MATLAB (Problems 1-3)

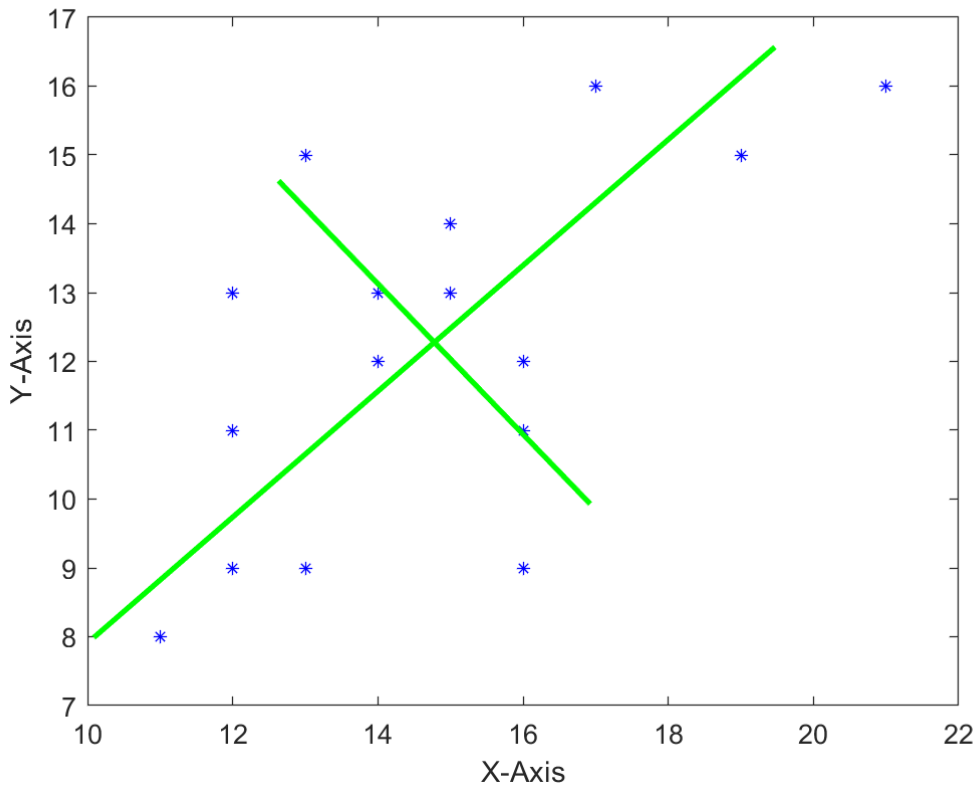
Output:

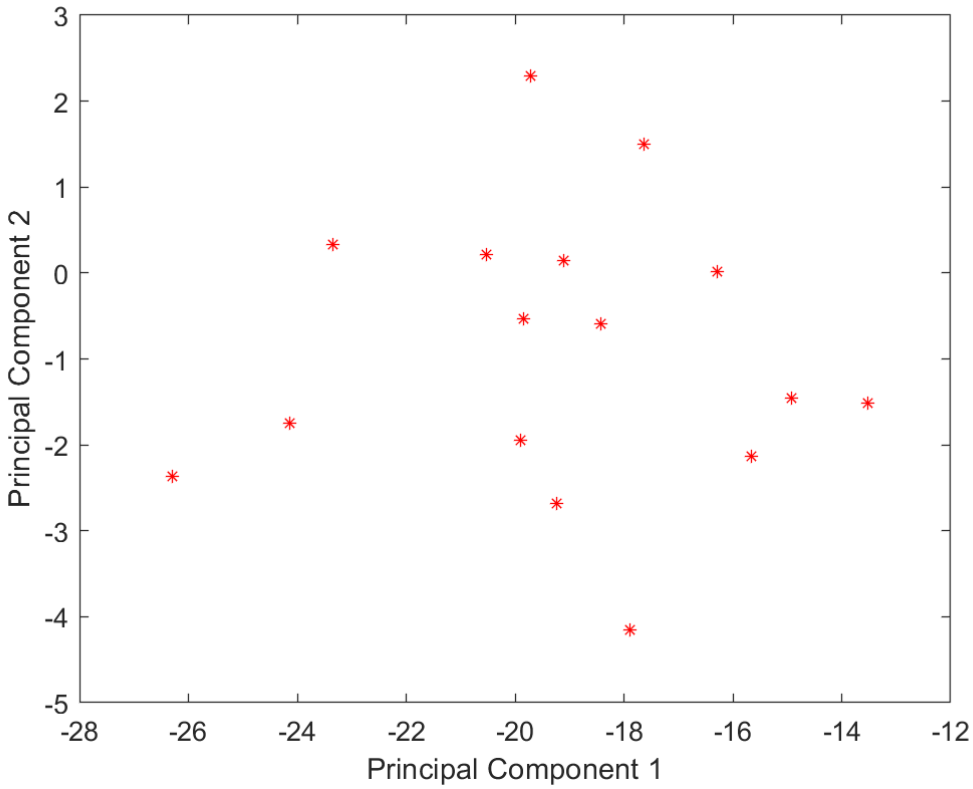
```
------------------------------------------------------------
Oscar Martinez          Homework 3: Problems 1-3          STA 5106
------------------------------------------------------------
-----Problem 1-----
X1 =

5     0     9     3
3     6     8     9
4     4     9     6
0     3     1     8
2     8     2     3


y =

20
17
32
10
12


b =

2.4413
0.3949
0.9165
0.7156

-----Problem 2-----
X2'=

ans =

15    13
16    11
12    13
14    12
13     9
15    14
16    12
21    16
12     9
```

```
11     8
19    15
14    13
13    15
14    13
16    12
17    16
12    11
16     9

---Part (a)---
---Part (b)---
U =

-0.7380   -0.6748
-0.6748    0.7380


Cov_X2 =

6.6536    3.7712
3.7712    5.9771


Cov_Z =

10.1017   -0.0000
-0.0000    2.5290


Total_var_X2 =

12.6307


Total_var_Z =

12.6307


Ratio =

1.0000

-----Problem 3-----
---Part (a)---
---Part (b)---
```

```
b1 =

1.0742
-1.7688
0.1040
0.3633
-0.3864
0.4965
0.8623
1.3953
-0.0185
1.0064

---Part (c)---
SSE =

33.6282
```

Figures:





Code:

```
%Problems 1–3
clc
clear

diary MATLAB_Output_OM.txt

%Introduction
fprintf('————————————————————————————————————————————————————————————\n')
    ;
fprintf('Oscar Martinez \t Homework 3: Problems 1–3 \t STA 5106\n');
fprintf('————————————————————————————————————————————————————————————\n')
    ;

%——————Problem 1:———————
fprintf('———————Problem 1———————');
X1=[5 0 9 3; 3 6 8 9; 4 4 9 6; 0 3 1 8; 2 8 2 3]
y=[20 17 32 10 12]'
```

```
b=multilinreg(X1,y)

%------Problem 2:------
fprintf('------Problem 2------\n');
X2=[15 16 12 14 13 15 16 21 12 11 19 14 13 14 16 17 12 16;
13 11 13 12 09 14 12 16 09 08 15 13 15 13 12 16 11 09];
fprintf(' X2^T=\n ');
X2'

fprintf('---Part (a)---\n');
%First Plot
figure(1);
plot(X2(1,:),X2(2,:),'b*', 'markersize', 5)
xlabel('X-Axis');
ylabel('Y-Axis');


fprintf('---Part (b)---');
%PCA analysis
[m, n] = size(X2);

% 1. Compute sample covariance
C = cov(X2')

% 2. SVD on C
[U, S, V] = svd(C);
U

% 3. select the first 2 columns of U
U1 = U(:,1:2);

% 4. Define Z
Z = U1'*X2;

% plot the transformed data
figure(2);
plot(Z(1,:), Z(2,:), 'r*', 'markersize', 5);
xlabel('Principal Component 1');
ylabel('Principal Component 2');
% axis([-110 -85 -1 6]);
% axis equal;

% plot the principal directions on the origianl data
m_X2 = mean(X2');
figure(1);
```

```
hold on;
plot(2*sqrt(S(1,1))*[-U(1, 1) U(1, 1)]+m_X2(1), ...
2*sqrt(S(1,1))*[-U(2, 1) U(2, 1)]+m_X2(2), 'g', 'linewidth', 2);
plot(2*sqrt(S(2,2))*[-U(1, 2) U(1, 2)]+m_X2(1), ...
2*sqrt(S(2,2))*[-U(2, 2) U(2, 2)]+m_X2(2), 'g', 'linewidth', 2);
hold off;
% compare the covariance and the total variance
Cov_X2 = cov(X2'),
Cov_Z = cov(Z'),
Total_var_X2 = trace(Cov_X2),
Total_var_Z = trace(Cov_Z),

Ratio=Total_var_Z/Total_var_X2
%------Problem 3:-------
fprintf('-------Problem 3-------\n');
clear

%Import Data
load hw3_3_data.mat

%-----Part (a)-----
fprintf('-----Part (a)-----\n');
%%% Perform the PCA analysis %%%
[m, n] = size(X);

% i. Compute sample covariance
C = cov(X);

% ii. SVD on C
[U, S, V] = svd(C);
%U is 100x100, was formerly 200x200 before making C the covariance of X
%transpose

% iii. select the first D=10 columns of U
U1 = U(:,1:10); %100x10

% iv. Define X3 (Prompt's X1)
X3 = X*U1; %Want this to be 200x10

%-----Part (b)-----
fprintf('-----Part (b)-----');
b1=multilinreg(X3,y)

%-----Part (c)-----
fprintf('-----Part (c)-----');
```

```
E=y-X3*b1;
SSE=norm(E,2)

%closing output
diary off
```

Python (Problems 4-6)

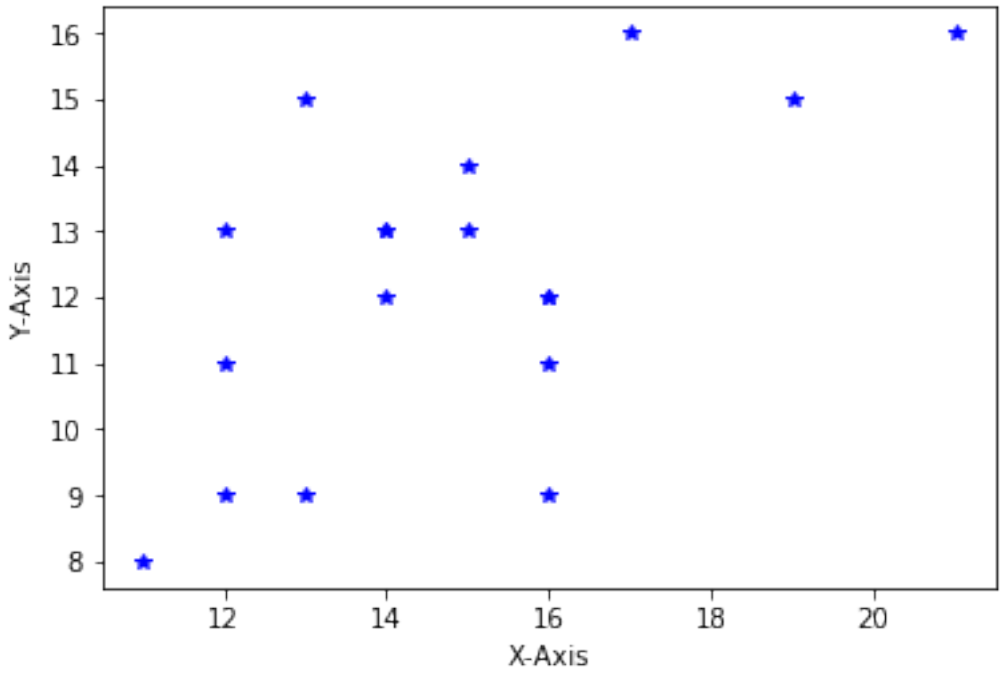
```
In [9]: #Problem 4
"-----Problem 4-----"
from numpy import *
set_printoptions(precision=4)

X=mat("5. 0. 9. 3.; 3. 6. 8. 9.; 4. 4. 9. 6.; 0. 3. 1. 8.; 2. 8. 2. 3.")
y=mat("20. 17. 32. 10. 12.").T

def backsub(X, y):
l = shape(X)
n = l[1]
b = zeros((n,1))
b[n-1, 0] = y[n-1, 0]/X[n-1, n-1]
for j in range(n-1,0,-1):
b[j-1,0] = (y[j-1,0] - dot(X[j-1, range(j,n)], b[range(j,n),0]))/X[j-1, j-1]
return b

def house(x):
m = size(x)
mu = linalg.norm(x)
v = x.copy()
if mu != 0:
c = x[0] + sign(x[0])*mu
v[1:m+1] = v[1:m+1]/c
v[0] = 1
return v

def rowhouse(X,v):
X = mat(X)
v = mat(v)
X = X - 2*v*v.T/(v.T*v)*X
return X

def householder(X0):
X = mat(X0.copy())
```

```
m, n = shape(X)
v = mat(zeros((m,1)))
for j in range(1, n+1):
v[j-1:m] = house(X[j-1:m,j-1])
X[j-1:m,j-1:n] = rowhouse(X[j-1:m,j-1:n], v[j-1:m])
return X

def multilinreg(X0,y0):
X = X0.copy()
y = y0.copy()
m, n = shape(X0)
for j in range(1,n+1):
v = house(X[j-1:m,j-1])
X[j-1:m,j-1:n] = rowhouse(X[j-1:m,j-1:n], v)
beta = -2.*(v.T*y[j-1:m])/(v.T*v)
y[j-1:m] = y[j-1:m] + v*beta

b = backsub(X,y)
return b

print("b=")
print(multilinreg(X,y))

b=
[[2.4413]
[0.3949]
[0.9165]
[0.7156]]


In [12]: #Problem 5
from matplotlib import pyplot
import scipy
print("-----Problem 5-----")
X=mat("15. 16. 12. 14. 13. 15. 16. 21. 12. 11. 19. 14. 13. 14. 16. 17. 12. 16.;
print("X=")
print(X)

x=X

#Part a
print("-----Part (a)-----")

## plot the raw data
pyplot.figure(0)
```

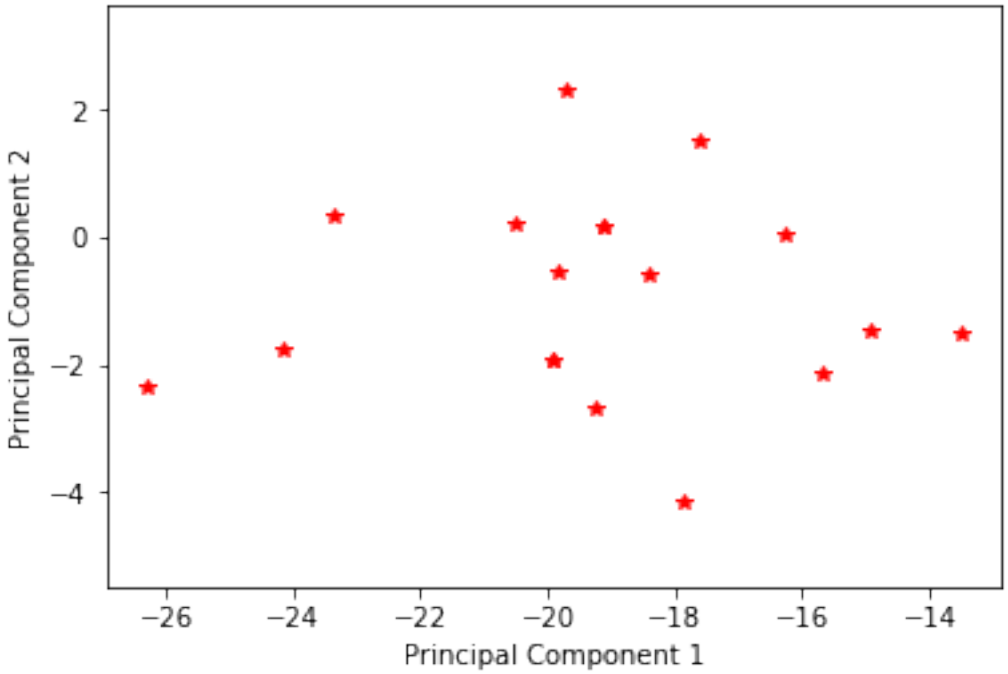
```
pyplot.plot(x[0,:], x[1,:], 'b*')
pyplot.xlabel('X-Axis')
pyplot.ylabel('Y-Axis');
pyplot.show()

#Part b
print("-----Part (b)-----")
#### Perform the PCA analysis ###
m, n = shape(x)
print(m)
print(n)

# 1. Compute sample covariance
C = cov(x)
print("C=")
print(C)

# 2. SVD on C
U, S, Vh = linalg.svd(C)
V = Vh.T

# 3. select the first 2 columns of U
U1 = U[:,0:2]
print("U1=")
print(U1)

# 4. Define Z
Z = U1.T*x;

## plot the transformed data
pyplot.figure(2)
pyplot.plot(Z[0,:], Z[1,:], 'r*')
pyplot.axis([-110, -85, -1, 6])
pyplot.axis('equal')
pyplot.xlabel('Principal Component 1')
pyplot.ylabel('Principal Component 2')

## add the principal directions
pyplot.figure(1)
pyplot.plot(x[0,:], x[1,:], 'b*')
pyplot.xlabel('X-Axis ')
pyplot.ylabel('Y-Axis ');
m_x = x.mean(axis=1)
TP1 = array(2*sqrt(S[0])*mat([-U[0, 0], U[0, 0]])+m_x[0])
TP2 = array(2*sqrt(S[0])*mat([-U[1, 0], U[1, 0]])+m_x[1])
```

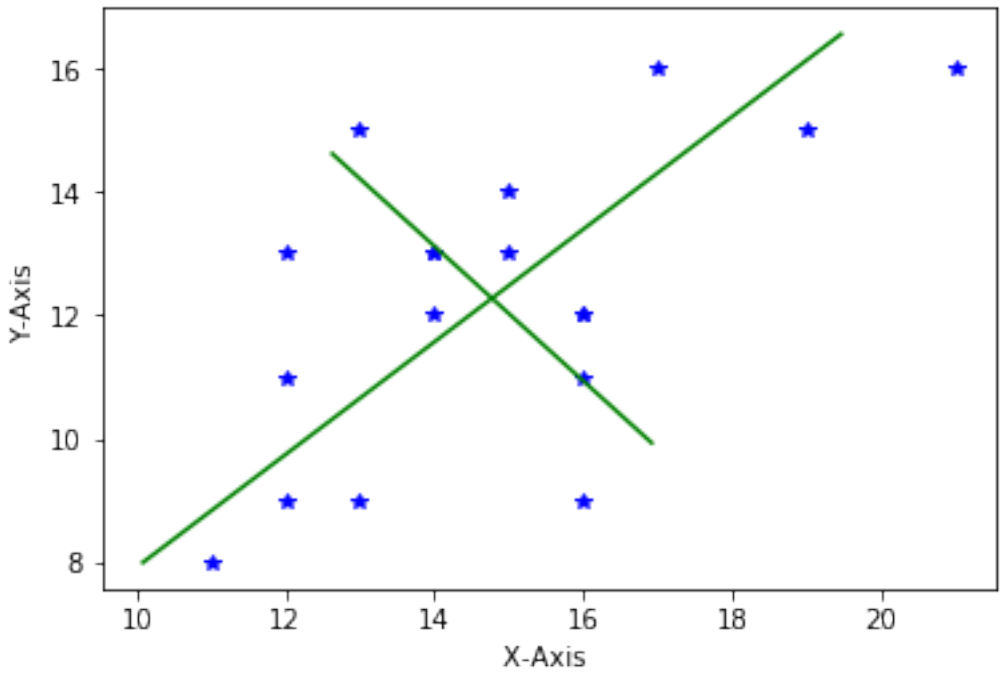
```
TP3 = array(2*sqrt(S[1])*mat([-U[0, 1], U[0, 1]])+m_x[0])
TP4 = array(2*sqrt(S[1])*mat([-U[1, 1], U[1, 1]])+m_x[1])
pyplot.plot(TP1[0], TP2[0], 'g-')
pyplot.plot(TP3[0], TP4[0], 'g-')

pyplot.show()

## compare the covariance and the total variance
Cov_x = cov(x)
Cov_Z = cov(Z)
Total_var_x = trace(Cov_x)
Total_var_Z = trace(Cov_Z)
Ratio=Total_var_x/Total_var_Z

#print('total_var_x = ' + repr(Total_var_x), 'total_var_z = ' + repr(Total_var_Z

print('total_var_x = %5.2f' %Total_var_x, 'total_var_Z = %5.2f' %Total_var_Z, 'R
```

```
-----Problem 5-----
X=
[[15. 16. 12. 14. 13. 15. 16. 21. 12. 11. 19. 14. 13. 14. 16. 17. 12. 16.]
[13. 11. 13. 12.  9. 14. 12. 16.  9.  8. 15. 13. 15. 13. 12. 16. 11.  9.]]
-----Part (a)-----
```



```
-----Part (b)-----
2
18
C=
[[6.6536 3.7712]
[3.7712 5.9771]]
U1=
[[-0.738  -0.6748]
[-0.6748  0.738 ]]
```






```
total_var_x = 12.63 total_var_Z = 12.63 Ratio is: 1.0000
```

```
In [9]: #Problem 5
print("-----Problem 5-----")
set_printoptions(precision=4)

#Importing Data
import scipy.io
mat_contents = scipy.io.loadmat('hw3_3_data.mat')
X = mat(mat_contents['X'])
y = mat(mat_contents['y'])

#Part a
print("----Part a----")

#### Perform the PCA analysis ###
m, n = shape(X)
print('X-m=', m, 'X-n=', n) #200x200

# i. Compute sample covariance
C = cov(X.T)
a, b = shape(C)
```

```
print('C-m=', a, 'C-n=', b) #100x100

# ii. SVD on C
U, S, Vh = linalg.svd(C)
V = Vh.T

# iii. select the first D=10 U
U1 = V[:,0:10]
o, p = shape(U1)
print('C-m=', o, 'C-n=', p) #100x10

# iv. Define X3 (Prompt's X1)
X3 = X*U1;
q, r = shape(X3)
print('X3-m=', q, 'X3-n=', r) #200x10


#Part b
print("----Part b----")
b=multilinreg(X3,y)
print('b=')
print(b)

#Part c
print("----Part c----")
E=y-X3*b
print("SSE = ",linalg.norm(E))
```

```
-----Problem 5-----
----Part a----
X-m= 200 X-n= 100
C-m= 100 C-n= 100
C-m= 100 C-n= 10
X3-m= 200 X3-n= 10
----Part b----
b=
[[ 1.0742]
[-1.7688]
[ 0.104 ]
[ 0.3633]
[-0.3864]
[ 0.4965]
[ 0.8623]
[ 1.3953]
[-0.0185]
```

```
[ 1.0064]]
----Part c----
SSE =  33.628167484025035
```