# Foundations for Mathematics

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# 1. Introduction

The rigorous study of mathematics is concerned with statements and the proofs of these statements. In order to build statements which are more complicated, there is a necessity to use results, statements, and concepts which have been previously encountered. Often times, understanding 'black-box' concepts in mathematics amounts to nothing more than adequately defining the pre-requisites, and studying these in detail. The idea of this course in 'Foundations' is to acquaint the interested reader with the concepts which allow for the construction of mathematical statements.

Aside from basic mathematical literacy, and a desire to understand these concepts, we make no assumption of the experience of the reader.

## 1.1 Format

This course hopes to cover the basics necessary to fully appreciate the content presented in other courses, and all of the content covered can be both stripped back and built further. We will provide references for those who wish to go further.

# 2. Logic

The field of logic is at the heart of mathematics, and essentially forms the language with which we can make mathematical statements. Logic is a deep, and well-explored area – we will explore only the beginnings of the subject.

## 2.1 Propositional logic

Propositional logic is concerned with the relationships and connections between statements which are either true or false. If we have two statements $A$ and $B$, we will consider how the respective truth of each of these statements influence the truth of statements such as,

$$A \text{ and } B, \quad A \text{ or } B, \quad \text{not } A, \quad \text{if } A \text{ then } B$$

Example 2.1. Suppose we have statement $A$ as 'I wake up late', and statement $B$ as 'I miss my train'. Think about how these statements are related to eachother. For instance, we could consider,

$$\text{if } A \text{ then } B$$

or, substituting for our plain english versions,

$$\text{if I wake up late then I miss my train}$$

which seems to make sense.

It is also interesting to consider,

$$\text{if } B \text{ then } A$$

or,

$$\text{if I miss my train then I woke up late}$$

making a small alteration in the statement to account for the english past tense. Is this statement true?

We will be concerned mostly with three relationships between statements, and it is useful to map out these relationships in what we call a truth table. Throughout the following subsections, we will be considering two statements $A$ and $B$. In order to keep the tables readable, we will now label true by $T$, and false by $F$.

### 2.1.1 Conjunction

We can, as seen, create a compound statement from statements $A$ and $B$ as,

$$A \text{ and } B$$

which is a process we sometimes refer to as conjunction.

> NOTATION. It is common in logic to denote the operation of statement conjuction by the operator $\wedge$, i.e., $A \wedge B$, although we will do fine with 'and'.

Suppose I make a small change to the document, including some new maths,

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

# 3. Set Theory

# 4. Real Analysis