Taller de Área de superficie de solidos de revolución

PARTE II

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Calculo Integral

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Area eje x

7. $y = \sqrt{1 + 4x}$, $1 \leq x \leq 5$

$x = \frac{y^2 - 1}{4} \rightarrow \frac{dx^2}{d^2y} + 1 = \left(\frac{\cancel{2}y}{\cancel{4}}\right)^2 + 1 = \frac{y^2}{4} + 1 = \frac{y^2 + 4}{4}$

$\rightarrow 2\pi \int y \sqrt{\frac{y^2 + 4}{4}}\, dy =$ $u = y^2 + 4$ $\quad u = (1)^2 + 4 = 5$

$du = 2y\,dy$ $\quad u = (5)^2 + 5 = 30$ $\quad \rightarrow \pi \int_5^{30} \sqrt{\frac{u}{4}}\, du$

$\frac{du}{2} = y\,dy$

$\rightarrow \frac{\pi}{2} \int_5^{30} \sqrt{u}\, du = \frac{\pi}{\cancel{2}} \left(\frac{\cancel{2}}{3} u^{3/2}\right)\Big|_5^{30} = \pi \left[\left(\frac{(\sqrt{30})^3}{3}\right) - \left(\frac{(\sqrt{5})^2}{3}\right)\right]$

$\rightarrow \pi \left(\frac{30\sqrt{30} - 5\sqrt{5}}{3}\right) = \pi \left(10\sqrt{30} - \frac{5\sqrt{5}}{3}\right)$

10. $y = \frac{x^3}{6} + \frac{1}{2x}$, $\frac{1}{2} \leq x \leq 1$

$dy = \frac{x^2}{2} - \frac{1}{2x^2}\, dx \rightarrow \frac{dy^2}{d^2x} + 1 = \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2 + 1$

$= \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^2} + 1 = \frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^2} = \left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2$

$\rightarrow 2\pi \int_{1/2}^{1} \left(\frac{x^3}{6} + \frac{1}{2x}\right) \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}\, dx = 2\pi \int_{1/2}^{1} \left(\frac{x^3}{6} + \frac{1}{2x}\right)\left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx$

$\rightarrow 2\pi \int_{1/2}^{1} \frac{x^5}{12} + \frac{x}{12} + \frac{x}{4} + \frac{1}{4x^3}\, dx \rightarrow 2\pi \int_{1/2}^{1} \frac{x^5 + x}{12} + \frac{3x}{12} + \frac{1}{12x^3}\, dx$

$$\rightarrow 2\pi \int_{1/2}^{1} \left(\frac{1}{12}\right)\left(x^3 + x + 3x + \frac{1}{x}\right) dx \rightarrow \frac{\pi}{6}\left(\int_{1/2}^{1} x^3\, dx + \int_{1/2}^{1} 4x\, dx + \int_{1/2}^{1} \frac{dx}{x}\right)$$

$$\rightarrow \frac{\pi}{6}\left(\frac{x^4}{4} + 2x^2 + \operatorname{Ln}|x|\right)\Bigg|_{1/2}^{1} \Rightarrow \frac{\pi}{6}\left[\left(\frac{1}{4} + 2 + \operatorname{Ln}|1|\right) - \left(\frac{\frac{1}{16}}{4} + \frac{2}{4} + \operatorname{Ln}\left|\frac{1}{2}\right|\right)\right]$$

$$\rightarrow \frac{\pi}{6}\left(\frac{9}{4} - \frac{1}{64} - \frac{1}{2} - \operatorname{Ln}\left|\frac{1}{2}\right|\right) \rightarrow \frac{\pi}{6}\left(\frac{16\cdot 9}{64} - \frac{1}{64} - \frac{32}{64} - \operatorname{Ln}\left|\frac{1}{2}\right|\right)$$

$$\rightarrow \frac{\pi}{6}\left(\frac{144}{64} - \frac{33}{64} - \operatorname{Ln}\left|\frac{1}{2}\right|\right) = \frac{\pi}{6}\left(\frac{111}{64} - \operatorname{Ln}\left|\frac{1}{2}\right|\right)$$

11. $x = \frac{1}{3}(y^2+2)^{3/2}, \quad y \in [1,2]$

$$dx = \frac{\cancel{3}}{\cancel{2}}\left(\frac{1}{\cancel{3}}\right)(y^2+2)^{1/2}(\cancel{2}y) = y\sqrt{y^2+2}$$

$$\rightarrow 2\pi \int_{1}^{2} \left(\frac{\left(\sqrt{y^2+2}\right)^3}{3}\right)\sqrt{\left(y\sqrt{y^2+2}\right)^2 + 1}\; dy$$

$$f(1) = 2\pi\left(\frac{(\sqrt{3})^3}{3}\right)\sqrt{(\sqrt{3})^2 + 1} \rightarrow 2\pi\left(\frac{3\sqrt{3}}{3}\right)\sqrt{4} = 4\sqrt{3}\pi$$

$$f(2) = 2\pi\left(\frac{(\sqrt{6})^3}{3}\right)\sqrt{(2\sqrt{6})^2 + 1} \rightarrow 2\pi\left(\frac{6\sqrt{6}}{3}\right)\sqrt{4(6)+1} = 20\sqrt{6}\pi$$

$$\rightarrow \int_{1}^{2} 4\sqrt{3}\pi + \frac{20\sqrt{6}\pi - 4\sqrt{3}\pi}{2-1}(x-1)\, dx$$

$$\rightarrow \int_{1}^{2} \cancel{4\sqrt{3}\pi} + 20\sqrt{6}\pi - \cancel{4\sqrt{3}\pi}(x-1)\, dx \rightarrow \int_{1}^{2} 20\sqrt{6}\pi\,(x-1)\,dx$$

$$\rightarrow 20\sqrt{6}\pi\left(\frac{x^2}{2} - x\right)\Bigg|_{1}^{2} = 20\sqrt{6}\pi\left[\left(\cancel{\frac{4}{2} - 2}\right)^{0} - \left(\frac{1}{2} - 1\right)\right]$$

$$\rightarrow 10\sqrt{6}\pi \quad //$$