STA 4102/5106: Homework Assignment #4

(Wednesday, September 17)
Due: Wednesday, September 24

1. Write a Matlab program to compute a root of the following function using: (i) simple iterations and (ii) Newton-Raphson method:

$$f(x) = (0.9\sin(x) - x).$$

Choose $x = \pi/4$ as the starting value and $\varepsilon = 10^{-6}$ in the absolute change stopping threshold. Plot the two converging sequences on the same plot (with Command "hold on").

2. In the case of Newton-Raphson method for root finding, the sequences converge to a root only linearly (i.e. the order of convergence is one) if that root has a multiplicity greater than one. For roots with multiplicity greater than one, Newton-Raphson satisfies the equation

$$E_{i+1} \sim KE_i,\ E_i = |x_i - x_{i-1}| \text{ and } K = (m-1)/m,$$

where m is the multiplicity of the root. Write a matlab program to find the multiplicity of root $x = -0.5$ of the polynomial

$$x^5 - 4.5x^4 + 4.55x^3 + 2.675x^2 - 3.3x - 1.4375.$$

Choose $x = -0.6$ as the starting value and $\varepsilon = 10^{-6}$ in the absolute change stopping threshold. From the converging sequence first find the value of K and using that find the value of m.

3. (STA 5106 Students Only) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed samples from a logistic distribution with the probability density function

$$f(x \mid \theta) = \frac{\exp(-(x-\theta))}{(1+\exp(-(x-\theta)))^2}.$$

Given the values of $X_1, X_2, \ldots, X_n$ in "hw4_3_data" from the blackboard website, our goal is to find the maximum likelihood estimate (MLE) of θ, using the following steps:

(a) Derive an expression for the log likelihood function

$$l(\theta) = \sum_{i=1}^{n} \log(f(X_i \mid \theta)),$$

such that the MLE is given by

$$\hat{\theta} = \arg\max_{\theta} l(\theta).$$

(b) Find the expression for $\dot{l}(\theta)$ and $\ddot{l}(\theta)$, the first and the second derivatives of l with respect to θ. Verify that $\ddot{l}(\theta) < 0$.

(c) Use Newton-Raphson method to find a root of $\dot{l}(\theta)$. Choose $\theta_0 = 7$ as the starting value and $\varepsilon = 10^{-6}$ in the absolute change stopping threshold.