

STA 5106: Homework Assignment #5

(Thursday, September 26)

Due: Thursday, October 3

1. Write a Matlab program to compute a root of the following function using: (i) simple iterations and (ii) Newton-Raphson method:

$$f(x) = (0.9\sin(x) - x).$$

Choose $x = \pi/4$ as the starting value and $\varepsilon = 10^{-6}$ in the absolute change stopping threshold. Plot the two converging sequences on the same plot (with Command “hold on”).

2. In the case of Newton-Raphson method for root finding, the sequences converge to a root only linearly (i.e. the order of convergence is one) if that root has a multiplicity greater than one. For roots with multiplicity greater than one, Newton-Raphson satisfies the equation

$$E_{i+1} \sim KE_i, E_i = |x_i - x_{i-1}| \text{ and } K = (m - 1)/m,$$

where m is the multiplicity of the root. Write a Matlab program to find the multiplicity of root $x = -0.5$ of the polynomial

$$x^5 - 4.5x^4 + 4.55x^3 + 2.675x^2 - 3.3x - 1.4375.$$

Choose $x = -0.6$ as the starting value and $\varepsilon = 10^{-6}$ in the absolute change stopping threshold. From the converging sequence first find the value of K and using that find the value of m .

3. Let X_1, X_2, \dots, X_n be independent and identically distributed samples from a logistic distribution with the probability density function

$$f(x | \theta) = \frac{\exp(-(x - \theta))}{(1 + \exp(-(x - \theta)))^2}.$$

Given the values of X_1, X_2, \dots, X_n in “hw5_3_data” from the blackboard website, our goal is to find the maximum likelihood estimate (MLE) of θ , using the following steps:

(a) Derive an expression for the log likelihood function

$$l(\theta) = \sum_{i=1}^n \log(f(X_i | \theta)),$$

such that the MLE is given by

$$\hat{\theta} = \arg \max_{\theta} l(\theta).$$

(b) Find the expression for $\dot{l}(\theta)$ and $\ddot{l}(\theta)$, the first and the second derivatives of l with respect to θ . Verify that $\ddot{l}(\theta) < 0$.

(c) Use Newton-Raphson method to find a root of $\dot{l}(\theta)$. Choose $\theta_0 = 7$ as the starting value and $\varepsilon = 10^{-6}$ in the absolute change stopping threshold.

4, 5 (Optional): Use Python program to finish Problems 1, 2.