STA 4102/5106: Final Project

(Monday, November 24)

Due: Friday, December 12 at Noon

Turn in a **hardcopy** to the Instructor's Office (**106B OSB**)

1. **Goal:** Our goal is to use dynamic programming method to reconstruct a binary function from a noisy observation.

2. **Problem Statement:** Let x = (x_1, …, x_n) be a given binary and Markovian sequence. In particular,

$$x_1 = \begin{cases} 0 & \text{with probability } 0.5 \\ 1 & \text{with probability } 0.5 \end{cases}$$

and

$$x_i = \begin{cases} x_{i-1} & \text{with probability } p \\ 1 - x_{i-1} & \text{with probability } 1 - p \end{cases}$$

Let y = (y_1, …, y_n) be a noisy observation of x. That is, for any i = 1, …, n,

$$y_i = x_i + e_i,$$

where $e_i \sim N(0, \sigma^2)$. Assuming p and σ^2 are known, our goal is to reconstruct x from y by the Maximum A Posterior (MAP) method:

$$\{z_i\} = \underset{\{x_i\}}{\operatorname{argmax}} \Pr\{\{x_i\} \mid \{y_i\}\} = \arg \max_{\{x_i\}} \log(\Pr\{\{x_i\} \mid \{y_i\}\})$$
$$= \underset{\{x_i\}}{\operatorname{argmax}} \sum_{i=1}^{N} \left(-\frac{(y_i - x_i)^2}{2\sigma^2}\right) + \sum_{i=2}^{N} \log(1_{x_i = x_{i-1}} p + 1_{x_i \neq x_{i-1}} (1 - p))$$

3. **Methodology:** Perform the following steps:
   (a) State the general idea behind dynamic programming.
   (b) Simulate x for given p.
   (c) Simulate y conditioned on x for given σ^2.
   (d) Use dynamic programming method to estimate MAP z = (z_1, …, z_n).

4. **Experimental Results:**
   (a) Implement the dynamic programming in Matlab. Choose p = 0.99 and σ = 1. Plot x, y and the estimate z. Compute success rate in this reconstruction.
   (b) Repeat step (a) 5 times, and report the averaged success rate.
   (c) Perform reconstruction for p = 0.9, 0.8, 0.7, 0.6 with σ = 1. Plot averaged (over 5 repetitions) success rate w.r.t. p.
   (d) Perform reconstruction for σ = 0.5, 1, 2, 5 with p = 0.99. Plot averaged (over 5 repetitions) success rate w.r.t. σ.

5. **Report:** Prepare a full report of your experiments including introduction, methodology, Matlab programs, results, and conclusions.