Taller Longitud de Arco

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Calculo Integral

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PARTE I:

I. Calcular la longitud del arco de la curva dada entre los valores dados

1. $30xy^3 - y^8 = 15$; $A = \left(\frac{8}{15}, 1\right)$ y $B = \left(\frac{271}{240}, 2\right)$

$y^3(30x - y^5) = 15$

$30x = \left(\frac{15}{y^3} + y^5\right) \rightarrow x = \left(\frac{1}{30}\right)\left(\frac{15}{y^3} + y^5\right) \rightarrow x = \frac{1}{2y^3} + \frac{y^5}{30}$

$\frac{dx}{dy} = \left(\frac{1}{2y^3} + \frac{y^5}{30}\right)' = \frac{(1)'(2y^3) - (1)(2y^3)'}{(2y^3)^2} + \frac{5}{30}y^4$

$= \frac{-\cancel{6}y^2}{\cancel{4}y^6} + \frac{y^4}{6} = \frac{y^4}{6} - \frac{3}{2y^4}$ // $\frac{dx^2}{d^2y} = \left(\frac{y^4}{6} - \frac{3}{2y^4}\right)^2 = \frac{y^8}{36} - 2\left(\frac{y^4}{6} \cdot \frac{3}{2y^4}\right) + \frac{9}{4y^8}$

$= \frac{y^8}{36} - \frac{1}{2} + \frac{9}{4y^8}$, Longitud del arco $= \int_1^2 \sqrt{1 + \left(\frac{y^8}{36} - \frac{1}{2} + \frac{9}{4y^8}\right)}\,dy$

cuadrado perfecto

$= \int_1^2 \sqrt{\frac{y^8}{36} + \frac{1}{2} + \frac{9}{4y^8}}\,dy = \int_1^2 \sqrt{\left(\frac{y^4}{6} + \frac{3}{2y^4}\right)^2}\,dy = \frac{1}{6}\int_1^2 y^4\,dy + \frac{3}{2}\int_1^2 y^{-4}\,dy$

$= \left[\frac{1}{6}\left(\frac{y^5}{5}\right) + \frac{3}{2}\left(\frac{y^{-3}}{-3}\right)\right]\Big|_1^2 = \left[\frac{y^5}{30} - \frac{1}{2y^3}\right]\Big|_1^2 = \left[\left(\frac{2^5}{30} - \frac{1}{2(2)^3}\right) - \left(\frac{1^5}{30} - \frac{1}{2(1)^3}\right)\right]$

$= \frac{32}{30} - \frac{1}{16} - \frac{1}{30} + \frac{1}{2} = \frac{31}{30} - \frac{1}{16} + \frac{1}{2} = \frac{31}{30} + \frac{7}{16} = \frac{496 + 210}{480}$

$\frac{706}{480} = \frac{353}{240}$ Unidades //

3. $y = \ln\left(\frac{e^x+1}{e^x-1}\right)$; $a \le x \le b$; $a, x > 0$

$$dy = \frac{1}{\left(\frac{e^x+1}{e^x-1}\right)} \cdot \left(\frac{(e^x+1)'(e^x-1)-(e^x+1)(e^x-1)'}{(e^x-1)^2}\right) dx$$

$$= \frac{\cancel{e^x-1}}{e^x+1}\left(\frac{e^x(e^x-1)-(e^x+1)e^x}{(e^x-1)^{\cancel{2}}}\right) dx$$

$$= \frac{e^x}{e^x+1}\left(\frac{(e^x-1)}{(e^x-1)} - \frac{e^x+1}{(e^x-1)}\right) dx = \frac{e^x}{e^x+1} - \frac{e^x}{(e^x-1)} dx$$

$$\frac{dy^2}{d^2x} = \left(\frac{e^x}{(e^x+1)} - \frac{e^x}{(e^x-1)}\right)^2 = \left(\frac{e^x(e^x-1)-(e^x(e^x+1))}{(e^x+1)(e^x-1)}\right)^2$$

$$= \left(\frac{e^{2x}-e^x-e^{2x}-e^x}{e^{2x}-1}\right)^2 = \left(\frac{-2e^x}{e^{2x}-1}\right)^2 = \frac{4e^{2x}}{(e^{2x}-1)^2}$$

Nota=

$(e^{2x}-1)^2 = e^{4x} - 2e^{2x} + 1$

Longitud de Arco =

$$\int_a^b \sqrt{1+\left(\frac{4e^{2x}}{(e^{2x}-1)^2}\right)}\, dx = \int_a^b \sqrt{\frac{(e^{2x}-1)^2+4e^{2x}}{(e^{2x}-1)^2}}\, dx$$

$$= \int_a^b \frac{\sqrt{(e^{2x}-1)^2+4e^{2x}}}{e^{2x}-1}\, dx = \int_a^b \frac{\sqrt{(e^{4x}-2e^{2x}+1)+4e^{2x}}}{e^{2x}-1}\, dx$$

$$= \int_a^b \frac{\sqrt{(e^{4x}+2e^{2x}+1)}}{e^{2x}-1}\, dx = \int_a^b \frac{\sqrt{(e^{2x}+1)^2}}{e^{2x}-1}\, dx$$

$$= \int_a^b \frac{e^{2x}+1}{(e^{2x}-1)}\,dx = \underbrace{\int_a^b \frac{e^{2x}}{(e^{2x}-1)}\,dx}_{1} + \underbrace{\int_a^b \frac{dx}{(e^{2x}-1)}}_{2}$$

$$1)\ \int_a^b \frac{e^{2x}}{e^{2x}-1}\,dx \rightarrow \begin{array}{l} u = e^{2x}-1 \\ du = 2e^{2x}\,dx \end{array} \rightarrow \frac{1}{2}\int_a^b \frac{2e^{2x}}{e^{2x}-1}\,dx \rightarrow \frac{1}{2}\int_a^b \frac{du}{u} = \frac{\ln|e^{2x}-1|}{2}\Bigg|_a^b$$

$$2)\ \int_a^b \frac{1}{e^{2x}-1}\,dx \quad \text{Dado que } d\left(\frac{1}{e^{2x}-1}\right) = -2e^{-2x} \rightarrow \int_a^b \frac{1}{e^{2x}-1}\left(\frac{-2e^{-2x}}{-2e^{-2x}}\right)dx$$

$$\rightarrow \int_a^b \frac{\cancel{2}\,e^{-2x}\,dx}{\cancel{2}\,(1-e^{-2x})} \Rightarrow \begin{array}{l} u = 1-e^{-2x} \\ du = -e^{-2x}(-2)\,dx \\ \hookrightarrow = 2e^{-2x}\,dx \end{array} \rightarrow \frac{1}{2}\int_a^b \frac{du}{u} = \frac{\ln|1-e^{-2x}|}{2}\Bigg|_a^b$$

$$1)+2) = \frac{1}{2}\left(\ln|e^{2x}-1| + \ln|1-e^{-2x}|\right)\Bigg|_a^b$$

$$= \frac{1}{2}\left(\ln|e^{2x}-1| + \ln\left|1-\frac{1}{e^{2x}}\right|\right)\Bigg|_a^b = \frac{1}{2}\left(\ln|e^{2x}-1| + \ln|e^{2x}-1| - \ln e^{2x}\right)\Bigg|_a^b$$

$$= \frac{1}{2}\left(2\ln|e^{2x}-1| - 2x\right)\Bigg|_a^b = \ln|e^{2x}-1| - x\Bigg|_a^b$$

$$= \ln|e^{2b}-1| - b - \ln|e^{2a}-1| + a = \ln\left|\frac{e^{2b}-1}{e^{2a}-1}\right| + a - b\ ;\ a>0$$

6. $8y = x^4 + 2x^{-2}$, $1 \le x \le 2$

$y = \frac{x^4}{8} + \frac{1}{4x^2}$, $dy = \frac{x^3}{2} - \frac{1}{2x^3}dx$, $\frac{dy^2}{d^2x} = \frac{x^6}{4} - \frac{1}{2} + \frac{1}{4x^6}$

$\frac{dy^2}{d^2x} + 1 = \frac{x^6}{4} - \frac{1}{2} + 1 + \frac{1}{4x^6} = \frac{x^6}{4} + \frac{1}{2} + \frac{1}{4x^6}$

Longitud desde 1 a 2 = $\int_1^2 \sqrt{\left(\frac{x^6}{4} + \frac{1}{2} + \frac{1}{4x^6}\right)}\,dx = \int_1^2 \sqrt{\left(\frac{x^3}{2} + \frac{1}{2x^3}\right)^2}\,dx$

$= \int_1^2 \frac{x^3}{2}\,dx + \int_1^2 \frac{1}{2x^3}\,dx = \frac{x^4}{8} - \frac{1}{4x^2}\Big|_1^2 = \left(\frac{16}{8} - \frac{1}{16}\right) - \left(\frac{1}{8} - \frac{1}{4}\right)$

$= \left(2 - \frac{1}{16}\right) - \left(-\frac{1}{8}\right) = \frac{31}{16} + \frac{2}{16} = \frac{33}{16}$ unidades

9. $y = \frac{x^3}{6} + \frac{1}{2x}$; $A\left(\frac{1}{2}, \frac{49}{48}\right)$ y $B = \left(1, \frac{2}{3}\right)$

$dy = \frac{x^2}{2} - \frac{1}{2x^2}dx$; $\frac{dy^2}{d^2x} = \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2 = \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}$

$\frac{dy^2}{d^2x} + 1 = \frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4} = \left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2$

Longitud de arco = $\int_{\frac{1}{2}}^{1} \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}\,dx = \frac{x^3}{6} - \frac{1}{2x}\Big|_{1/2}^{1}$

$= \left(\frac{1}{6} - \frac{1}{2}\right) - \left(\frac{\left(\frac{1}{8}\right)}{6} - \frac{1}{2\left(\frac{1}{2}\right)}\right) = \left(-\frac{1}{3}\right) - \left(\frac{1}{48} - 1\right) = -\frac{1}{3} + \frac{47}{48}$

$= \frac{141 - 48}{144} = \frac{93}{144} = \frac{31}{48}$

17. $y = \frac{x^2}{2} - \frac{\ln|x|}{4}$; $2 \leq x \leq 4$

$dy = x - \frac{1}{4x} dx$; $\frac{dy^2}{d^2x} = \left(x - \frac{1}{4x}\right)^2 + 1 = x^2 - \frac{1}{2} + \frac{1}{16x^2} + 1$

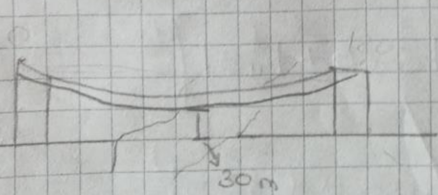
$= x^2 + \frac{1}{2} + \frac{1}{16x^2} = \left(x + \frac{1}{4x}\right)^2$

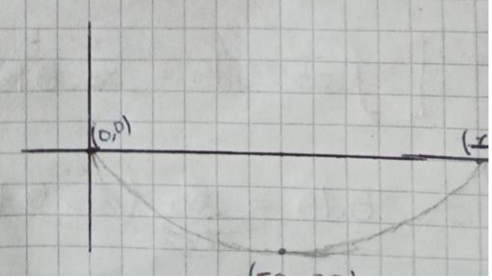
Longitud de Arco = $\int_2^4 \sqrt{\left(x + \frac{1}{4x}\right)^2}\, dx = \frac{x^2}{2} + \frac{\ln|x|}{4}\Big|_2^4$

$= \left(8 + \frac{\ln|4|}{4}\right) - \left(2 + \frac{\ln|2|}{4}\right) = 6 + \frac{\ln|2|}{4}$

II)

3. Los cables de soporte de un puente colgante sobre un río, tiene
sus apoyos anclados a la misma altura. Por ley natural, t
forma parabólica, con una luz de 120 metros, estando el
punto más bajo de los cables a un nivel de 30 metros
los puntos de soporte. Si la densidad lineal del cable es
de 14 (Kg/m), hallar el peso total de uno de los cables.

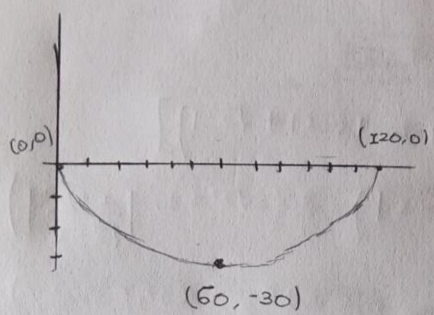




PARTE II:

Taller

①



$f(x) = A(x-0)(x-120)$

Cuando $x = 60$, $f(x) = -30$

$A(60)(60-120) = -30$

$A(-3600) = -30 \rightarrow A = \frac{1}{120}$

$\frac{1}{120}(x^2 - 120x) = \frac{x^2}{120} - x$

$\rightarrow f'(x) = \frac{x}{60} - 1 \rightarrow [f'(x)]^2 + 1 = \left(\frac{x}{60} - 1\right)^2 + 1$

Longitud de arco $= \int_0^{120} \sqrt{\left(\frac{x}{60} - 1\right)^2 + 1}\, dx \rightarrow u = \frac{x}{60} - 1$; $u = \frac{0}{60} - 1 = -1$

$du = \frac{dx}{60}$; $u = \frac{120}{60} - 1 = 1$

$60du = dx$

$\int_{-1}^{1} \sqrt{u^2 + 1}\; 60du \rightarrow$ $a^2 = 1$, $u = \tan\theta$, $du = \sec^2\theta\, d\theta$ $\rightarrow \int \sqrt{\tan^2\theta + 1}\; 60 \sec^2\theta\, d\theta$

$\rightarrow \int \sec\theta\, 60 \sec^2\theta\, d\theta \rightarrow$ $u = \sec\theta$, $du = \tan\theta \sec\theta\, d\theta$, $dv = 60 \sec^2\theta\, d\theta$, $v = 60 \tan\theta$ $= 60 \tan\theta \sec\theta - \int 60 \tan\theta \tan\theta \sec\theta\, d\theta$

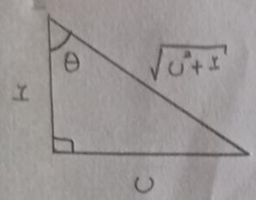
$\rightarrow \int \sec\theta\, 60 \sec^2\theta\, d\theta = 60 \tan\theta \sec\theta - \int 60(\sec^2\theta - 1) \sec\theta\, d\theta$

$\rightarrow 60 \int \sec^3\theta\, d\theta = 60 \tan\theta \sec\theta - \int 60 \sec^3\theta\, d\theta + 60 \int \sec\theta\, d\theta$

$\rightarrow 2 \cdot 60 \int \sec^3\theta\, d\theta = 60 \tan\theta \sec\theta + 60 \text{Ln}|\sec\theta + \tan\theta|$

$\rightarrow 60 \int \sec^3\theta\, d\theta = 30(\tan\theta \sec\theta + \text{Ln}|\sec\theta + \tan\theta|)$

Arbol binario



$\tan\theta = u$

$\sec\theta = \sqrt{u^2+1}$

$\rightarrow \frac{30}{}\left(u\sqrt{u^2+1} + \ln\left|u + \sqrt{u^2+1}\right|\right)\Big|_{-1}^{1}$

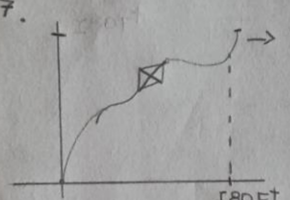
$= \left[30\left(1\sqrt{2} + \ln|1+\sqrt{2}|\right) - 30\left((-1)\sqrt{2} + \ln|(-1)+\sqrt{2}|\right)\right]$

$= 30\left(\sqrt{2} + \ln|1+\sqrt{2}| + \sqrt{2} - \ln|\sqrt{2}-1|\right) = 30\left(2\sqrt{2} + \ln\left|\frac{1+\sqrt{2}}{\sqrt{2}-1}\right|\right)$

$\approx 137{,}73523m \rightarrow \rho = \frac{m}{L} \Rightarrow m = \rho L \rightarrow m = 14\ kg/m\ (137{,}73523\ m)$

$R| = 1928{,}29\ kg$

7\.



$\rightarrow y = 150 - \frac{(x-50)^2}{40} \rightarrow dy = -\frac{2}{40}(x-50)_{dx} = \frac{-x}{20} + \frac{5}{2}dx$

$= \frac{5}{2} - \frac{x}{20}dx$

$\frac{dy^2}{d^2x} + 1 = \left(\frac{5}{2} - \frac{x}{20}\right)^2 + 1 = \left(\frac{25}{4} - \frac{1}{4} + \frac{x^2}{400}\right) + 1$

fácil — difícil

$\rightarrow \int_0^{180} \sqrt{\left(\frac{5}{2} - \frac{x}{20}\right)^2 + 1}\, dx \Rightarrow u = \frac{5}{2} - \frac{x}{20}$; $u = \frac{5}{2} - \frac{180}{20} = -\frac{13}{2}$

$du = -\frac{dx}{20}$; $u = \frac{5}{2} - 0$

$dx = -20du$

$\rightarrow \int_{\frac{5}{2}}^{-\frac{13}{2}} \sqrt{u^2+1}\,(-20du) \Rightarrow 20\int_{-\frac{13}{2}}^{\frac{5}{2}} \sqrt{u^2+1}\,du \rightarrow$

$a^2 = 1$

$u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$$\rightarrow 20\int \sqrt{\tan^2\theta + 1}\,\sec^2\theta\, d\theta = 20\int \sec^3\theta\, d\theta \Rightarrow$$

$u = \sec\theta$

$du = \tan\theta\sec\theta\, d\theta$

$dv = 20\sec^2\theta\, d\theta$

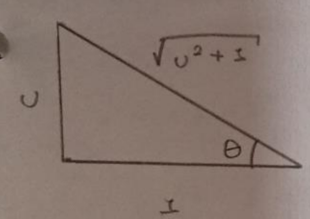
$v = 20\tan\theta$

$$\rightarrow 20\int \sec^3\theta\, d\theta = 20\tan\theta\sec\theta - \int 20\tan^2\theta\sec\theta\, d\theta$$

$$\rightarrow 20\int \sec^3\theta\, d\theta = 20\tan\theta\sec\theta - 20\int \sec^3\theta\, d\theta + 20\int \sec\theta\, d\theta$$

$$\rightarrow 20\int \sec^3\theta\, d\theta = 10\left(\tan\theta\sec\theta + \ln|\sec\theta + \tan\theta|\right)$$

$$10\left(u\sqrt{u^2+1} + \ln\left|\sqrt{u^2+1} + u\right|\right)\Big|_{-13/2}^{5/2}$$



$$\rightarrow \left[10\left(\frac{5}{2}\sqrt{\frac{25}{4}+\frac{4}{4}} + \ln\left|\sqrt{\frac{29}{4}} + \frac{5}{2}\right|\right)\right] - \cdots$$

$$\cdots - \left[10\left(-\frac{13}{2}\sqrt{\frac{169}{4}+\frac{4}{4}} + \ln\left|\sqrt{\frac{173}{4}} - \frac{13}{2}\right|\right)\right]$$

$$\rightarrow \left[10\left(\frac{5\sqrt{29}}{2} + \ln\left|\frac{\sqrt{29}+5}{2}\right|\right)\right] - \left[10\left(-\frac{13\sqrt{173}}{2} + \ln\left|\frac{\sqrt{173}-13}{2}\right|\right)\right]$$

$$\rightarrow \left(25\sqrt{29} + 65\sqrt{173} + 10\ln\left|\frac{\sqrt{29}+5}{\sqrt{173}-13}\right|\right) \approx 1031,7511$$