MODELO DE REGRESIÓN LINEAL MÚLTIPLE

Conjunto de datos: SWISS

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1.INTRODUCCIÓN

1.Introducción

```
> head(conjunto_swiss)
             Fertility Agriculture Examination Education Catholic Infant.Mortality
Courtelary     927.112         197         173       139      115              257
Delemont     13615.935        7390         983      1475    13901             3637
Franches-Mnt  7919.850        3399         428       428     7997             1730
Moutier       1990.560         847         278       162      783              471
Neuveville    1814.840        1027         401       354      122              486
Porrentruy    4906.928        2276         580       451     5840             1715
```

1.1. Descripción de las variables

- **Fertility:** Es la media estandarizada común de fertilidad.

```
> typeof(Fertility)
[1] "double"
```

- **Agriculture:** Número de hombres agricultores.

```
> typeof(Agriculture)
[1] "double"
```

- **Examination:** Número de reclutas que reciben la calificación más alta en el examen del ejército.

```
> typeof(Examination)
[1] "double"
```

- **Education:** Número de habitantes con estudios superiores.

```
> typeof(Education)
[1] "double"
```

- **Catholic:** Número de católicos.

```
> typeof(Catholic)
[1] "double"
```

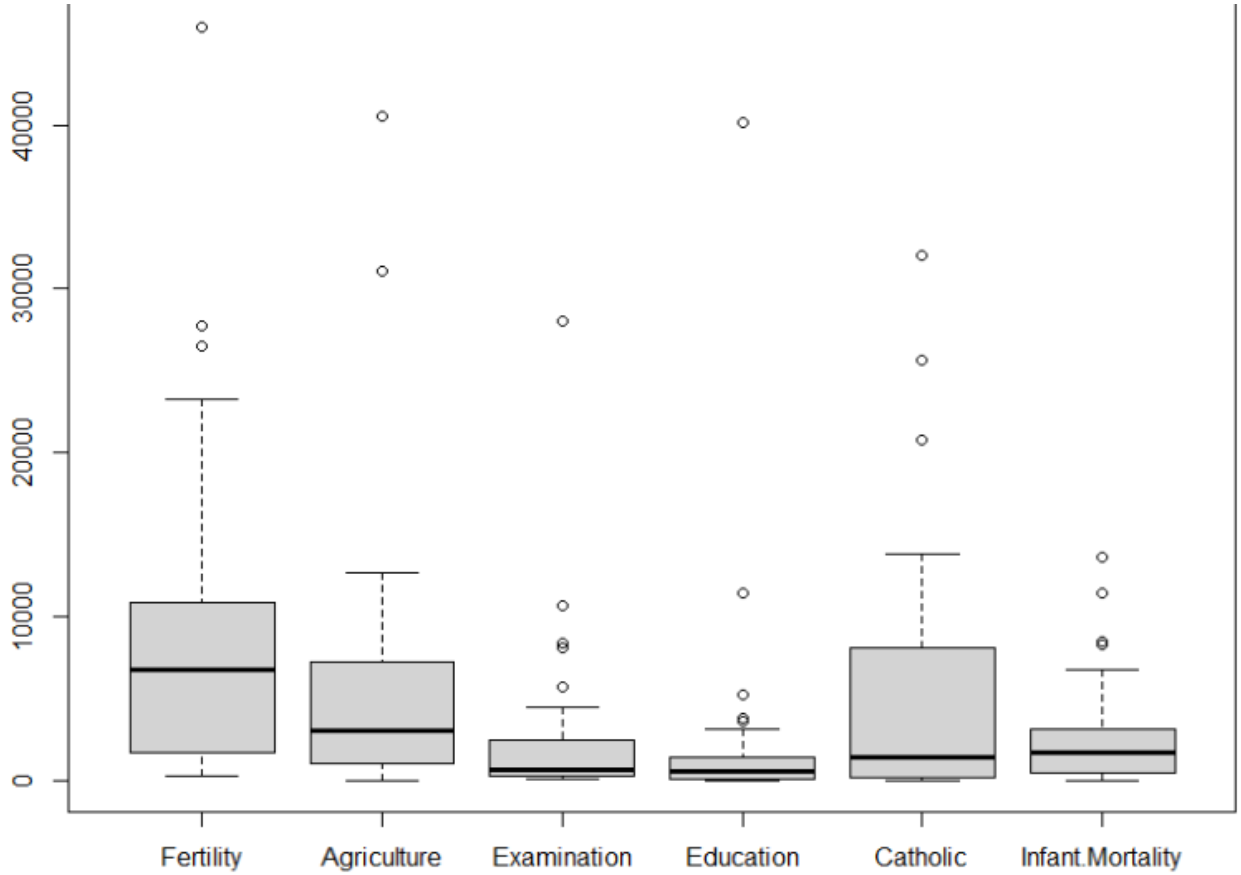
- **Infant.Mortality:** Niños que viven menos de un año.

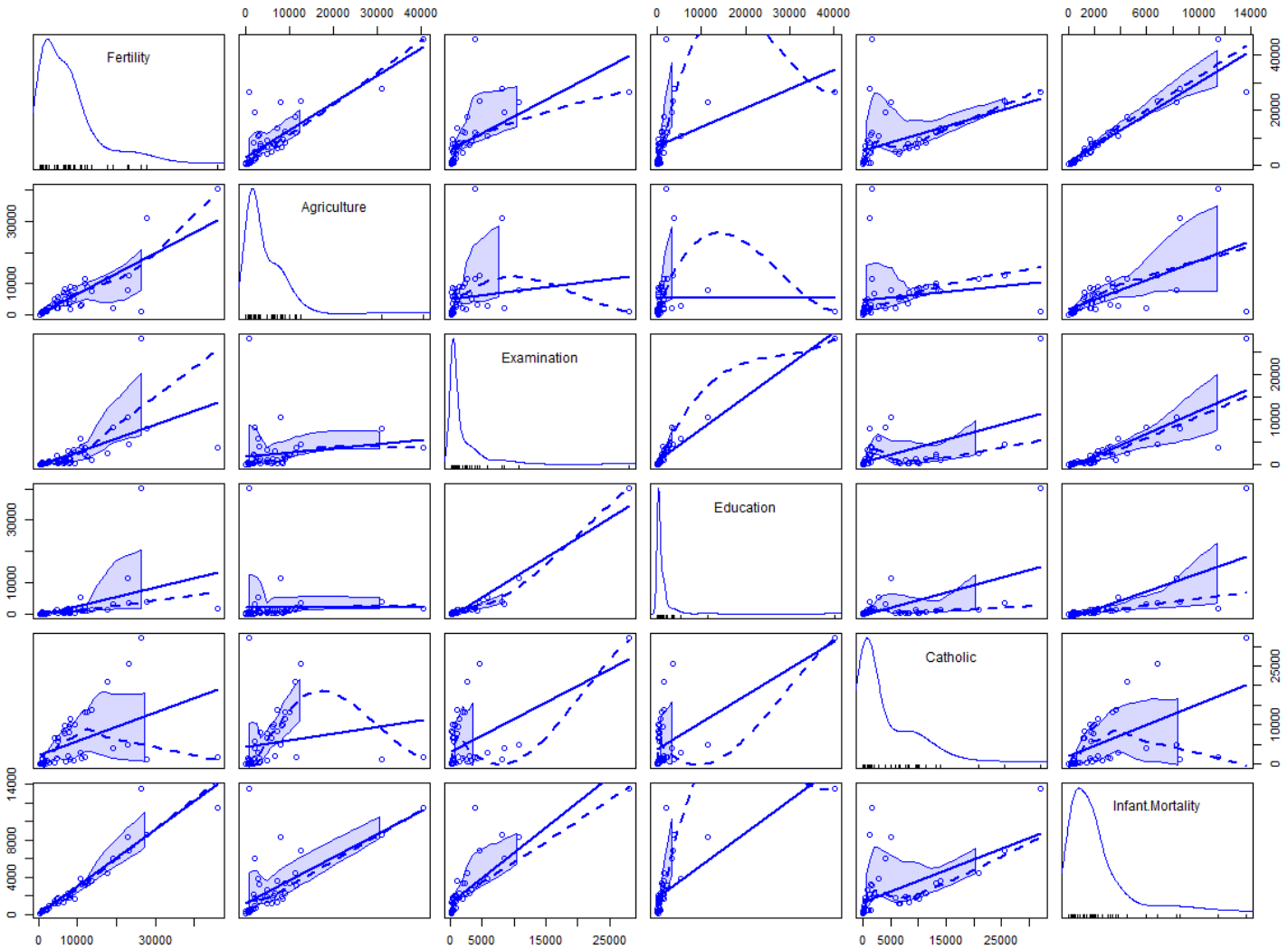
```
> typeof(Infant)
[1] "double"
```

2. ESTUDIO Y EVALUACIÓN DEL MODELO COMPLETO

2.Estudio y evaluación del modelo completo

```
> summary(conjunto_swiss)
   Fertility        Agriculture     Examination      Education        Catholic     Infant.Mortality
 Min.   :  308.4   Min.   :   86   Min.   :  173   Min.   :   21   Min.   :   12   Min.   :   61.0
 1st Qu.: 1755.0   1st Qu.: 1117   1st Qu.:  330   1st Qu.:  175   1st Qu.:  201   1st Qu.:  478.5
 Median : 6763.7   Median : 3095   Median :  708   Median :  586   Median : 1463   Median : 1715.0
 Mean   : 8386.6   Mean   : 5508   Mean   : 2379   Mean   : 2008   Mean   : 5130   Mean   : 2493.6
 3rd Qu.:10927.5   3rd Qu.: 7287   3rd Qu.: 2516   3rd Qu.: 1484   3rd Qu.: 8168   3rd Qu.: 3152.0
 Max.   :45908.6   Max.   :40489   Max.   :28012   Max.   :40126   Max.   :32055   Max.   :13628.0
```


0
10000
20000
30000
40000
Fertility
Agriculture
Examination
Education
Catholic
Infant.Mortality


Fertility
Agriculture
Examination
Education
Catholic
Infant.Mortality

3. SELECCIÓN DEL MEJOR MODELO

3.Selección del mejor modelo

Definimos las variables explicativas y la variable respuesta:

```
> Fertility<-conjunto_swiss$Fertility
> Agriculture<-conjunto_swiss$Agriculture
> Examination<-conjunto_swiss$Examination
> Education<-conjunto_swiss$Education
> Catholic<-conjunto_swiss$Catholic
> Infant<-conjunto_swiss$Infant.Mortality
```

3.Selección del mejor modelo

En forma matricial nos queda la siguiente ecuación de regresión:

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} \text{ con } i \in \{1, ..., 47\}$$
$$Y_{47x1} = X_{47x6}\beta_{6x1} + \varepsilon_{47x1}$$

3.Selección del mejor modelo

Nuestro modelo y sus coeficientes son:

```
> model<-lm(Fertility~Agriculture+Examination+Education+Catholic+Infant, data=conjunto_swiss)
> model$coefficients
 (Intercept)  Agriculture  Examination    Education     Catholic       Infant
117.18755122   0.06906232  -0.44990220  -0.35570581   0.01914249   3.83994829
```

3.Selección del mejor modelo

Realizamos el siguiente contraste de hipótesis:

$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$

$H_1 : \beta_i = 0$

3.Selección del mejor modelo

```
> summary(model)

Call:
lm(formula = Fertility ~ Agriculture + Examination + Education +
    Catholic + Infant, data = conjunto_swiss)

Residuals:
    Min      1Q  Median      3Q     Max
-2269.2  -476.5   -67.7   406.1  2346.0

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 117.18755  195.25119   0.600 0.551684
Agriculture   0.06906    0.04651   1.485 0.145189
Examination  -0.44990    0.17573  -2.560 0.014244 *
Education    -0.35571    0.09509  -3.741 0.000562 ***
Catholic      0.01914    0.02647   0.723 0.473683
Infant        3.83995    0.21926  17.513  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 902 on 41 degrees of freedom
Multiple R-squared:  0.9911,    Adjusted R-squared:   0.99
F-statistic: 915.4 on 5 and 41 DF,  p-value: < 2.2e-16
```

3.1.Métodos basados en pruebas

3.1.1) Método Forward

En cada iteración, vamos añadiendo la **variable explicativa** que cumpla:

$$p\text{-}valor < \alpha_{critic} = 0{,}05$$

Primera Iteración

Infant.Mortality: $Pr(>t) < 2 \cdot 10^{-16} < \alpha_{critic} = 0{,}05$

	Df	Sum of Sq	RSS	AIC	F value	Pr>F
<none>			3756931492	857,25		
Agriculture	1	2490455546	1266475947	808,14	88,490	$3{,}406 \cdot 10^{-12}$
Examination	1	1397162087	2359769405	837,39	26,643	$5{,}359 \cdot 10^{-6}$
Education	1	785611536	2971319956	848,22	11,898	0,001231
Catholic	1	820867437	2936064056	847,66	12,581	0,000924
Infant.Mortality	1	3341249575	415681917	755,78	361,710	$< 2{,}2 \cdot 10^{-16}$

Segunda Iteración

Examination: $Pr(>t) < 2{,}2 \cdot 10^{-16} < \alpha_{critic} = 0{,}05$

	Df	Sum of Sq	RSS	AIC	F value	Pr>F
<none>			41568197	755,78		
Agriculture	1	298539564	117142353	698,25	112,1349	$1{,}109 \cdot 10^{-13}$
Examination	1	367054893	48627024	643,93	332,1284	$< 2{,}2 \cdot 10^{-16}$
Education	1	365564388	50117529	658,35	320,9423	$< 2{,}2 \cdot 10^{-16}$
Catholic	1	17595127	398086790	755,75	1,9448	0,1702

Observación

Aunque había otra variable explicativa con, aparentemente el mismo *p-valor* (en este caso era *Education*) nos hemos decantado por *Examination* ya que su *valor F* era más grande y sabemos que cuanto más grande es, más pequeño es el *p-valor*.

Tercera Iteración

Education: $Pr(> t) < 0{,}0002196 < \alpha_{critic} = 0{,}05$

	Df	Sum of Sq	RSS	AIC	F value	Pr>F
<none>			48627024	656,93		
Agriculture	1	2961590	45665434	655,98	2,7887	0,1021947
Education	1	13358152	35268872	643,83	16,2863	0,0002196
Catholic	1	1941217	46685807	657,01	1,7880	0,1882059

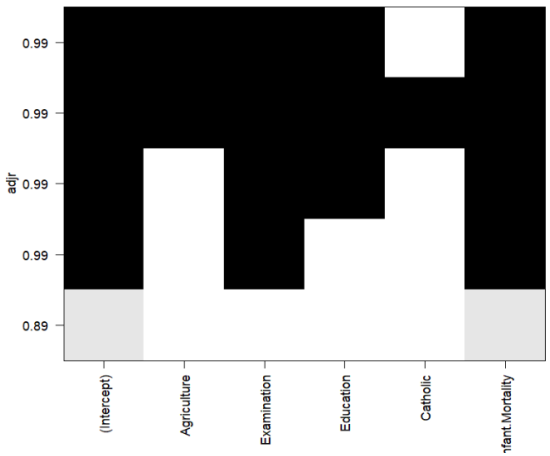
3.2.Métodos basados en criterios

Los posibles modelos son:

```
> summary(model)

Call:
lm(formula = Fertility ~ Agriculture + Examination + Education +
    Catholic + Infant, data = conjunto_swiss)

Residuals:
    Min      1Q  Median      3Q     Max
-2269.2  -476.5   -67.7   406.1  2346.0

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 117.18755  195.25119   0.600 0.551684
Agriculture   0.06906    0.04651   1.485 0.145189
Examination  -0.44990    0.17573  -2.560 0.014244 *
Education    -0.35571    0.09509  -3.741 0.000562 ***
Catholic      0.01914    0.02647   0.723 0.473683
Infant        3.83995    0.21926  17.513  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 902 on 41 degrees of freedom
Multiple R-squared:  0.9911,    Adjusted R-squared:   0.99
F-statistic: 915.4 on 5 and 41 DF,  p-value: < 2.2e-16
```

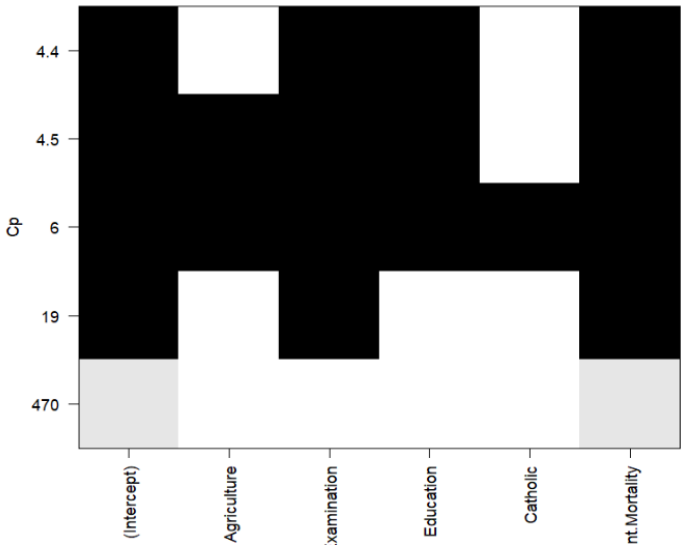
1. $Fertility \sim Infant$
2. $Fertility \sim Examination + Infant$
3. $Fertility \sim Examination + Education + Infant$
4. $Fertility \sim Agriculture + Education + Examination + Infant$
5. $Fertility \sim Agriculture + Examination + Education + Catholic + Infant$

3.2.1.R cuadrado ajustado



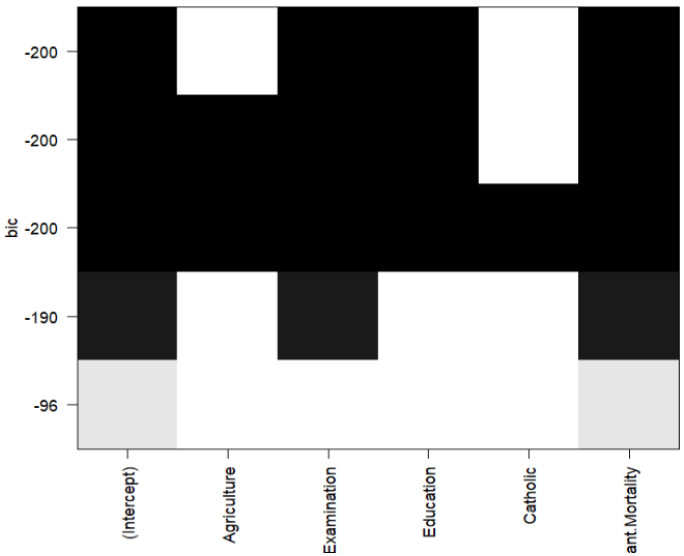

```
> MR2adj<-summary(models)$adjr2
> MR2adj
[1] 0.8868972 0.9864684 0.9899572 0.9901522 0.990039
> which.max(MR2adj)
[1] 4
> plot(models,scale="adjr")
```

3.2.2.Cp de Mallows




```
> MCp<-summary(models)$cp
> MCp
[1] 467.953565  18.772028   4.353017   4.522777   6.00000
> which.min(MCp)
[1] 3
> plot(models,scale="Cp")
```

3.2.3.Criterio de Información de Bayes (BIC)




```
> MBIC<-summary(models)$bic
> MBIC
[1]  -95.76727 -192.76694 -204.01138 -202.18854 -198.93389
> which.min(MBIC)
[1] 3
> plot(models,scale="bic")
> install.packages("MASS")
```

3.2.4.Criterio de Información de Akaike (AIC)

```
> #AIC
> SCOPE<-(~.)
> stepAIC(modelo_completo, scope=SCOPE,k=2)
Start:  AIC=645.21
Fertility ~ Agriculture + Examination + Education + Catholic +
    Infant

             Df Sum of Sq       RSS    AIC
- Catholic    1    425301  33780501 643.81
<none>                    33355200 645.21
- Agriculture 1   1793764  35148964 645.67
- Examination 1   5332289  38687490 650.18
- Education   1  11383630  44738830 657.01
- Infant      1 249523972 282879173 743.69

Step:  AIC=643.81
Fertility ~ Agriculture + Examination + Education + Infant

             Df Sum of Sq       RSS    AIC
<none>                    33779724 643.81
- Agriculture 1   1489148  35268872 643.83
+ Catholic    1    425447  33354278 645.21
- Examination 1   8432237  42211961 652.28
- Education   1  11885710  45665434 655.98
- Infant      1 310151705 343931429 750.87

Call:
lm(formula = Fertility ~ Agriculture + Examination + Education +
    Infant, data = conjunto_swiss)

Coefficients:
(Intercept)  Agriculture  Examination    Education       Infant
  169.64188      0.06116     -0.50652     -0.32488      3.90494
```

3. Selección del mejor modelo

Como hemos obtenido dos modelos candidatos a mejor modelo, realizamos un contraste de hipótesis:

```
> modelo_mejor<-lm(Fertility~Examination+Education+Infant)
> modelo_mejor2<-lm(Fertility~Agriculture+Examination+Education+Infant)
> anova(modelo_mejor, modelo_mejor2)
Analysis of Variance Table

Model 1: Fertility ~ Examination + Education + Infant
Model 2: Fertility ~ Agriculture + Examination + Education + Infant
  Res.Df      RSS Df Sum of Sq      F Pr(>F)
1     43 35268872
2     42 33779724  1   1489148 1.8515 0.1809
```

El mejor modelo es:

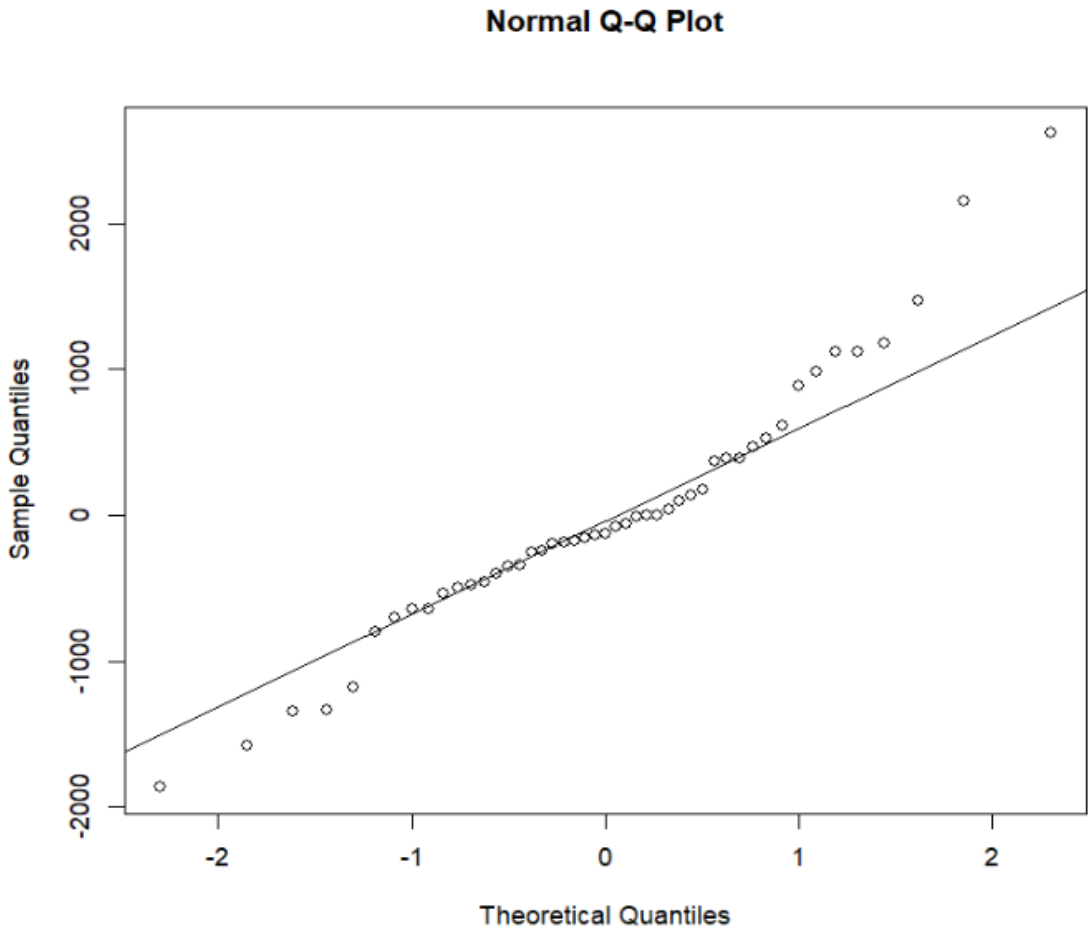
$$Fertility \sim Examination + Education + Infant$$

4. DIAGNÓSTICO

4.1.Linealidad, normalidad y homocedasticidad

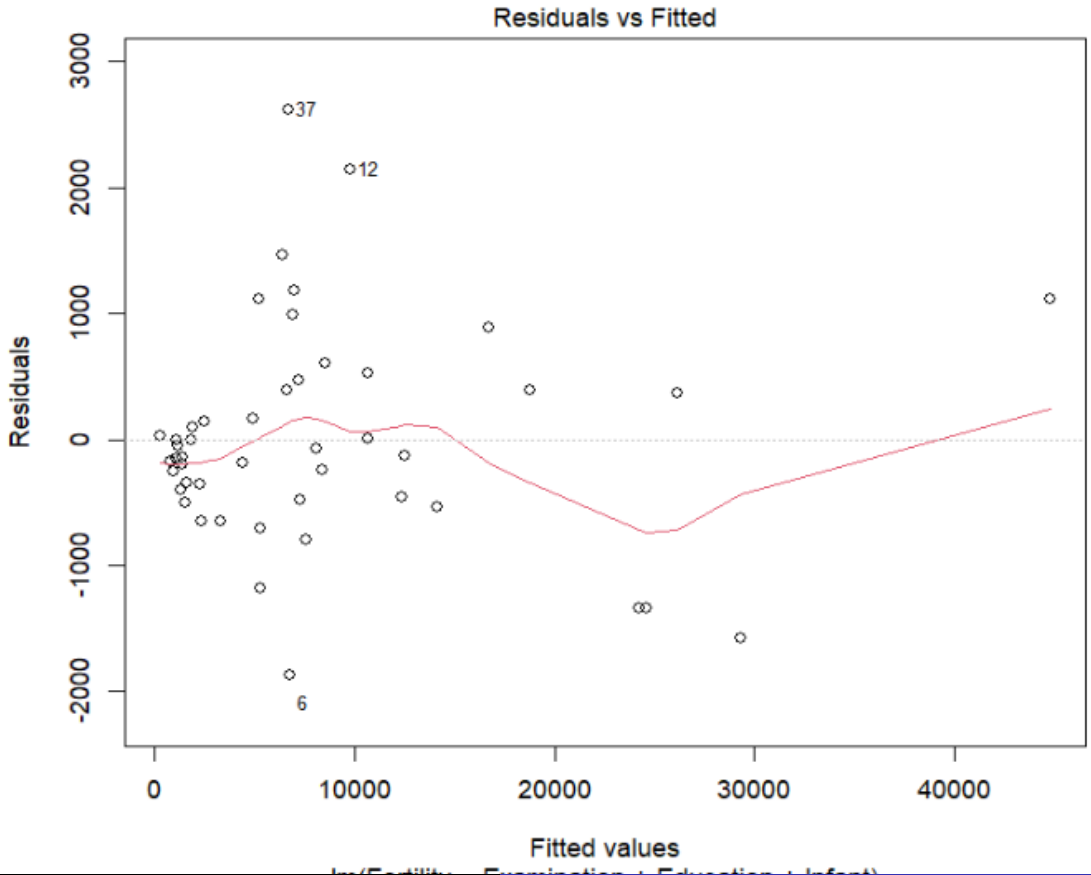
- **Linealidad.**
- **Normalidad.**

```
> #Normalidad
> #Vamos a usar el test de Shapiro-Wilk
> shapiro.test(resid(modelo_mejor))

        Shapiro-Wilk normality test

data:  resid(modelo_mejor)
W = 0.95708, p-value = 0.08238
```


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
2000
1000
0
-1000
-2000
-2
-1
0
1
2

- **Homocedastidad.**

```
        studentized Breusch-Pagan test

data:  modelo_mejor
BP = 3.3069, df = 3, p-value = 0.3467
```



4.2.Autocorrelación

Como el p-valor es menor que 0.05 existe autocorrelación entre los errores.

```
> durbinWatsonTest(bestmodel)
 lag Autocorrelation D-W Statistic p-value
   1       0.04244051       1.90737   0.702
 Alternative hypothesis: rho != 0
```

4.3 ESTUDIO DE OUTLIERS, INFLUYENTES Y LEVERAGE

4.3.1.Outliers

Aplicamos el método de Bonferroni con un nivel de significación del 0.05 para detectar outliers:

```
> outlierTest(bestmodel)
No Studentized residuals with Bonferroni p < 0.05
Largest |rstudent|:
       rstudent unadjusted p-value Bonferroni p
Sierre 3.261806          0.0022017       0.10348
```

4.3.1.Outliers

Obtenemos la observación Sierre (la número 37) como outlier.

```
> bestmodel_sin_37<-lm(Fertility~Examination+Education+Infant, data=conjunto_swiss_sin37)
> bestmodel_sin_37

Call:
lm(formula = Fertility ~ Examination + Education + Infant, data = conjunto_swiss_sin37)

Coefficients:
(Intercept)  Examination    Education       Infant
    89.8321      -0.5522      -0.3667       4.1261
```

4.3.1.Outliers

Como el p-valor es menor que 0.05 existe autocorrelación entre los errores.

```
> durbinWatsonTest(bestmodel_sin_37)
 lag Autocorrelation D-W Statistic p-value
   1      -0.1036541       2.200708   0.486
 Alternative hypothesis: rho != 0
```

4.3.1.Outliers

Como el p-valor es menor que 0.05 existe normalidad.

```
> shapiro.test(resid(bestmodel_sin_37))

        Shapiro-Wilk normality test

data:  resid(bestmodel_sin_37)
W = 0.96724, p-value = 0.2182
```

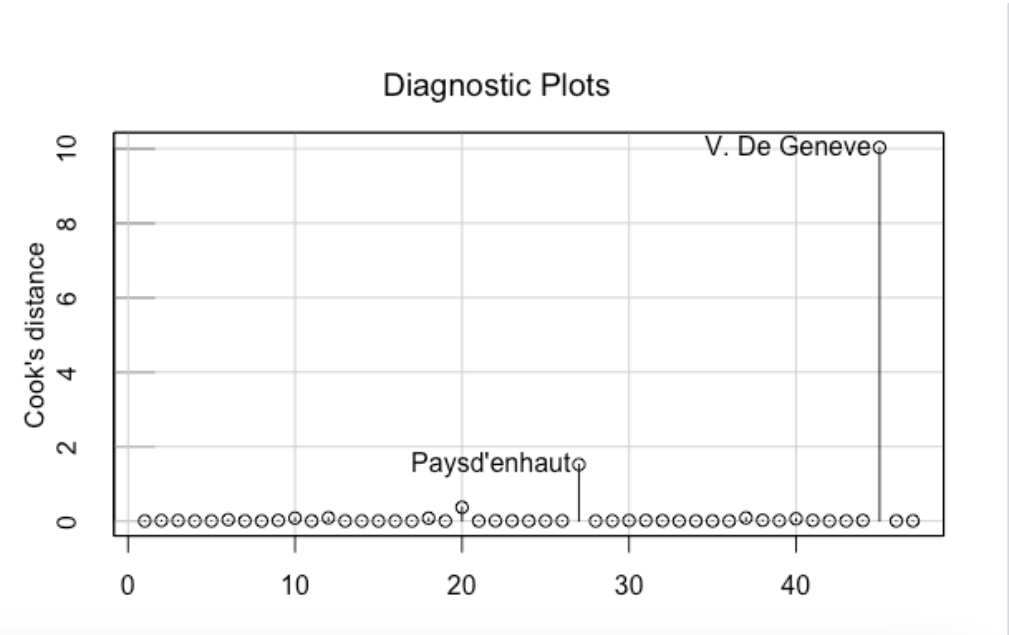
4.3.1.Outliers

Perdemos la homocedasticidad pues el p-valor es menor que 0.05

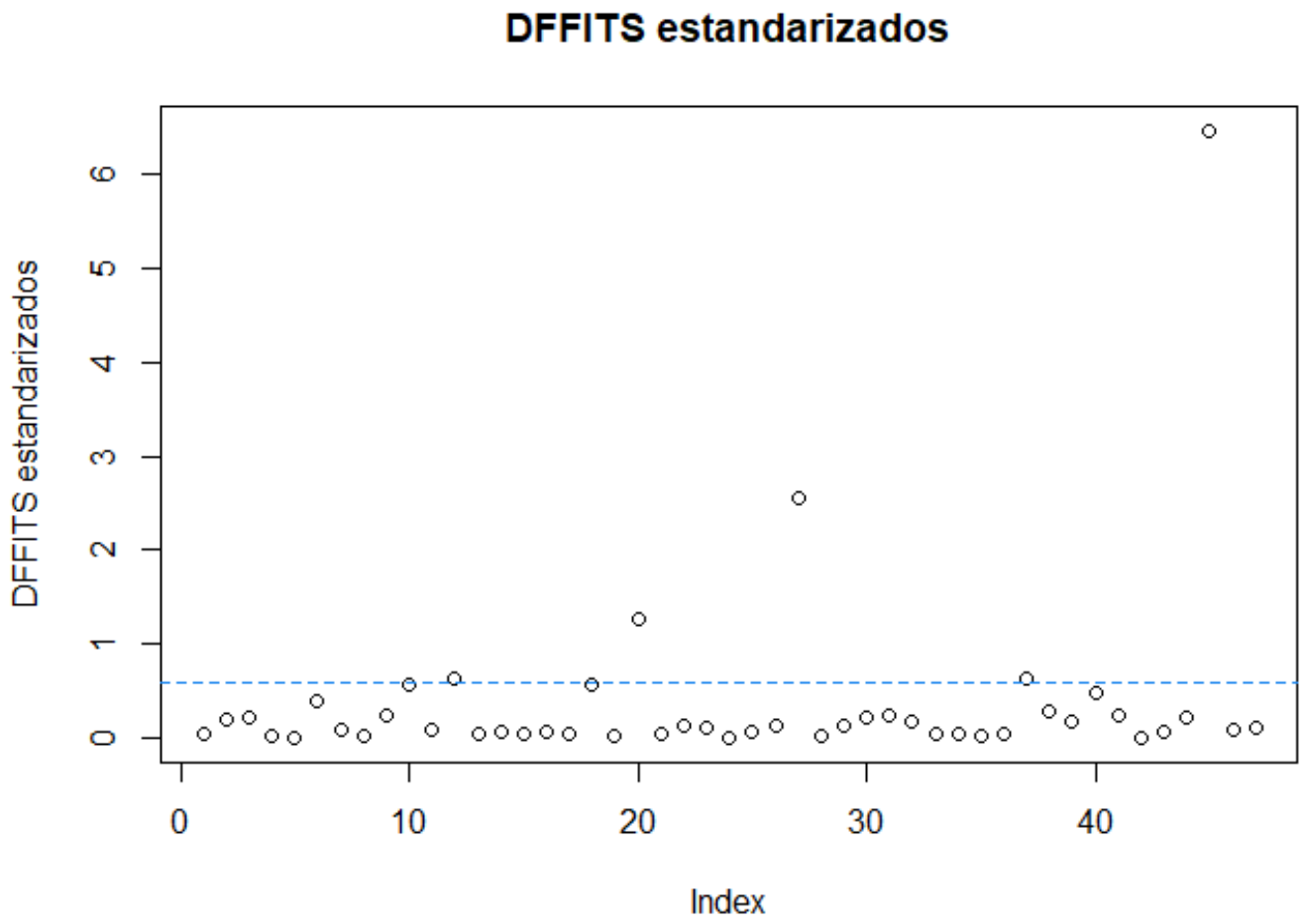
```
> bptest(bestmodel_sin_37)

        studentized Breusch-Pagan test

data:  bestmodel_sin_37
BP = 7.9765, df = 3, p-value = 0.0465
```

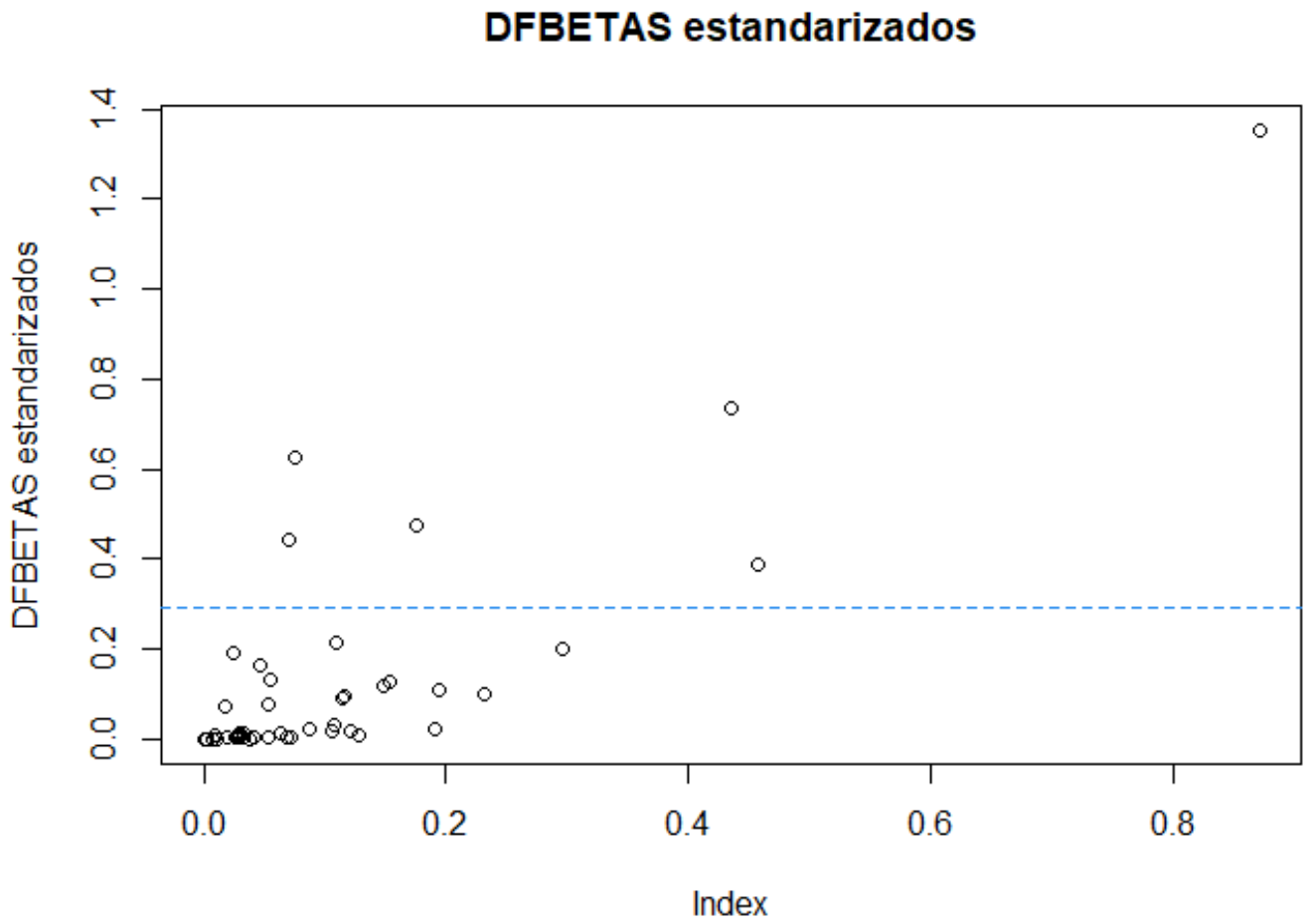
4.3.2.Observaciones influyentes

Para ver las observaciones influyentes hemos utilizado tres métodos distintos: la distancia de Cook, los DFFITS y los DFBETAS. Estos son los resultados que hemos obtenido:



4.3.2.Observaciones influyentes




DFBETAS estandarizados
DFBETAS estandarizados
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.0
0.2
0.4
0.6
0.8
Index

Estudio del modelo sin la observación V. de Geneve.

```
        Shapiro-Wilk normality test

data:  resid(posible_modelo_sin45)
W = 0.94078, p-value = 0.02094
```

```
> bptest(posible_modelo_sin45)

        studentized Breusch-Pagan test

data:  posible_modelo_sin45
BP = 1.6853, df = 3, p-value = 0.6402
```

```
> durbinWatsonTest(posible_modelo_sin45)
 lag Autocorrelation D-W Statistic p-value
   1      0.06527015      1.863147   0.584
 Alternative hypothesis: rho != 0
```

Estudio del modelo sin la observación Paysd'enhaut.

```
> shapiro.test(resid(posible_modelo_sin27))

        Shapiro-Wilk normality test

data:  resid(posible_modelo_sin27)
W = 0.9159, p-value = 0.002724
```

```
> bptest(posible_modelo_sin27)

        studentized Breusch-Pagan test

data:  posible_modelo_sin27
BP = 2.0947, df = 3, p-value = 0.553
```

```
> durbinWatsonTest(posible_modelo_sin27)
 lag Autocorrelation D-W Statistic p-value
   1       0.1193181       1.74736   0.346
 Alternative hypothesis: rho != 0
```

Estudio del modelo sin la observación Lavaux.

```
> shapiro.test(resid(posible_modelo_sin20))

        Shapiro-Wilk normality test

data:  resid(posible_modelo_sin20)
W = 0.94157, p-value = 0.02241
```

```
> bptest(posible_modelo_sin20)

        studentized Breusch-Pagan test

data:  posible_modelo_sin20
BP = 1.9598, df = 3, p-value = 0.5808
```

```
> durbinWatsonTest(posible_modelo_sin20)
 lag Autocorrelation D-W Statistic p-value
   1      0.02323095      1.947479   0.784
 Alternative hypothesis: rho != 0
```

Estudio del modelo sin la observación Porrentruy.

```
> shapiro.test(resid(posible_modelo_sin6))

        Shapiro-Wilk normality test

data:  resid(posible_modelo_sin6)
W = 0.94645, p-value = 0.03429
```

```
> bptest(posible_modelo_sin6)

        studentized Breusch-Pagan test

data:  posible_modelo_sin6
BP = 3.4924, df = 3, p-value = 0.3217
```

```
> durbinWatsonTest(posible_modelo_sin6)
 lag Autocorrelation D-W Statistic p-value
   1      0.02629031      1.936485   0.738
 Alternative hypothesis: rho != 0
```

Estudio del modelo sin la observación Sarine.

```
> shapiro.test(resid(posible_modelo_sin10))

        Shapiro-Wilk normality test

data:  resid(posible_modelo_sin10)
W = 0.95504, p-value = 0.0735
```

```
> bptest(posible_modelo_sin10)

        studentized Breusch-Pagan test

data:  posible_modelo_sin10
BP = 2.1482, df = 3, p-value = 0.5422
```

```
> durbinWatsonTest(posible_modelo_sin10)
 lag Autocorrelation D-W Statistic p-value
   1      0.04657698      1.899036    0.74
 Alternative hypothesis: rho != 0
```

Estudio del modelo sin la observación La Chauxdfnd.

```
> shapiro.test(resid(posible_modelo_sin40))

        Shapiro-Wilk normality test

data:  resid(posible_modelo_sin40)
W = 0.95488, p-value = 0.07243
```

```
> bptest(posible_modelo_sin40)

        studentized Breusch-Pagan test

data:  posible_modelo_sin40
BP = 3.0309, df = 3, p-value = 0.3869
```

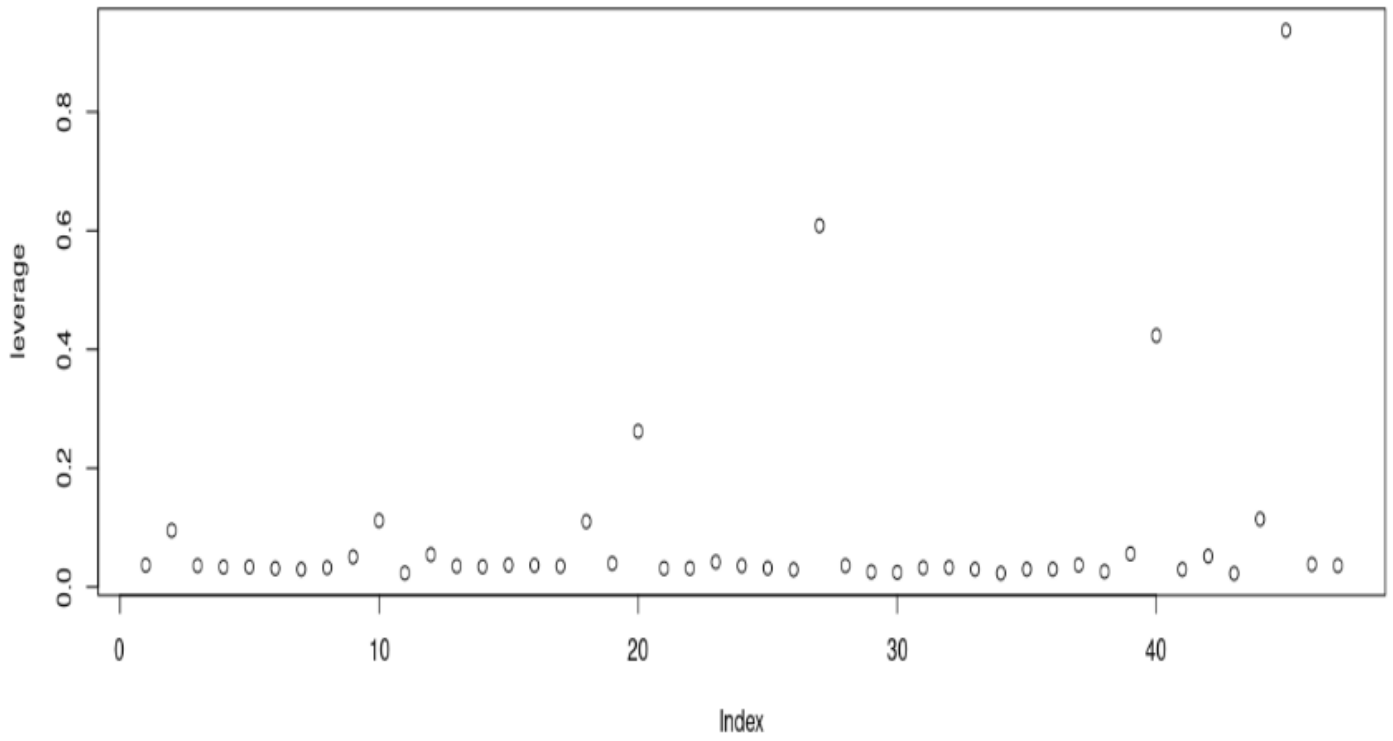
```
> durbinWatsonTest(posible_modelo_sin40)
 lag Autocorrelation D-W Statistic p-value
   1      0.05081376      1.890029   0.688
 Alternative hypothesis: rho != 0
>
```

4.4.Colinealidad

Como el valor del determinante es muy grande, descartamos la opción de que la matriz sea casi singular. Concluímos que no existe multicolinealidad.

```
> X<-cbind(rep(1,length(Fertility)),Examination,Education,Infant)
> det(t(X)%*%X)
[1] 6.218467e+26
```

4.4.1.Leverage



5. CALCULAR EL ERROR DE TEST

5.Calcular el Error de test

Separaremos los datos entre 70 % entrenamiento y 30 % test y realizaremos la validación cruzada.

```
> model.exh
Subset selection object
Call: regsubsets.formula(Fertility ~ ., data = conjunto_swiss[train,
    1:6], method = "exhaustive")
5 Variables  (and intercept)
                 Forced in Forced out
Agriculture          FALSE      FALSE
Examination          FALSE      FALSE
Education            FALSE      FALSE
Catholic             FALSE      FALSE
Infant.Mortality     FALSE      FALSE
1 subsets of each size up to 5
Selection Algorithm: exhaustive
```

```
> summary(model.exh) #todos los modelos posibles para los `predictores
Subset selection object
Call: regsubsets.formula(Fertility ~ ., data = conjunto_swiss[train,
    1:6], method = "exhaustive")
5 Variables  (and intercept)
                 Forced in Forced out
Agriculture          FALSE      FALSE
Examination          FALSE      FALSE
Education            FALSE      FALSE
Catholic             FALSE      FALSE
Infant.Mortality     FALSE      FALSE
1 subsets of each size up to 5
Selection Algorithm: exhaustive
         Agriculture Examination Education Catholic Infant.Mortality
1  ( 1 ) " "         " "         " "       " "      "*"
2  ( 1 ) "*"         " "         " "       " "      "*"
3  ( 1 ) "*"         " "         "*"       " "      "*"
4  ( 1 ) "*"         "*"         "*"       " "      "*"
5  ( 1 ) "*"         "*"         "*"       "*"      "*"
```

```
> val.errors
[1] 50899007 20279332  2991010  3047738  3051125
> coef(model.exh, which.min(val.errors))
     (Intercept)      Agriculture        Education Infant.Mortality
     -39.7102481        0.2150309       -0.4950186        3.3301860
```

El mejor modelo es: $Fertility \sim Agriculture + Education + Infant$

6. CONCLUSIÓN

6.1.Nuevas observaciones

Creamos un nuevo data.frame() con 3 nuevas provincias suizas

```
> newdataframe
  Fertility Agriculture Education Examination Catholic Infant
1     13.20       40.31        23       20.11    50.31  62.31
2     50.30       52.54         2       12.19    36.54  21.00
3     18.93       39.99         5       20.51    27.89  24.70
```

6.1.Nuevas observaciones

El valor estimado de la fertilidad y sus intervalos de confianza al 95 %:

	Yhat	S_yhat	IntervConfInf	IntervConfSup
1	37673.40	177.1037	37233.36	38113.45
2	12901.66	178.3231	12458.59	13344.74
3	13973.17	178.1431	13530.53	14415.80

6.2.Conclusión final

Mejor modelo,

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i, \quad i = 1, \cdots, 46$$

siendo $\beta_0 = 145{,}57623$, $\beta_1 = -0{,}63120$, $\beta_2 = -0{,}33337$ y $\beta_3 = 4{,}18837$

6.2.Conclusión final

Su tabla ANOVA es:

```
> anova(bestmodel_sin_10)
Analysis of Variance Table

Response: Fertility
             Df     Sum Sq    Mean Sq F value    Pr(>F)
Examination   1 1326184684 1326184684 1675.57 < 2.2e-16 ***
Education     1  608156072  608156072  768.38 < 2.2e-16 ***
Infant        1 1563443859 1563443859 1975.34 < 2.2e-16 ***
Residuals    42   33242274     791483
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

6.2.Conclusión final

Su valor R^2 es 0.9899132. En el summary observamos:

```
> summary(bestmodel_sin_10)

Call:
lm(formula = Fertility ~ Examination + Education + Infant, data = conjunto_swiss_sin10)

Residuals:
    Min      1Q  Median      3Q     Max
-1905.3  -464.5  -124.2   376.7  2575.7

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 145.57623  179.19163   0.812 0.421140
Examination  -0.63120    0.14011  -4.505 5.22e-05 ***
Education    -0.33337    0.08314  -4.010 0.000244 ***
Infant        4.18837    0.09424  44.445  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 889.7 on 42 degrees of freedom
Multiple R-squared:  0.9906,    Adjusted R-squared:  0.9899
F-statistic:  1473 on 3 and 42 DF,  p-value: < 2.2e-16
```

6.2 Conclusión final

También hemos observado que un modelo igual de bueno es el que relaciona las variables 'Agriculture', 'Education' e 'Infant'.

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i, \quad i = 1, \cdots, 47$$

siendo $\beta_0 = 849{,}82333$, $\beta_1 = 0{,}84578$, $\beta_2 = 1{,}60544$ y $\beta_3 = -0{,}46890$

6.2.Conclusión final

Su tabla ANOVA es:

```
Analysis of Variance Table

Response: Fertility
             Df     Sum Sq    Mean Sq F value    Pr(>F)
Agriculture   1 2490455546 2490455546 311.369 < 2.2e-16 ***
Examination   1  897597291  897597291 112.222 1.457e-13 ***
Education     1   24947226   24947226   3.119   0.08448 .
Residuals    43  343931429    7998405
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

6.2.Conclusión final

Su valor R^2 es 0.9947118. En el summary observamos:

```
Call:
lm(formula = Fertility ~ Agriculture + Examination + Education,
    data = conjunto_swiss)

Residuals:
     Min       1Q   Median       3Q      Max
-10623.2  -1071.9   -614.9   1121.7   6186.5

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 849.82333  557.81880   1.523   0.1350
Agriculture   0.84578    0.06492  13.029  < 2e-16 ***
Examination   1.60544    0.35824   4.482 5.42e-05 ***
Education    -0.46890    0.26550  -1.766   0.0845 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2828 on 43 degrees of freedom
Multiple R-squared:  0.9085,    Adjusted R-squared:  0.9021
F-statistic: 142.2 on 3 and 43 DF,  p-value: < 2.2e-16
```