Taller de Área de superficie de solidos de revolución

PARTE I

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Calculo Integral

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$f(y=0) = \text{Arctan}(0) = 0$

$f(y=\frac{\pi}{3}) = \text{Arctan}(\frac{\pi}{3}) = 0{,}8084$

Area de superficie $2\pi \int_0^{\pi/3} \tan x \sqrt{(\sec^2 x)^2 + 1}\, dx$

$y = \tan x,\ 0 \leq x \leq \pi/3$

↳ Integral difícil de resolver

$dy = \sec^2 x\, dx$

Regla del trapecio

$\frac{dy^2}{d^2x} + 1 = (\sec^2 x)^2 + 1$

$$\int_a^b f(x)dx \approx \int_a^b P_1(x)dx \approx \int_a^b \left[f(a) + \frac{f(b)-f(a)}{b-a}(x-a)\right]dx$$

$\rightarrow f(0) = \tan(0)\sqrt{\sec^4(0)+1}\,(2\pi) = 0$

$\rightarrow f(\pi/3) = \tan(\pi/3)\sqrt{\sec^4(\pi/3)+1}\,(2\pi) = 2\sqrt{51}\,\pi$

$$\approx 2\pi \int_0^{\pi/3} \left[0 + \frac{2\sqrt{51}\,\pi - 0}{\frac{\pi}{3} - 0}(x-0)\right] dx \approx 2\pi \int \frac{6\sqrt{51}\,\cancel{\pi}}{\cancel{\pi}}(x)\, dx$$

$$\approx 6\sqrt{51}\,\pi x^2 \Big|_0^{\pi/3} \approx 140{,}9660 (\text{Unidades})^2$$

Area de superficie (Eje y) = $\qquad 0{,}80844879263$

① $y = \tan x \;,\; 0 \leq x \leq \pi/3$

$f(y) = x = \operatorname{Arctan}(y)$

$dx = \dfrac{dy}{1+y^2}$

$$\rightarrow 2\pi \int_0^{\pi/3} \operatorname{Arctan}(y) \sqrt{\left(\frac{1}{1+y^2}\right)^2 + 1}\, dy$$

$$\rightarrow 2\pi \int_0^{\pi/3} \frac{\operatorname{Arctan}(y)}{1+y^2} \sqrt{\left(\frac{1}{1+y^2}\right)^2 + 1}\, dy$$

$u = \operatorname{Arctan}(y)$

$du = \dfrac{dy}{1+y^2} \quad \rightarrow 2\pi \displaystyle\int_0^{0{,}8084} \sqrt{u^2+1}\, du \rightarrow$ $\quad 1 = a$, $u = \tan\theta$, $du = \sec^2\theta\, d\theta$

$u = \operatorname{Arctan}(0) = 0$

$u = \operatorname{Arctan}(\pi/3) = 0{,}8084$

$$\rightarrow 2\pi \int \sqrt{\tan^2\theta + 1}\, \sec^2\theta\, d\theta = 2\pi \int \sec^3\theta\, d\theta \rightarrow$$

$u = \sec\theta$

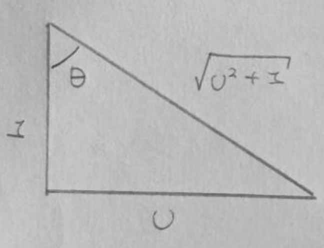
$du = \tan\theta \sec\theta$

$dv = 2\pi \sec^2\theta$

$v = 2\pi \tan\theta$

$$\rightarrow 2\pi \int \sec^3\theta\, d\theta = 2\pi \sec\theta \tan\theta - \int 2\pi \tan^2\theta \sec\theta\, d\theta$$

$$\rightarrow 2\pi \int \sec^3\theta\, d\theta = \frac{\cancel{2}\pi \sec\theta \tan\theta + \cancel{2}\pi \ln|\sec\theta + \tan\theta|}{\cancel{2}}$$



$$\rightarrow \pi\left(\sqrt{u^2+1}\cdot u + \ln\left|u + \sqrt{u^2+1}\right|\right)\Big|_0^{0{,}8084}$$

$\approx 5{,}5884\ u^2$

Ej: x =

$x = \ln|2y+1| \ , \ 0 \leq y \leq 1$

$$y = \frac{e^x - 1}{2} \rightarrow dy = \frac{1}{2}(e^x)\,dx \rightarrow \frac{dy^2}{d^2x} = \left(\frac{e^x}{2}\right)^2 + 1$$

$$Y_1 = \frac{e^0 - 1}{2} = 0$$

$$Y_2 = \frac{e^1 - 1}{2} = \frac{e-1}{2}$$

$$\int_0^{\frac{e-1}{2}} \left(\frac{e^x - 1}{2}\right)\sqrt{\left(\frac{e^x}{2}\right)^2 + 1}\,dx = \frac{1}{2}\int_0^{\frac{e-1}{2}} e^x - 1\sqrt{\frac{e^{2x}}{4} + \frac{4}{4}}\,dx$$

$$\rightarrow \frac{1}{4}\int_0^{\frac{e-1}{2}} e^x - 1\sqrt{e^{2x} + 4}\,dx \rightarrow \underbrace{\frac{1}{4}\int_0^{\frac{e-1}{2}} e^x\sqrt{(e^x)^2 + 4}\,dx}_{1} - \underbrace{\frac{1}{4}\int_0^{\frac{e-1}{2}} \sqrt{(e^x)^2 + 4}\,dx}_{2}$$

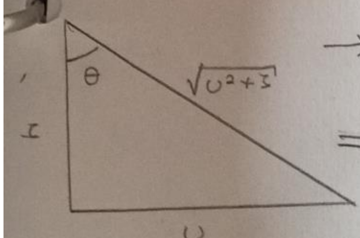
(1)

$$\rightarrow u = e^x$$

$$du = e^x dx \rightarrow \frac{1}{4}\int \sqrt{u^2 + 1}\,du \rightarrow \begin{matrix} 1 = a^2 \\ u = \tan\theta \\ du = \sec^2\theta\, d\theta \end{matrix}$$

$$\rightarrow \frac{1}{4}\int \sec^3\theta\, d\theta = \frac{\sec\theta\tan\theta}{4} - \int \tan^2\theta\sec\theta\, d\theta = \ldots$$

$$\ldots = \frac{\sec\theta\tan\theta}{4} - \int \frac{\sec^3\theta\, d\theta}{4} + \int \frac{\sec\theta\, d\theta}{4}$$



$$\rightarrow \frac{1}{4}\int \sec^3\theta\, d\theta = \frac{1}{8}\left(\tan\theta\sec\theta + \ln|\tan\theta + \sec\theta|\right)$$

$$\Longrightarrow \frac{1}{8}\left(u\sqrt{u^2+1} + \ln|u + \sqrt{u^2+1}|\right)$$

$$\Longrightarrow \frac{1}{8}\left(e^x\sqrt{e^{2x}+1} + \ln|e^x + \sqrt{e^{2x}+1}|\right)\Big|_0^{\frac{e-1}{2}}$$

$$\rightarrow \left[\frac{1}{8}\left(e^{\left(\frac{e-1}{2}\right)}\sqrt{e^{(e-1)} + 1} + \ln\left|e^{\left(\frac{e-1}{2}\right)} + \sqrt{e^{(e-1)} + 1}\right|\right)\right] - \ldots$$

$$\ldots - \left[\frac{1}{8}\left(1\sqrt{2} + \ln|1 + \sqrt{2}|\right)\right] \approx 0{,}6691\ u^2$$

② $\frac{1}{4}\int_0^{\frac{e-1}{2}} \underbrace{\sqrt{e^{2x}+4}}_{g(x)}\,dx$

$g(0) = \sqrt{e^0+4} = \sqrt{5}$

$g\left(\frac{e-1}{2}\right) = \sqrt{e^{2\left(\frac{e-1}{2}\right)}+4} \approx 3{,}09434$

$$= \frac{1}{4}\int_0^{\frac{e-1}{2}} \sqrt{5} + \frac{\sqrt{e^{e-1}+4}-\sqrt{5}}{\frac{e-1}{2}-0}(x-0)\,dx$$

$$= \frac{1}{4}\int_0^{\frac{e-1}{2}} \sqrt{5} + \frac{2x}{(e-1)/\left(\sqrt{e^{e-1}+4}-\sqrt{5}\right)}\,dx = \frac{1}{4}\left(\sqrt{5}x + \frac{x^2}{(e-1)/\left(\sqrt{e^{e-1}+4}-\sqrt{5}\right)}\right)\Bigg|_0^{\frac{e-1}{2}}$$

$\approx 0{,}605401$

① $\approx 0{,}669141$

② $\approx 0{,}605401$

$\boxed{③ \approx 0{,}06374}$ $\rightarrow R = 0{,}0637\ u^2$

$4.\ x = \ln|2y+1|,\ 0 \leqslant y \leqslant 1$

$$dx = \frac{1}{2y+1} \cdot 2\,dy \quad \Big| \quad \frac{dx^2}{d^2y} + 1 = \left(\frac{2}{2y+1}\right)^2 + 1 \to \frac{4+(2y+1)^2}{(2y+1)^2}$$

$$2\pi \int_0^1 \ln|2y+1| \sqrt{\left(\frac{4+(2y+1)^2}{(2y+1)^2}\right)}\,dy \to 2\pi \int_0^1 \frac{\ln|2y+1|}{2y+1} \sqrt{4+(2y+1)^2}\,dy$$

$u = 2y+1$

$du = 2dy \to dy = \frac{du}{2}$

$u = 2(1)+1 = 3$

$u = 2(0)+1 = 1$

$$\to \pi \int_1^3 \frac{\ln|u|}{u} \sqrt{4+(u)^2}\,du$$

$\hookrightarrow$ Integral complicada de resolver

$$f(1) = \frac{\ln|1|}{1} \sqrt{4+1^2}\,(\pi) = 0$$

$$f(3) = \frac{\ln|3|}{3} \sqrt{4+9}\,(\pi) = \frac{1}{3} \ln|3| \sqrt{13}\,\pi$$

$$\to \approx \pi \int_1^3 0 + \frac{\frac{\sqrt{13}\ln|3|\pi}{3} - 0}{3-1}(x)\,dx \to \pi \int_1^3 \frac{\ln|3|\sqrt{13}\,\pi}{6} x\,dx$$

$$\approx \pi \cdot \frac{\ln|3|\sqrt{13} \cdot \pi}{3} x^2 \Big|_1^3 = \left[\left(\frac{\pi^2 \ln|3|\sqrt{13} \cdot 9}{3}\right) - \left(\frac{\pi^2 \ln|3|\sqrt{13}}{3}\right)\right]$$

$$\approx \frac{8\pi^2\sqrt{13}\,\ln|3|}{3} = 243{,}3990$$