STA 5106: Homework Assignment #4

(Thursday, September 19)
Due: Thursday, September 26

1. PCA and Images: Consider the problem of analysis of images. Each (gray scale) image can be thought of as a matrix of numbers, say $I \in \mathbf{R}^{m_1 x m_2}$. We can rewrite this matrix as a long vector $X \in \mathbf{R}^{m_1 m_2}$. Setting n = $m_1 m_2$, we want to use PCA to reduce dimension from n to d. For the data file provided to you on the website perform PCA and present the following results:

(a) Show images of the first three principal directions of the data. That is, take the vectors U_1, U_2, and U_3 and display them as images. (Use the commands below to form images from vectors.)
(b) Take the first image in the data, and show its projection onto the principal subspace for d = 50 and d = 100. The projection of the first image into first d components is:

$$\sum_{i=1}^{d} (X(1,:) * U_i) * U_i.$$

Load the data file using “load hw4_1_data”. This will give you a 200 × 644 matrix where each row of this matrix is a vector form of an image with $m_1 = 28$ and $m_2 = 23$. So there are 200 images in this dataset.
For a 644 length vector v you can form and display it as an image using:

```
I = reshape(v,28,23);
imagesc(I);
colormap(gray)
axis equal;
```

2. LDA: Consider a labeled data set X with the following properties: there are m = 5 classes, each class has k = 10 observations, and each observation is a vector of size n = 3. Therefore, X can be thought of as three-dimensional array with dimensions 3 × 5 × 10. In Matlab, X(:, i, j) denotes the jth observation vector of ith class.
Given this data, perform a linear discriminant analysis of the data for d = 1, and find the projection U ∈ $\mathbf{R}^{n\times d}$ that is optimal for separating observed classes. You can use the **eigs** function in matlab to perform generalized eigen decomposition. For the resulting U:

(a) Plot the original data using command **plot3**.
(b) State U.
(c) Project the data X into Z, and plot the observations of Z.

Download X in “hw4_2_data” from the class website.

3. LLE: In the LLE framework, we minimize the error for a point $\vec{X}$ with K neighbors $\vec{\eta}_j$

$$\left|\vec{X}-\sum_{j=1}^{K} W_j \vec{\eta}_j\right|^2 = \left|\sum_{j=1}^{K} W_j \left(\vec{X}-\vec{\eta}_j\right)\right|^2 = \sum_{jk} W_j W_k G_{jk}$$

where the Gram matrix

$$G_{jk} = \left(\vec{X}-\vec{\eta}_j\right)\cdot\left(\vec{X}-\vec{\eta}_k\right)$$

Prove that the optimal reconstruction weights are

$$W_j = \frac{\sum_k [G^{-1}]_{jk}}{\sum_{lm} [G^{-1}]_{lm}}$$