<!--Introduction to Modelling in Physics-->

Wildfire Modelling

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Contents

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Introduction

Generalized Heat Equation

$$\frac{\partial T}{\partial t} = \underbrace{\alpha \nabla^2 T}_{\text{Diffusion}} - \underbrace{v_{wind} \cdot \nabla T}_{Advection} + \underbrace{\dot{Q}_{comb}}_{Source} - \underbrace{h(T - T_{amb})}_{Losses}$$

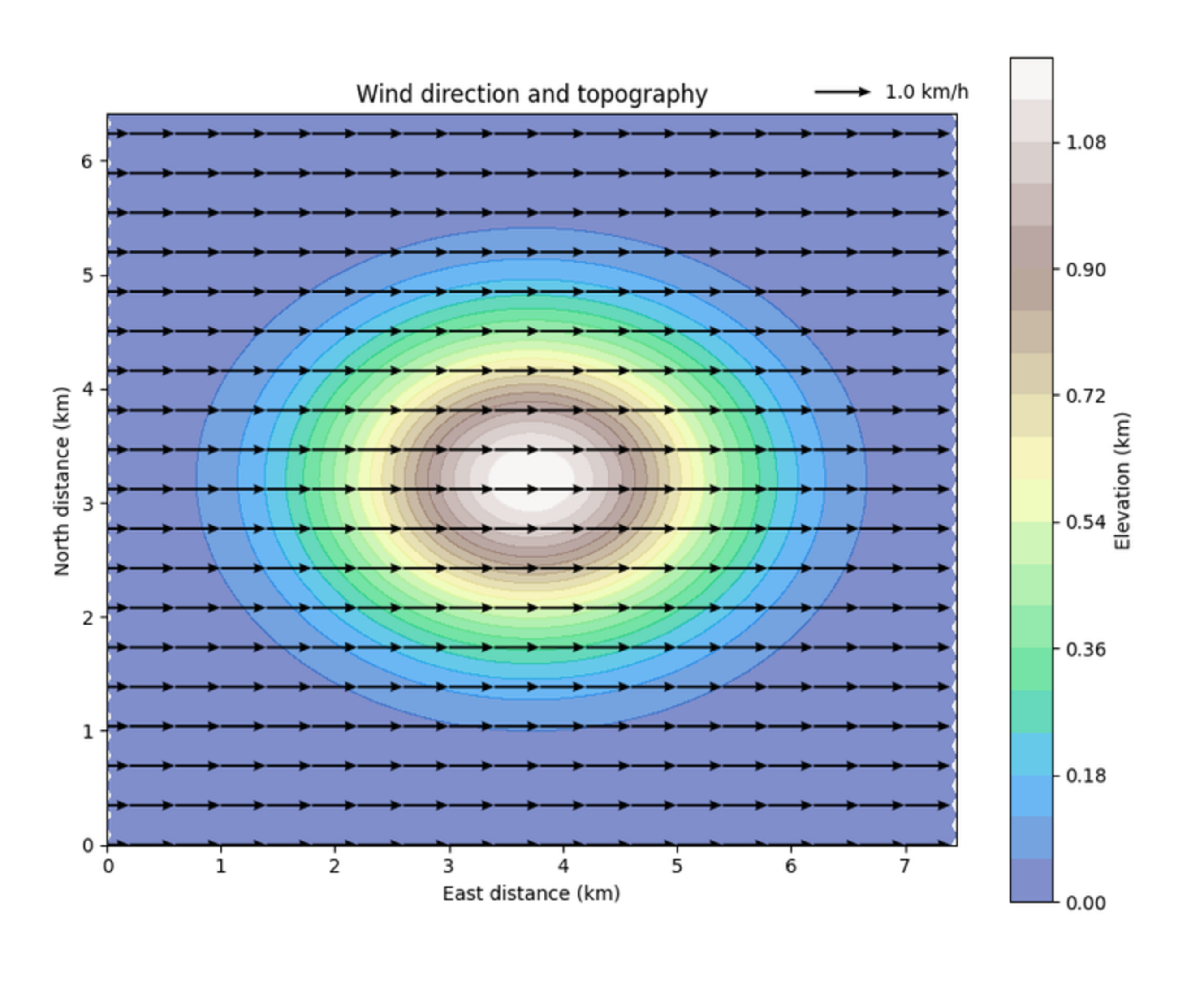
Parameters to consider

Terrain

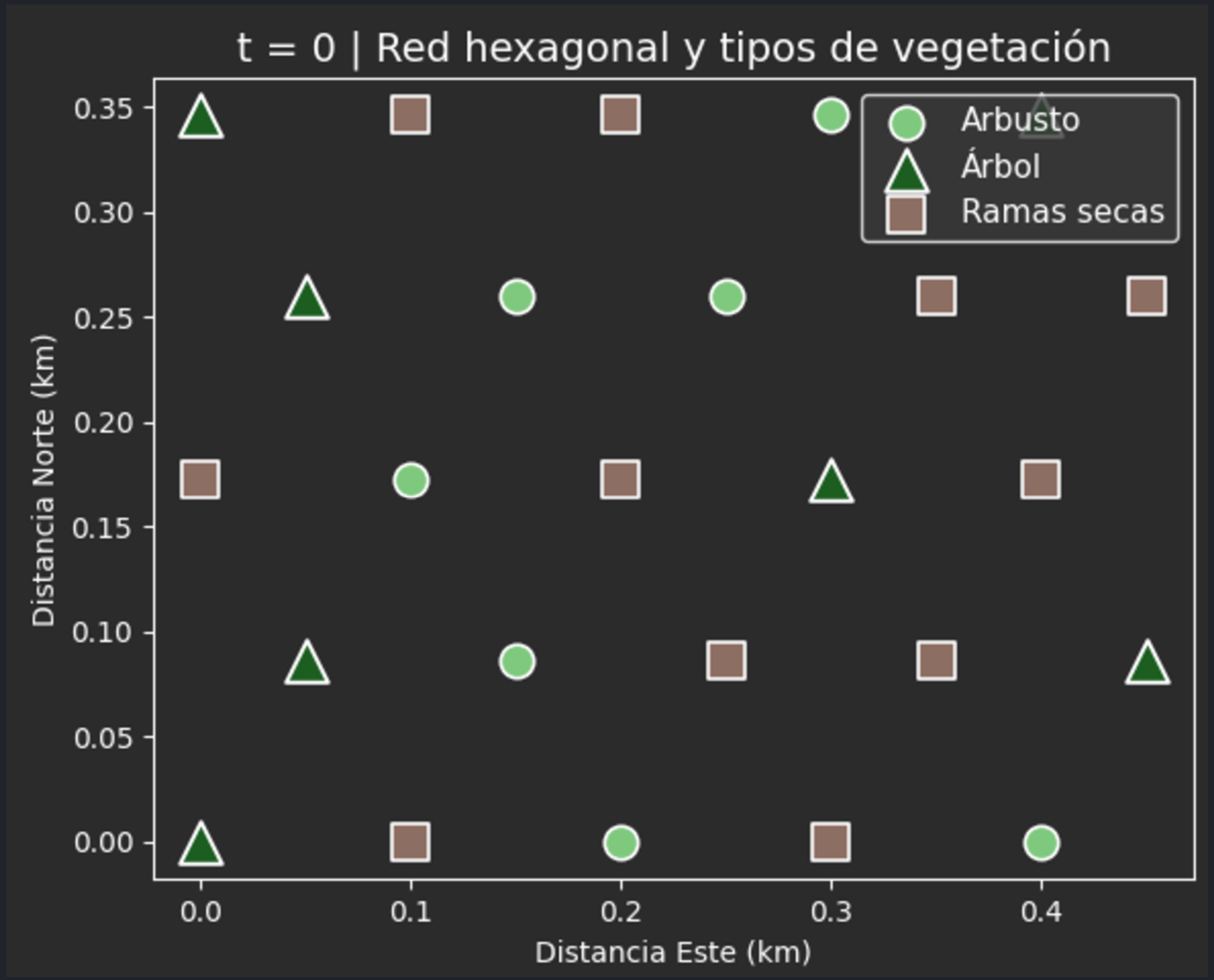
- Elevation
- Slope
- Moisture
- Wind

Vegetation

- Thermal diffusivity α
- Ignition temperature T_{ign}
- Cooling rate C
- Heat release Q
- Fuel consumption rate k



Grid Definition



- Hexagonal grid definition.
- Vegetation types:
  - Tree.
  - Shrub.
  - Dry branches.
- We parametrize each vegetation type.
- We assign a type of vegetation to each cell.

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Vegetation Parameters

Parameter	Tree	Shrub	Dry branches
Thermal diffusivity	↓	—	↑
Ignition temperature (K)	↑	—	↓
Cooling rate (h^-1)	↓	—	↑
Heat release coefficient (K/h)	↑	—	↓
Fuel consumption rate (h^-1)	↓	—	↑

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Theoretical Framework

Governing equation ⟷ $$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \alpha \nabla^2 T + S(T)$$

Definition of the Laplacian Operator ⟷ $$\mathcal{L} \equiv \alpha \nabla^2 - \mathbf{v} \cdot \nabla$$

Resulting Equation ⟷ $$\frac{\partial T}{\partial t} = \mathcal{L}[T] + S(T)$$

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Spatial Discretization

Laplacian on a hexagonal grid:

$$\nabla^2 T_i \approx \frac{2}{3h^2} \sum_{j \in \text{vecinos}(i)} (T_j - T_i)$$

Advection term:

$$(\mathbf{v} \cdot \nabla T)_i \approx \sum_{j \in \text{vecinos}(i)} \frac{\mathbf{v} \cdot \mathbf{e}_{ij}}{N_{dir} \cdot h} (T_j - T_i)$$

Relation for a cell:

$$T_{i,j}^{n+1} - \sum_{k=1}^{6} C_{ij}(T_k^{n+1} - T_{i,j}^{n+1}) = T_{i,j}^{n} + \sum_{k=1}^{6} C_{ij}(T_k^{n} - T_{i,j}^{n}) + \Delta t S_i^n$$

Temporal Discretization

Time derivate

$$\frac{T^{n+1} - T^n}{\Delta t} = \frac{1}{2}(\mathcal{L}[T^{n+1}] + \mathcal{L}[T^n]) + S^n$$

Grouping terms

$$\left(1 - \frac{\Delta t}{2}\mathcal{L}\right) T^{n+1} = \left(1 + \frac{\Delta t}{2}\mathcal{L}\right) T^n + \Delta t S^n$$

Transfer coefficient: $C_{ij} = \frac{1}{2}(\lambda - \gamma_{ij})$

Fourier number (diffusion): $\lambda = \frac{\alpha \cdot \Delta t}{3h^2}$

Courant number (advection): $\gamma_{ij} = \frac{\Delta t \cdot |v|}{h}$ }

Definition of the Matrix System

$$T_{i,j}^{n+1} - \sum_{k=1}^{6} C_{ij}(T_k^{n+1} - T_{i,j}^{n+1}) = T_{i,j}^{n} + \sum_{k=1}^{6} C_{ij}(T_k^{n} - T_{i,j}^{n}) + \Delta t S_i^n$$

$$LHS \cdot \vec{T}^{n+1} = RHS \cdot \vec{T}^{n} + \overline{S}$$

<u>Left-Hand Side</u>

$$LHS = \begin{pmatrix} D & V & V & \cdots \\ V & D & V & \cdots \\ V & V & D & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad \begin{cases} D = 1 + \sum_{k \in vec} C_{ij} \\ V = -C_{ij} \end{cases}$$

<u>Right-Hand Side</u>

$$RHS = \begin{pmatrix} D & V & V & \cdots \\ V & D & V & \cdots \\ V & V & D & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad \begin{cases} D = 1 - \sum_{k \in vec} C_{ij} \\ V = C_{ij} \end{cases}$$

<u>Note</u>: Each matrix is heptadiagonal, consisting of one main diagonal (D) and six neighbour (V).

Time Step Computation

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$$LHS \cdot \vec{T}^{n+1} = RHS \cdot \vec{T}^{n} + \vec{S}$$

1. Ignition probability computation: $\sigma\left(T_i^n\right) = \dfrac{1}{1 + e^{\left(-\frac{T_i^n - T_{ign}}{\Delta T_{ign}}\right)}}$

2. Source term computation: $S_i^n = Q_i F_i^n \sigma\left(T_i^n\right) \Delta t$

3. Fuel update: $F_i^{n+1} = F_i^n \left(1 - k_i \sigma\left(T_i^n\right) \Delta t\right)$

4. Right-hand update: $B^n = RHS \cdot \overrightarrow{T^n} + \overrightarrow{S^n}$

5. Set: $A = LHS$

The system becomes:

$$A \cdot \vec{T}^{n+1} = B^n$$

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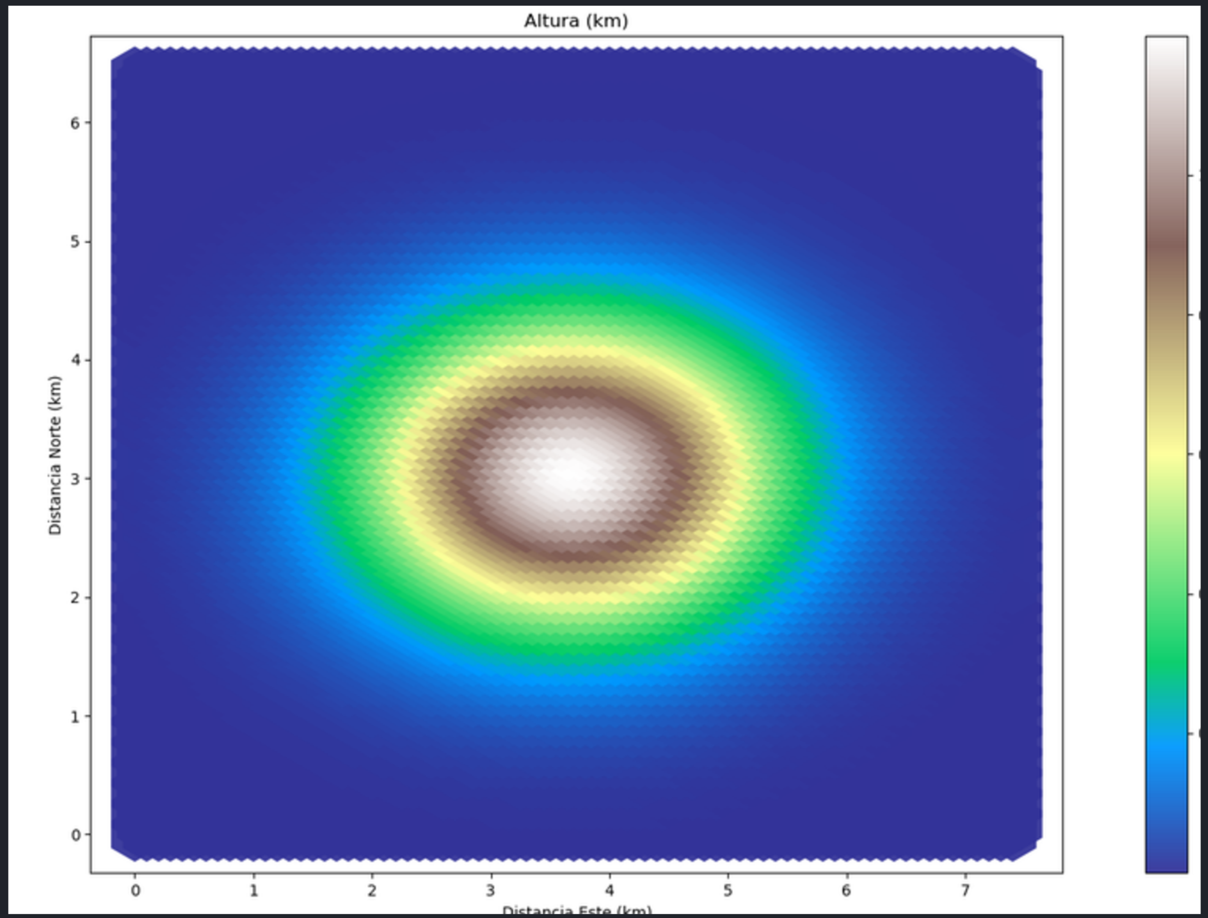
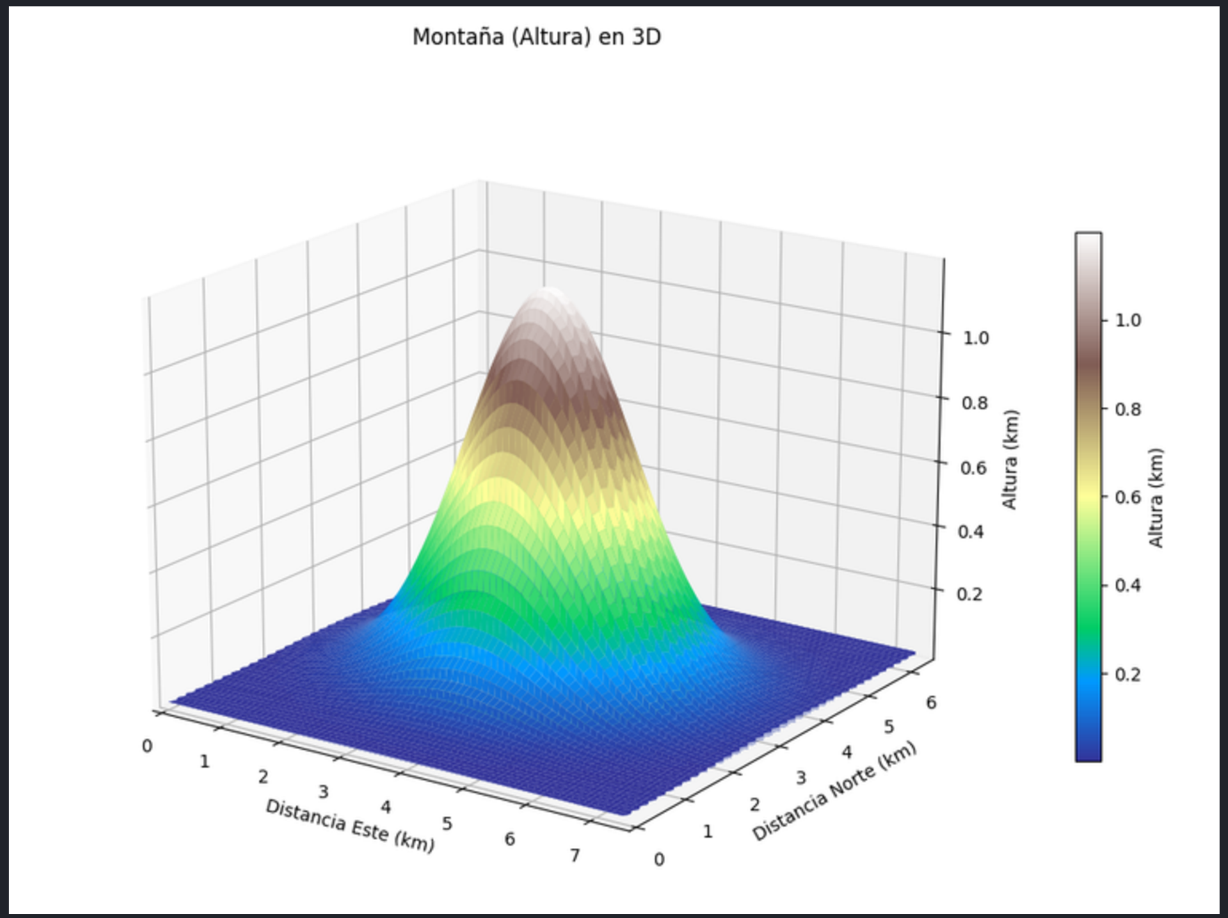
Elevation

Gaussian mountain definition:

$$H\left(x, y\right) = H_{max} e^{-\left(\frac{\left(x - x_0\right)^2}{2\sigma_x^2} + \frac{\left(y - y_0\right)^2}{2\sigma_y^2}\right)}$$





Slope

It is approximated using finite differences:

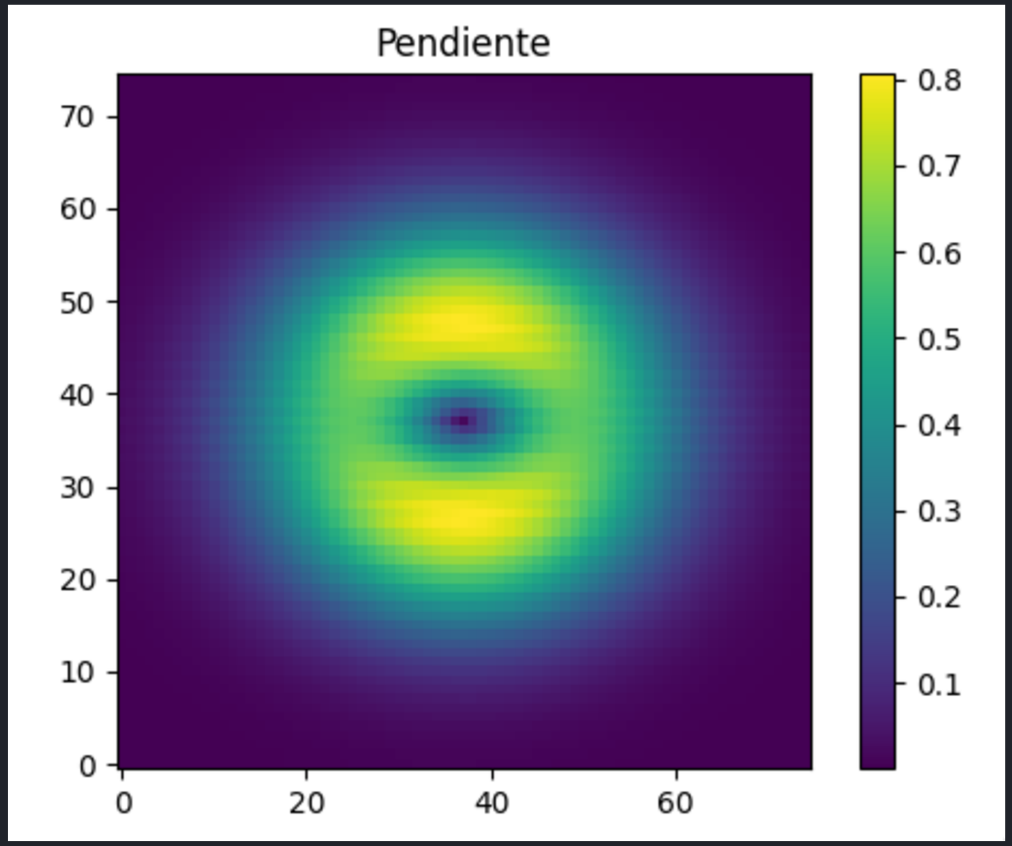
$$P(x,y) = \left(\left(\frac{dH(x,y)}{dx}\right)^2 + \left(\frac{dH(x,y)}{dy}\right)^2\right)^{\frac{1}{2}} \longrightarrow P_{ij} \approx \frac{\Delta H_{ij}}{h} = \frac{H_j - H_i}{h}$$

Slope factor (advection):

$$v_{ij}^{pend} = min\left(c_{pend} P_{ij}, v_{max}^{pend}\right)$$

Effective advection velocity:

$$v_{ij}^{eff} = \left(v^{viento} \cdot e_{ij}\right) + v_{ij}^{pend}$$



Moisture

Height normalization: $H_{norm} = \dfrac{H - H_{min}}{H_{max} - H_{min}} \in [0, 1]$

Moisture is defined as: $M(H_{norm}) = M_{cumbre} + (M_{valle} - M_{cumbre})(1 - H_{norm})^p$

Parameters modified by moisture:

- Ignition temperature:

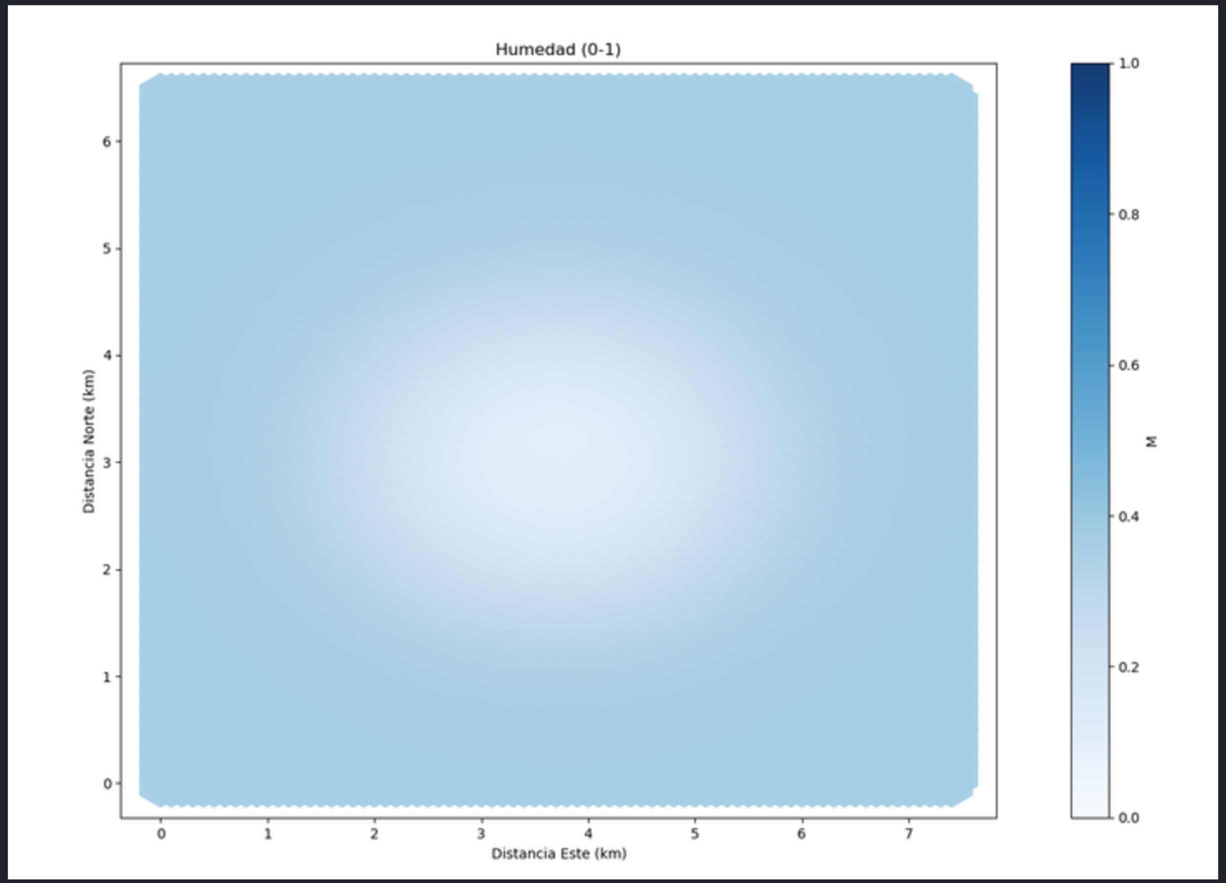
$$T_{ign}(r) = T_{ign,base} + k_{ign} M(r)$$

- Cooling rate:

$$C(r) = C_{base}(r)(1 + k_C M(r))$$

- Heat release rate:

$$Q(r) = Q_{base}(r)(1 - k_Q M(r))$$



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