

1. The Weibull distribution is often used to model the duration of various market phenomenon. The distribution function of the Weibull is:

$$F(t) = 1 - \exp(-e^{\alpha} t^{\beta})$$

for $\beta > 0$ and $t > 0$. The Weibull reduces to the Exponential under the restriction $\beta = 1$. The parameter α is the scale parameter and β is the shape parameter. Assume that $\alpha = X_i \gamma$. This makes the scale parameter a linear function of observable regressors, X_i .

Consider the MLS data set. Let T_i denote the observed variable, “time on the market,” t_i denote the latent variable “time to sale,” and J_i denote a binary indicating whether a house is “sold.” If a house is sold, then the observed value of “time on the market” equals the latent value of “time to sale.” That is, $t_i = T_i$. If a house is unsold, then all we know is that “time to sale” exceeds “time on the market.” That is, $t_i > T_i$. This is censoring of the upper tail with an observation specific censoring threshold. Note that this is not a Tobit model. Assume that the latent variable, “time to sale,” follows the Weibull distribution.

- Find the likelihood function for γ and β .
- Find the score equations for γ and β .
- Find the Hessian matrix and show that any root to the score equations is a global MLE.
- Estimate this model using the MLS data, where X_i contains the regressors vacancy and log of asking price.

2. The *Octave* data file *martins.mat* contains information on female labor supply in Portugal. The log of the wage (in escudos per hour) is denoted LWAGE. It is observed when the binary variable EMPLOY equals one, and is missing otherwise. Years of education is denoted EDU, potential work experience is PEXP, and the number of children under 21 years of age is CHD. An interaction term for PEXP and CHD is denoted PEXPCHD. Use this data to estimate the wage equation

$$\text{LWAGE} = \alpha + \beta \text{EDU} + \delta \text{PEXP} + \gamma \text{PEXP}^2 + \theta \text{PEXPCHD} + \lambda \text{PEXP}^2 \text{CHD} + \varepsilon$$

for the common (non-zero) threshold specification.

- Construct a 95 percent confidence interval for β .
- Construct a 95 percent confidence interval for the marginal effect of a one year increase in potential work experience on the hourly wage. Evaluate potential work experience and number of children under 21 at their median values. Evaluate wages at the median value for the observed subsample.

Estimate the model using wages rather than log of wages as the dependent variable.

- Compare the estimates of β for the linear and semi-log models.
- Compare the fit of the linear and semi-log models (in terms of the maximized value of the log-likelihood function).