40, 32, 19], we here roughly follow the work by [40] since it is efficient for sparse bundle adjustment problem.

4.3 Visual-inertial Odometry Pipeline Summary

In this section we summarize our visual-inertial odometry pipeline (see Figure 4-1). We also analysis the computational cost of our visual inertial odometry system in this section.

An initialisation step is needed for both nominal state and error state. After system obtains measurements (e.g., 3-vector from accelerometer, 3-vector from gyroscope) from IMU sensor, nominal state and error state are then predicted using Equation (4.14) to (4.25), and system will update corresponding covariance matrix using Equation (4.43).

Now if camera gives estimated camera pose in this turn, system then applies a *correction step* as we describe in Section 4.1.3. After injecting error state from nominal state, we obtain the estimated true state which contains camera pose. System then reset the error state, and further improving the odometry estimation quality by applying a keyframe bundle adjustment (keyframe BA) if this camera frame is specified as key frame by visual odometry as we discuss in Section 4.2.3. Finally system output the estimated camera pose and start next round after receiving IMU measurements.

In our ESKF framework, computational complexity remains constant since we only keep the current states and covariance, and the size of states and covariance matrix are fixed during estimation process. To certify, we design an experiment (see Section TODO**??**) to show that computational time remains unchanged when running a single IMU integration process using ESKF. As we discuss in Section 2.2, the computational complexity is

$$O(m^2 \cdot n) \tag{4.69}$$

where m is the number of key frame, and n is the number of landmarks. We claim that our odometry can also be extended to large scale since the number of landmarks can be enlarged and this in general is the key factor of limiting the scalability of SLAM-like system. Moreover it is also more beneficial to obtain high estimation accuracy by increasing the number of landmarks than number of key frames [30].

We do not build a environment map due to the time limitation of this master thesis. However though a few more map optimization steps are needed, the key frame BA we propose in Section 4.2.3 gives a direct result of optimized landmarks, which are the main components of mapping. We explore this in future work chapter (Chapter 6).

All together, in this chapter we suggest a error-state kalman filter (ESKF) based visual-inertial odometry. The system accurately estimates the camera-IMU platform pose by fusing the measurements from IMU and camera sensors. By using a loosely-coupled approach, system runs ESKF in a constant computational complexity and needs no special initialization step, therefore it is suitable for localization of mobile robot in real-time. This odometry can also be extended to large scaled scene, and/or an efficient SLAM system.

Chapter 5

Experiments

In this chapter, we first introduce our synthetic dataset, this dataset contains IMU measurements and corresponding visual frames, it also provides the ground truth IMU-camera pose to be used in evaluation stage. We then perform some experiments to show that our visual-inertial odometry has good accuracy and low computational cost in several different experimental settings.

5.1 Synthetic Dataset

We use a synthetic IMU-camera dataset in this master thesis because it has following advantages rather than real dataset

- Noises in synthetic dataset is controllable. Though we have considered the noise model in our work, the types of noises in real data varies. Besides, denoising from such as IMU sensor is beyond the content of this master thesis.

- Synchronization between IMU and camera is annoying. Though we know the frequency of camera and IMU from datasheet or introduction, the real output might still differ, e.g., the first image frame might correspond to fourth IMU measurement in first trial but changes to fifth IMU measurement in next trial. In synthetic data, we could use aligned IMU and camera data.

(a) Trajectory 003 (b) Trajectory 004

Figure 5-1: Visualization of trajectory **003** and trajectory **004**.

Name	Overall Duration $[s]$	Travelled Distance $[m]$	Description
001	10.0	TODO	Pure movement without rotation
002	10.0	0.0	Pure rotation without translation
003	30.0	118.99	Random straight trajectory
004	120.0	257.71	Random circle-like trajectory

Table 5.1: Trajectories we create for experiments, the datasets are named after the corresponding trajectories. Noted that **001** and **002** are ideal trajectories to test the correctness and performance of ESKF IMU integration. The trajectory **003** and **004** try to simulate the trajectories of micro helicopter by setting the similar velocity and pose.

- Synthetic data is more flexible. We could generate the special trajectory, i.e., a pure rotation sequence. However it is hard to create such a sequence in real platform.
- Synthetic data can provide accurate ground truth data at each timestamp. In real platform, a evaluation of position drift usually only happens at end point, e.g., setting the start point as same as end point and measure the drift. Motion capture system can provide ground truth data at each timestamp, however accuracy of such system usually is not high as synthetic data.
- Numerous calibration steps are saved. In real equipment, it is normal to calibrate the device again after several uses. We can ignore the calibration process in simulated platforms.

Also to best of our knowledge, there is no open sourced IMU-camera synthetic data set, hence we decide to generate our own IMU-camera synthetic dataset.

Trajectory We start to generate synthetic data by first defining the trajectory of our IMU-camera platform. As we explain before, we assume the camera and IMU sensor share an identical pose in all our experiments for simplification. We have created four different types of trajectory showed in Table 5.1. Dataset **001** and Dataset **002** is used to evaluate the performance of ESKF IMU integration as we use ground-truth data in *correction step*; And Dataset **003** and Dataset **004** test the performance of visual-inertial odometry which **004** has longer travelled distance. We have made following several general assumptions of trajectories,

- We assume the initial position of trajectory as $(0, 0, 0)$ in global frame, and initial orientation as quaternion $(1, 0, 0, 0)$ which points at the positive z axis of global coordinate system.
- We assume the second derivative of position and orientation remains constant within two successive sensor samplings.
- We assume the second derivative of position and orientation obeys a normal distribution with additional white Gaussian noise.

we illustrate trajectory **003** and **004** in Figure 5-1.

Synthetic IMU data We then use IMUSim [41] to simulate IMU data by our customized trajectory. IMUSim is a powerful python-based open-sourced IMU simulation tool, which models a wide range of real-world environments and/or external noises. To obtain more realistic IMU data, we set the output frequency of IMU as 100 [Hz], sensitivity of gyroscope is 1200 [deg/s] and sensitivity of accelerometer is 4 gravity. The whole IMU model is noise-free with some additional white Gaussian noise. Overall we have 3-vector gyroscope readings, 3-vector accelerometer readings, 3-vector ground truth position, 4-vector ground truth orientation from IMUSim for single IMU sampling.

Synthetic visual data Blender [36] is the main tool that creates virtual scene for our experiments.

5.2 Some other experiments

Chapter 6

Summary, Discussion and Future Works

Appendix A

The Derivation of Error-state Kinematic Equations

We here derive Equation (4.21) and Equation (4.22) in Chapter 4.

In order to simplify our notation, we define

$$\boldsymbol{a}_n \triangleq \boldsymbol{a}_m - \boldsymbol{a}_{bn} \tag{A.1}$$

$$\boldsymbol{\omega}_n \triangleq \boldsymbol{\omega}_m - \boldsymbol{\omega}_{bn} \tag{A.2}$$

$$\boldsymbol{a}_e \triangleq -\boldsymbol{a}_{be} - \boldsymbol{a}_n \tag{A.3}$$

$$\boldsymbol{\omega}_e \triangleq -\boldsymbol{\omega}_{be} - \boldsymbol{\omega}_n \tag{A.4}$$

We derive Equation (4.21) by

$$\dot{\boldsymbol{v}}_e = \dot{\boldsymbol{v}}_t - \dot{\boldsymbol{v}}_n \tag{A.5}$$

$$= (\mathbf{R}_t \boldsymbol{a}_t + \boldsymbol{g}_t) - (\mathbf{R}_n \boldsymbol{a}_n + \boldsymbol{g}_n) \tag{A.6}$$

we then uses small signal approximation for R_t by Equation (3.29) and Equation (3.31), which leads to

$$\mathbf{R}_t = \mathbf{R}_n(\mathbf{1} + [\boldsymbol{\theta}_n]_\times) + O(\|\Delta\boldsymbol{\theta}_e\|^2) \tag{A.7}$$

we omit the second order of angular term and followed by Equation (4.15), we obtain

$$\dot{\boldsymbol{v}}_e = (\mathbf{R}_n(\mathbf{1} + [\boldsymbol{\theta}_n]_\times)\boldsymbol{a}_t + \boldsymbol{g}_t) - (\mathbf{R}_n \boldsymbol{a}_n + \boldsymbol{g}_n) \tag{A.8}$$

$$= \mathbf{R}_n(\mathbf{1} + [\boldsymbol{\theta}_n]_\times)(\boldsymbol{a}_n + \boldsymbol{a}_e) + \boldsymbol{g}_e \tag{A.9}$$

By applying the property of skew-symmetric matrix $[\boldsymbol{a}]_\times \boldsymbol{b} = [\boldsymbol{b}]_\times \boldsymbol{a}$, and recalling (A.1), (A.2). We have

$$\dot{\boldsymbol{v}}_e = \mathbf{R}_n \left[\boldsymbol{a}_m - \boldsymbol{a}_{bn}\right]_\times - \mathbf{R}_n \boldsymbol{a}_{be} + \boldsymbol{g}_e - \mathbf{R}_n \boldsymbol{a}_n \tag{A.10}$$

which is exactly Equation (4.21).

We then derive Equation (4.22) by

$$\dot{\boldsymbol{q}}_e = \frac{1}{2}(\boldsymbol{q}_e \otimes \boldsymbol{\omega}_t - \boldsymbol{\omega}_n \otimes \boldsymbol{q}_e) \tag{A.11}$$

and we have pure quaternion $\dot{\boldsymbol{\theta}}_e = 2\dot{\boldsymbol{q}}_e$. By expanding all terms in above equations, we have

$$\dot{\boldsymbol{\theta}}_e = -\left[\boldsymbol{\omega}_n\right]_\times \boldsymbol{\theta}_e + \boldsymbol{\omega}_e + O(\|\Delta\boldsymbol{\theta}_e\|^2) \tag{A.12}$$

By omitting all second-oder terms and recalling (A.3), (A.4), we obtain

$$\dot{\boldsymbol{\theta}}_e = \left[\boldsymbol{\omega}_m - \boldsymbol{\omega}_{bn}\right]_\times - \boldsymbol{\omega}_{be} - \boldsymbol{\omega}_n \tag{A.13}$$

which is Equation (4.22).

Appendix B

Integration Methods

B.1 Runge-Kutta Numerical Integration Methods

Runge-Kutta methods aims to give a approximate solutions of ordinary differential equations, e.g., our time-derivative state $\dot{\boldsymbol{x}}$, which is

$$\dot{\boldsymbol{x}} = f(t, \boldsymbol{x}) \tag{B.1}$$

The solution give in Equation (4.14) - (4.19) by simplest version of Runge-Kutta methods, e.g., assuming $\dot{\boldsymbol{x}}$ over each time period Δt, which gives us

$$\boldsymbol{x}_{n+1} = \boldsymbol{x}_n + f(t_n, \boldsymbol{x}_n)\Delta t \tag{B.2}$$

More complicated Runge-Kutta methods please refers to [39].

B.2 Closed-form Integration Methods

We first gives a clean version of Equation (4.22), which is

$$\dot{\boldsymbol{\theta}}_e = -\left[\boldsymbol{\omega}\right]_\times \boldsymbol{\theta}_e \tag{B.3}$$

we update $\boldsymbol{\theta}_e$ by

$$\begin{aligned}
\boldsymbol{\theta}'_e &= \boldsymbol{\theta}_e + \dot{\boldsymbol{\theta}}_e \Delta t && \text{(B.4)}\\
&= \boldsymbol{\theta}_e - \left[\boldsymbol{\omega}\right]_\times \boldsymbol{\theta}_e \Delta t && \text{(B.5)}\\
&= \mathbf{R}\{-\boldsymbol{\omega}\Delta t\} && \text{(B.6)}\\
&= \mathbf{R}\{\boldsymbol{\omega}\Delta t\}^T && \text{(B.7)}
\end{aligned}$$

which we apply Equation (3.29) and Equation (3.31) from Equation (B.5) to (B.6). This integration is a closed-form integration.

Appendix C

Approximation Methods

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