STA 4102/5106: Homework Assignment #8

(Wednesday, November 5)
Due: Wednesday, November 12

Useful commands on random sample generation:

rand: uniform on [0, 1]
randn: normal with mean 0, variance 1
exprnd: exponential distribution

1. Write a matlab program to estimate the following moments of a standard normal random variable using a classical Monte Carlo method:

(a) Mean: $\mu = \int x f(x) dx.$

(b) Variance: $\sigma^2 = \int (x-\mu)^2 f(x) dx.$

(c) Kurtosis: $\kappa = \dfrac{\int (x-\mu)^4 f(x) dx.}{\sigma^4}$

In each case, plot the convergence of the estimator and its (sample) variance as a function of the sample size.

2. Write matlab programs to estimate the quantity $\theta = Pr\{X < 1\}$, where X is an exponential random variable with mean 1, using two different Monte Carlo estimators. In other words, the random samples used in estimators should have different probability densities. Plot both the estimates as functions of n, the number of samples generated.

(Hint: $\theta = \int_0^1 \exp(-x) dx$).

3. (**STA 5106 Students Only**) Suppose that X is exponential with mean 1, and given that $X = x$, Y is exponential with mean x. Design a Monte Carlo algorithm using variance reduction via conditioning to estimate $Pr\{XY \le 3\}$. Implement the algorithm and plot the estimated probability versus the sample size.