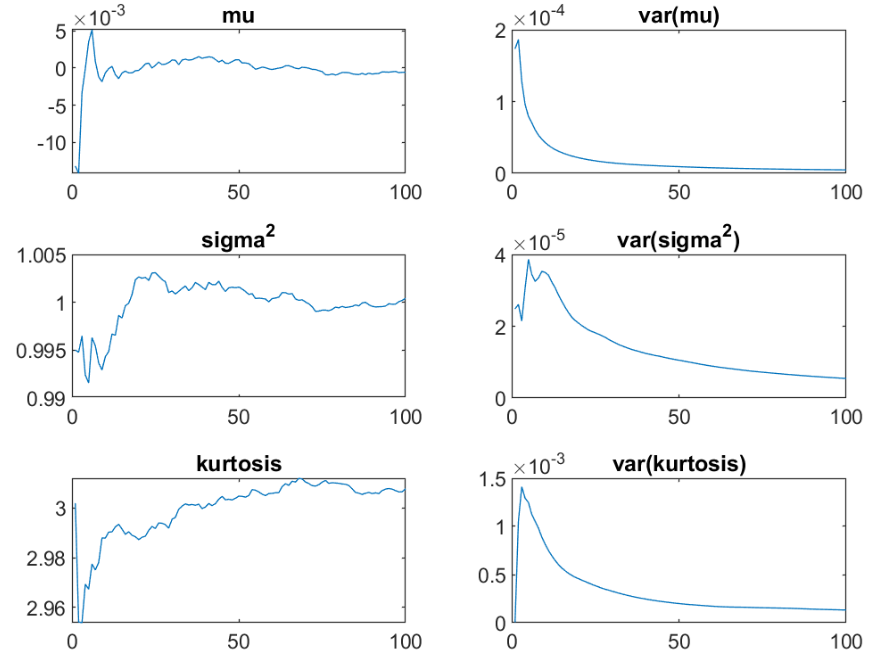
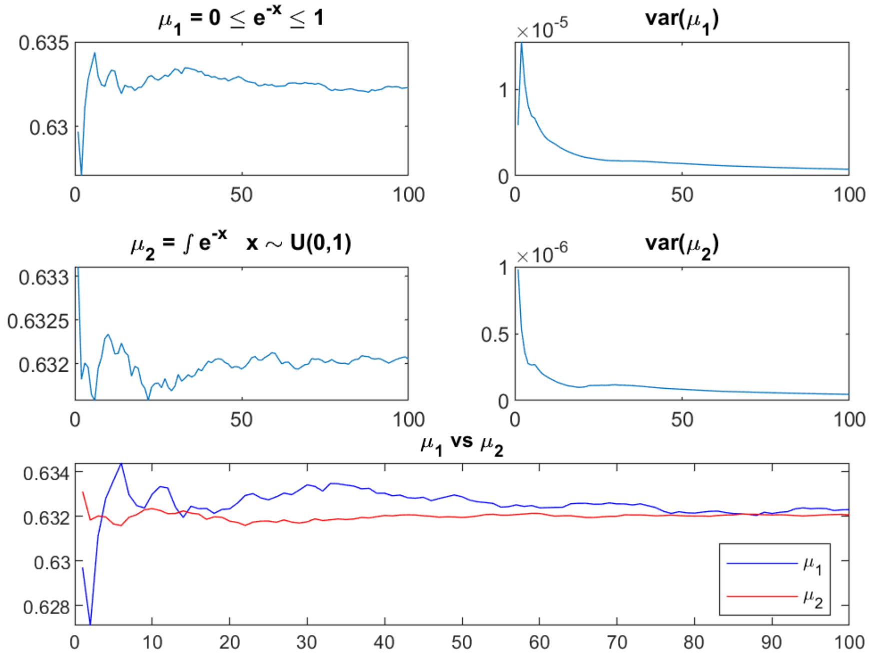
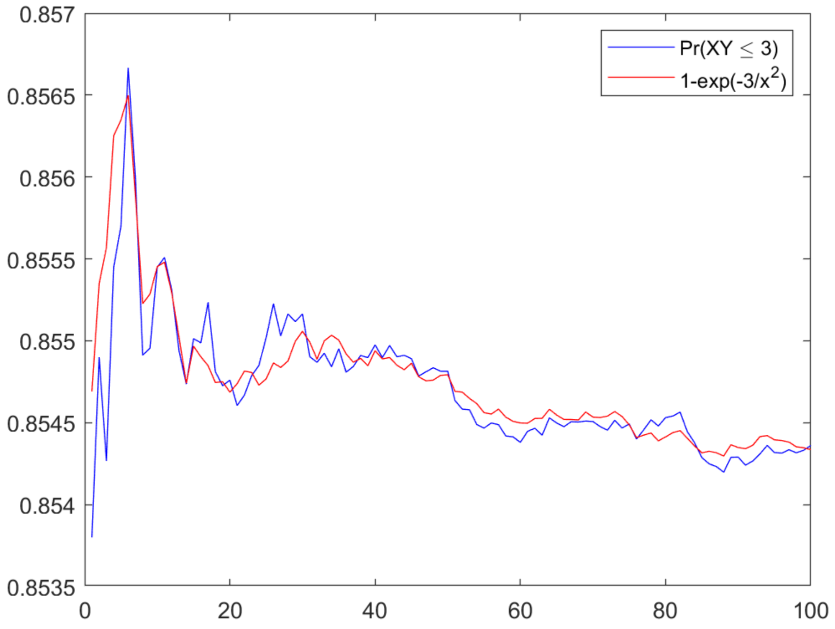
Problem 1



Problem 2



Problem 3



Red seems to be more robust as in exhibiting less variance. Running the variance command on MATLAB confirms this belief.

MATLAB Code for Problems 1, 2, 3:

```
clc
clear
%Diary
dfile ='MATLAB_Output_OM.txt';
if exist(dfile, 'file') ; delete(dfile); end
diary(dfile)
diary on

%Introduction
fprintf('----------------------------------------------------------------------\n');
fprintf('\t Oscar Martinez \t HW 9 \t STA 5106\n');
fprintf('----------------------------------------------------------------------\n');

```

```
%————Problem 1:————
fprintf('————————Problem 1————————\n');

K = 100; %Iterations
M = 10000; %Sample Size
for i = 1:K;
    rng(i + 10); %Set seed
    x(:,i) = randn(1,M); %Create x_i ~iid~ N(0,1); i = 1,..,M
    y = reshape(x,1,[]); %Reshape for larger means
    mu(i) = mean(y); %Sample Mean
    sig2(i) = mean((y—mu(i)).^2); %Sample Variance
    kurt(i) = mean((y—mu(i)).^4)/(sig2(i)^2); %Sample Kurtosis

    %Sample Estimator Variances
    Vmu(i) = sum( ( mu(1:i) — 0 ).^2 )/i; %Var of x bar
    Vsig2(i) = sum( ( sig2(1:i) — 1 ).^2 )/i; %Var of s
    Vkurt(i) = sum( ( kurt(1:i) — 3 ).^2 )/i; %Var of s kurt
end

%Plot
figure(1);
subplot(3,2,1);
plot(1:K, mu);
title('mu');
subplot(3,2,2);
plot(1:K, Vmu);
title('var(mu)');
subplot(3,2,3);
plot(1:K, sig2);
title('sigma^2');
subplot(3,2,4);
plot(1:K, Vsig2);
title('var(sigma^2)');
subplot(3,2,5);
plot(1:K, kurt);
title('kurtosis');
subplot(3,2,6);
plot(1:K, Vkurt);
title('var(kurtosis)');

%————Problem 2:————
fprintf('————————Problem 2————————\n');
clear

K = 100; %Iterations
```

```
M = 10000; %Sample Size
for i = 1:K;
    rng(i + 10); %Set seed
    T = 1—exp(—1); %true probability

    %MC estimator 1: Exp
    x(:,i) = exprnd(1,1,M); %Create x_i ~iid~ exp(1); i = 1,..,M
    y = reshape(x,1,[]); %Reshape for aggregate mean
    mu1(i) = sum( (y <= 1 & y >= 0) )/size(y,2); %Sample Mean 1
    Vmu1(i) = sum( ( mu1(1:i) — T ).^2 )/i; %Var of x bar 1

    %MC estimator 2: Unif
    u(:,i) = rand(1,M); %Create x_i ~iid~ U(0,1); i = 1,..,M
    z = reshape(u,1,[]); %Reshape for aggregate mean
    mu2(i) = sum(exp(—z))/size(z,2); %Sample mean 2
    Vmu2(i) = sum( ( mu2(1:i) — T ).^2 )/i; %Var of xbar 2
end

%Plot
figure(2);
subplot(3,2,1);
plot(1:K, mu1);
title('\mu_1 = 0 \leq e^{—x} \leq 1');
subplot(3,2,2);
plot(1:K, Vmu1);
title('var(\mu_1)');
subplot(3,2,3);
plot(1:K, mu2);
title('\mu_2 = \int e^{—x}   x \sim U(0,1)');
subplot(3,2,4);
plot(1:K, Vmu2);
title('var(\mu_2)');
subplot(3,2,[5,6]);
plot(1:K, mu1, 'b', 1:K, mu2, 'r')
title('\mu_1 vs \mu_2');
legend('\mu_1', '\mu_2','Location', 'southeast');

%————Problem 3:————
fprintf('——————————Problem 3——————————\n');
clear

K = 100; %Iterations
M = 10000; %Sample Size

%Set R.V.'s
```

```
for i = 1:K;
    rng(i + 10); %Set seed
    x(:,i) = exprnd(1,1,M);
    P1 = reshape(x,1,[]);
    y(:,i) = exprnd(x(:,i));
    z(:,i) = x(:,i).*y(:,i);
    P3 = reshape(z<=3,1,[]);
    mu1(i) = (sum(P3))/size(P3,2); %Pr XY<=3
    mu2(i) = mean(1—exp(—3./(P1.^2))); %Cond
end

%Plot
figure(3)
plot(1:K, mu1, 'b', 1:K, mu2, 'r')
legend('Pr(XY \leq 3)', '1—exp(—3/x^2)')

diary off
```

Problem 4

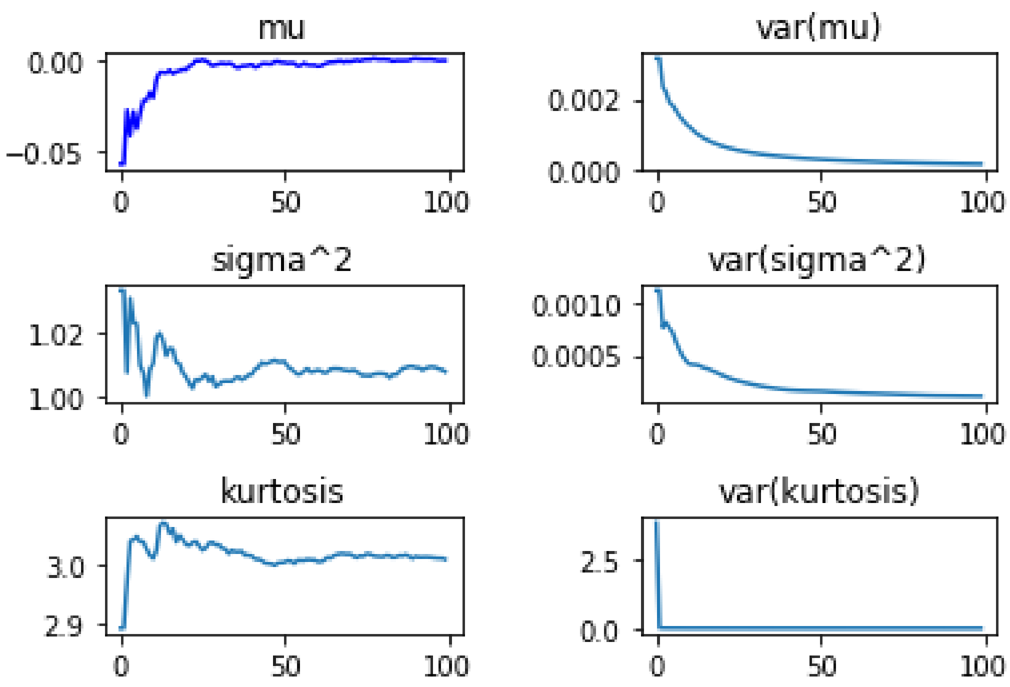
[65]:
```
          #-----------------------------Problem␣
↪1---------------------------------
          import numpy as np
          from matplotlib import pyplot
          import random
          print('Problem 1')
          K = 100
          M = 1000
          random.seed(1)
          x = np.zeros((M,K))
          mu = np.zeros((1,K))
          sig2 = np.zeros((1,K))
          kurt = np.zeros((1,K))
          Vmu = np.zeros((1,K))
          Vsig2 = np.zeros((1,K))
          Vkurt = np.zeros((1,K))

          for j in range(K):
          x[:,j] = np.array( [random.normalvariate(0,1) for x in␣
↪range(M)] )
          if j == 0:
          mu[0,j] = np.mean(x[:,0])
```

```
        sig2[0,j] = np.mean( ( x[:,0] - mu[0,j] )**2 )
        kurt[0,j] = np.mean( ( ( x[:,j] - mu[0,j] )**4 )/( sig2[0,j]**2 ) )
        Vmu[0,j] =  (mu[0,0] - 0)**2
        Vsig2[0,j] =  (sig2[0,0] - 1)**2
        Vkurt[0,j] =  (sig2[0,0] - 3)**2
        else:
        mu[0,j] = np.mean(x[:,0:j])
        sig2[0,j] = np.mean( ( x[:,0:j] - mu[0,j] )**2 )
        kurt[0,j] = np.mean( ( ( x[:,0:j] - mu[0,j] )**4 )/( sig2[0,j]**2 ) )
        Vmu[0,j] = ( np.sum( (mu[0,0:j+1] - 0)**2 ) )/(j+1)
        Vsig2[0,j] = ( np.sum( (sig2[0,0:j+1] - 1)**2 ) )/(j+1)
        Vkurt[0,j] = ( np.sum( (kurt[0,0:j+1] - 3)**2 ) )/(j+1)

        t = range(0,K)
        #Plot
        pyplot.figure(1);
        pyplot.subplot(3,2,1);
        pyplot.plot(t, mu[0,:], 'b');
        pyplot.title('mu');
        pyplot.subplot(3,2,2);
        pyplot.plot(t, Vmu[0,:]);
        pyplot.title('var(mu)');
        pyplot.subplot(3,2,3);
        pyplot.plot(t, sig2[0,:]);
        pyplot.title('sigma^2');
        pyplot.subplot(3,2,4);
        pyplot.plot(t, Vsig2[0,:]);
        pyplot.title('var(sigma^2)');
        pyplot.subplot(3,2,5);
        pyplot.plot(t, kurt[0,:]);
        pyplot.title('kurtosis');
        pyplot.subplot(3,2,6);
        pyplot.plot(t, Vkurt[0,:]);
        pyplot.title('var(kurtosis)');
        pyplot.subplots_adjust(hspace=1,wspace=0.5)
        pyplot.figure(num=None, figsize=(10, 10), dpi=140, facecolor='w', edgecolor='k')
        pyplot.show()
```

```
Problem 1
```



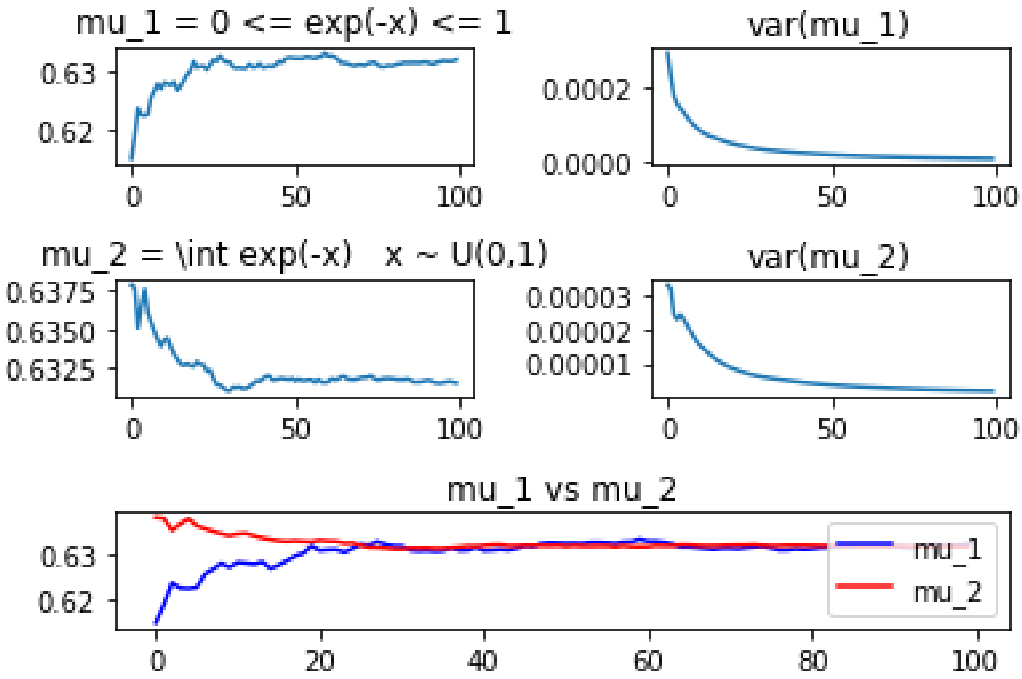
Problem 5

[66]:

```
        #-----------------------------Problem␣
↪2----------------------------------
        import numpy as np
        from matplotlib import pyplot
        import random
        print('Problem 2')
        K = 100
        M = 1000
        random.seed(1)
        x = np.zeros((M,K))
        y = np.zeros((M,K))
        mu1 = np.zeros((1,K))
        mu2 = np.zeros((1,K))
        sig2 = np.zeros((1,K))
        kurt = np.zeros((1,K))
        Vmu1 = np.zeros((1,K))
        Vmu2 = np.zeros((1,K))
        Vsig2 = np.zeros((1,K))
        Vkurt = np.zeros((1,K))
        Tr = 1-np.exp(-1) #True Value
        for j in range(K):
        x[:,j] = np.array( [random.expovariate(1) for x in range(M)] )
        y[:,j] = np.array( [random.uniform(0,1) for x in range(M)] )
        mu1[0,j] = ( np.sum( (x[:,0:j+1] >= 0) & (x[:,0:j+1] <= 1)) )/(np.size(x[:,0:j+1]))
        Vmu1[0,j] = (np.sum( ( mu1[0,0:j+1] - Tr )**2 ) )/(j+1)
        mu2[0,j] = (np.sum( np.exp( -y[:,0:j+1] )  ) )/(np.size(x[:,0:j+1]))
        Vmu2[0,j] = (np.sum( ( mu2[0,0:j+1] - Tr )**2 ) )/(j+1)

        t = range(0,K)
        #Plot
        pyplot.subplot(3,2,1);
        pyplot.plot(t, mu1[0,:]);
        pyplot.title('mu_1 = 0 <= exp(-x) <= 1');
        pyplot.subplot(3,2,2);
        pyplot.plot(t, Vmu1[0,:]);
        pyplot.title('var(mu_1)');
        pyplot.subplot(3,2,3);
        pyplot.plot(t, mu2[0,:]);
        pyplot.title('mu_2 = \int exp(-x)   x ~ U(0,1)');
        pyplot.subplot(3,2,4);
        pyplot.plot(t, Vmu2[0,:]);
        pyplot.title('var(mu_2)');
        pyplot.subplot(3,1,3);
        pyplot.plot(t, mu1[0,:], 'b', t, mu2[0,:], 'r')
        pyplot.title('mu_1 vs mu_2');
        pyplot.legend(('mu_1', 'mu_2'),loc='lower right');
        pyplot.subplots_adjust(hspace=1,wspace=0.5)
        pyplot.figure(num=2, figsize=(10, 10), dpi=140, facecolor='w', edgecolor='k')
        pyplot.show()
```

Problem 2



Problem 6

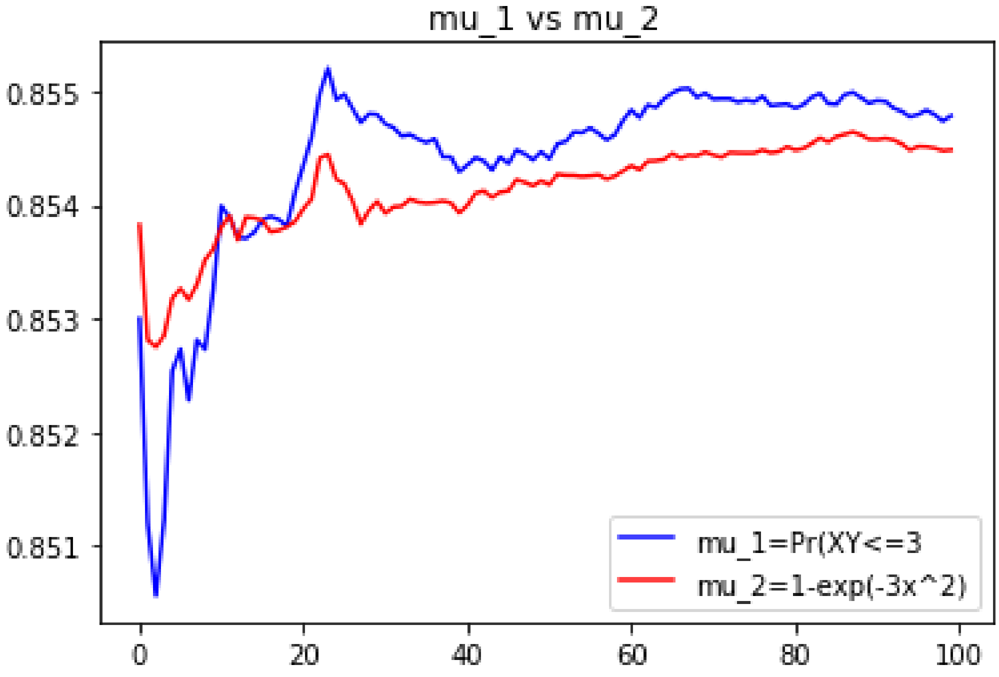
[67]:
```
        #-----------------------------Problem␣
↪3----------------------------------
        import numpy as np
        from matplotlib import pyplot
        import random
        print('\nProblem 3')
        K = 100
        M = 10000
        random.seed(1)
        x = np.zeros((M,K))
        y = np.zeros((M,K))
        z = np.zeros((M,K))
        P3 = np.zeros((M,K))
        mu1 = np.zeros((1,K))
        mu2 = np.zeros((1,K))

        for j in range(K):
        x[:,j] = np.array( [random.expovariate(1) for x in range(M)] )
        for i in range(M):
```

```
        y[i,j] = random.expovariate(1/x[i,j])
        z[i,j] = x[i,j]*y[i,j]
        P3[i,j] = (z[i,j]<=3)
        mu1[0,j] = ( np.sum( P3[:,0:j+1] ) )/( np.size(P3[:,0:j+1]) )
        mu2[0,j] = ( np.sum( 1-np.exp(-3/( x[:,0:j+1]**2 ) ) ) )/( np.
↪size(x[:,0:j+1]) )

        t = range(0,K)
        #Plot
        pyplot.plot(t, mu1[0,:], 'b', t, mu2[0,:], 'r')
        pyplot.title('mu_1 vs mu_2');
        pyplot.legend(('mu_1=Pr(XY<=3', 'mu_2=1-exp(-3x^2)'),loc='lower␣
↪right');
        pyplot.subplots_adjust(hspace=1,wspace=0.5)
        pyplot.figure(num=2, figsize=(10, 10), dpi=140, facecolor='w',␣
↪edgecolor='k')
        pyplot.show()
```

```
Problem 3
```


mu_1 vs mu_2
mu_1=Pr(XY<=3
mu_2=1-exp(-3x^2)