STA 4102/5106: Homework Assignment #7

(Wednesday, October 29)
Due: Wednesday, November 5

1. Write a matlab program to implement a uniform random number generator using a multiplicative congruential method with m = $2^{13} - 1$ and a = 17. Generate 500 numbers for the starting point $x_0 = 100$. You can use the **mod** command in matlab for evaluating the modulus function. For the sequence $\boldsymbol{u_i = x_i / m}$:
   (a) Plot a histogram to display the results.
   (b) Calculate the coefficient correlation of the pairs of successive number u_i and u_{i+1}. (You can use **corrcoef** in matlab).
   (c) Plot the pairs (u_i, u_{i+1}) on a 2D plot.

2. Repeat Problem 1 with $a = 85$.

3. Derive and implement a method to generate samples of a Weibull random variable whose probability distribution function is given by:
$$F(x) = 1 - \exp(-\alpha\, x^{\beta}), \quad 0 < x < \infty$$
   Run your program to simulate 1000 values of Weibull random variable with $\alpha = 1$ and $\beta = 0.5$. Draw the histogram of the simulated values with 100 bins.

4. **(STA 5106 Students Only)** Let X_i, $i = 1, 2, \ldots, n$ be a sequence of independent and identically distributed exponential random variables with mean 1. Define a random variable Z_i, for $i > 1$
$$Z_i = \begin{cases} 1 & \text{if } X_i \geq \max(X_1, X_2, \ldots, X_{i-1}) \\ 0 & \text{otherwise} \end{cases}$$
   We will assume that $Z_1 = 1$. In the sequence Z_i, $i = 1, 2, \ldots, n,$ we are interested in how many times $Z_i = 1$ are separated by k time points, where $k = 0, 1, \ldots, 8$.
   (a) Write a matlab program to simulate the sequence Z_i for $n = 200$.
   (b) Compute the number of times $Z_i = 1$ are separated by k time points. Call this random value Y_k.
   (c) Repeat the above simulation $m = 3000$ times. Draw the histogram the values of Y_k and compute the mean of those simulated values. Compare the histogram of Y_k with the probability mass function of a Poisson random variable with mean $1/(k+1)$.