The Harmonic Algorithm

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"**HARMONIC**, NOUN: *A NOTE PRODUCED ON A MUSICAL INSTRUMENT AS AN OVERTONE*"

"**HARMONIC**, ADJECTIVE: *RELATING TO OR CHARACTERISED BY HARMONY*"

"**ALGORITHM**, NOUN: *A PROCESS OR SET OF RULES TO BE FOLLOWED IN CALCULATIONS OR OTHER PROBLEM-SOLVING OPERATIONS*"

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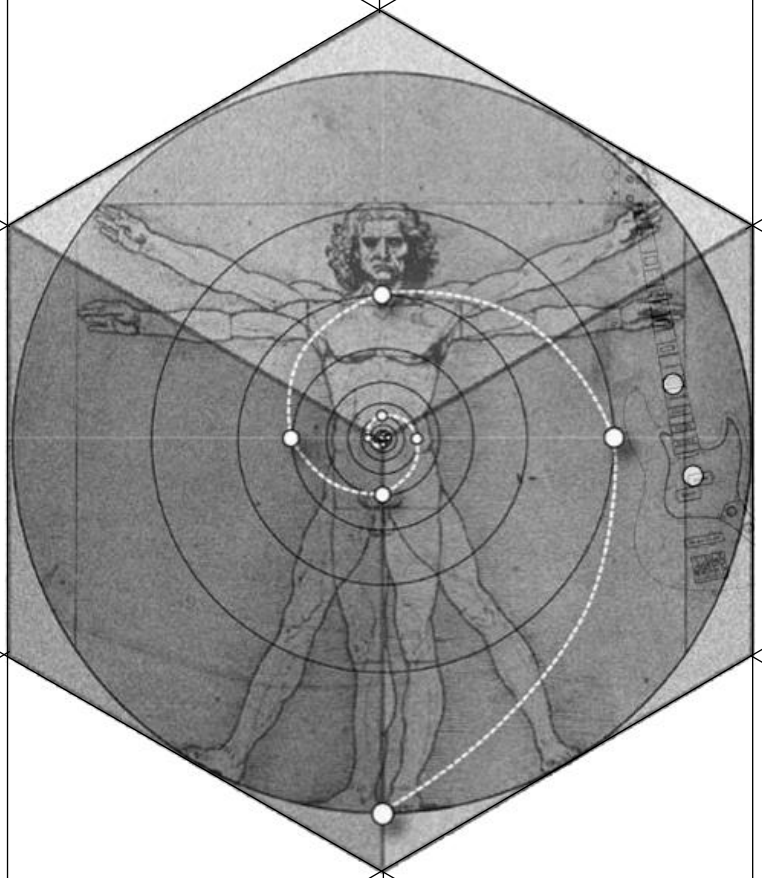


Foreword

This work as a whole is interested specifically in the nature of practice; it's primary purpose is to create a practising process which will lead to new operationally functional understandings and developments which are tangibly useful in performance. Other peripheral aims include targeting specific desirable side benefits of the practising process such as gaining an intimate knowledge of the technical and harmonic possibilities available on the instrument, creating a repeatable process for determining the complete tonal 'harmonic' potential of a given stringed instrument and contributing a valid development to the niche field of performance in general.

A necessity in achieving the objectives set for this project is the incorporation of practise itself as an integral part of the method. The project is designed with this in mind and this document will serve to frame, focus, document and expand on the discoveries and developments made throughout the practising process. There will be a further 'reflective' document, which will reflect on and discuss the process retrospectively. In order to facilitate the most effective results inside the scope of the work and to generate the most profound discoveries in performance, a significantly exhaustive and methodical analysis will be carried out on the harmonic potential of the instrument when played in this style. This is the titular 'Harmonic Algorithm', which is not so much a data in-data out algorithm but rather a "process or set of rules" by which the desired effect can be achieved. An exhaustive method for this process has been chosen due to the presence of overtones on a strung instrument as a by-product of its physical sound production method (as opposed to the more logically practical possibilities presented by stopped notes). This unintuitive sound production method (further compounded by the preparations made to the instrument discussed in Section One) may result in potential harmonic possibilities becoming obfuscated or obscure; thus, this process will allow all possibilities within the defined limits of the analysis to be clearly determined.

In contrast to these highly theoretical analysis sections, the latter sections of the work will be primarily concerned with documenting and presenting the practical applications and possibilities presented by the prior results. This will be heavily led by and centred on musical examples demonstrating practical extrapolations from the analytical sections. Musical examples will present the concepts in a visual form as well as providing audio recordings to demonstrate the sounds achieved. The exploration will be open ended and inquisitive within the realms of the work's scope and will specifically focus on building a fundamental and practical basis for the application of the research into a performance setting.

Of relevant note in this work is the distinction in definition between 'bass playing' (referring to providing the musical/compositional bass role, primarily in an ensemble setting) and the playing of 'a bass' (referring to the playing of a bass family instrument abstractly, with no specific implication towards a given musical role). This work will view the bass guitar as an instrument of creative freedom, both as a multiple voiced 'self-accompanying' solo instrument (such as a piano or fingerpicked guitar) and as an ensemble instrument of extended possibility. Advanced or original techniques utilised which go beyond the scope of conventional bass playing will be discussed in Section One.

The reasoning behind the decision to call this process an 'algorithm' is partially but not solely artistic; it highlights an additional functional purpose. Although the task is being approached from my own personal perspective and developmental requirements as a performer, to be truly useful the process needs to be clearly defined enough that it presents a deeper level of insight or practical value for other performers approaching this technical area. In order to achieve this, the process will be developed in such a way that it can be repeated to create the same outcome for any individual in his own specific context of usage. Whether or not this occurs is unimportant, as hopefully the clarity in documentation throughout which this will provide will result in a work that is interesting and revealing in its process. With this in mind, three specific requirements will be established for the analytical sections:

1. *A clearly defined process by which the desired information can be extrapolated.*
2. *Organisation of the raw data into an accessible and useful format by a process of practical musical consideration.*
3. *Exploration and development of the practicalities presented by the process and results of objectives '1.' And '2.'*

Ultimately, music requires a human component. This statement will permeate the work throughout and is specifically poignant in the following analytically focussed chapters. The reason being that music doesn't come from analysis; rather analysis comes from music. It is important to keep in mind that the purpose of analysis is not to search for any new or innovative musical discovery but to reveal possibilities which can be used in practice, throughout the human creative process of exploration and discovery.

Contents

Tuning & Preparations

This section will discuss the tuning methods and instrumental preparations that I have chosen to use personally in the course of the present work. Extended tunings and preparations are not specifically required for application of the work's research. In addition to standard tuning also being covered, the work is designed so that it's method can be re-purposed for application to any chosen tuning system.

Tuning

The fourths based tuning of an electric bass is firmly established and having been adopted from double bass, even predates the instrument itself. In my own professional practice much of the work is indeed performed on a standardly tuned instrument. With this in mind, no argument against the use of standard tuning will be presented and the work will incorporate standard tuning into its method.

However, the strength of standard tuning should by no means eliminate possibility for exploration and utilisation of alternatives. In this study, the practicality of another tuning system which I have adopted in order to allow myself to perform extended technique playing more effectively will be the primary vessel for application of the analysis.

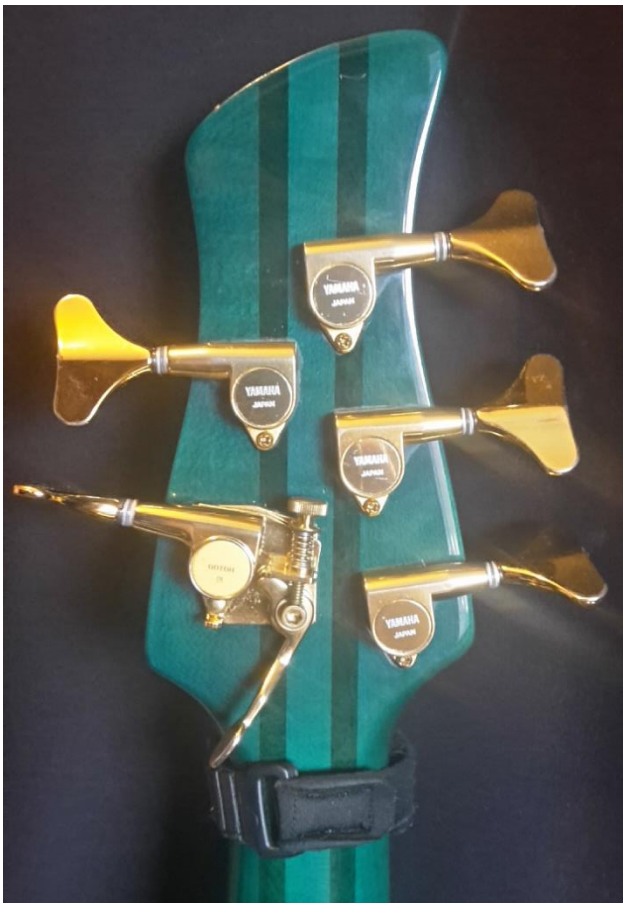
The tuning in question is 'E, A, e, G, b/C (the b/C representing a dynamically re-tuneable string and a lower case letter representing a deviation from standard tuning). This tuning will hereby be referred to as Electric Contrabass Cittern tuning or 'ecbc', for no reason aside from differentiation from standard tuning and a long running joke relating to performance of this instrument within a folk music context. The original purpose for this tuning was in development of two handed tapping techniques, as the top three strings when b/C is set to B not only form a minor triad (E, G, B) but also allow access to any major or minor triadic inversion with no more than a 2 fret span whilst retaining a 'standard' E and A string for bass movement. Through juxtaposition of basic triads over various bass tones, a large number of useful harmonic variations are possible. The present study takes this principle and expands it further, adopting overtones rather than tapped stopped notes. An interesting point of note regarding this tuning is that as is it built from irregular intervals (fourth, fifth, flat third, natural third). It cannot be called an 'open' tuning but it is neither based on any specific intervallic logic. Rather than being unwieldy, this format presents some very useful musical possibilities especially when working with smaller 'cells' of notes such as tetrachords (diatonic four note clusters which can be derived from diatonic or pentatonic modes). This area will not be discussed in detail in the process of this work but is of value to mention for its potential application when utilising artificial harmonics.

The overtone series' available in this tuning were not chosen in a calculated manner, however the triadic nature of the tuning presents a rich and expanded 'palate' of pitches to work with. The re-tuner (discussed below) was added to move the upper three strings from a minor triad in root position into a major triad in first inversion, in the process presenting an alternative 'composite' overtone series with its own unique possibilities (discussed in Section Two).

Preparations

In order to access a greater range of overtones on my instrument, I utilise a tool called a 'Hipshot D-Tuner'. It is a small lever system intended for dropping a low E string to D or C in order to play in these common re-tunings without stopping to tune re-tune. In the application for the present work the re-tuner is placed on the highest string rather than lowest, allowing access to another series of natural harmonics without having to manually retune the instrument.

Various alternatives to the re-tuning format were tried before settling on the final version. The re-tuner is capable of a maximum of three positions, and originally the plan was to retune between B, C and D. Much analysis was performed on the D position in preparation for this but unfortunately proved to be impossible as the re-tuner lacked the range to reach a whole tone above the C position. Another three position alternative (A#/B/C) was attempted, however while it was possible and provided some interesting diminished and dominant options, it was unwieldy to perform with and required an extreme level of upkeep that did not suit live performance. In the end, B/C was chosen for its suitability in performance, its musical practicality (the musical effect itself of moving smoothly between the B/C, F#/G and D#/E overtones should not be overlooked) and enhancement to the overall harmonic potential that this format provides.



Other preparations that should be mentioned include a soft muting device placed at the nut end of the neck. This kind of tool (or similar things such as hairbands) are commonly used by extended technique players to dampen strings that are not in use. When performing with harmonics the tool gains an additional function: when placed in a mildly muted position it changes the character of harmonics from resonant and sustained to short and percussive, somewhat reminiscent of that of orchestral pitched percussion instruments.

The pickup system is also worthy of mention for its practicality in performance. Rather than purely magnetic pickups, I use a blend of piezo and magnetic (Fender Jazz style bridge pickup). This gives two specific benefits:

1. A piezo pickup provides a much more level representation of the frequency range when compared to magnetic pickups. By primarily using the piezo signal with some magnetic pickup blended in for 'body', the instrument amplifies without accompaniment (or blends in an acoustic situation) far more effectively and in a more musically pleasing manner. The
2. The most convenient position to hold an E-Bow (discussed below) on most electric basses is directly over the neck pickup, however with magnetic pickups this results in extremely loud and distorted signals. By using mostly or entirely piezo signal, it is possible to perform using clean or distorted sounds at usable volume levels, without the need for dynamic volume adjustments.

Temperament & Tension

This section will discuss the mathematical minutia of stringing and fine-tuning the prepared instrument (discussed in the previous subsection). While these details may seem trivial or insignificant, it is my belief that attention to these kinds of minuscule details greatly contributes towards the subjective 'magic' that can be tangibly felt when experiencing great music. The creation of subjectively great music is in some form the ultimate aim of all developing musicians.

For standard tuning it may not be necessary (depending on personal preference) to consider these factors in the same level of detail. In my personal use case of standard tuning, the playability, intonation and tone of stopped notes is of most importance, with overtones being utilised for extra texture and ornamentation. For this reason, for standard tuning I choose my string type and gauge based on general playability/tone and tune the instrument to equal temperament for the most in tune intonation for stopped notes.

In 'ecbc' tuning, I have calculated the optimum temperament and tension for best playability and most in tune overtones. These calculations will be discussed below.

Temperament

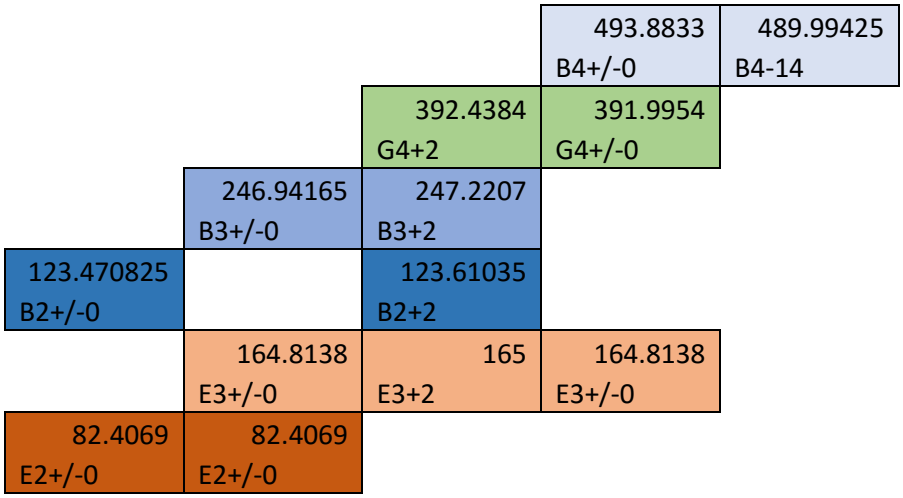
The topic of equal versus alternative temperament is a deep and far reaching one which spans far wider than the realms of this work. In the most basic layman's terms I can present, the reason that temperament is a concern to us is as follows:

> *The chromatic scale is created by stacking twelve consecutive perfect fifths (thus, the circle of fifths, representing tonal distance). In naturally occurring mathematics, twelve perfect fifths would equal slightly more than 7 octaves. The result of this is that one key would be perfectly in tune while others would be progressively more out of tune (depending on tonal distance). Equal temperament slightly flattens the fifth in order to bring all key signatures into an equally (yet acceptably) state of out-of-tune-ness.*

This means that while stopped tones are more reliably in tune and more importantly, equally usable in every key signature, it also means that the overtones of equally tempered notes (most noticeably the thirds) are slightly out of tune. The table below shows the notes and overtones in standard tuning, with reference to the pitch in Hertz as well as the note number and fine tuning in cents. Unisons are highlighted for clarity and presented side by side below the main chart.

Pitch measurements of Fundamentals and Overtones in Equally Tempered tuning

String	Fund.	OT1	OT2	OT3	OT4
C	130.8128 C2+/-0	261.6256 C3+/-0	392.4384 G4+2	523.2512 C4+/-0	654.064 E5-14
b	123.470825 B2+/-0	246.94165 B3+/-0	370.412475 F#4+2	493.8833 B4+/-0	617.354125 D#4-14
G	97.99885 G2+/-0	195.9977 G3+/-0	293.99655 D3+2	391.9954 G4+/-0	489.99425 B4-14
e	82.4069 E2+/-0	164.8138 E3+/-0	247.2207 B3+2	329.6276 E4+/-0	412.0345 G#4-14
A	55 A1+/-0	110 A2+/-0	165 E3+2	220 A3+/-0	275 C#3-14
E	41.20345 E1+/-0	82.4069 E2+/-0	123.61035 B2+2	164.8138 E3+/-0	206.01725 G#3-14



The system for naming overtones is relatively simple: N1 is the first overtone above the fundamental (octave), N2 is the second (fifth), N3 the third (second octave) and N4 the fourth (major third). The chart shows clearly the minor variances between overtones and the pitches of notes that they represent. It also highlights discrepancies between overtones representing the same pitch. For the most part, the differences are acceptable, however, the discrepancy between the unison occurring at G4 and b3 can be very noticeable. My personal recommendation for how to solve this is to marginally raise the G string (while being careful not to sharpen the already slightly sharp G2 overtone too much) and also very slightly dropping the b/C string only on its B position. As fine tuning for each individual position is possible, marginally flattening the B position without affecting the intonation of the C overtones is possible.

In practice, this can be done by ear or by becoming familiar with the correct pitch value readings for the desired result from the display of a floor tuner. Below is the modified table, taking these factors into consideration.

To tune by ear, tune to equal temperament and fine tune until the intonation conforms to the following three rules.

1. e2 and b1 should sound in tune
2. G4 and b3 should sound in tune
3. G3 and C2 should sound in tune

This will provide a generally pleasing level of in-tune-ness. However, you may find that this renders the b4 overtone too flat to use effectively. To utilise this overtone more effectively, the B string can be tuned to equal temperament or marginally above (stopping when the b1-3 overtones begin to sound out of tune). Combination with G4 should be avoided in this configuration. If a more precisely intonated G2 overtone is desired, the G string should be tuned to equal temperament (or marginally below) and the G4 overtone should be generally avoided, especially in combination with b3.

It is recommended that players learn what considerations regarding intonation are necessary for any given situation and to make adjustments on an ad-hoc basis to suit performative requirements.

At this point, it would be possible to digress in exploration of the vast field of possibilities in this area. There is much interest to be found in exploration of the different timbres and textures which different temperaments can create, as well as the increasingly popular notion of adopting alternative temperaments such as ‘A=432’ and so forth. For the purposes of this study however, we will stop only having gone so far as to suggest some minor alterations for a more favourable result, as well as achieving a greater appreciation of the impact and importance of this level of attention to detail with regards to our instrument and its sound production method.

For the purpose of completion, the following chart presents the same information for a ‘standard’ tuned instrument:

Ref. Pitch	Fundamental	OT1	OT2	OT3	OT4
G	97.99 G2+/-0	195.99 G3+/-0	293.99 D3+2	391.99 G4+/-0	489.98 B4-14
D	73.42 D1+/-0	146.84 D2+/-0	220.26 A3+2	293.68 D3+/-0	367.1 F#4-14
A	55 A1+/-0	110 A2+/-0	165 E3+2	220 A3+/-0	275 C#3-14
E	41.20 E1+/-0	82.41 E2+/-0	123.61 B2+2	164.81 E3+/-0	206.02 G#3-14

Tension

Tension concerns both playability and tone. Higher tensioned strings will have a very different feel to play compared to strings of lower tension and changes in tension will also have a direct effect on the tonal quality of an instrument. Sudden changes in string tension will also result in drastic changes to the action (neck relief) of the instrument, which will have many effects on playability and tone. Any overall tension change should be considered as part of a complete re-adjustment of the instrument.

Regarding tone, there is much hearsay in existence debating 'light tension vs heavy tension', 'round core vs hex core', 'neck through vs bolt on', 'stainless steel vs nickel' and so forth. In reality, almost all electric instruments are capable of producing and amplifying a good range of harmonics and the basic factors which will have a direct effect are much more general: use of suitable compression and EQ, pickup location (most commonly favouring of the bridge pickup in order to avoid harmonic nodes existing directly over the pickup) and regularly changing strings. For these reasons I would suggest finding a brand of string that suits your playing style and to become intimately comfortable with the tonal nuance and feel of those strings in particular.

The strings used on the two instruments utilised in the present study are Status Graphite 'Hot Wire' Round Core Long Scale Round-Wounds ('ecbc') and Thomastik-Infeld Jazz Flatwounds ('standard'). I find both of these brands to be a favourable balance of playability and tonality and the Status Graphite strings additionally offer the advantage of being able to hand pick the required gauges for custom string sets. The Status strings also offer both round core and hex core options. Theoretically, round core strings provide a lower and more evenly distributed vibrational tension for proportionately higher mass; this should (theoretically) produce stronger 'even' overtones, which should result in enhanced usable harmonics. Practically, I have found both types of strings to be excellent for use on an overtone-focussed instrument; I would recommend to try both and discover which suits your own playing best. Hex core strings also (theoretically) boast a longer lifespan, due to the lower likelihood of the wrapping slipping on the angular core.

The situation when tension becomes immediately practical is regarding playability. Generally and simply: for optimum playability, all strings should be evenly tensioned. This will provide not just consistent feel and response over the entire instrument, but will also provide the most consistent tonal quality. For standard tuning, any good quality set of strings available will already have carefully considered and well balanced tension over the set. There are many resources available online for calculating string tension. I have included the ones used personally in the 'References & Bibliography' section. The included chart shows string gauges chosen (from available options) and tension for the 'ecbc' instrument:

String	Frequency	Gauge	Tension
b/C	130.81/146.83	.30	40.83/51.44
G	98.00	.40	40.74
e	82.41	.50	45.01
A	55.00	.75	45.11
E	41.20	.100	45.01

Extended Techniques

This section will provide an overview and clarification of the primary sound production methods utilised in the process and applications of this work. These subcategories should not be considered separate 'techniques' but as aspects of 'technique'. They are all nothing more than different methods for creating different sounds and should not be used in an isolated manner but in various degrees of combination and manipulation, in order to create the overall sound desired by the artist.

Harmonics

The titular technique on which the content this work is derived from. The most general description of a harmonic is 'the sounding of an overtone above the root'. In the case of this study I am only considering the first four overtones (octave, fifth, further octave and major third from the fundamental). The reason for this is firstly to limit the scope of the work to a manageable level and secondly because these four overtones are mostly in tune with equal temperament and resting near frets, are feasibly accessible by most utilised tone production methods including tapping. Further harmonics (the next two being an octave of the fifth, flat seventh and octave of major third) are possible to achieve but less accessible and do not add enough extended harmonic possibility beyond the first four to warrant the exponential increase in analytical time that they would require to include.

In the context of bass playing technique, harmonics are best considered as a subtractive sound production method and close attention should be paid to effectively muting unwanted overtones, as these will reduce the clarity of the desired tone. Harmonics generally require two points contact to perform but can be performed by one point of contact if the correct level of percussive pressure is applied by the node'ing finger. The primary methods of activating harmonics are as follows:

Two independent points of contact.

The first point of contact creates a 'node' through which only string vibrations with a zero point can pass. This will eliminate the fundamental and undesired overtones on activation of the string. The second point of contact is used to activate the string. After activation all points of contact can be removed and the string will continue to sound.

Two connected points of contact.

It is possible to node and activate the string with one hand only (usually the right hand), using the forefinger or thumb to node and any further available fingers to activate. I prefer to use the forefinger as a node so that the thumb can remain free to articulate bass tones. It is possible to perform fast passages of harmonics by using alternate picking between the ring and little finger. It is also possible (although difficult) to perform harmonics in this way on two adjacent strings, by pairing off the fore and ring fingers and middle and little fingers. In

combination with a stopped note by the thumb, full triadic harmonic structures can be performed using only the right hand.

One point of contact (tapped harmonics).

By applying a carefully controlled amount of percussive pressure to the node point, a harmonic can be activated using only one point of contact. This can be performed by either hand although the left hand is generally more difficult to apply consistent pressure with. Both hands can be used together to perform in the style of a pitched percussion instrument and this effect can be accentuated further by muting at the nut or bridge ends of the string (any loose fabric can work and there are also several products available to purchase which will perform the task more effectively).

Additionally to these methods, harmonics can be performed 'naturally' (overtones of the open string) or 'artificially' (overtones of a stopped note). There is no practical difference in the two methods beyond the additional technical considerations presented by the requirement to stop the 'artificial' note and the greater range of accessible pitches that can be achieved by using this method. In this work I am primarily interested in expanding the scope of 'natural' harmonics.

Compression is a useful tool when utilising harmonics because as you ascend the harmonic series there is a natural reduction in volume for each successive overtone. I personally prefer a dual band compressor (separately processing low and high frequencies) because electric basses are primarily designed to produce low end frequencies and can do so very powerfully; this can result in the volume of the overtone content being reduced or 'ducked' with the lows as the lower notes pass the compressor's threshold.

One Point Articulations (Tapping)

The term 'tapping' generally describes the act of sound production by stopping a string at the fret with a controlled level of pressure, thus sounding the note while using only one point of contact. Tapping can equally be performed by the left and right hands and the hands can either work together to create one musical 'part' or independently in a pianistic/self-accompanying manner in order to create polyphonic counterpoint, homophonic, or other musically rich textures.

Along with the use of harmonics, tapping constitutes a fundamentally important aspect for my application of extended technique. The two sound production methods cross over frequently (as in the case of tapped and 'artificial' harmonics) and the utility of each is enhanced by the other.

It is suitable to introduce the concept of 'Three Point Playing' here, although it could easily occupy its own subtitle. In the context of extended bass performance, 'Three Point Playing' describes the act of performing three voices or musical roles simultaneously (with only two hands). This is achieved in a number of ways including sustaining a note with one finger while

playing another voice with remaining fingers, activating natural harmonics while performing other musical parts and delegating 'middle' voices between the left and right hand subject to practicality. It is an ongoing area of consideration which is entirely case by case, and should be considered and developed by the artist subject to his/her own musical requirements. An example of a piece performed using tapping in a 'three point' style is provided in Audio Example 1. Specifically of note is the solo section from just after 2:20, where the bass performs the solo, rhythmic chord accompaniment and bassline all together.

Two Point Articulations (Finger Playing)

While not necessarily extended in that 'normal' bass playing also falls under this subtitle, this category includes all conventional bass playing styles as well as slapping. The subtitle of "Two Point Articulations" describes the action of using one hand to stop a note at the fret or fingerboard and the other to activate the string.

Slapping is worthy of note because while it is largely treated as a separate style, it is usually treated so more for its historic stylistic implications than its timbral qualities. In the purpose of this work I will consider slap techniques simply as a timbral choice in the process of sound production. Appropriate application of the technique is left to the discretion of the artist.

When used in combination with tapping, the thumb can be very effectively utilised as a point of contact for bass parts. It is common and tempting to tap the bass parts, but this can provide for poor separation of voices as well as an unrefined tonal characteristic. By using the thumb to activate the lower strings, much more control and finesse is available to the player.

It is also useful on extended range instruments to adopt some techniques from the 'Fingerstyle' methods utilised by folk guitarists. These advanced finger picking patterns are rhythmically pleasing and when practised are easy and fluid to execute. Some adaption is required to bring this style to a bass, for example when there are not enough strings available it is necessary to perform the articulations of two strings on a single string. When practised, these alterations to the technique quickly become second nature. Audio Example 2 demonstrates a piece in which the bass reproduces the guitar fingerpicking patterns note for note for the entire duration of the composition. The bass moves between finger picking and tapping techniques throughout the piece to suit the dynamic level, while the final 'traditional' section is performed in a melodic manner.

Muting

Muting technique for electric bass guitar is incredibly important as the instrument is capable of creating extremely loud low end frequencies. If left unregulated these frequencies will sound sympathetically or be activated by movement of the instrument, greatly diminishing the musical value of the sound produced. The importance of effective muting technique is further accentuated when utilising extended techniques, due partly to the increased intricacy and finesse of the performance style and partly to the nature of spreading available cognitive attention more thinly throughout the many extended considerations necessary.

Broadly, muting discussion can be divided between the left and right hands:

Left

By playing the instrument with the soft pads of the fingers resting against unplayed strings, much unwanted resonance can be eliminated. With practise and experience, the fingers of this hand can be used on demand to kill a string's vibrations. A further muting technique can be utilised by the left hand in order to give a greater control of timbral quality and attack: By resting the middle, ring or little finger gently on the string directly above the stopped note, upper harmonic content can be reduced and control over the attack and decay granted. This method was first popularised by Francis Rocco Prestia, and allows for fine timbral control as well as careful shaping of the notes. This technique can also be utilised effectively while tapping in order to control the sharp attack of tapped notes.

Right

While playing, resting the thumb against or on the low strings will silence them while they are not required. This method of muting is very effective when combined with the left hand's soft pads resting on the upper strings, and both in combination will eliminate almost all unwanted noise. A further utility of right hand muting is the use of the palm close to the bridge, to provide fine tone control. The application of this is very similar to the timbral application of muting discussed in regards to the left hand, and the two techniques can be used together. Palm muting is also effective when tapping and can provide a tone closer to finger playing which is often preferable to the sharp attack of tapped notes.

Re-tuner Movements

The use of re-tuners in performance is a somewhat new, if moderately unknown area of instrumental performance, which has begun to be popularised by players such as Michael Manring who has been developing performance using this area of technique for many years. His use case is much more deep than the present example which only uses a single re-tuner placed on the highest string (by contrast he has them attached to either end of every string). The harmonic capability of an instrument can be increased further by introducing additional re-tuners to the higher strings, however this can be limited by the instrument itself as some basses headstocks are too small to fit more than one or two.

The practical implications of this technical enhancement are closely connected with the concept of 'Three Point Playing' in that activating the lever requires a point of contact, rendering that hand unable to perform other musical roles during the action. The lever does however present an additional musical application; when the open string or a harmonic is ringing it is possible to move pitch with an extremely smooth portamento motion. With consideration for the practicality of performance (such as using tapping to perform another musical part with the other hand while the lever is activated) it is possible to create very pleasing melodic effects in this way.

E-Bow

The E-Bow is a small magnetic field generator intended for activating electric guitar strings in a manner somewhat relatable to how classical stringed instruments are bowed. While it is intended for use on 6 string electric guitar and its design is ill-suited for application to electric bass, it is still an effective and powerful tool when utilised effectively. Michael Manring has historically been a strong advocate and innovator in E-Bow use on electric bass, even going as far as developing compositions using two E-Bows at the same time.

The E-Bow is held in the picking hand and with practise can be effectively supported by just the little and ring fingers. This leaves the pointing and middle fingers as well as thumb available for utilising other playing techniques. It is possible to sustain notes with the E-Bow while playing the instrument otherwise normally. The E-Bow can also be held 'in reserve' by these fingers and applied only when necessary.

A distinctive characteristic of the E-Bow is that when held over a magnetic pickup, the signal is distorted and boosted to an extreme level (5-10x normal volume). While this distortion can have a favourable effect, the volume boost is very unwieldy in practice and renders the tool of little use when used in combination with other techniques. However, when used in combination with a piezo pickup this shortcoming is sidestepped, as a piezo pickup will not distort and boost in the same manner as a magnetic one. Moreso, by blending 5-10% of the

magnetic pickup into the piezo, a controlled and favourable level of distortion and boost can be achieved. With use of the piezo pickup, the tonal vocabulary of the E-Bow is changed from a somewhat gimmicky and unwieldy 'trick' technique into an extremely useful and broad tonal spectrum of possible utility.

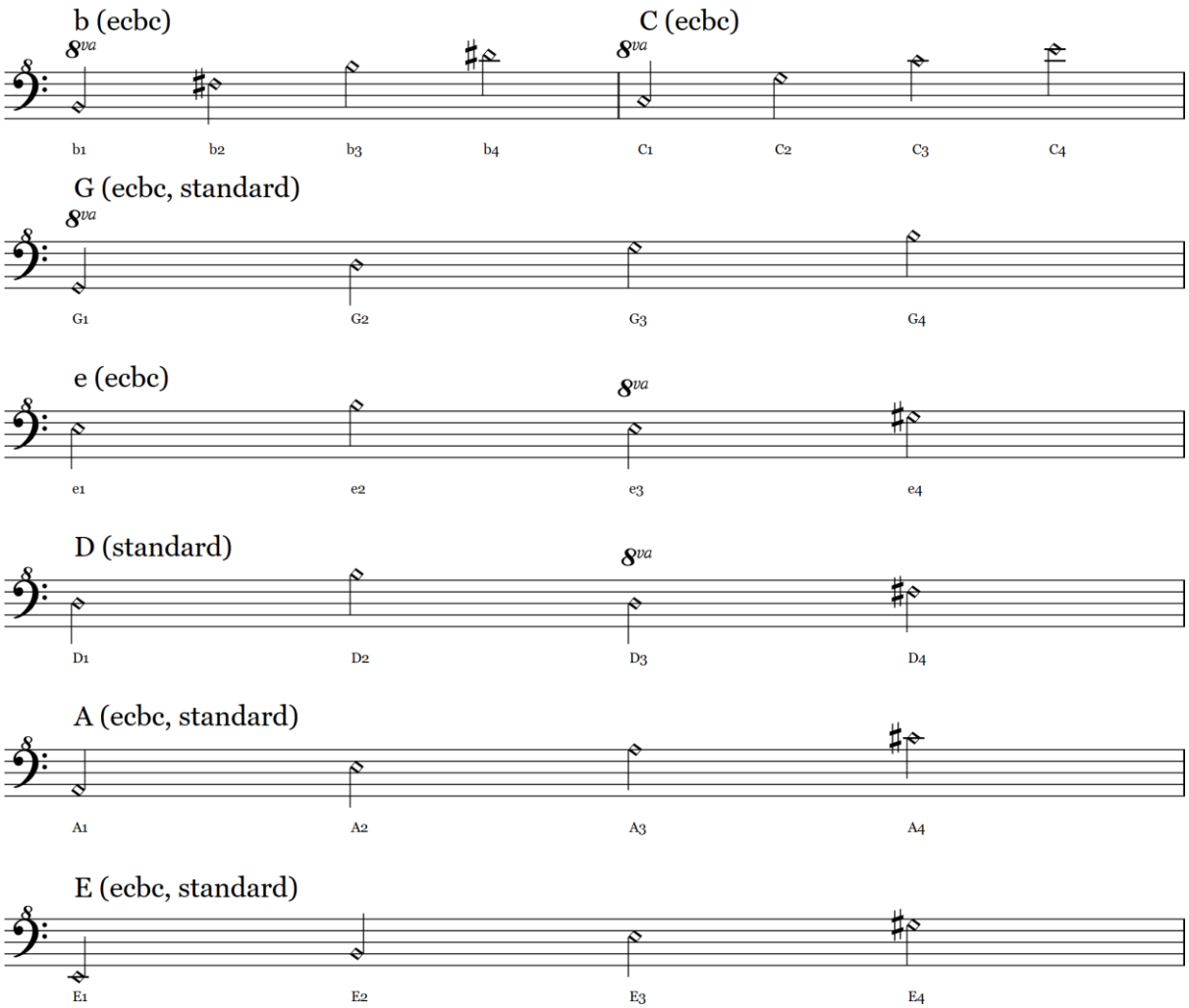
A challenge faced by bass players when using an E-Bow is that the tool is designed to rest on strings spaced as on an electric six string guitar. This means that when used on an electric bass it must be placed manually and precisely without the use of this reference aid. However, in combination with a piezo pickup as described above, the pickup of the bass can be used as an 'anchor' that with practise will provide enough stability to apply the E-Bow effectively. It is worthy to note that some players without piezo pickups avoid the volume/distortion shortcoming by applying the E-bow over the neck. While this is possible, I find the tonal and practical aspects of playing over the pickup to be preferable.

This brings about the conclusion of the introductory discussions section. The following chapter will discuss preliminary concerts and concepts which are important in the development of the 'Harmonic Algorithm' concept.

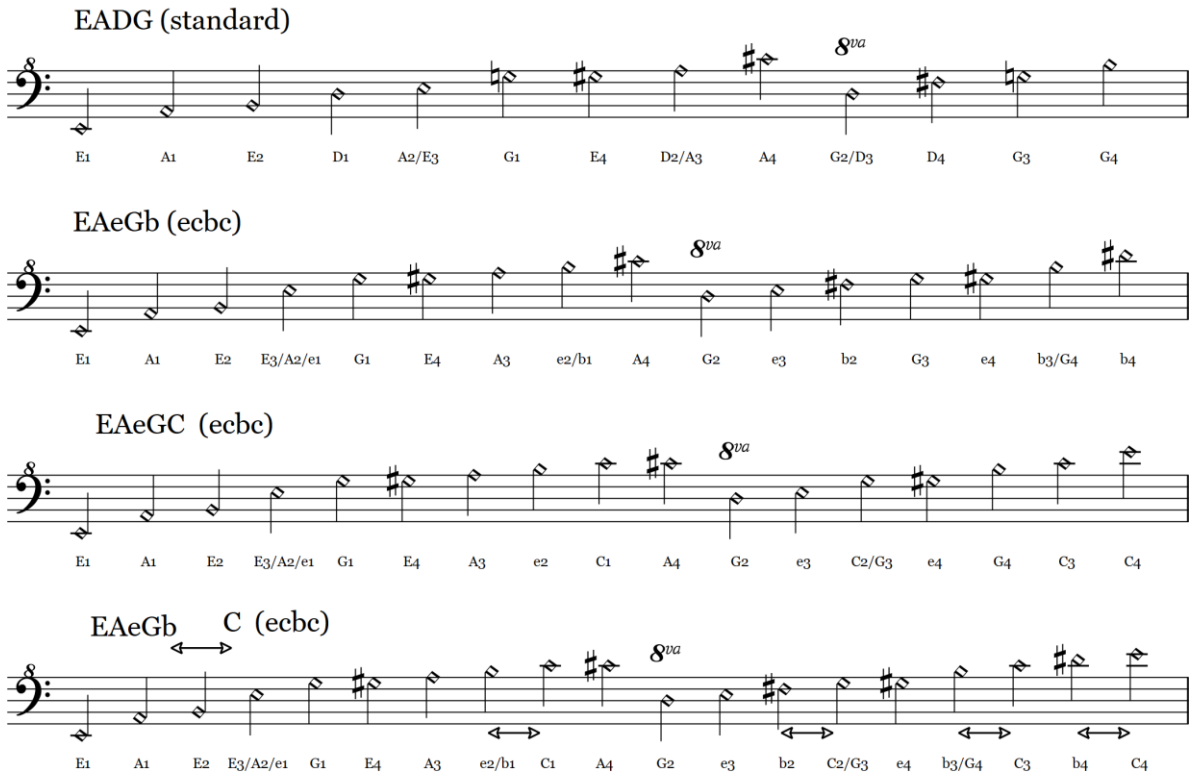
Composite Overtone Series'

A 'Composite Overtone Series' is exactly that; a combination of different overtone series which together form a singular combined series. This allows instruments with multiple strings to perform melodic and harmonic repertoire with overtones that goes far beyond the possibilities of the basic overtone series.

The fundamental starting point for this is to consider all the overtones available on each individual string. These are presented below for both 'ecbc' tuning as well as standard, from highest string descending to lowest. Fundamental notes which sit inside the standard bass guitar paradigm of ascending fourths are capitalised while notes sitting outside this convention are in lower case. Under each note is also a reference to which overtone of which string is being considered. These will play an important technical and practical function as the study grows in depth. Only overtones 1-4 have been considered for each string, for practical & technical reasons discussed in the introductory chapters as well as for the purposes of keeping this study's scope within the realms of possibility.



Next, the overtones are compiled into overtone series for each of the different tunings considered in this study. For 'ecbc' each possible re-tuning is presented, as well as both together in a complete form with reference made to available portamento melodic movements.



These Composite Overtone Series can be considered the 'chromatic scale' of this study and form the basis of the Harmonic Algorithm concept which will be explored fully in the next chapter. The harmonic possibilities available can be expanded through integration with the performance techniques discussed in previous chapters, the overtone series themselves can be supplemented by the introduction of artificial harmonics. These additional considerations will not be considered in the following analytical chapters, then will be re-introduced later in a practical 'per use case' basis in the subsequent development chapters.

The Harmonic Algorithm

The titular 'Harmonic Algorithm' is a concept that is simple in nature and deep in scope. In its simplest possible definition it is a chart which sets out all of the possible combinations of three overtones over all twelve possible chromatic stopped tones as bass notes, in order to provide the harmonic implications of each different overtone structure over each bass note. Triads of overtones were chosen rather than double stops or combinations of four or more for the following reasons:

1. The smallest possible number of overtones required for every result to make a 'chord' by definition. In many cases the bass tone is also present in the overtone structure, which with only two overtones would result in a double stop rather than a chord.
2. The largest number of overtones possible to be played on a four stringed instrument when including a stopped tone.
3. Simple to spot when removal of one overtone would result in a more favourable triad (two overtones plus stopped bass tone) in interpretation of the results.
4. Enough harmonic complexity to generate almost any (if not all) possibilities inside widely accepted functional harmonic implication.

These factors combine to create a generally practical number of overtones with which to work, in both a theoretical and practical sense. The basic process of creation will be defined in the following paragraphs, in order to provide deeper insight into the practical value of the results.

The beginning of the process was to take each of the Composite Overtone Series in question and derive from it every possible combination of three notes. It is of relevance that overtones of the same note in different octaves have not been counted as different for the purposes of this study. Only harmonic implication is being considered in the clinical environment of the spreadsheet, and considerations such as which octave to voice a specific overtone in are left to the discretion of the composer. For practical purposes of application and musical judgement, location of overtones contained in each structure are defined in the rightmost column of each row. This information is drawn from the Composite Overtone Series given in the previous section and also includes reference to cases where there is more than one possible choice for production of a given overtone. There are no impossible or 'theoretical' combinations of overtones considered in the study (ie. more than one overtone required per string) so with the use of this column, everything should be immediately playable.

After the list of all possible combinations of notes was created, this was set against all twelve possible stopped bass tones. The decision to set the overtone structures over chromatic bass notes is firstly to exponentially expand the number of functional possibilities provided to us by the analysis and secondly to provide us with a range of possibilities for performance inside of and modulation through alternative key signatures which would ordinarily be impossible (or cryptic possibilities) when using overtones alone.

Inside the format of this framework, the functionality of each overtone voicing over each root has been filled in manually. In the classification of harmonic functionality, every harmonic

structure has been considered to be in root position and inversions of each overtone structure have not been considered. The exhaustive nature of the data collection process eliminates the need to consider whether a given voicing functions as an inversion, as functionality of each possible overtone structure is shown over each possible chromatic root. As a result of this, all possible inversions will already be presented in the data. Naturally, players are also advised to use their own creative judgement in root motion when utilising the results of this study in practice.

For the purposes of this study, the labelling system for functionality of the overtone structures has been modified in a few minor ways from the 'standard' systems that you might find in a real book or in day to day use (however, it is questionable whether any system can be considered standardised in reality given that so many variants exist). The purpose for this is to remove any ambiguity in what notes the voicing contains; it is quite common to write harmonic function in a way that either allows for interpretation or discretion by the performer, or in a way as to present the most 'streamlined' chord symbols for faster interpretation. The foremost factor considered in this study when designating harmonic implication is concrete unambiguity of what notes are present in any given voicing. Furthermore, different designations for the same higher extension (such as sus4 and add11) have no necessary implication on the octave that these notes are presented at, but rather contain other information (such as whether the tone is added or substituted).

Chromatic tone clusters containing two or more consecutive semitones have been eliminated from consideration prior to analysis. For the purposes of this work, they are considered too unmusical for practical use. Voicings where the root stopped tone is present in the overtone structure are outlined in black. All further necessary information regarding interpretation of the harmonic implication of the overtone structures in question is presented in the included key

The pages below present the charts for the three tunings in use for this study (the linked 'ecbc' tunings of EAeGb and EAeGC as well as 'standard' EADG).

The spreadsheets contain additional insight, presented in the shading of cells in varying colours. This will be defined and explored in the following section on Harmonic Extrapolation.

b9	Contains b9
sus2	Substitute 2 for 3
add9	Contains 9
9	Contains 3, b7 & 9
m	Flattened 3
#9	Contains #9
no3	No 3 present
sus4	Substitute 4 for 3
add11	Contains 11
#11	Contains #11
b5	Substitute b5/#11 for 5
no5	No 5 present
#5	Substitute b13 for 5
b13	Contains b13
6	Contains 6
b7	Contains b7
7	Contains 3, b7 & 5 unless specified
-	Contains b3, b7 & 5 unless specified
Δ	Contains 7 (replace b7 if - Δ)
o	Contains b3 & b7
ø	Contains b3, b5 & b7
o7	Contains b3, b5 & 6
+	Contains 3 & #5

[See accompanying document ‘EAeGb’ or spreadsheet ‘The Harmonic Algorithm’]

[See accompanying document ‘EAeGb’ or spreadsheet ‘The Harmonic Algorithm’]

[See accompanying document ‘EAeGC’ or spreadsheet ‘The Harmonic Algorithm’]

[See accompanying document ‘EAeGC’ or spreadsheet ‘The Harmonic Algorithm’]

[See accompanying document ‘EADG’ or spreadsheet ‘The Harmonic Algorithm’]

[See accompanying document ‘EADG’ or spreadsheet ‘The Harmonic Algorithm’]

Harmonic Extrapolation

In the opening statement of this work, the fundamental objective was stated as:
"to create a process which will lead to new operationally functional understandings and developments which are tangibly useful in performance". The method by which this work has set out to achieve its objective is to exhaustively uncover all possibilities in a theoretical manner, so that obfuscated or obscure possibilities which may not be obvious through practical 'exploratory' approaches to discovery will be revealed.

Three clearly defined requirements of meeting this objective were made, and the presentation of the analytical results in the previous section brings us to the conclusion of requirement one:

1. *A clearly defined process by which the desired information can be extrapolated.*

The present chapter will begin to address the second requirement:

2. *Organisation of the raw data into an accessible and useful format by a process of practical musical consideration.*

Before extracting information from the charts above, extra insight was added to them via colour coding. The meaning of these different shadings are presented in the following key:

Simple Structure
Tone Cluster Structure
Complex Structure
Melodic Minor Structure
Impractical Structure
[outlined] Contains Root
Chromatic Tone Cluster
Duplication across tunings

Requirements for the 'simple' (green), 'complex' (blue) and 'impractical' (orange) structure categories are defined below. The 'simple' and 'complex' structures are further separated by a lighter or darker highlight, as noted in the bullet points.

Cells where the overtone structure contains the note of the stopped bass tone are outlined, while cells where clusters of 3 or more semitones would be present are highlighted red.

Overtone structures which are present in both EAeGb and EAeGC are highlighted grey in the rightmost column.

Simple –

- Formally acknowledged 'common' harmonic structures, without consideration for tonality.
- 'Common' harmonic structures with no more than a single omission or basic extension.
- Basic musically pleasing structures such as simple tone clusters (darker highlighted rows).

Complex –

- Harmonically viable structures containing desirable higher extensions
- Musically viable structures missing key harmonic components.
- Tonic, altered or other 'concrete' melodic minor structures (darker highlight).

Impractical –

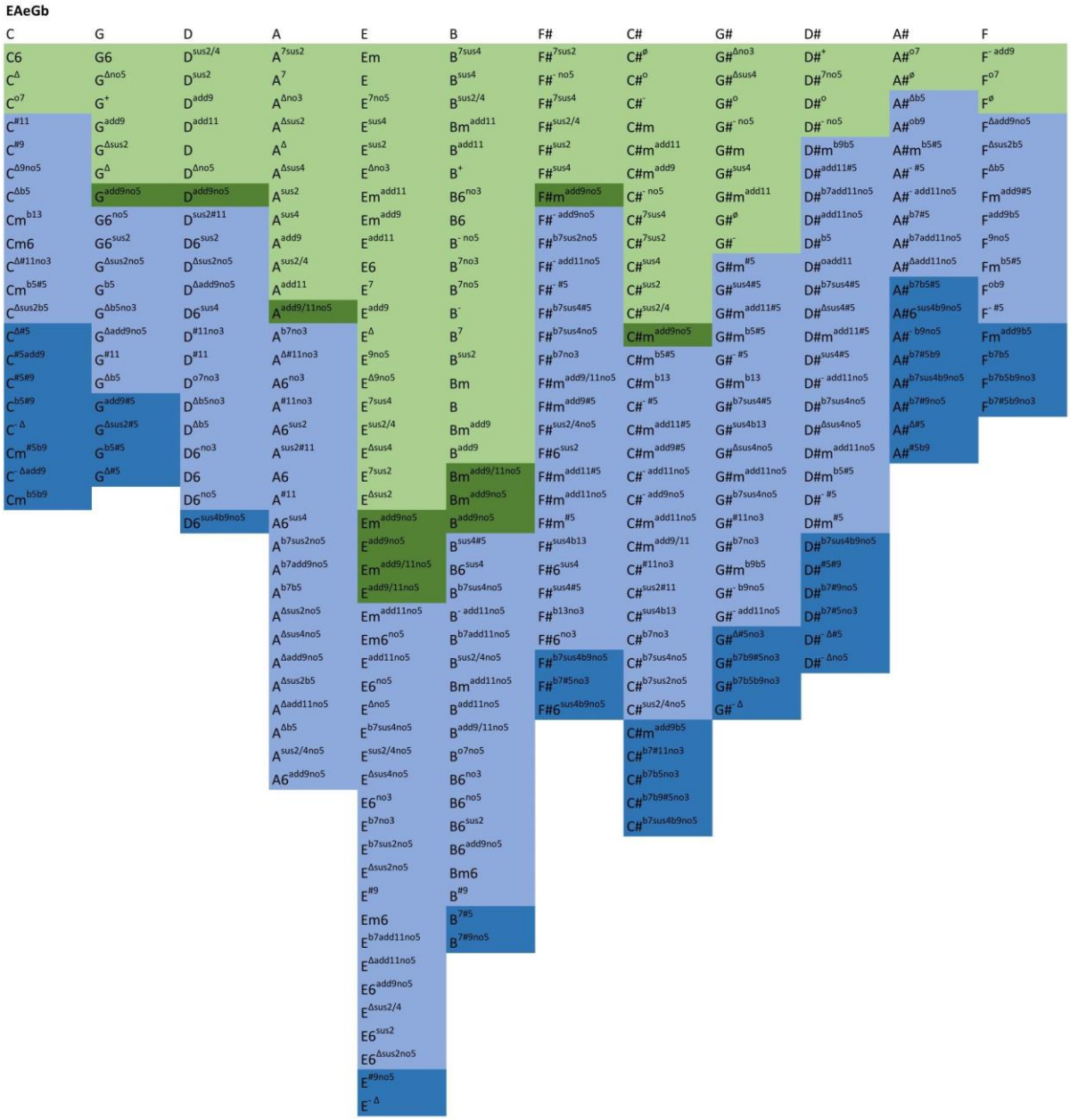
- Structures containing multiple dissonant higher extensions without harmonic grounding.
- Particularly unmusical structures.

The following charts show the raw data organised into possible harmonic vocabulary for all twelve chromatic keys, first for ‘ecbc’ and then for ‘standard’. The harmonic possibilities are presented modally inside each key. The following points should be considered in interpretation of the information:

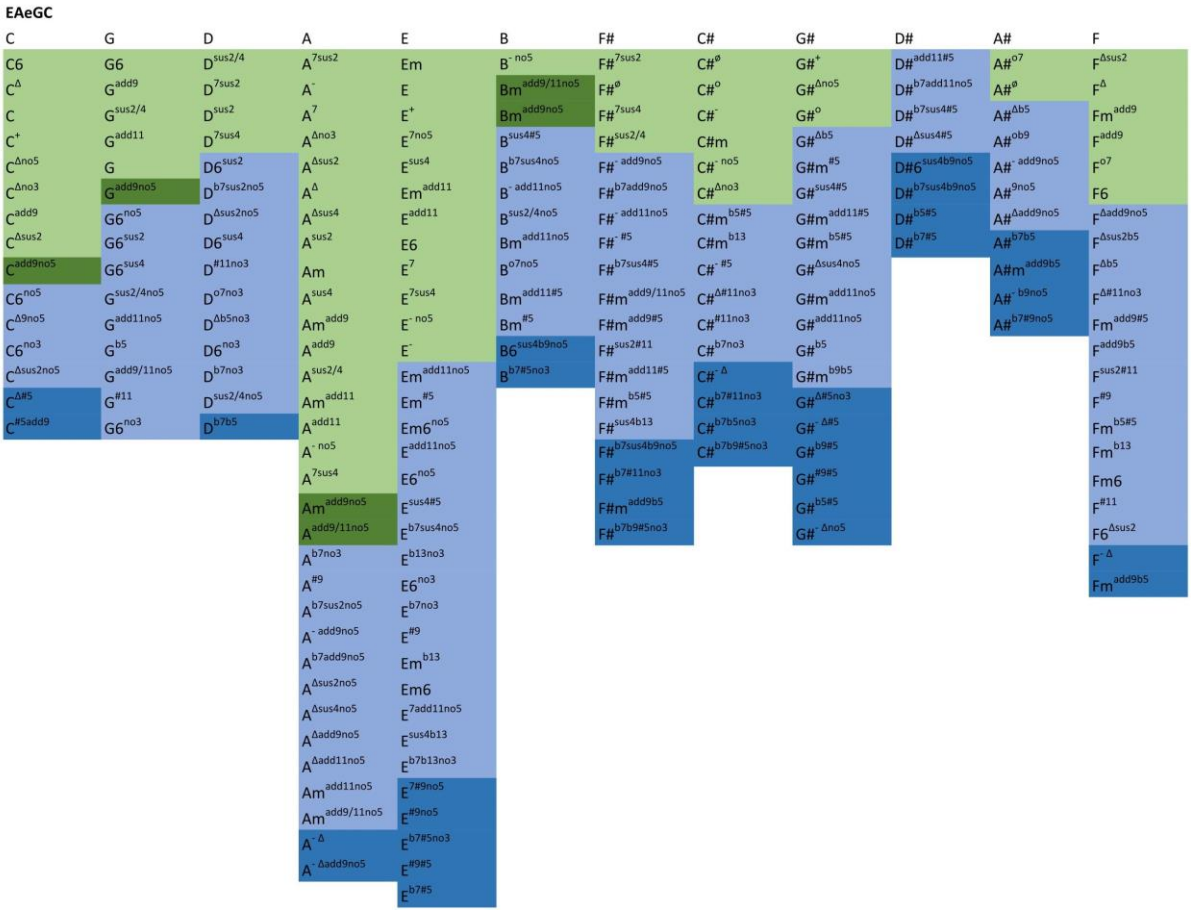
- In ‘ecbc’ tuning, options for each mode are presented separately side by side for EAeGb and EAeGC. This is to allow for easier decision making with regard to the position of the re-tuner lever in a practical use case.
- Some keys in ‘standard’ tuning have been omitted, due to lack of practical possibilities.
- Not all Altered possibilities are shown. The selection shown includes all different options for available Altered tensions, which can be can be chosen from and combined at the discretion of the artist.
- There are many more options available through omission of unwanted overtones. These possibilities are easily discerned inside the triadic possibilities presented, and can be utilised at the discretion of the artist.

‘ecbc’

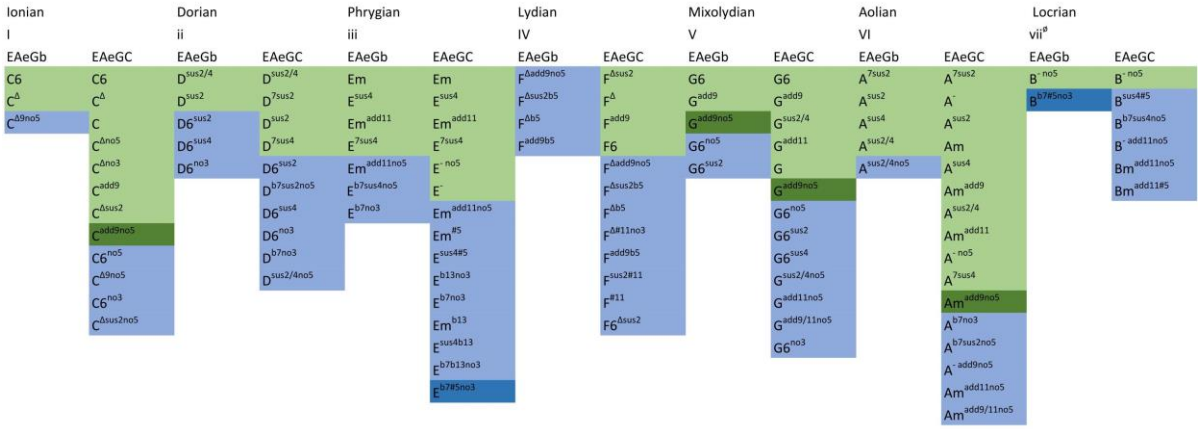
All Keys (EAeGb):



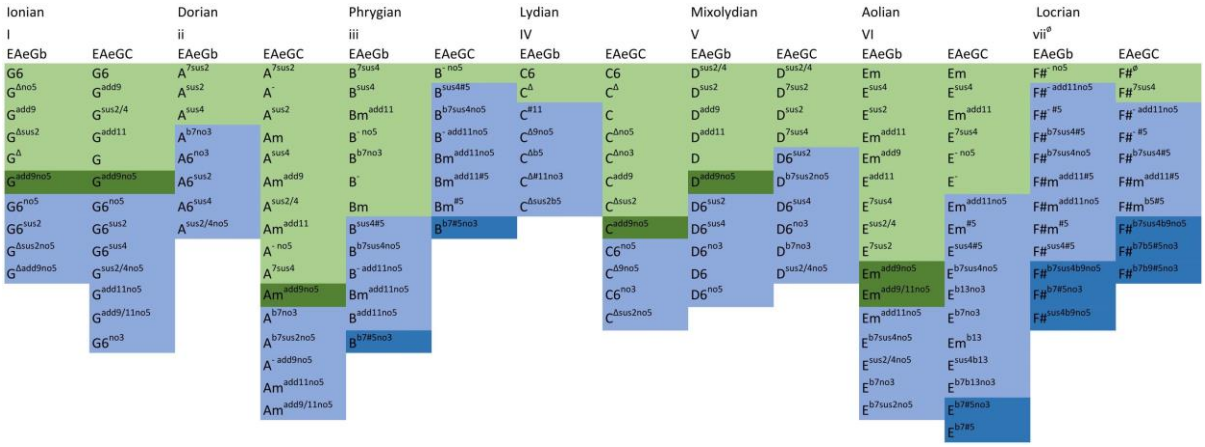
All Keys (EAeGC):



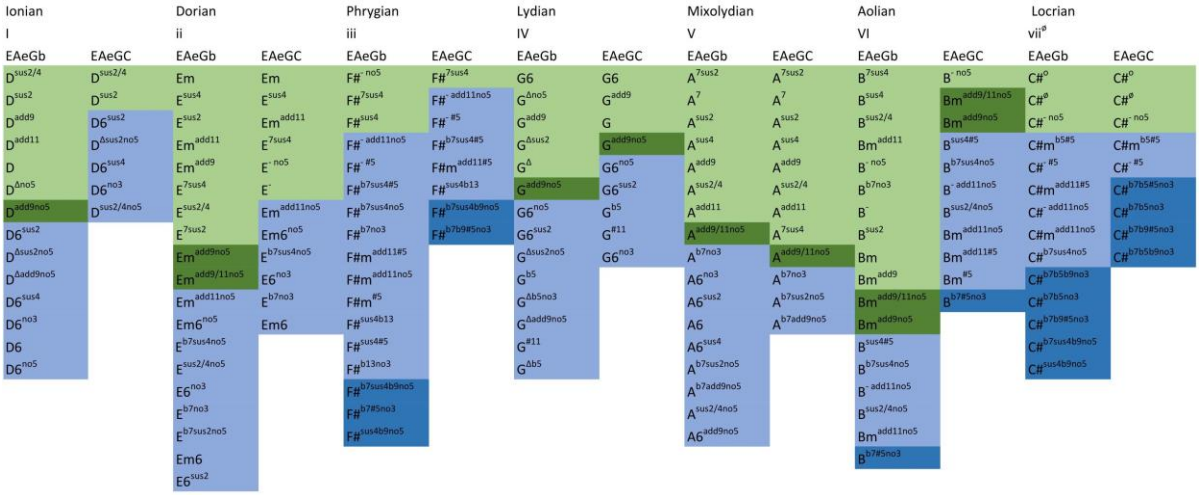
C Major:



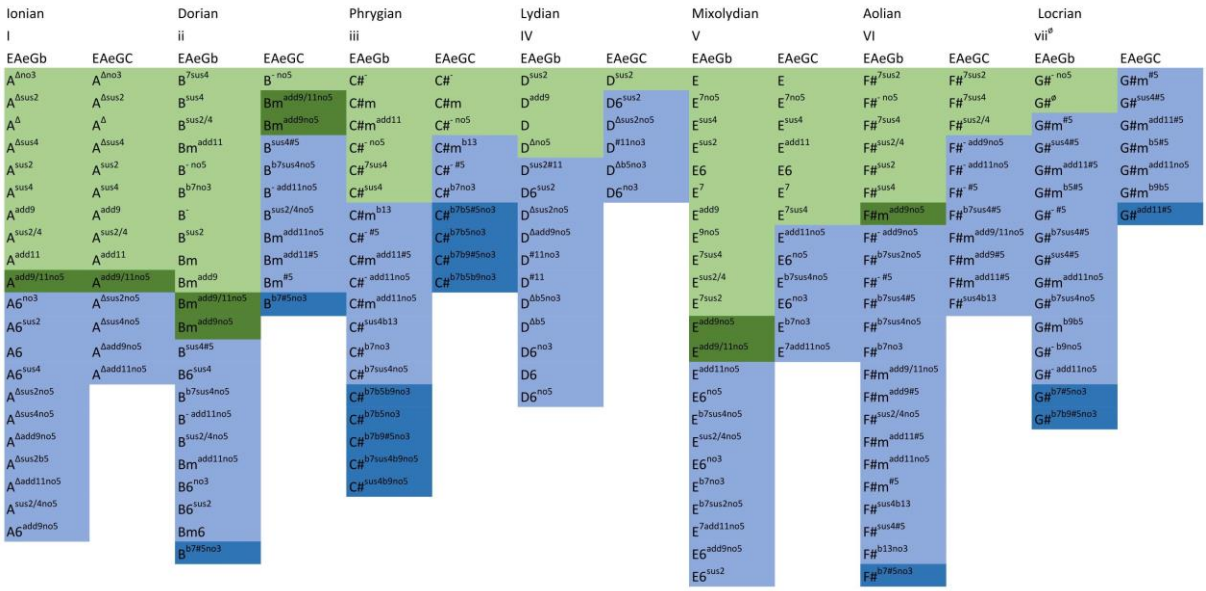
G Major:



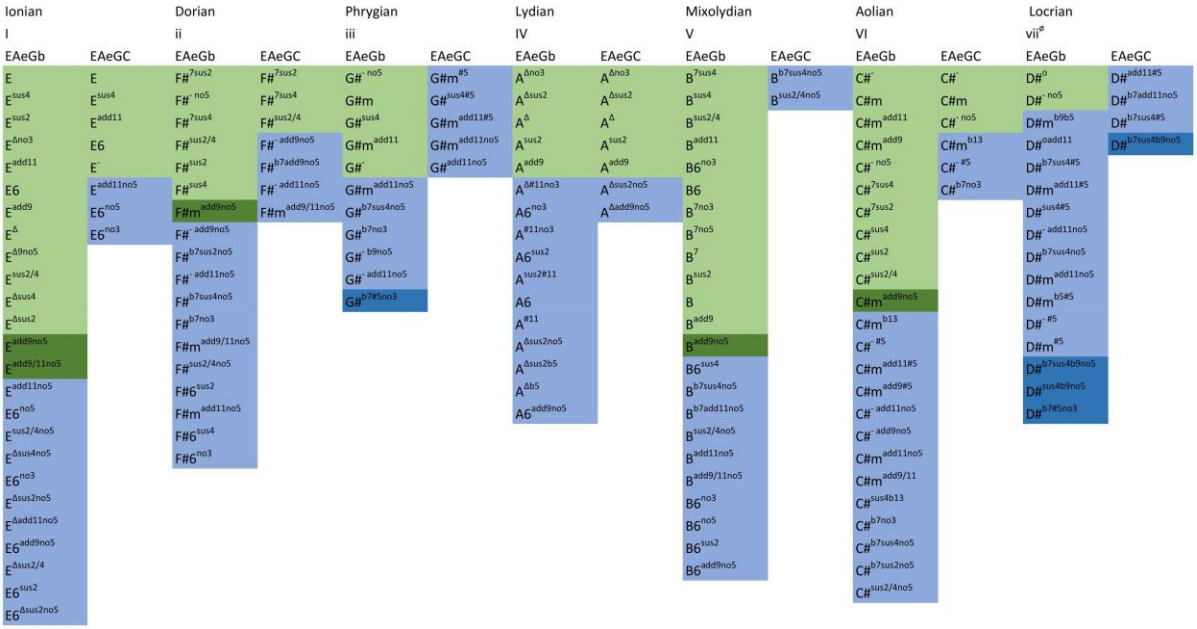
D Major:



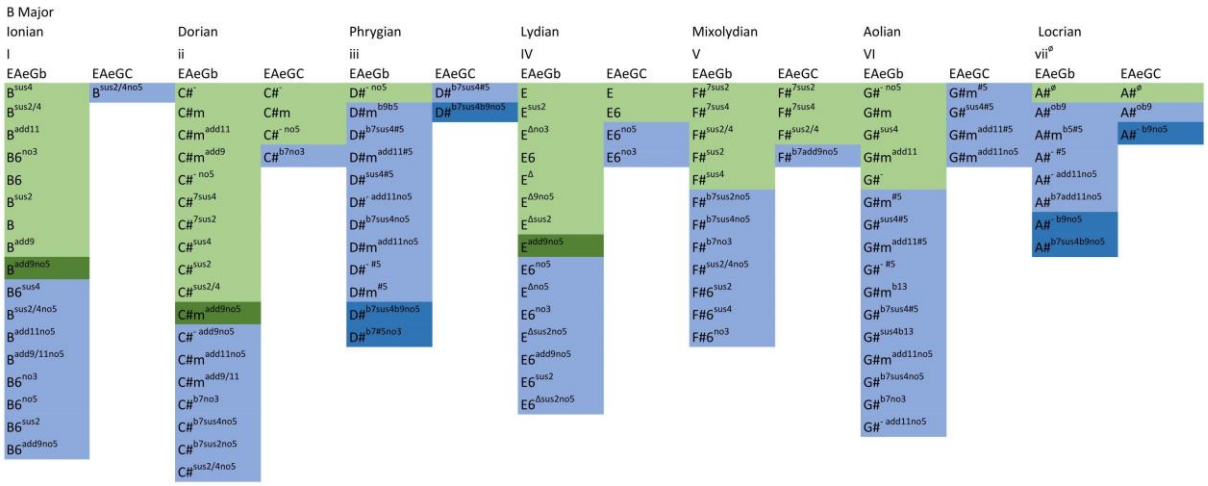
A Major:



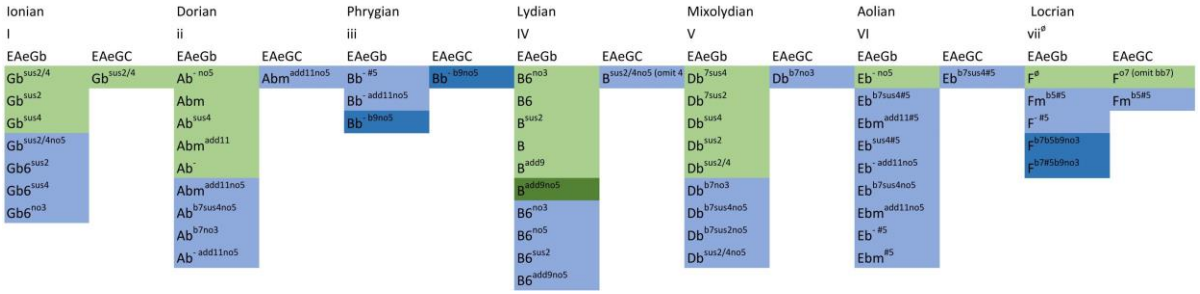
E Major:



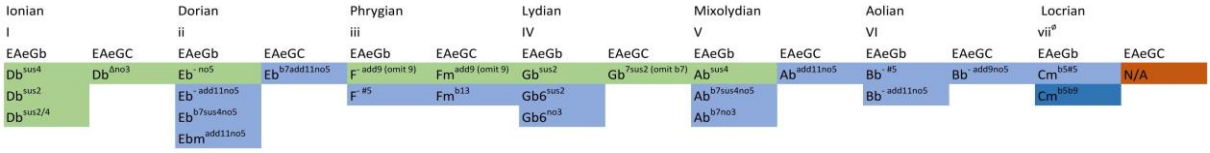
B Major:



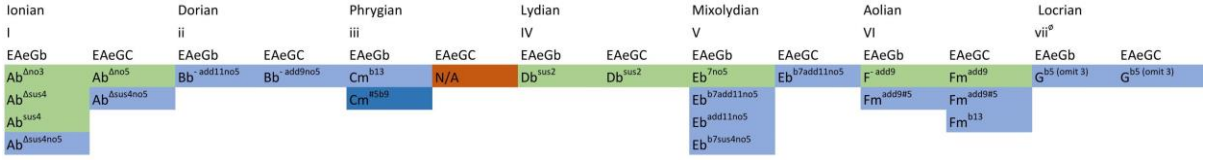
Gb Major:



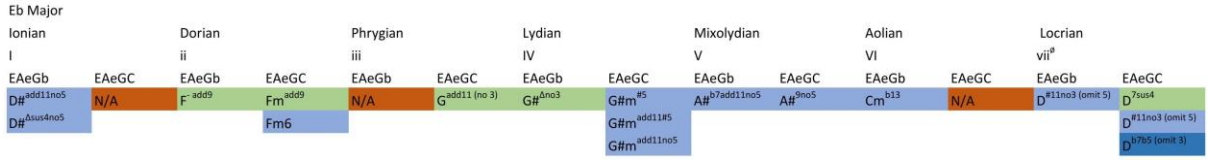
Db Major:



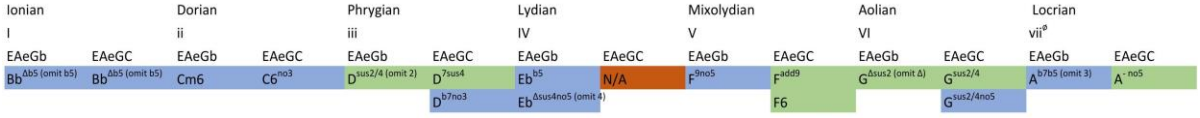
Ab Major:



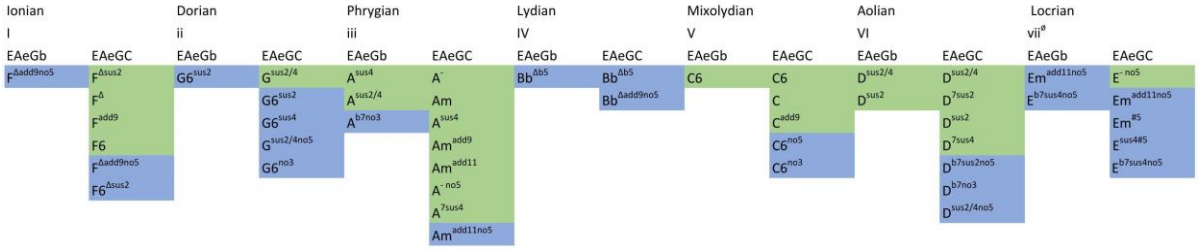
Eb Major:



Bb Major:

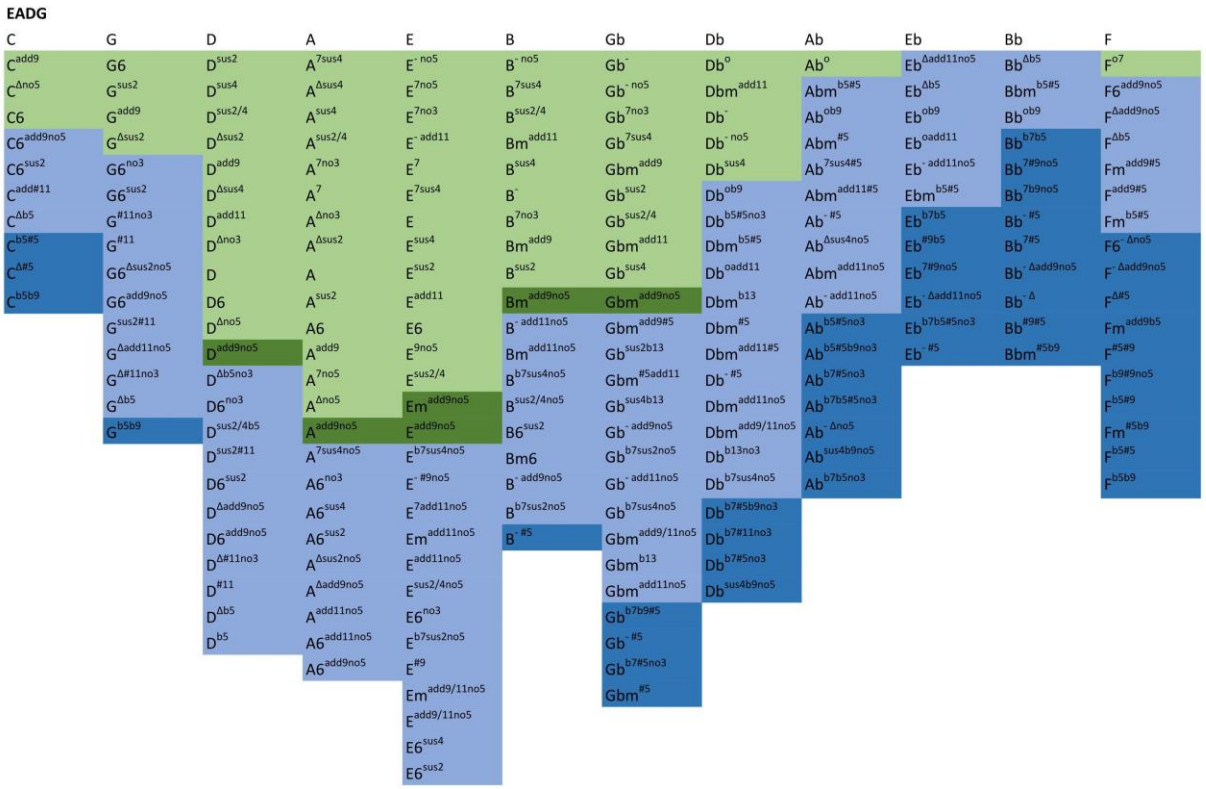


F Major:



'standard'

All Keys



C Major:

EADG

Ionian	Dorian	Phrygian	Lydian	Mixolydian	Aolian	Locrian
I	ii	iii	IV	V	VI	vii^{ø}
C^{add9}	D^{sus2}	E^{-no5}	$F6^{add9no5}$	G6	A^{7sus4}	B^{-no5}
$C^{\Delta no5}$	D^{sus4}	E^{-add11}	$F^{\Delta add9no5}$	G^{sus2}	A^{sus2}	$B^{-add11no5}$
C6	$D^{sus2/4}$	E^{7sus4}	$F^{\Delta b5}$	G^{add9}	$A^{7sus4no5}$	$Bm^{add11no5}$
$C6^{add9no5}$	$D6^{no3}$	E^{sus4}		G6^{no3}		$B^{b7sus4no5}$
$C6^{sus2}$	$D6^{sus2}$	$E^{7sus4no5}$		$G6^{sus2}$		$B^{-\#5}$
		$Em^{add11no5}$		$G6^{add9no5}$		
				$G^{\Delta add11no5}$		

G Major:

EADG

Ionian	Dorian	Phrygian	Lydian	Mixolydian	Aolian	Locrian
I	ii	iii	IV	V	VI	vii^{ø}
G6	A^{7sus4}	B^{-no5}	C^{add9}	D^{sus2}	E^{-no5}	F#$^{-no5}$
G^{sus2}	A^{sus4}	B^{7sus4}	C$^{\Delta no5}$	D^{sus4}	E^{7no3}	F#m$^{\#5add11}$
G^{add9}	A$^{sus2/4}$	Bm^{add11}	C6	D$^{sus2/4}$	E^{-add11}	F#$^{-add11no5}$
G$^{\Delta sus2}$	A^{7no3}	B^{sus4}	C6^{add9no5}	D^{add9}	E^{7sus4}	F#^{b7sus4no5}
G6^{no3}	A^{sus2}	B^{-}	C6^{sus2}	D^{add11}	E^{sus4}	F#$^{b7b9\#5}$
G6^{sus2}	A^{7sus4no5}	B$^{-add11no5}$	C$^{add\#11}$	D	E^{sus2}	F#$^{-\#5}$
G6$^{\Delta sus2no5}$	A6^{no3}	Bm^{add11no5}	C$^{\Delta b5}$	D6	E$^{sus2/4}$	F#$^{b7\#5no3}$
G6^{add9no5}	A6^{sus4}	B^{b7sus4no5}		D^{add9no5}	Em^{add9no5}	F#m$^{\#5}$
G$^{\Delta add11no5}$	A6^{sus2}	B$^{-\#5}$		D6^{no3}	E^{7sus4no5}	
				D6^{sus2}	Em^{add11no5}	
					E$^{sus2/4no5}$	
					E^{b7sus2no5}	
					Em$^{add9/11no5}$	

D Major:

EADG

Ionian	Dorian	Phrygian	Lydian	Mixolydian	Aolian	Locrian
I	ii	iii	IV	V	VI	vii^{ø}
D^{sus2}	E$^{-\ no5}$	F#$^{-}$	G6	A^{7sus4}	B$^{-\ no5}$	C#^{o}
D^{sus4}	E^{7no3}	F#$^{-\ no5}$	G^{sus2}	A^{sus4}	B^{7sus4}	C#$^{-\ no5}$
D$^{sus2/4}$	E$^{-\ add11}$	F#^{7no3}	G^{add9}	A$^{sus2/4}$	B$^{sus2/4}$	C#$^{b5\#5no3}$
D$^{\Delta sus2}$	E^{7sus4}	F#^{7sus4}	G$^{\Delta sus2}$	A^{7no3}	Bm^{add11}	C#m$^{b5\#5}$
D^{add9}	E^{sus4}	F#m^{add11}	G6^{no3}	A^{7}	B^{sus4}	C#m$^{\#5}$
D$^{\Delta sus4}$	E^{sus2}	F#^{sus4}	G6^{sus2}	A	B^{-}	C#$^{-\ \#5}$
D^{add11}	E$^{sus2/4}$	F#m$^{\#5add11}$	G$^{\#11no3}$	A^{sus2}	B^{7no3}	C#^{b7sus4no5}
D$^{\Delta no3}$	Em^{add9no5}	F#^{sus4b13}	G$^{\#11}$	A6	Bm^{add9}	C#$^{b7\#5no3}$
D	E^{7sus4no5}	F#$^{-\ add11no5}$	G6$^{\Delta sus2no5}$	A^{add9}	B^{sus2}	C#$^{b7\#5no3}$
D6	Em^{add11no5}	F#^{b7sus4no5}	G6^{add9no5}	A^{7no5}	Bm^{add9no5}	C#^{sus4b9no5}
D$^{\Delta no5}$	E$^{sus2/4no5}$	F#m^{b13}	G$^{sus2\#11}$	A^{add9no5}	B$^{-\ add11no5}$	
D^{add9no5}	E6^{no3}	F#m^{add11no5}	G$^{\Delta\#11no3}$	A^{7sus4no5}	Bm^{add11no5}	
D6^{no3}	E^{b7sus2no5}	F#$^{b7b9\#5}$	G$^{\Delta b5}$	A6^{no3}	B^{b7sus4no5}	
D6^{sus2}	Em$^{add9/11no5}$	F#$^{-\ \#5}$		A6^{sus4}	B$^{sus2/4no5}$	
D$^{\Delta add9no5}$	E6^{sus4}	F#$^{b7\#5no3}$		A6^{sus2}	B$^{-\ add9no5}$	
D6^{add9no5}	E6^{sus2}	F#m$^{\#5}$		A^{add11no5}	B^{b7sus2no5}	
				A6^{add11no5}	B$^{-\ \#5}$	
				A6^{add9no5}		

A Major:

EADG

Ionian	Dorian	Phrygian	Lydian	Mixolydian	Aolian	Locrian
I	ii	iii	IV	V	VI	vii^{ø}
A$^{\Delta sus4}$	B$^{-\ no5}$	C#m^{add11}	D^{sus2}	E^{7no5}	F#$^{-}$	G#^{o}
A^{sus4}	B^{7sus4}	C#$^{-}$	D$^{\Delta sus2}$	E^{7no3}	F#$^{-\ no5}$	G#m$^{b5\#5}$
A$^{sus2/4}$	B$^{sus2/4}$	C#$^{-\ no5}$	D^{add9}	E^{7}	F#^{7no3}	G#^{ob9}
A$^{\Delta no3}$	Bm^{add11}	C#^{sus4}	D$^{\Delta no3}$	E^{7sus4}	F#^{7sus4}	G#m$^{\#5}$
A$^{\Delta sus2}$	B^{sus4}	C#m^{b13}	D	E	F#m^{add9}	G#$^{7sus4\#5}$
A	B^{-}	C#m$^{add11\#5}$	D6	E^{sus4}	F#^{sus2}	G#m$^{add11\#5}$
A^{sus2}	B^{7no3}	C#$^{-\ \#5}$	D$^{\Delta no5}$	E^{sus2}	F#$^{sus2/4}$	G#$^{-\ \#5}$
A6	Bm^{add9}	C#m^{add11no5}	D^{add9no5}	E^{add11}	F#m^{add11}	G#m^{add11no5}
A^{add9}	B^{sus2}	C#^{b13no3}	D$^{\Delta b5no3}$	E6	F#^{sus4}	G#$^{-\ add11no5}$
A$^{\Delta no5}$	Bm^{add9no5}	C#^{b7sus4no5}	D6^{no3}	E^{9no5}	F#m^{add9no5}	G#$^{b5\#5no3}$
A^{add9no5}	B$^{-\ add11no5}$	C#$^{b7\#5b9no3}$	D$^{sus2\#11}$	E$^{sus2/4}$	F#m$^{add9\#5}$	G#$^{b5\#5b9no3}$
A6^{no3}	Bm^{add11no5}	C#$^{b7\#5no3}$	D6^{sus2}	E^{add9no5}	F#^{sus2b13}	G#$^{b7\#5no3}$
A6^{sus4}	B^{b7sus4no5}		D$^{\Delta add9no5}$	E^{7sus4no5}	F#m$^{\#5add11}$	G#$^{b7b5\#5no3}$
A6^{sus2}	B$^{sus2/4no5}$		D6^{add9no5}	E^{7add11no5}	F#^{sus4b13}	G#^{sus4b9no5}
A$^{\Delta sus2no5}$	B6^{sus2}		D$^{\Delta\#11no3}$	E^{add11no5}	F#$^{-\ add9no5}$	G#^{b7b5no3}
A$^{\Delta add9no5}$	Bm6		D$^{\#11}$	E$^{sus2/4no5}$	F#^{b7sus2no5}	
A^{add11no5}	B$^{-\ add9no5}$		D$^{\Delta b5}$	E6^{no3}	F#$^{-\ add11no5}$	
A6^{add11no5}	B^{b7sus2no5}		D^{b5}	E^{b7sus2no5}	F#^{b7sus4no5}	
A6^{add9no5}				E$^{add9/11no5}$	F#m$^{add9/11no5}$	
				E6^{sus4}	F#m^{b13}	
				E6^{sus2}	F#m^{add11no5}	
					F#$^{-\ \#5}$	
					F#$^{b7\#5no3}$	
					F#m$^{\#5}$	

E Major:

EADG

Ionian	Dorian	Phrygian	Lydian	Mixolydian	Aolian	Locrian
I	ii	iii	IV	V	VI	vii^{ø}
E	F#$^{-}$	G#m$^{\#5}$	A$^{\Delta no3}$	B^{7sus4}	C#m^{add11}	D#^{ob9}
E^{sus4}	F#$^{-\ no5}$	G#$^{7sus4\#5}$	A$^{\Delta sus2}$	B$^{sus2/4}$	C#$^{-}$	D#^{oadd11}
E^{sus2}	F#^{7no3}	G#m$^{add11\#5}$	A	B^{sus4}	C#$^{-\ no5}$	D#$^{-\ add11no5}$
E^{add11}	F#^{7sus4}	G#$^{-\ \#5}$	A^{sus2}	B^{7no3}	C#^{sus4}	D#m$^{b5\#5}$
E6	F#m^{add9}	G#m^{add11no5}	A6	B^{sus2}	C#m^{b13}	D#$^{b7b5\#5no3}$
E^{9no5}	F#^{sus2}	G#$^{-\ add11no5}$	A^{add9}	B^{b7sus4no5}	C#m$^{\#5}$	D#$^{-\ \#5}$
E$^{sus2/4}$	F#$^{sus2/4}$	G#$^{b7\#5no3}$	A$^{\Delta no5}$	B$^{sus2/4no5}$	C#m$^{add11\#5}$	
E^{add9no5}	F#m^{add11}	G#^{sus4b9no5}	A^{add9no5}	B6^{sus2}	C#$^{-\ \#5}$	
E^{add11no5}	F#^{sus4}		A6^{no3}	B^{b7sus2no5}	C#m^{add11no5}	
E$^{sus2/4no5}$	F#m^{add9no5}		A6^{sus2}		C#m$^{add9/11no5}$	
E6^{no3}	F#$^{-\ add9no5}$		A$^{\Delta sus2no5}$		C#^{b13no3}	
E$^{add9/11no5}$	F#^{b7sus2no5}		A$^{\Delta add9no5}$		C#^{b7sus4no5}	
E6^{sus4}	F#$^{-\ add11no5}$		A6^{add9no5}		C#$^{b7\#5no3}$	
E6^{sus2}	F#^{b7sus4no5}					
	F#m$^{add9/11no5}$					
	F#m^{add11no5}					

B Major:

EADG

Ionian	Dorian	Phrygian	Lydian	Mixolydian	Aolian	Locrian
I	ii	iii	IV	V	VI	vii^{ø}
B$^{sus2/4}$	C#m^{add11}	D#$^{-\ add11no5}$	E	F#^{7no3}	G#m$^{\#5}$	A#m$^{b5\#5}$
B^{sus4}	C#$^{-}$	D#$^{-\ \#5}$	E^{sus2}	F#^{7sus4}	G#$^{7sus4\#5}$	A#^{ob9}
B^{sus2}	C#$^{-\ no5}$		E6	F#^{sus2}	G#m$^{add11\#5}$	A#$^{-\ \#5}$
B$^{sus2/4no5}$	C#^{sus4}		E^{9no5}	F#$^{sus2/4}$	G#$^{-\ \#5}$	A#m$^{\#5b9}$
B6^{sus2}	C#m^{add11no5}		E^{add9no5}	F#^{sus4}	G#m^{add11no5}	
	C#m$^{add9/11no5}$		E6^{no3}	F#^{b7sus2no5}	G#$^{-\ add11no5}$	
	C#^{b7sus4no5}		E6^{sus2}	F#^{b7sus4no5}	G#$^{b7\#5no3}$	

Gb Major:

EADG

Ionian	Dorian	Phrygian	Lydian	Mixolydian	Aolian	Locrian
I	ii	iii	IV	V	VI	vii^{ø}
Gb^{sus2}	Abm^{add11no5}	Bb$^{-\ \#5}$	B^{sus2}	Db^{sus4}	Eb$^{-\ add11no5}$	F$^{o7\ (omit\ bb7)}$
Gb$^{sus2/4}$	Ab$^{-\ add11no5}$	Bbm$^{\#5b9}$	B6^{sus2}	Db^{b7sus4no5}	Eb$^{-\ \#5}$	Fm$^{b5\#5}$
Gb^{sus4}						Fm$^{\#5b9}$

Db Major:

[Omitted due to lack of harmonic options]

Ab Major:

[Omitted due to lack of harmonic options]

Eb Major:

[Omitted due to lack of harmonic options]

Bb Major:

EADG

Ionian	Dorian	Phrygian	Lydian	Mixolydian	Aolian	Locrian
I	ii	iii	IV	V	VI	vii^{ø}
Bb$^{\Delta b5\ (omit\ b5)}$	C6^{sus2}	D^{sus4}	Eb$^{\Delta b5}$	F6^{add9no5}	G^{sus2}	A^{7sus4no5}

F Major:

EADG

Ionian	Dorian	Phrygian	Lydian	Mixolydian	Aolian	Locrian
I	ii	iii	IV	V	VI	vii^{ø}
F6^{add9no5}	G^{sus2}	A^{7sus4}	Bb$^{\Delta b5}$	C^{add9}	D^{sus2}	E$^{-\ no5}$
F$^{\Delta add9no5}$	G6^{no3}	A^{sus4}		C6	D^{sus4}	E^{b7sus4no5}
	G6^{sus2}	A$^{sus2/4}$		C6^{add9no5}	D$^{sus2/4}$	
		A^{7no3}		C6^{sus2}	D^{add11}	
		A^{sus2}				
		A^{7sus4no5}				

These results demonstrate a 'practical' musical concept applicable to performance using these techniques that could be referred to as 'semi-diatonicism'. What this essentially means is that while overtone performance styles can be used in all keys, they are more (or less) effective or simply different in application, when utilised in different keys. While this doesn't have a profound effect on the study it is worthwhile to keep it in mind, as it is a fundamental characteristic of performing with overtones.

While the above results have been presented in their entirety for the logical and immediate practical interpretation of the data, this work does not attempt to function as a 'chord book'. This extrapolation is essentially just an example of one of the many possible routes to go down with regard to treatment of the raw information. One of the underlying objectives of this work from a personal point of view was to create something of a 'compositional system' or at least a thorough presentation of the physical possibilities of this method of sound creation, in a way that they can be explored, arranged and developed in a musically insightful way, without simply 'feeling around' on the instrument for ideas. It is recommended that individuals explore the data and develop their own systems, concepts and methodologies in extracting the data. Whatever the method, the end result is the same: the creation and internalisation of a practical approach to musicality which stretches beyond what may be found by pure discovery.

The following 'developmental' chapters will focus on application of the practical possibilities derived throughout the previous chapters.

Melody in Practice

As we move into the practical development sections of this work, this chapter, which focusses on melodic implementation of overtone, will build the bridge between objectives:

1. *Organisation of the raw data into an accessible and useful format by a process of practical musical consideration.*

and

3. *Exploration and development of the practicalities presented by the process and results of objectives '1.' And '2.'*

Before applying the prior chapter's harmonic analysis to practice, it is important to remember that the underlying basis of this study is musicality and that musicality is intrinsically linked with melody. Even the sound of a harmonic progression is defined by the internal melodies which weave throughout, and to command cadence and modulation effectively is to understand and utilise the musicality contained in these internal structures rather than just treating them as 'blocks' of sound (as which they are presented in the results of the analysis).

Thus, in order to fully utilise the musicality contained in the results of the analysis, we must appreciate, understand and internalise the fundamentals that it is built on as well as develop a fluid and expressive grasp of melody inside this technical area. Performance of melody with overtones is not so simple as the 'linear' application of melody for stopped notes, and requires study and internalisation of patterns often containing little internal logic. To do this, we will revisit the concept of Composite Overtone Series and derive the fundamentals of melody and harmony in a clinical way which can be internalised, before putting these fundamentals into practise in an expressive context.

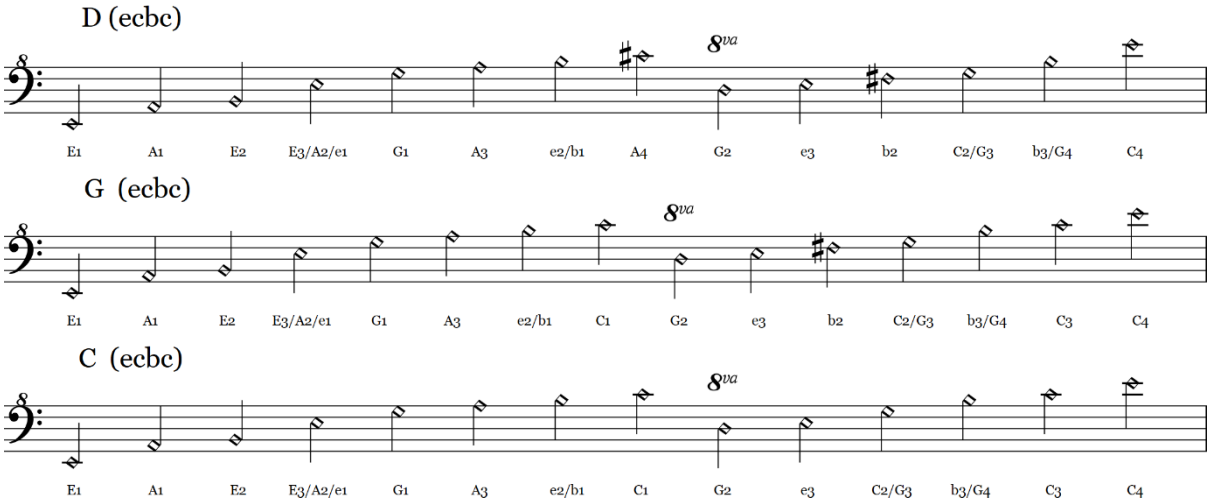
Inside the constraints of this study, the key signatures of D, G and C will be focussed on. The reason that these keys have been chosen specifically is primarily (aside from keeping the scope of the study inside reasonable bounds) for the proliferation of the key signatures of D and G in British and Celtic folk music (a primary use case in practice for this research) and the widespread adoption of C as a harmonic 'starting point' in many musical fields. Choosing only these three closely related keys is on the understanding that although many more interesting harmonic possibilities will be initially overlooked, in order to truly harness the musicality unearthed by the analysis it must be thoroughly studied and internalised at a fundamental level. These three key signatures form a strong basis from which in the future to expand around the circle of fifths from, into new and more varied possibilities for harmonic modulation.

Note that from this section forward, while manuscript examples will be given for both tunings under analysis in this study, audio examples will only be given for the instrument being immediately studied ('Electric Contrabass Cittern').

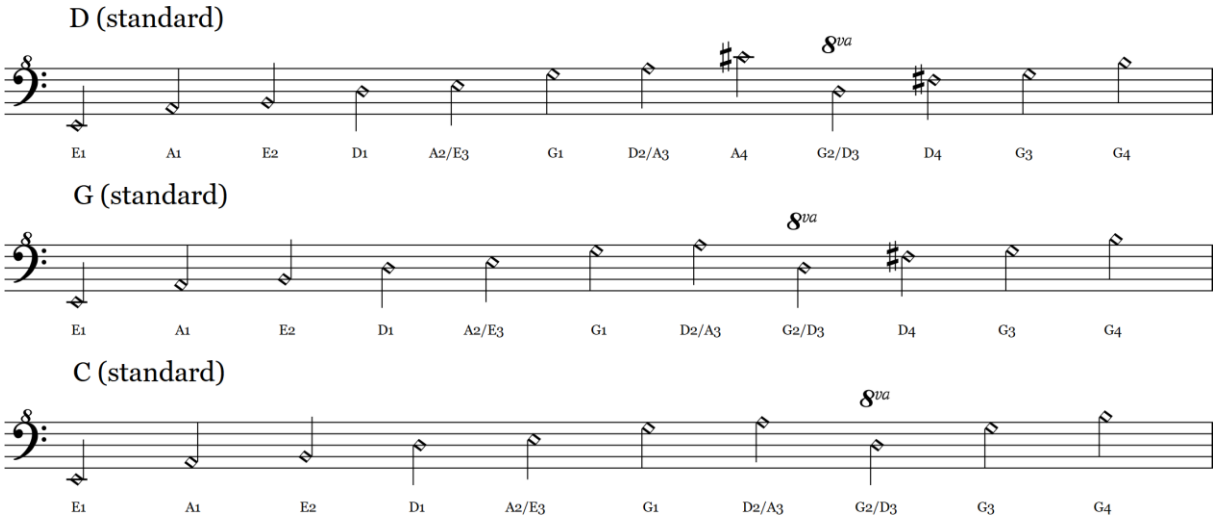
Key Signatures

The fundamental building block of the vast majority of melody is the diatonic key signature. When performing using overtone techniques, key signatures (scales) are treated more like a brass instrument, playing the available range from top to bottom of range rather than the pattern based methods employed by most fretted instrument players.

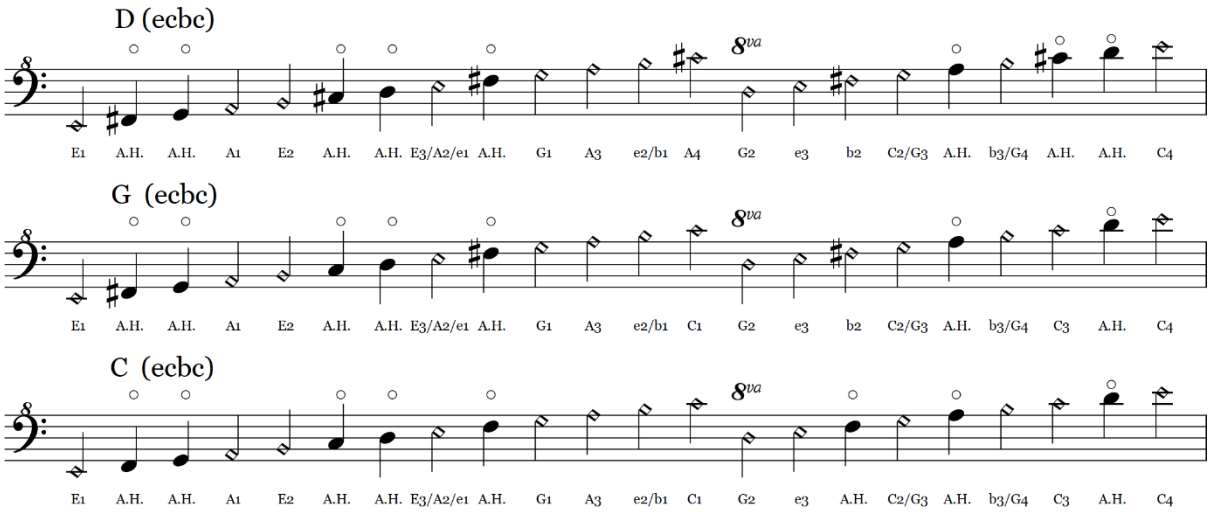
Firstly, here are the Composite Overtone Series showing only overtones matching pitches appearing in each respective key, both tunings being considered:



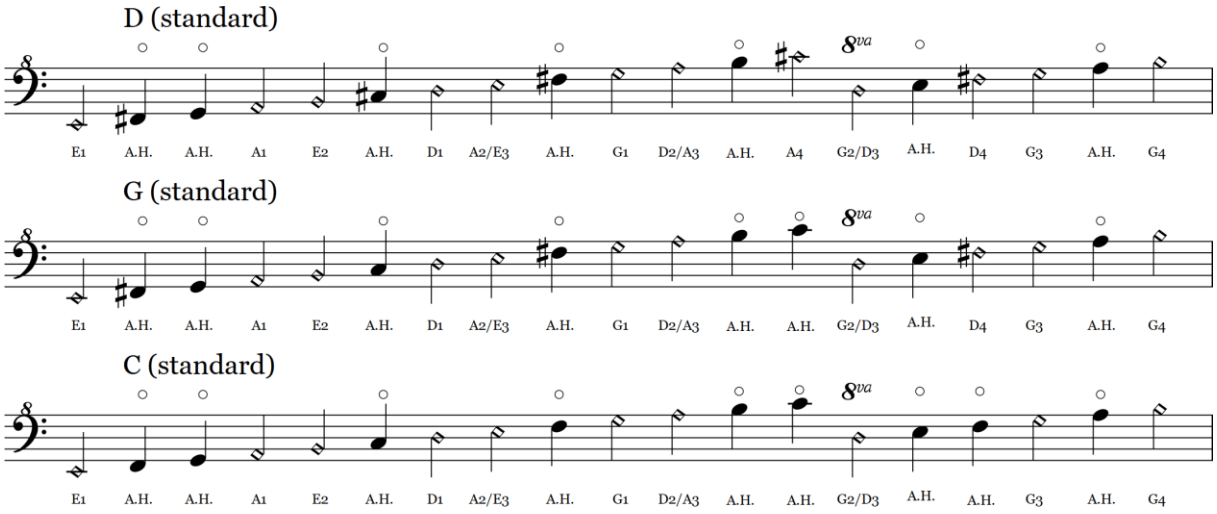
Audio Examples 3 - 5



Next, the scales are filled out by adding the missing notes from these key signatures using artificial harmonics:



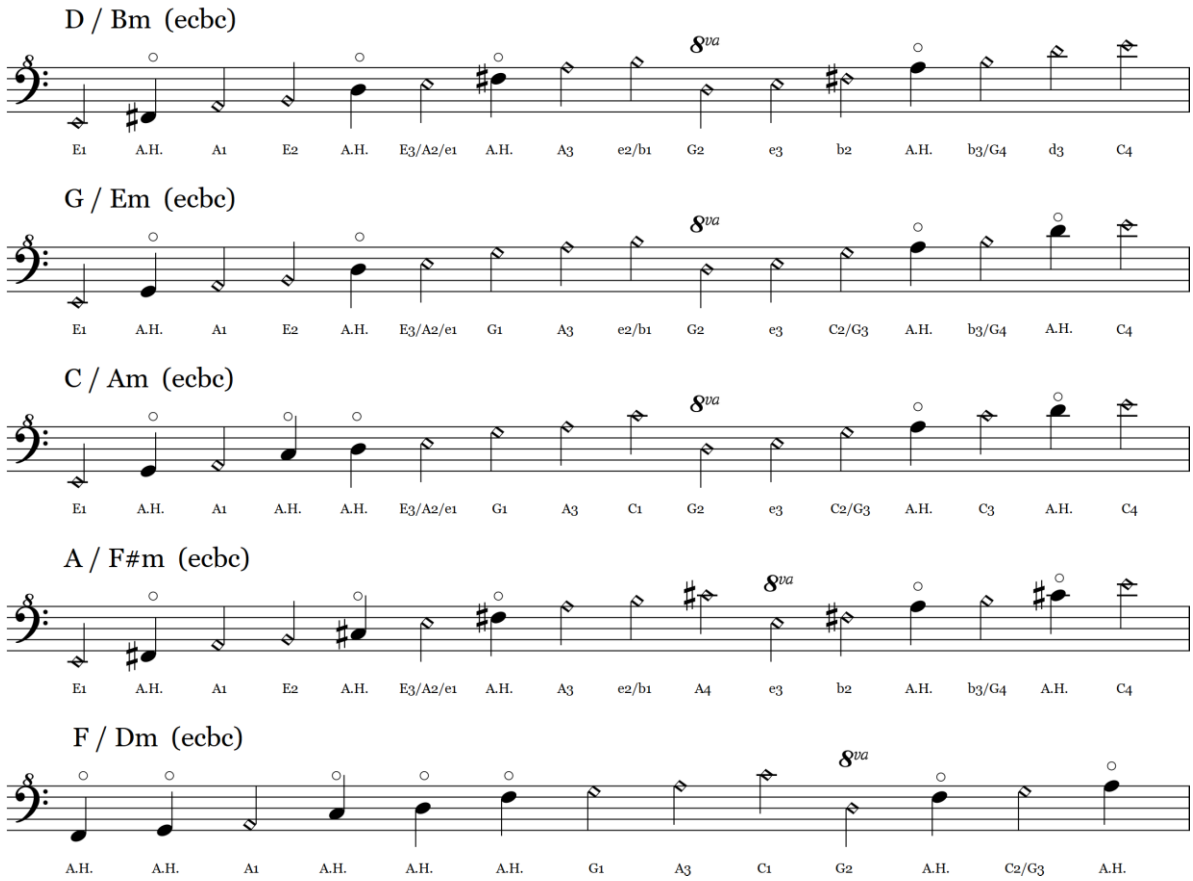
Audio Examples 6 - 8



Further Fundamentals

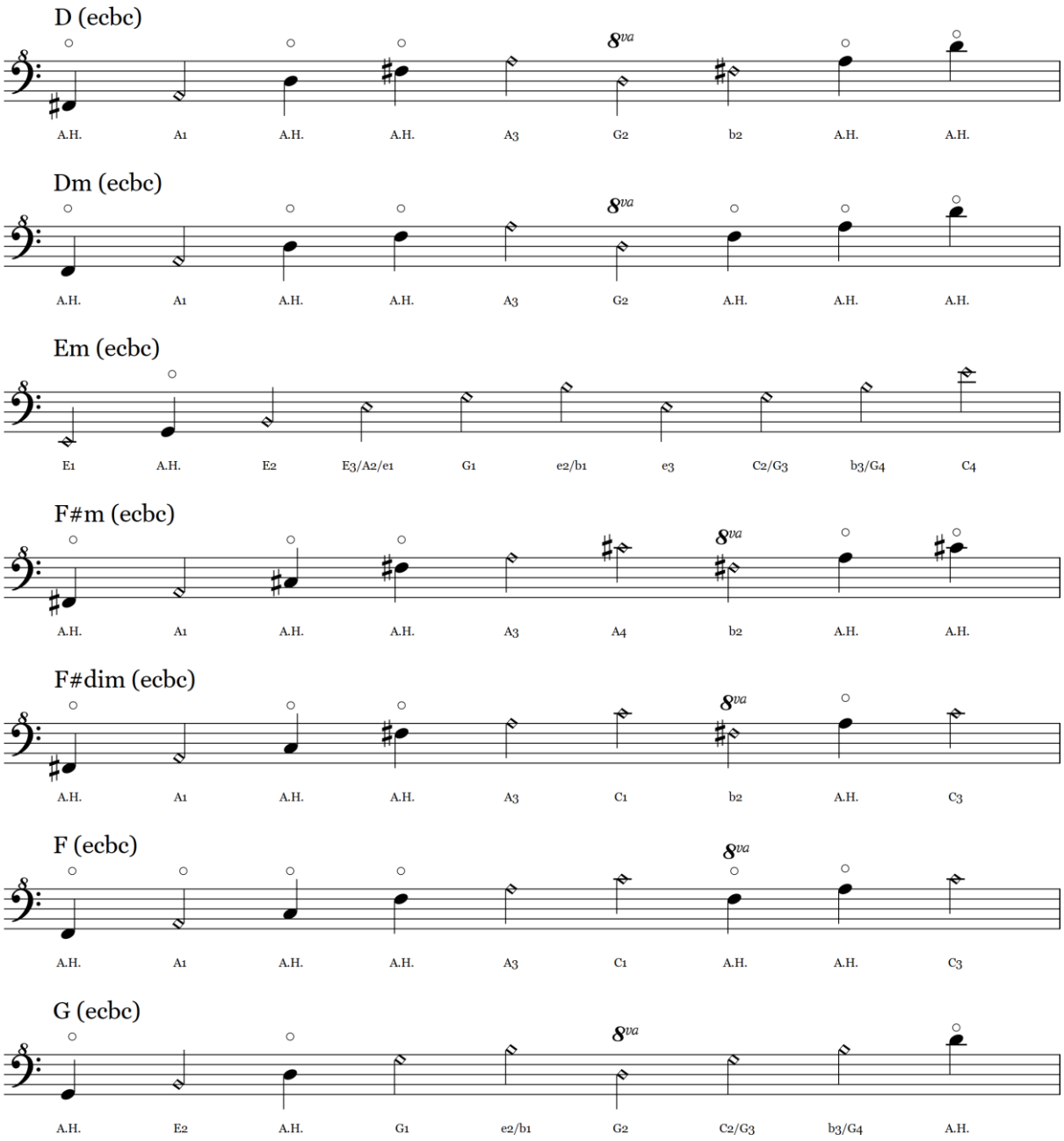
Beyond key signatures, there are various other fundamental building blocks of melody that should be internalised. The next of these: Pentatonic scales, are also a very important component of melodic musicality, linked both to human developmental perception of music and also theoretically connected to the overtone series (both these areas stretching far beyond the practical realms of this study).

Inside each key signature there are three major (or minor) pentatonic scales. Due to overlap between closely related keys, there are five pentatonic scales available to us in total. They are presented below for both tunings:

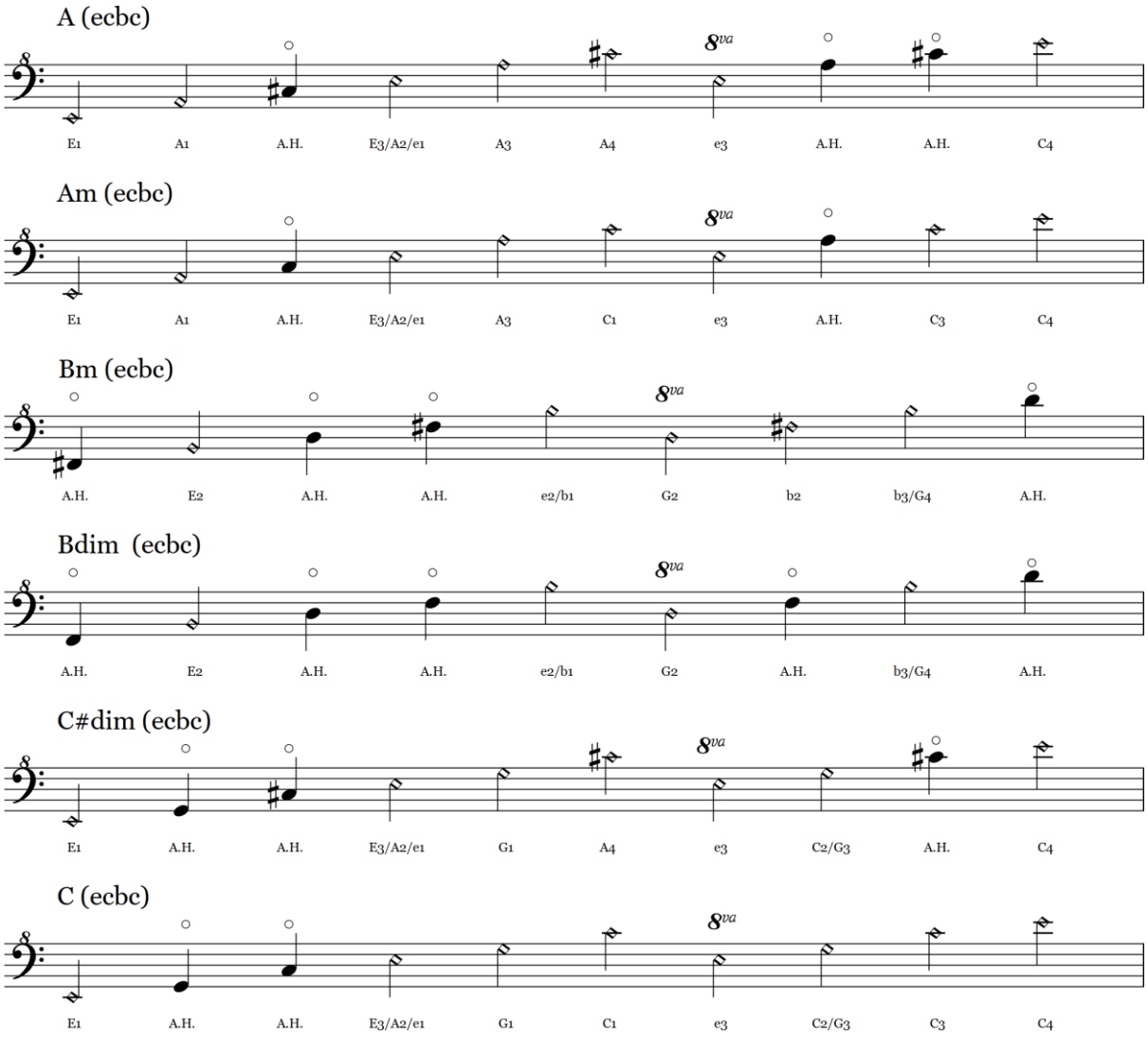


Audio Examples 9 - 13

The primary ‘connective tissue’ between melody and harmony is arpeggios (chords broken into individual notes). Below, all diatonic thirds based triads for the keys of D, G and C are presented for both tunings:

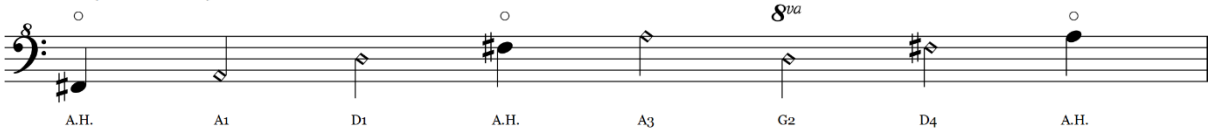


Audio Examples 14 - 20

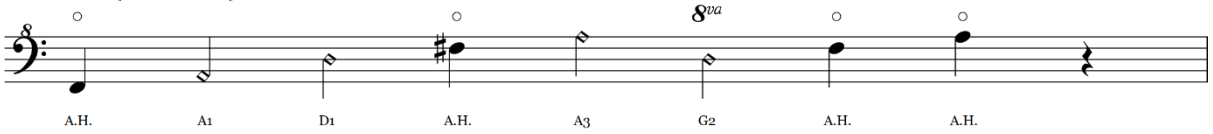


Audio Examples 21 - 26

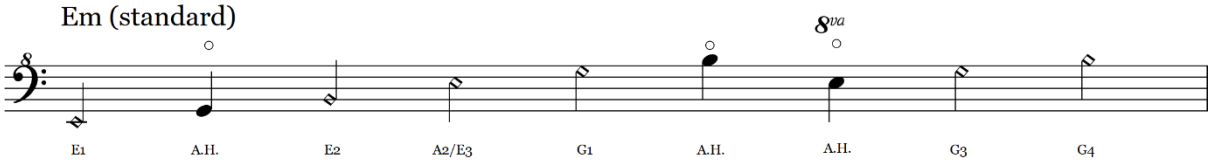
D (standard)



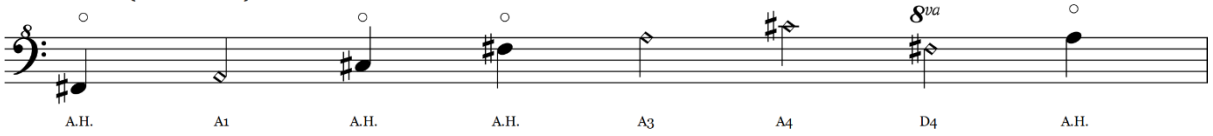
Dm (standard)



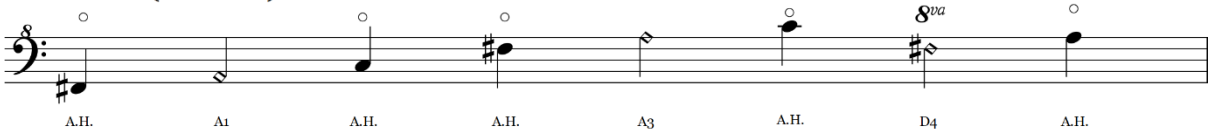
Em (standard)



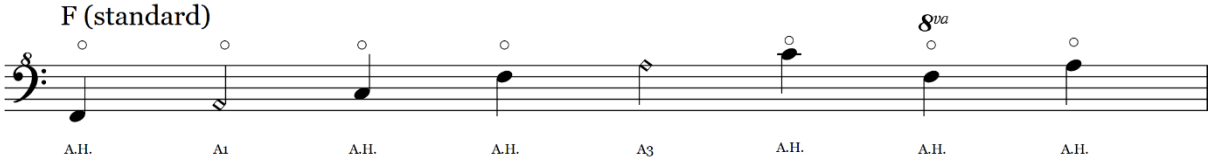
F#m (standard)



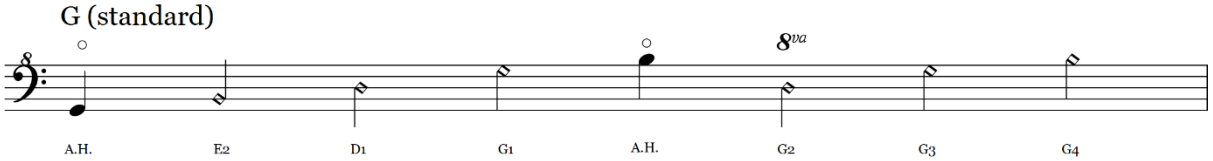
F#dim (standard)

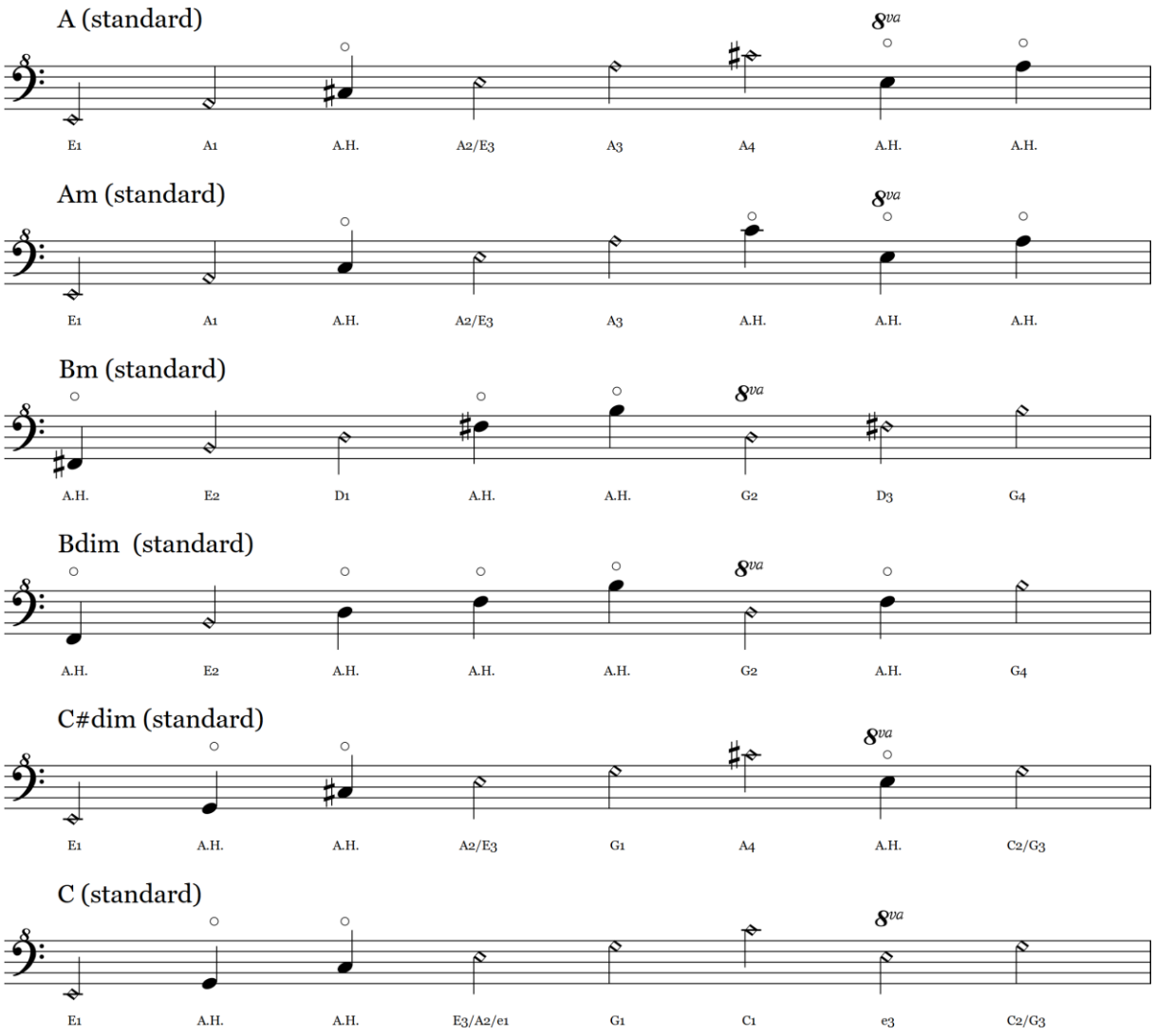


F (standard)



G (standard)





This concludes the content of this section. The exercises are intended to provide insight and knowledge into an individual's musicality and instrumental technique through the practising process, and thus will not be discussed in any more detail at present. For commentary on insights gained throughout my personal practising process, see the 'Reflections' document.

Harmony in Practice

The ultimate aim of this study is not to exhaustively internalise all harmonic possibilities into our playing but to determine according to preference and subjectivity what is practical and what is unnecessary in the process of creating an expanded personal vocabulary of harmonic possibilities.

The next step in this process is to take the analytical information that we have available from the 'Harmonic Algorithm' chapter and put it into practise. This will allow our ears and our 'muscle memory' to become familiar with the feel and sound of the new harmonic vocabulary.

While the overarching theme of applying theoretical work to practise in Sections Three and Four is closely connected, the nature of each section's approach differs. Whereas the previous section was developed from the 'Composite Overtone Series' concept and aimed to provide a strong, practical grasp of the fundamentals underpinning this studies musical content, the present chapter aims to creatively explore the subject matter of the titular 'Harmonic Algorithm' concept.

Because the nature of this chapter is rooted in personal exploration of individual technique and musicality, much of the most significant knowledge and understanding exists in the process of the method. Because of this, logic dictates that providing 'ready-made' examples would be counter-intuitive. For this reason, from this point onwards I will only show information relating to my personal studies of the 'ecbc' instrument. In order to best illuminate the practical process of uncovering this knowledge I will provide insight into the 'working method' used to derive the results. It is recommended that a player of a standard tuned instrument take insight from these workings and explore their own instrument in a similar manner, in order to gain the most benefit for their musicianship.

Diatonic Movement

The most fundamental and populous harmonic movements in musical repertoire are chord changes inside one key signature. An effective and logical method for internalising these movements in both a technical and musical way is by practising moving fluidly by various intervals inside each key.

For each interval, there are many different possibilities for voicing of overtones. In the interest of internalising a fluid 'basic' harmonic repertoire, the present study will usually opt for the most immediately 'obvious', 'effective' or otherwise most simple option. Fluidity in approaching the chord from a technical perspective will also be considered. Considering the wider goals of practising, it is recommended that the player explores and internalises an ever increasing range of voicing choices.

Only intervals from a second through to fourth are presented. The reason being that intervals beyond this correspond with inverted versions of these same movements (second/seventh, third/sixth, fourth/fifth). As we are presenting each of these intervals in ascending and descending forms, the need to explicitly present the higher intervals is not necessary.

The Harmonic Algorithm charts from the Harmonic Extrapolation section were used to derive the possibilities for all examples in this chapter. Rather than choose one specific option, I found it preferable to choose the few most suitable looking options and then to experiment with them in order to find the most suitable choices in practice. The charts demonstrating the different options considered are shown below (and throughout this section). The audio examples don't necessarily follow exactly one path through the multiple option progressions, but utilise insight provided by the different possibilities presented in order to move through the harmony with the most fluid degree of musicality.

The first intervallic movement is the 'second'. Harmonic movements in seconds are commonly referred to as Chord Scales.

D						
EAeGb	EAeGC	EAeGb	EAeGC	EAeGb/C	EAeGb	EAeGb/C
D	E^{-}	$F\#^{-\ no5}$	G^{add9}	A^{7}	B^{-}	$C\#^{o}$
D^{add9}		$F\#^{sus4}$	G	A^{add9}	Bm	$C\#^{ø}$

G						
EAeGb	EAeGC	EAeGb	EAeGb	EAeGC	EAeGb/C	EAeGC
G^{Δ}	A^{-}	B^{-}	C^{Δ}	D^{7sus4}	Em	$F\#^{ø}$
	Am	Bm	$C^{\#11}$	EAeGb	E^{-}	
			EAeGC	D		
			C^{Δ}			
			C			

C						
EAeGb/C	EAeGC	EAeGb	EAeGC	EAeGC	EAeGC	EAeGC
C^{Δ}	D^{7sus2}	Em	F^{Δ}	G	A^{-}	$B^{-\ no5}$
C		E^{-}	F^{add9}	G^{add9}	Am	

Audio Examples 27-32

The interval of the 'third' is not the strongest harmonic movement in terms of functionality, but is characterful and important in its own right, especially when considering the relationship between relative major and minor keys.

D

EAeGb	EAeGb	EAeGb/C	EAeGb/C	EAeGb	EAeGC	EAeGb
D	$F\#^{7sus4}$	A^{7}	$C\#^{o}$	Em	G^{add9}	B^{-}
D^{add9}	$F\#^{-\ \#5}$	A^{add11}	$C\#^{ø}$	Em^{add9}	G	Bm

G

EAeGC	EAeGb	EAeGb	EAeGC	EAeGC	EAeGC	EAeGb
G^{add9}	B^{-}	D^{add9}	$F\#^{ø}$	A^{-}	C^{Δ}	Em
G	Bm	D		Am	C	Em^{add9}
				Am^{add9}	C^{add9}	EAeGC
						Em
						E^{-}

C

EAeGC	EAeGb	EAeGC	EAeGb/C	EAeGb/C	EAeGC	EAeGC
C^{Δ}	Em	G^{add9}	$B^{-\ no5}$	D^{sus2}	F^{Δ}	A^{-}
C	Em^{add9}	G		D^{7sus4}	F^{add9}	Am

Audio Examples 33-38

The intervals of fourth and fifth are very functionally important in tonal music due to the strength and character of their resolution (or 'cadence').

D

EAeGb	EAeGC	EAeGb/C	EAeGb	EAeGb	EAeGb	EAeGC
D	G^{add9}	$C\#^{o}$	$F\#^{-\ no5}$	B^{-}	Em	A^{7}
D^{add9}	G	$C\#^{ø}$	$F\#^{7sus4}$	Bm	Em^{add9}	A^{add9}
			EAeGC		EAeGC	EAeGb
			$F\#^{7sus4}$		Em	A^{7}
					E^{-}	A^{add9}

G

EAeGC	EAeGb	EAeGC	EAeGb	EAeGb/C	EAeGC	EAeGb
G^{add9}	C^{Δ}	$F\#^{ø}$	B^{-}	Em	A^{-}	D^{add9}
G	$C^{\#11}$		Bm	E^{-}	Am	D
					A^{7sus4}	

C

EAeGC	EAeGC	EAeGb/C	EAeGb/C	EAeGC	EAeGC	EAeGC
C^{Δ}	F^{Δ}	$B^{-\ no5}$	Em	Am	D^{7sus2}	G
C	F^{add9}		E^{-}	Am^{add9}	D^{7sus4}	G^{add9}
C^{add9}				A^{7sus4}		

Audio Examples 39-44

Cadence

The next step in building harmonic repertoire is to move outside of the key signature and look at chord progressions centred around resolution.

The following examples give Two-Five-One progressions landing on every major and minor triadic chord contained inside the key signatures under consideration. The concept of each chord of the key having its own dominant chord is referred to in classical terminology as 'Secondary Dominants'. In these examples, secondary dominant chords are given a 'tertiary' subdominant to complete the (primarily) Jazz concept of the Two-Five-One cadence. Many interesting harmonic movements, resolutions and modulations can be created from these 'peripheral' harmonic options, especially when placing the outside chords into different inversions.

These examples were derived in the same manner as the previous diatonic exercises. The different voicing options considered for each example are listed below. In some cases where they provided interesting alternative options, tritone substitutions have been made. Voicings with the strongest harmonic implication have been chosen for these examples.

D

EAeGC	EAeGC	EAeGb
E^{-}	A^{7}	D
		$D^{\Delta no5}$

Dm

EAeGC	EAeGb	EAeGC
$E^{-\,no5}$	$D\#^{7no5}$	D^{sus2}
$E^{7\#9no5}$	$D\#^{b7\#9no5}$	D^{b7no3}
$E^{b7\#5}$		

Em

EAeGC	EAeGb	EAeGC
$F\#^{ø}$	B^{7}	Em
	$B^{7\#5}$	E^{-}
	$B^{7\#9no5}$	EAeGb
		$E^{-\,\Delta}$

F

EAeGC	EAeGb	EAeGC
$C\#^{ø}$	$C^{\#9}$	F^{Δ}

F#m

EAeGb	EAeGb	EAeGb
$G\#^{ø}$	$C\#^{7sus4}$	$F\#^{-\,no5}$
EAeGC	$C\#^{b7b5no3}$	
D^{b7b5}		

G

EAeGC	EAeGb	EAeGC
A^{-}	D	G

A

EAeGb	EAeGb	EAeGC
B^{-}	E^{7}	A^{Δ}
	E^{9no5}	
	$E^{\#9}$	

Am

EAeGC	EAeGb	EAeGC
$B^{-\,no5}$	$E^{\#9no5}$	A^{-}
$F^{ø}$	$A\#^{b7\#5b9}$	Am
	$A\#^{b7\#9no5}$	$A^{-\,\Delta}$

Bm		
EAeGb/C	EAeGb/C	EAeGb
C#^{ø}	F#^{7sus4}	B^{-}
	F#^{7sus2}	Bm

C		
EAeGC	EAeGb/C	EAeGC
D^{7sus4}	G	C
D^{b7no3}		

C#m		
EAeGb	EAeGb	EAeGC
D#^{o}	G#$^{b7b9\#5no3}$	C#$^{-}$
D#^{b7sus4b9no5}	EAeGC	C#m
D#$^{\#5\#9}$	G#$^{\#9\#5}$	C#$^{-\Delta}$

Audio Examples 45-55

While the 'half diminished' chord seen above is a naturally occurring structure inside diatonic harmony, we can find other interesting possibilities for resolution through the use of 'synthetic' structures such as the fully diminished chord. The fully diminished or 'diminished seventh' chord is constructed by stacked thirds. Because of its stacked intervallic nature, every inversion of the chord can be considered a root position in its own right. This means that the diminished chord can resolve equally strongly from any of its tones as if that was the root of the chord. For these reasons, diminished chords are excellent tools for modulation and allow for a great range of interesting root motions, harmonic textures and tonal leaps. Owing to its unique properties and high level of internal tension, diminished chords can resolve in a satisfying manner to almost any stable harmonic structure. The most common resolution however is by ascending one semitone into the resolved structure.

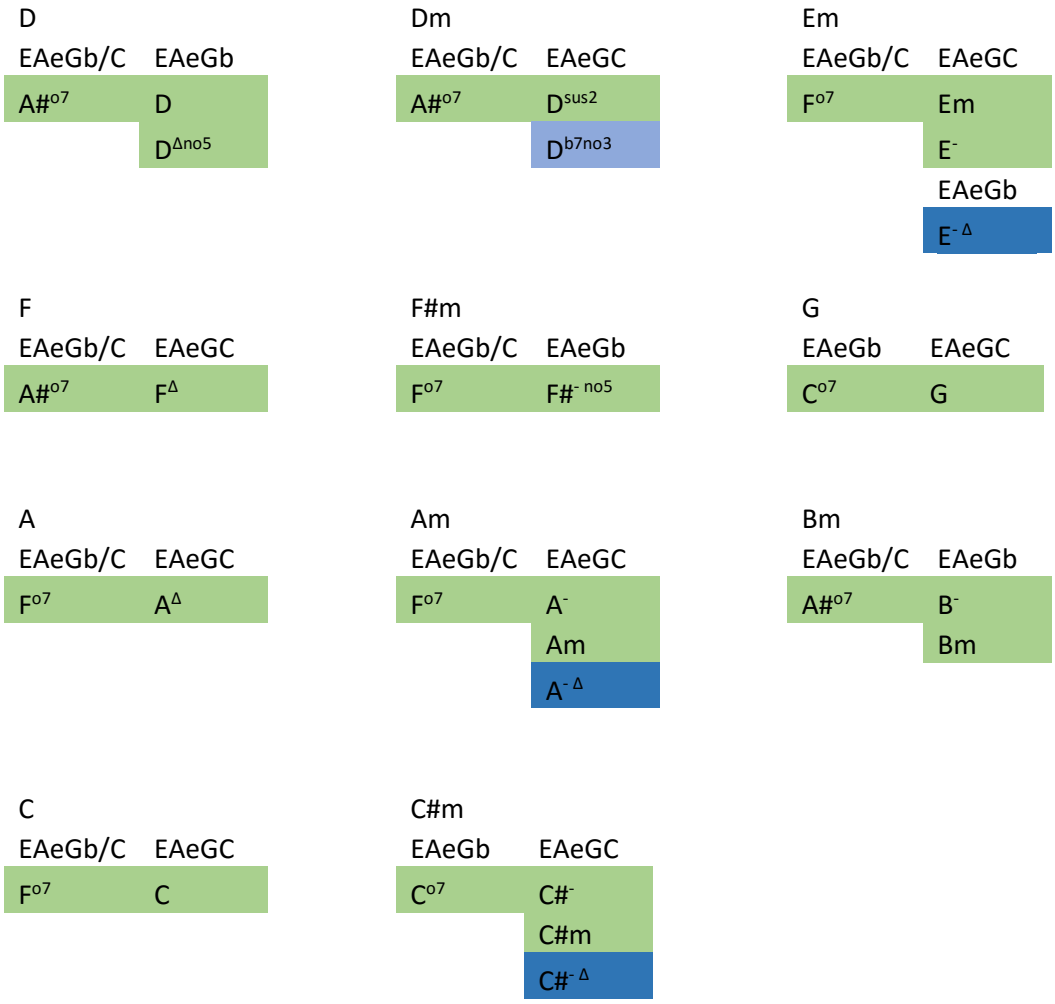
By searching inside The Harmonic Algorithm's data, we can find that there are three possibilities for diminished seventh chords: C, F and A# (Bb). By writing out the complete notes for these three structures, we can observe another interesting phenomenon of diminished chords.

C	Eb (D#)	Gb (F#)	A
F	Ab (G#)	B	D
A# (Bb)	C#	E	G

Inside these three structures, all twelve notes of the chromatic scale are present. This is because as a result of their stacked nature there are only three 'variations' of diminished seventh chords in existence, everything else being an inversion of these three and each inversion functioning as a root.

The following diminished progressions use the same options for resolution as the Two-Five-One progressions above. The diminished chord chosen for each is simply whichever of the three possibilities listed that contains the note a semitone below the target for resolution.

You can immediately observe many options for interesting progression or modulation from these examples alone. Root motion was used to create smoother cadences.



Audio Examples 56-66

This concludes the body of this studies' theoretical and practical content. The next Section includes all the remaining commentary as well as any additional information required in order to bring the work to its logical completion.

Concluding Sections

Final Word

A primary consideration throughout this work from its conception through to completion has been how the process of practising can be incorporated fluidly and effectively into a primarily written document format. The method chosen was to create a methodology specifically tailored to develop a closely targeted area of technique, in a manner that strove to be truly original.

An important aspect of this work's nature is that it is by design a work in progress. The point at which it concludes is considered to represent a good balance of practical achievement inside the realistic scope set out to achieve by this work. A further two audio examples (67 & 68) have been provided demonstrating use of the concepts developed throughout this project in real world settings. Although audio examples 3-26 were presented ascended throughout this document, they have been performed descending in order to place emphasis on the upper harmonics.

This document itself aims only to provide a documentation of the method by which the aims of the study are achieved and does not provide any discussion or commentary further to this. The document alone may not necessarily capture the essence or the sheer scale of effort and time invested spent on both the analytical and practical aspects of the study. However, this is not necessarily a negative trait; the purpose of this document is to present this vast input of study and preparation in a concise and practical manner. The process of these aspects of the work will be discussed in more depth in the 'Reflections' document.

Audio Examples Reference

Preliminary Discussion Examples:

1. *Example in practice of 'Three Part Playing' using only tapping techniques: 'Tango' (live) by Russell Cottier & Oscar South.*
2. *Example in practice of emulating guitar finger picking patterns using a mix of tapping and picking techniques: 'Fingerpicking Jigs' by Oscar South.*

Melodic Development Examples (all utilising 'noded' two-point articulation):

3. *Key of D Major (descending) using only natural harmonics.*
4. *Key of G Major (descending) using only natural harmonics.*
5. *Key of C Major (descending) using only natural harmonics.*
6. *Key of D Major (descending) using natural and supplementary artificial harmonics.*
7. *Key of G Major (descending) using natural and supplementary artificial harmonics.*
8. *Key of C Major (descending) using natural and supplementary artificial harmonics.*
9. *Pentatonic scale of D Major / B Minor (descending) using natural and supplementary artificial harmonics.*
10. *Pentatonic scale of G Major / E Minor (descending) using natural and supplementary artificial harmonics.*
11. *Pentatonic scale of C Major / A Minor (descending) using natural and supplementary artificial harmonics.*
12. *Pentatonic scale of A Major / F# Minor (descending) using natural and supplementary artificial harmonics.*
13. *Pentatonic scale of F Major / D Minor (descending) using natural and supplementary artificial harmonics.*
14. *D Major Arpeggio (descending) using natural and supplementary artificial harmonics.*
15. *D Minor Arpeggio (descending) using natural and supplementary artificial harmonics.*
16. *E Minor Arpeggio (descending) using natural and supplementary artificial harmonics.*
17. *F# Minor Arpeggio (descending) using natural and supplementary artificial harmonics.*
18. *F# Diminished Arpeggio (descending) using natural and supplementary artificial harmonics.*
19. *F Major Arpeggio (descending) using natural and supplementary artificial harmonics.*
20. *G Major Arpeggio (descending) using natural and supplementary artificial harmonics.*
21. *A Major Arpeggio (descending) using natural and supplementary artificial harmonics.*
22. *A Minor Arpeggio (descending) using natural and supplementary artificial harmonics.*
23. *B Minor Arpeggio (descending) using natural and supplementary artificial harmonics.*
24. *B Diminished Arpeggio (descending) using natural and supplementary artificial harmonics.*
25. *C# Diminished Arpeggio (descending) using natural and supplementary artificial harmonics.*
26. *C Major Arpeggio (descending) using natural and supplementary artificial harmonics.*

Harmonic Development Examples (utilising a mix of tapped one-point articulation and 'noded' two-point articulation):

27. *Diatonic Chord Scale in D (ascending).*
28. *Diatonic Chord Scale in G (descending).*
29. *Diatonic Chord Scale in C (ascending).*
30. *Diatonic Chord Scale in D (descending).*
31. *Diatonic Chord Scale in G (ascending).*

32. *Diatonic Chord Scale in C (descending).*
33. *Diatonic Movement by thirds in D (ascending).*
34. *Diatonic Movement by thirds in D (descending).*
35. *Diatonic Movement by thirds in G (ascending).*
36. *Diatonic Movement by thirds in G (descending).*
37. *Diatonic Movement by thirds in C (ascending).*
38. *Diatonic Movement by thirds in C (descending).*
39. *Diatonic Movement by fourths in D (ascending).*
40. *Diatonic Movement by fourths in D (descending).*
41. *Diatonic Movement by fourths in G (ascending).*
42. *Diatonic Movement by fourths in G (descending).*
43. *Diatonic Movement by fourths in C (ascending).*
44. *Diatonic Movement by fourths in C (descending).*
45. *Two-Five-One Cadence resolving to D Major.*
46. *Two-Five-One Cadence resolving to D Minor.*
47. *Two-Five-One Cadence resolving to E Minor.*
48. *Two-Five-One Cadence resolving to F Major.*
49. *Two-Five-One Cadence resolving to F# Minor*
50. *Two-Five-One Cadence resolving to G Major.*
51. *Two-Five-One Cadence resolving to A Major.*
52. *Two-Five-One Cadence resolving to A Minor.*
53. *Two-Five-One Cadence resolving to B Minor.*
54. *Two-Five-One Cadence resolving to C Major.*
55. *Two-Five-One Cadence resolving to C# Minor.*
56. *Diminished Resolution to D Major.*
57. *Diminished Resolution to D Minor.*
58. *Diminished Resolution to E Minor.*
59. *Diminished Resolution to F Major.*
60. *Diminished Resolution to F# Minor*
61. *Diminished Resolution to G Major.*
62. *Diminished Resolution to A Major.*
63. *Diminished Resolution to A Minor.*
64. *Diminished Resolution to B Minor.*
65. *Diminished Resolution to C Major.*
66. *Diminished Resolution to C# Minor.*

Practical application of Harmonic Algorithm techniques in real world examples:

67. *Example A of techniques from The Harmonic Algorithm being used in context: 'The Last Red Lead of Autumn' by Elfin Bow. Unpublished on date of use (track unfinished).*
68. *Example B of techniques from The Harmonic Algorithm being used in context: 'The House Carpenter' by Russell Cottier. Unpublished on date of use (track unfinished).*

Bibliography & References

Barrett, E., Bolt, B. (2007) *Practice as Research: Approaches to Creative Arts Enquiry*, I B Tauris & Co Ltd

Bernstein, L. (1976) *The Unanswered Question*,
Retrieved (2016) from
http://www.openculture.com/2012/03/leonard_bernsteins_masterful_lectures_on_music.html

Candy, L. (2006) *Practice Based Research: A Guide*, CCS Report: 2006-V1.0 November, University of Technology Sydney

Gann, K. (1997) *Just Intonation Explained,*
Retrieved (2016) from
http://www.kylegann.com/tuning.html

International Piano Supply (1997-2002) *Cents to Hz Conversion Chart,*
Retrieved (2016) from
http://www.pianosupply.com/cents-hz/

Lapp, D., R., *The Physics of Music and Musical Instruments,* Tufts University Massachusetts
Retrieved (2016) from
http://kellerphysics.com/acoustics/Lapp.pdf

Levine, M. (1995) *The Jazz Theory Book,* Sher Music

Manring, M. (2005) *Soliloquy - Extended notes*, Manthing Music

Manring, M. (1998) *Michael Manring [Instructional DVD for Bass],* Hal Leonard Corporation

Music Calculator - Hertz
Retrieved (2016) from
http://musiccalculator.com/#convert-hertz

Oxford Dictionaries (2016) *Definitions for 'Harmonic' & 'Algorithm'*,
Retrieved (2016) from
http://www.oxforddictionaries.com/definition/english/algorithm

Pastorius, J. (2001) *Jaco Pastorius - Modern Electric Bass*, Manhattan Music Inc

Pear, D. (1999) *Bass Harmonics: New Concepts and Techniques,* Alfred Music

Prestia, R., Schaub, A. S. D. (1993) *Fingerstyle Funk with Francis Rocco Prestia*

Rawlins, R., Bahha, N. E. (2005) *Jazzology: The Encyclopedia of Jazz Theory for All Musicians,* Hal Leonard Corporation

Suits, B. H. (1998) *Frequencies for equal-tempered scale, A4 = 440 Hz*, Michigan Technological University,
Retrieved (2016) from
http://www.phy.mtu.edu/~suits/notefreqs.html

Suits, B. H. (1998) *Scales: Just vs Equal Temperament*, Michigan Technological University,
Retrieved (2016) from
http://www.phy.mtu.edu/~suits/notefreqs.html

Tension and Gauge Calculator
Retrieved (2016) from
http://www.stringbusters.com/stringfaqs.asp#Calculator

Tontechnik-Rechner – sengpielaudio, *Harmonics, Overtones, and the Fundamental, Calculations of Harmonics from Fundamental Frequency*
Retrieved (2016) from
http://www.sengpielaudio.com/calculator-harmonics.htm

Wooten, V. L. (2006) *The Music Lesson,* Berkley Publishing Group

Wooten, V. L., Bailey, S. (1993) *Victor Wooten & Steve Bailey: Bass Extremes*, Alfred Publishing Co.(UK)Ltd