Comparing Portfolio Optimizer from Black-Litterman Model and Markowitz

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Introduction

This research project implements Black-Litterman Model for portfolio optimizer along with Markowitz Optimizer. Optimize a portfolio is not an easy task since the target equation is not usually linear and that with high variance of finance world, the constraints are different from time to time. In this project, I implement the solution given by Black and Litterman to analyze the returns and weights from the same financial assets but different optimizers. Unlike Markowitz method assuming that investors know expected returns, Black-Litterman model use the CAPM model to calculate market equilibrium return and weight as the input. In addition, it incorporates subjective views from investors or portfolio managers into calculation. To make a comparison, there are two optimized portfolios, one with arbitrarily generated views, another one with views generated by machine learning algorithm.

Data Selection and Processing

The data randomly selected for this research project are Bitcoin, futures and ETF shown below. This monthly data set is between June 2010 and July 2021.
Following are the name of each component:

- Bitcoin (Currency)
- Crude Oil (Futures)
- Gold (Futures)
- Copper (Futures)
- Silver (Futures)
- Corn (Futures)
- S&P500 Index (Futures)
- Nasdaq 100 E-mini (Futures)
- 10-year US Treasury Notes (Futures)
- Invesco Emerging Market (ETF)
- Sovereign Debt (ETF)
- Vanguard Emerging Market (ETF)
- Vanguard High Dividend Yield (ETF)
- Vanguard Real Estate (ETF)
- S&P500 (ETF)
- Russell 2000 (ETF)
- S&P Oil & Gas Exploration & Production (ETF)

The log return is used more commonly than percentage return. Once the log return is found, the correlation matrix can be calculated. To deal with scaling difference, the correlation matrix is used instead of covariance matrix.

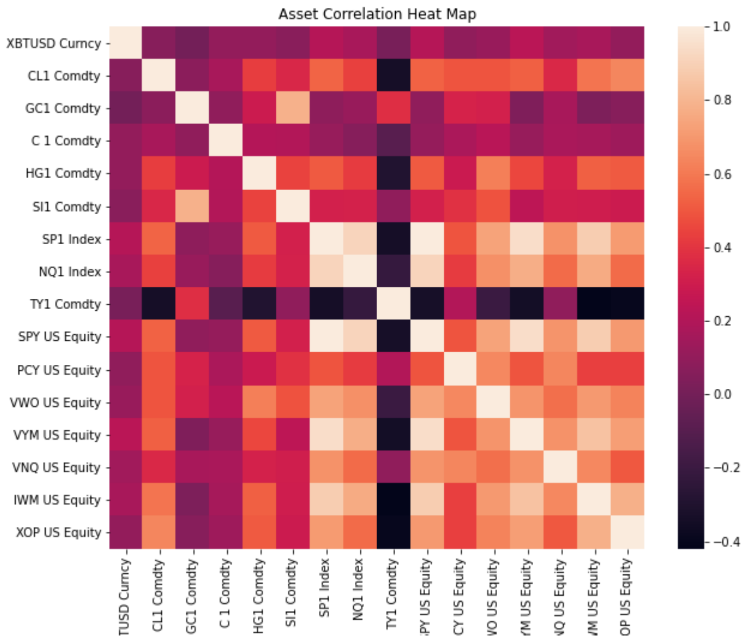


Fig. 1 Financial Asset Correlation Matrix

Principal Component Analysis

It is necessary to check if these randomly picked assets in the portfolio are uncorrelated, so that this is where Principal Component Analysis (PCA) comes in. The more principal components needed to represent 95% variance of the dataset, the more uncorrelated the assets are picked. Originally, the portfolio contains 16 components, but after conducting Principal Component Analysis, it showed that only 9 principal components are sufficient to represent the entire dataset. With this result, we know that optimized this portfolio will achieve diversification and reduce systematic risk. In figure 3 are the principal component analysis every year from 2010 through 2021. The analysis indicates that normally 5 to 6 principal components are enough to represent 95% variance of the original dataset.

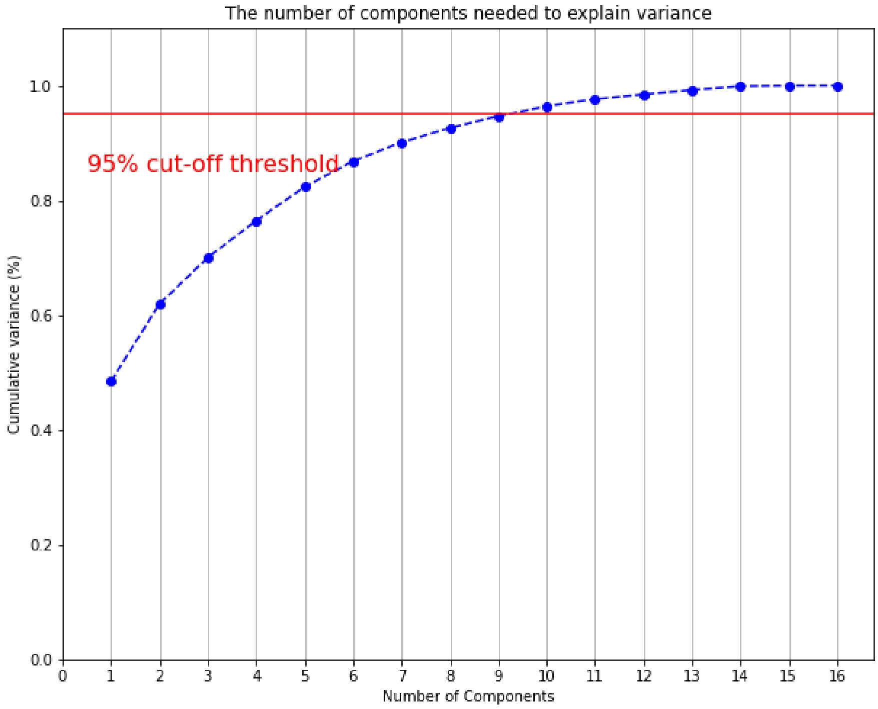


Fig.2 Variance Contribution for every Principal Component entire dataset

Fig.3 Rolling PCA

	0	1	2	3	4	5
0	-3.291290	-1.506131	1.947924	-0.908989	1.043034	0.001529
1	4.532145	-0.594880	-0.278120	-1.619613	-1.090220	0.695498
2	1.878523	2.068322	0.950896	0.305281	0.442454	0.051928
3	-0.711805	-1.950086	-1.248691	1.552052	-0.519205	-1.086721
4	3.695608	-0.394032	-1.989057	-0.477065	1.765227	-0.182377
5	-0.851799	1.418639	-1.539230	-0.808363	-0.806670	-1.169811
6	1.696286	-0.111125	0.640682	0.767418	-0.044881	-0.588742
7	-0.496182	-2.130347	-0.618538	0.708525	-0.610689	1.211632
8	1.383383	1.857747	2.712224	1.849707	-0.120623	0.279816
9	-3.309200	3.167870	-0.427990	-1.060865	-0.217181	0.279566
10	-3.136575	0.207031	-2.441046	0.854543	0.340872	0.856928
11	-1.389096	-2.033008	2.290947	-1.162632	-0.182118	-0.349246

(a) PCA from 2010 to 2011

	0	1	2	3
0	2.700791	4.800164	0.148324	-1.120826
1	9.339922	-1.976968	-1.183409	-1.048160
2	-7.900177	-0.945136	-1.199581	-0.926328
3	0.838047	0.144397	-1.111604	0.008817
4	1.014826	-1.835677	2.363555	0.625305
5	-4.409878	1.170052	-1.779282	1.643930
6	-2.366976	-1.029213	-0.549530	-0.820103
7	-0.621262	-1.742643	-0.279409	-0.000378
8	0.219799	0.715143	0.561659	0.575691
9	5.574805	0.238248	-0.460976	1.555087
10	-3.170130	-0.253473	1.491310	-0.393585
11	-1.219767	0.715105	1.998942	-0.099449

(b) PCA from 2011 to 2012

	0	1	2	3	4	5	6
0	-1.781161	-0.347277	-0.605068	-1.017929	-0.350654	1.067675	-0.431436
1	-2.018784	-0.615758	-1.313045	-1.061558	0.283517	-0.754219	0.550204
2	1.814153	1.170807	-1.679149	-0.355848	-0.532938	-0.343378	-0.331368
3	-0.489617	0.601474	-0.973680	-0.777626	-0.163956	0.536190	0.378710
4	0.315298	0.434398	-0.835938	0.392877	1.005580	-0.752332	-0.434857
5	-0.907783	-0.895798	1.858087	-1.016637	0.894898	0.276311	-0.345397
6	0.721127	1.446716	0.874871	0.295183	-0.722117	-0.020214	0.631275
7	-1.088242	0.317545	2.067502	-0.416792	-0.862514	-0.841969	-0.455529
8	0.673069	3.035776	0.765625	0.786162	0.727191	0.436519	0.346760
9	1.560849	-2.801539	0.484645	0.376453	0.027807	-0.008278	0.952460
10	4.423407	-1.382498	-0.122432	0.325336	-0.079748	0.244693	-0.587572
11	-3.222317	-0.963847	-0.521418	2.470379	-0.227067	0.159001	-0.273249

(c) PCA from 2012 to 2013

	0	1	2	3	4	5
0	-0.272408	3.616404	-0.864552	1.371477	-0.674161	-0.113051
1	0.141229	-2.306878	0.459634	-0.770803	-0.966452	0.211740
2	0.943751	-1.871290	0.672604	0.870589	0.333053	-0.472467
3	5.294336	0.426895	-0.681168	0.225171	0.673612	0.179220
4	-1.262856	-0.300071	-2.396405	-0.955965	-0.026150	1.119158
5	-0.692551	2.546662	2.597524	-0.499984	0.883182	0.672542
6	-2.093679	-1.618860	0.130789	1.958578	0.640624	0.189692
7	-0.988822	-0.088831	-1.239617	-1.559983	1.806191	-0.738095
8	-0.787245	-0.061453	0.814311	-0.383909	0.759363	-0.377574
9	0.675137	-1.432133	1.062645	-0.272848	-0.939604	0.363223
10	-0.658538	0.088962	-0.693049	1.775370	-0.742523	-0.365565
11	-0.298356	1.000593	0.137284	-1.757692	-1.747136	-0.668823

(d) PCA from 2013 to 2014

	0	1	2	3	4	5
0	0.517530	-2.337512	0.406505	-0.462654	-0.131635	-0.028319
1	0.115397	3.401447	-1.061228	-1.262157	0.539081	-0.268230
2	-0.271311	-2.570938	-0.685196	1.820063	0.836450	-0.745639
3	-0.927213	-0.953267	-1.248949	-0.366274	-1.015413	-0.222705
4	-0.657146	1.106825	-0.255908	1.509566	-0.115727	1.104756
5	-3.648266	-0.508756	2.544998	-0.442313	-0.202607	0.098461
6	3.789292	-1.235698	-0.614241	-0.178946	-0.140919	0.740071
7	-0.277151	0.805710	0.885923	0.435670	1.085146	0.541761
8	2.593698	0.066520	2.112939	-1.269612	0.186778	-0.572772
9	-0.087969	0.190553	0.080163	-0.410489	-0.703164	0.307722
10	0.803482	2.478808	0.204116	1.886996	-0.688001	-0.828776
11	-1.950342	-0.443693	-2.369122	-1.259850	0.350009	-0.126330

(e) PCA from 2014 to 2015

	0	1	2	3	4	5
0	-4.179722	-0.105692	1.359039	0.028553	-0.771817	-0.702641
1	-2.245485	-0.614583	-0.610883	0.414280	1.169090	0.322814
2	4.742268	-0.242508	-0.049886	0.364659	-0.676376	0.867060
3	-0.121149	-2.996063	-0.301254	-1.075365	-0.493790	-0.490567
4	-2.019522	-0.872125	-1.163519	1.643795	0.282137	-0.339228
5	-3.840840	0.944721	0.016406	-1.070864	0.317534	0.747032
6	-0.642801	1.661700	-0.791906	0.449829	-0.870147	0.425172
7	4.618436	0.314147	0.917770	-0.255285	0.915239	-0.625179
8	0.318090	1.366998	2.696046	0.626618	0.041184	0.051910
9	0.673341	-2.734755	0.427906	-0.299231	0.175411	0.620949
10	0.405867	2.999271	-1.090742	-0.934461	0.200720	-0.284962
11	2.291518	0.278888	-1.408978	0.107473	-0.289185	-0.592359

(f) PCA from 2015 to 2016

	0	1	2	3	4	5
0	0.182462	1.773944	-1.348442	1.300250	0.155972	-0.302041
1	0.032075	-0.258143	1.187651	0.238406	1.202302	0.147266
2	0.344545	2.101265	1.460156	-0.641174	-0.122862	-0.246261
3	5.032526	-0.983225	-0.236711	-0.128026	-0.188537	-0.261606
4	-0.289901	0.317107	-1.211225	-0.795322	0.876547	0.320867
5	-1.100646	-1.700947	0.977954	0.847802	-0.247178	0.127958
6	-1.365402	-1.433715	-0.929610	-0.439715	-0.074103	0.045165
7	-0.501091	0.954711	0.324349	0.737957	-0.311847	0.188154
8	-1.302335	0.267647	-0.516929	-0.173109	-0.625149	0.007895
9	-1.332080	-0.055008	0.147386	-1.011951	-0.199803	-0.676767
10	0.790267	0.345294	0.085130	-0.350869	-0.515177	0.990732
11	-0.490420	-1.328929	0.060291	0.415751	0.049836	-0.341362

(g) PCA from 2016 to 2017

	0	1	2	3	4	5	6
0	-0.234880	2.597304	-0.563067	-0.926287	-0.209239	0.164679	-0.268430
1	-0.029122	-1.685693	0.200595	0.894891	0.599917	-0.142180	-0.039000
2	-0.952655	1.032772	-0.041196	0.396660	-0.628883	-0.003014	0.028652
3	-0.770134	-0.239140	-0.577144	0.572961	-0.096726	0.178080	-0.115307
4	-0.572307	1.940367	0.021588	0.422655	0.354603	-0.872964	0.056774
5	-2.442706	-1.442517	1.667587	-0.958319	-0.551733	-0.630495	-0.117446
6	3.002158	0.603153	2.040862	-0.036521	-0.296804	0.562648	0.342412
7	2.089262	-0.750933	-0.639495	-1.025213	1.181998	-0.800044	-0.181343
8	0.483236	0.053897	-0.167052	1.287779	-0.150283	-0.503816	0.225540
9	-0.339393	-0.934217	-1.304248	-0.651654	-0.362343	0.377999	1.166111
10	1.301226	-1.213264	-0.936909	-0.009833	-1.140405	0.368396	-0.855964
11	-1.534684	0.038272	0.298478	0.032882	1.299899	1.300710	-0.242000

(h) PCA from 2017 to 2018

	0	1	2	3	4	5
0	-1.221670	-1.398685	0.648270	-1.415168	1.172496	0.206121
1	0.099667	-0.825626	-1.695399	0.467184	-0.703360	-0.501794
2	4.830516	-1.285552	-0.129666	0.128081	-0.375909	-0.102931
3	-0.670043	-0.693366	2.076535	2.035005	0.061975	0.389638
4	6.180315	1.434199	-1.025040	0.876866	0.972887	-0.016315
5	-5.429415	1.170585	0.422384	0.449582	-0.387401	-0.961028
6	-1.870920	-0.932642	-0.591250	-0.028088	-0.589154	0.201671
7	-0.846601	0.595739	0.411624	-0.646786	0.797456	-0.992959
8	-1.939954	-0.489190	-0.513555	-0.814331	-0.230724	0.338258
9	4.969335	1.143005	1.465876	-1.287474	-1.096965	0.085800
10	-3.603828	1.809621	-0.655585	-0.104955	-0.002762	1.164400
11	-0.497402	-0.528087	-0.414193	0.340083	0.381462	0.189138

(i) PCA from 2018 to 2019

	0	1	2	3
0	1.266823	-0.978423	2.344450	0.738699
1	-0.612930	0.733484	-2.248888	-0.085866
2	-1.206205	0.044691	-0.131543	-0.288138
3	-0.921286	0.740056	-1.754443	-0.811570
4	-2.695082	-0.079932	-1.091824	0.468805
5	1.685419	0.004789	1.864748	0.127385
6	5.119642	-1.685918	-0.660936	2.038826
7	14.693948	1.463217	-0.002490	-1.242648
8	-6.868525	4.568270	0.569826	0.142510
9	-4.810503	-3.855815	-0.179247	-1.836070
10	-2.184401	-0.895022	-0.985114	1.190115
11	-3.466900	-0.059396	2.275461	-0.442049

(j) PCA from 2019 to 2020

	0	1	2	3	4	5	6
0	1.954659	0.223716	-1.557351	0.182614	-0.532387	1.001766	1.134983
1	-4.808738	-1.240144	0.505431	1.016964	0.994743	-0.459161	-0.304476
2	-2.737680	-0.875404	-0.278244	-0.915162	-0.307973	0.349027	-0.584291
3	6.010358	0.252004	1.637073	0.716009	0.119738	-0.847976	-0.784102
4	1.455393	-0.379669	-2.094055	-1.006666	-0.658325	0.371099	-0.659531
5	-0.902363	-1.601176	-0.052639	-0.753557	-0.329075	-0.525646	0.267163
6	1.044511	-0.930043	2.567675	-1.686035	0.822256	0.358498	1.043106
7	-0.217620	0.094708	1.220742	0.337862	-0.251687	1.605332	-1.047494
8	1.652273	-0.656831	-2.275176	1.028663	1.872492	-0.082738	0.145027
9	-0.900284	2.285260	-0.721643	-1.569460	-0.174237	-1.448928	-0.166946
10	-0.616551	-1.215409	0.450876	1.787000	-1.708089	-0.715438	0.587494
11	-1.933958	4.042987	0.597311	0.861768	0.152543	0.394165	0.369066

(k) PCA from 2020 to 2021

The require variables for Markowitz portfolio optimizer are the expected rate of return, risk-free rate, and covariance matrix. Since we have 11 years of historical data inputted and processed, we can estimate the expected rate of return using historical return. Even though it is a poor forecast of the future returns, it is still a reasonable approach for Markowitz optimizer. Thus, the table shows the log return of each asset.

Expected Return	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1
µ	10.19%	-0.05%	0.32%	0.25%	0.23%	0.27%	1.05%	1.58%
Expected Return	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP
µ	0.06%	1.05%	0.02%	0.15%	0.78%	0.58%	0.93%	-0.51%

The risk free is set to be the average yield of 3-month US treasury bond which is 2.23%.

Markowitz Optimizer Summary

There are three portfolio optimizers in this research project in the need of comparing to Black-Litterman optimizer.

Tangency Portfolio:

Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1
W (%)	-0.78	0.66	-9.66	1.64	3.21	1.32	12.74	-23.39
Weight	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP
W (%)	97.86	28.03	-3.51	2.16	1.01	-16.87	5.40	0.18

One can see that the one of the drawbacks of Markowitz optimizer is that the extremely large position of long and short positions which is a highly risky move.

Minimum Variance Portfolio with cash position:

Portfolio expected return = 0.5%

Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1	Cash
W (%)	-0.61	0.51	-7.45	1.26	2.48	1.01	9.83	-18.05	159.1
Weight	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP	
W (%)	75.5	21.62	-2.71	1.67	0.78	-13.01	4.16	0.14	

Portfolio expected return = 5%

Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1	Cash
W (%)	0.97	-0.81	11.93	-2.02	-3.97	-1.62	-15.74	28.89	690.9
Weight	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP	
W (%)	-120.89	-34.62	4.33	-2.67	-1.25	20.84	-6.67	-0.23	

Portfolio expected return = 10%

Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1	Cash
W (%)	2.73	-2.27	33.46	-5.67	-11.14	-4.56	-44.14	81.05	1281
Weight	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP	
W (%)	-339.11	-97.12	12.15	-7.49	-3.51	58.45	-18.7	-0.64	

Portfolio expected return = 30%

Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1	Cash
W (%)	9.75	-8.12	119.57	-20.27	-39.81	-16.29	-157.75	289.66	3645
Weight	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP	
W (%)	-1211.9	-347.11	43.42	-26.77	-12.55	208.9	-66.83	-2.27	

If investors are aiming 30% return, then they have to hold a short position of -347% of total portfolio values of S&P 500 and cash positions are nearly 3645%. In addition, the variance is also extremely high which you will see in the following table. Such distorted positions are impossible to realize.

Maximum Return Portfolio with cash position:

Portfolio expected volatility = 0.5%

Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1	Cash
W (%)	0.47	-0.39	57.07	-0.97	-1.9	-0.78	-7.53	13.82	
Weight	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP	
W (%)	-57.84	-16.56	2.07	-1.28	-0.6	9.97	-3.19	-0.11	

Portfolio expected volatility = 5%

Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1	Cash
W (%)	4.65	-3.87	57.06	-9.67	-18.9	-7.77	-75.3	138.23	
Weight	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP	
W (%)	-578.4	-165.64	20.72	-12.77	-5.99	99.68	-31.89	-1.08	

Portfolio expected volatility = 10%

Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1	Cash
W (%)	9.3	-7.75	114.12	-19.35	-37.9	-15.55	-150.56	276.5	
Weight	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP	
W (%)	-1156.7	-331.3	41.4	-25.55	-11.98	199.4	-63.78	-2.17	

Portfolio expected volatility = 30%

Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1	Cash
W (%)	27.9	-23.2	342.4	-58.0	-113.9	-46.6	-451.7	829.4	
Weight	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP	
W (%)	-3470.1	-993.8	124.3	-76.6	-35.9	598.1	-191.3	-6.5	

Unlike minimum variance portfolio, the maximum return portfolio draws a result which looks more realistic, but the cash position is still above 100%.

Return and Variance

	Tangency Portfolio	Minimum Variance Portfolio				Maximum Return Portfolio			
		0.5%	5%	10%	30%	0.5%	5%	10%	30%
μ	-18.73	-1.71	2.79	7.79	27.79	1.35	13.3	26.5	79.53
Σ	173.43	0.65	1.04	2.93	10.5	0.5	5	10	30

As for equal-weighted portfolio, the return is 1.05.

Markowitz Optimizer Analysis

As you can see from the result, the long/short positions we get from Markowitz optimizer is impossible to construct. Even if the portfolios are constructed, the risks i.e., the variance and cash positions are also high. It is caused by the assumption of Markowitz optimization that investors knew the expected return to the components in Portfolio. It is clear that using historical return as an approximation yield an unsatisfactory result. Also, Markowitz optimization method might result in an opposite result against investors' views toward the market. For example, one might believe that silver futures will outperform copper, then the maximum return portfolio contradicts with this view as it requires shorting silver more than copper. This is where we are going to move on to Black-Litterman model which take views into consideration.

Black-Litterman Portfolio Optimization

Data and Variable

For Black-Litterman Portfolio Optimizer, the same data set is used. However, instead of using historical mean to find expected return rate, the equilibrium market rate is estimated. It is naturally to use market capitalization to estimate the market weight, but for futures which don't directly have market capitalization data, I used its open interest times last price to estimate its market capitalization data. The market cap numbers are taken from July 2010 to July 2011.

Market Cap	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1
W (%)	73.1	0.0021	0.0004	0.0001	8.47 E-05	2.07 E-06	0.018	0.12
	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP
W (%)	0.02	16.3	0.23	4.36	1.18	2.05	2.6	0.14

Now the market equilibrium return can be found. Then we move on to decide the views and its uncertainty (Ω)

Views

The following views are arbitrarily determined. I assumed that the prices of futures contracts which terminate next year are the expected rise of price, so that I can get the views on its return.

View 1: Crude oil futures will rise 26.5%

View 2: Corn futures will rise 2.2%

View 3: Gold futures will rise 3.7%

View 4: Copper will rise 20.9%

View 5: Silver will decrease 11.2%

Matrix P indicates the assets in our portfolio.

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since it is all absolute excess return instead of relative, all entries in matrix P are 1 representing respective asset. When you have relative view, then there are several criteria to meet. Every row must sum to 0. The asset increasing is represented in positive number while the asset decreasing is represented in negative number. But there is a variation about whether to use an equally weighting scheme or market capitalization weighted scheme.

Though the views are not backed by solid theories or equations, there are some researches lately about using Machine Learning to generate the views.

Q matrix on the other hand represent the degree of how much the assets in views vary. In the scenario above, the Q matrix looks like:

$$Q = \begin{bmatrix} 0.265 \\ 0.022 \\ 0.037 \\ 0.209 \\ -0.112 \end{bmatrix}$$

After Q and P are determined, we can calculate the uncertainty of views (Ω). One can build up the Ω matrix with the following equation provided in reference.

$$\Omega = \begin{bmatrix} P1\Sigma_R \, P_1{}^T & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & P5\Sigma_R \, P_5{}^T \end{bmatrix} = \begin{bmatrix} 0.0156 & 0 & 0 & 0 & 0 \\ 0 & 0.008 & 0 & 0 & 0 \\ 0 & 0 & 0.0022 & 0 & 0 \\ 0 & 0 & 0 & 0.0039 & 0 \\ 0 & 0 & 0 & 0 & 0.0087 \end{bmatrix}$$

where Σ_R is the covariance matrix of excess return.
There is another parameter tau (τ) which is a scaling measure that measure the uncertainty of market equilibrium expected return, in which mostly research found that it ranges from 0.01 to 0.05. Here I arbitrarily assume it to be 0.05. As for lambda (λ), it is a risk aversion parameter, and it could be calculated by dividing market variance from market excess return.

Black-Litterman Model Portfolio

The Black-Litterman Optimizer function is written in another independent file called BlackLitterman.py. It needs some inputs such as market equilibrium weight, risk aversion parameter, tau, covariance matrix of excess return, P, Q and omega. The function returns four matrices: expected return, Black-Litterman optimizer expected return, updated covariance matrix and updated weight where "updated" mean incorporating investors' views.

Expected Return

	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1
μ_{mkt} (%)	7.36	0.24	0.0035	0.21	0.18	0.18	0.22	0.2
	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP
μ_{mkt} (%)	0.001	0.22	0.077	0.18	0.22	0.18	0.25	0.36

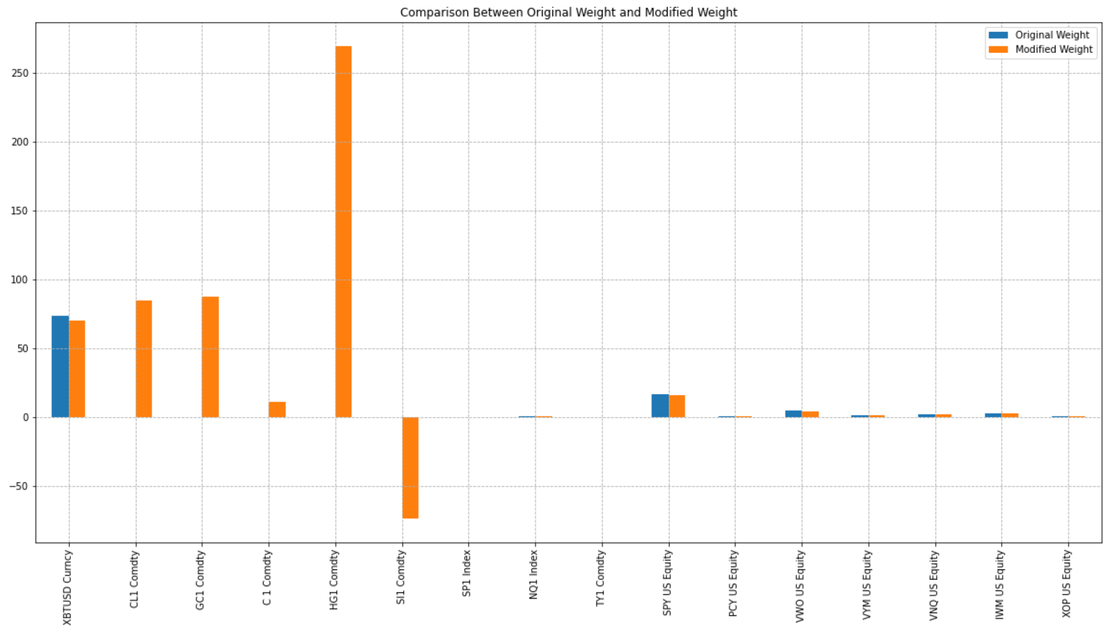
Modified Expected Return

	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1
μ_{BL} (%)	7.99	2.06	0.19	0.64	1.32	0.83	0.67	0.62
	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP
μ_{BL}(%)	-0.1	0.67	0.32	0.86	0.62	0.52	0.93	1.89

Updated Weight

BL Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1
W(%)	69.6	84.5	87.5	10.7	269.2	-74.2	0.017	0.12
	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP
W(%)	0.019	15.5	0.22	4.16	1.12	1.96	2.48	0.13

Here is the comparison graph between market weight and Black Litterman optimized weight.



Analysis

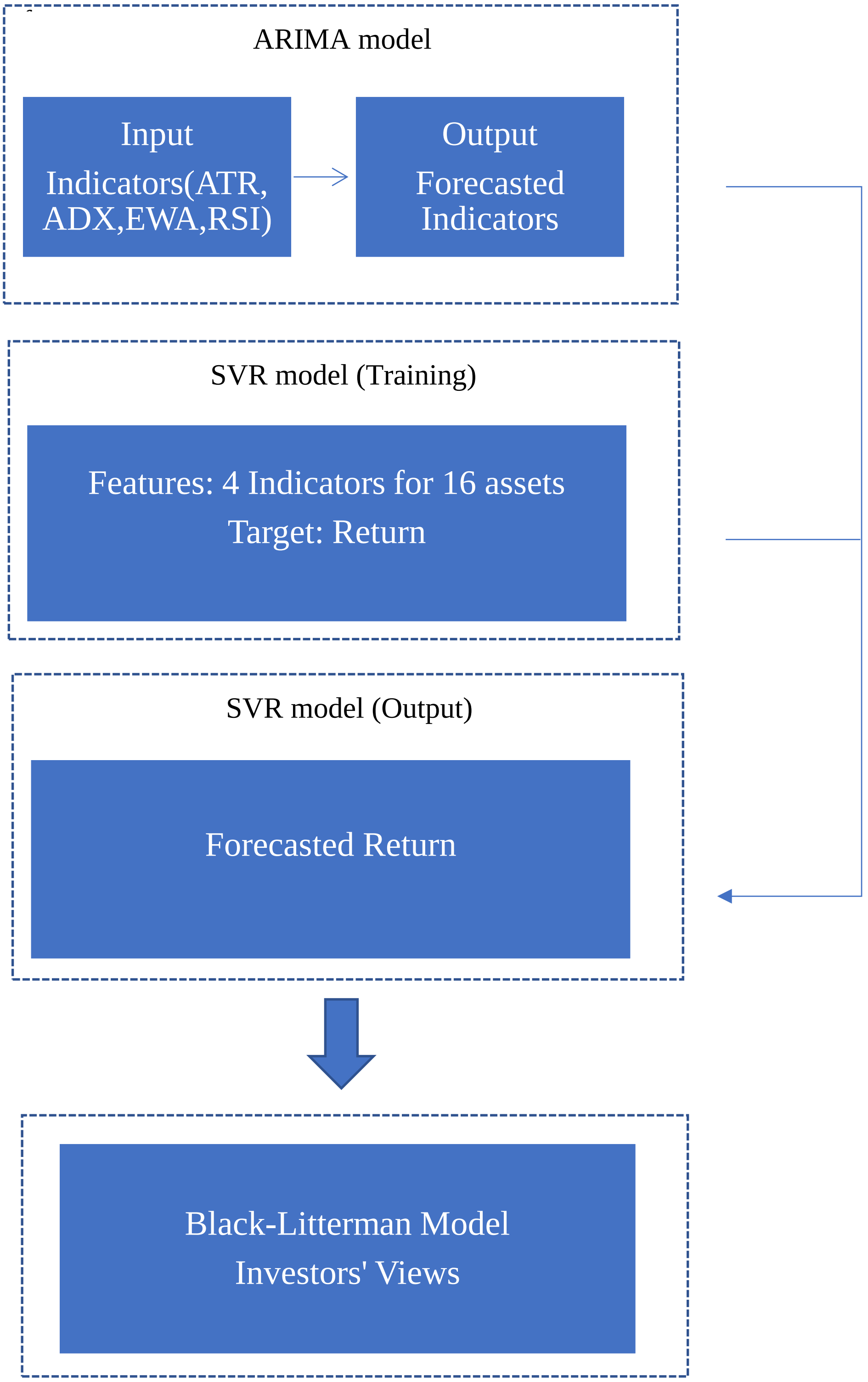
1. The updated return is higher than market expected return which meet the expectation when we try to optimize a portfolio. The reason for this is that the updated weights are allowed to take a short position unlike market weights which are all positive.

2. One can see that the updated weights fit our views where we expect price of crude oil, corn, gold, and copper to rise and silver to decrease. For those assets which we have bull opinion, we increase their weights and decrease those we have bear opinion. Having the views incorporated into the portfolio gives a more desirable result comparing to Markowitz optimizer.

3. From the graph above, we can see that most of the weights are focused on commodity futures. One of the possible reasons is that because of the views are only about them. If we have views about ETF or index futures, then their weights are going to rise. Despite their low weights relative to the entire portfolio, they still contribute almost 40% of the entire portfolio return.

Views generated by Support Vector Regression

Besides arbitrarily generate views, we can take advantage of machine learning algorithm to generate views. Here, I decided to use Support Vector Regression as the algorithm and use four indicators as features to forecast returns. After that, the percent change of the average returns are the views.



ARIMA & Indicators

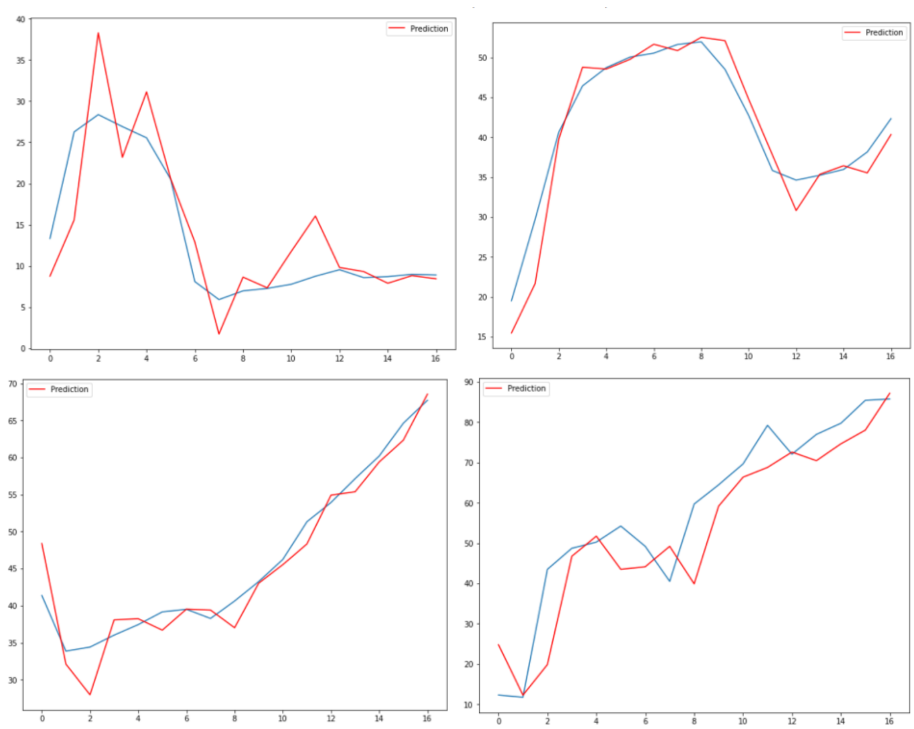
Four indicators are selected to train Autoregressive integrated moving average (ARIMA) time series model. First, Actual True Range (ATR) measures volatility for a specified period which I chose to look back 5 months. Second, Average Directional Index (ADX) is used to determine the strength of a trend. Then, Exponential Weighted Moving Average (EWA) monitors trend and gives higher weight to recent movement. Last, Relative Strength Index (RSI) is a momentum indicator used to measure the magnitude of recent price change.

		Indicator	Type
1	ATR	Average True Range	Volatility
2	ADX	Average Directional Index	Trend
3	EWA	Exponential Weighted Average	Trend
4	RSI	Relative Strength Index	Momentum

ARIMA is a statistical analysis model that uses time series data to either better understand the data set or to predict future trends. The ARIMA model takes 3 parameters, p, d, and q, which are defined as:

- p: the number of lag observations in the model where I used 5
- d: the number of times the raw observations are differenced where I used 1
- q: the size of the moving average window where I used 0

Following are some comparative graphs that show the predictive and the actual numbers. As you can see that the model has predicted the trend successfully.



Support Vector Regression

The model is fit by 4 historical indicators in 84 months and the log returns. The forecasted returns are predicted by the trained SVR model with forecasted 4 trading indicators as input.

Views

The views generated by ML model are in the following.

View 1: Crude oil futures will fall 40.4%

View 2: Corn futures will rise 4.7%

View 3: Gold futures will rise 8.9%

View 4: Copper will rise 13.7%

View 5: Silver will rise 18.1%

Expected Return

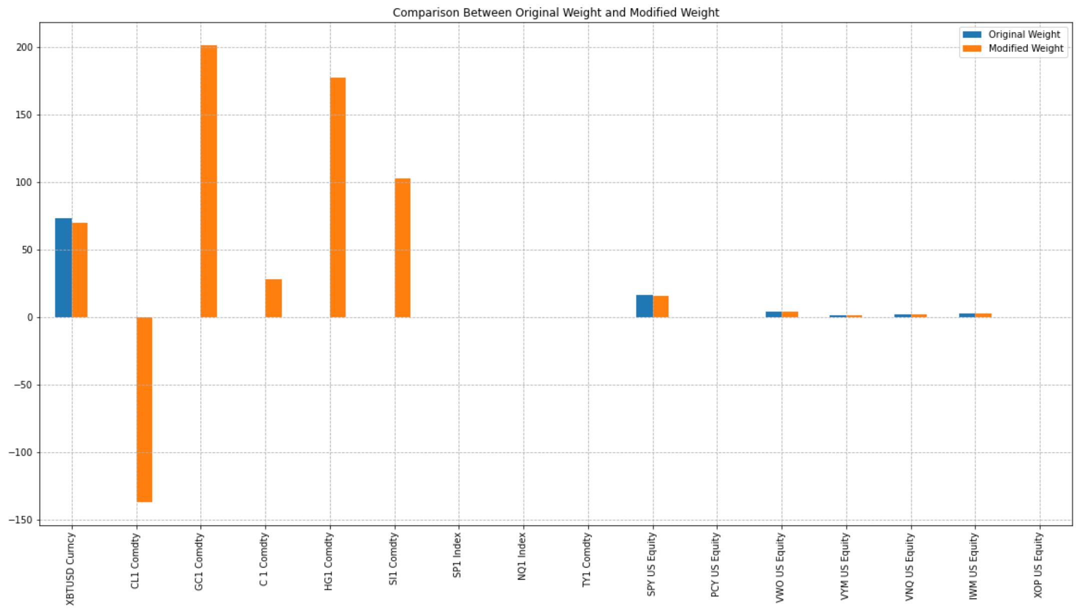
	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1
μ_{mkt} *(%)*	7.36	0.24	0.0035	0.21	0.18	0.18	0.22	0.2
	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP
μ_{mkt} *(%)*	0.001	0.22	0.077	0.18	0.22	0.18	0.25	0.36

Modified Expected Return

	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1
μ_{BL} *(%)*	7.67	-0.68	0.82	0.60	0.83	1.59	0.24	0.27
	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP
μ_{BL}*(%)*	0.09	0.24	0.13	0.50	0.18	0.29	0.22	0.18

Updated Weight

BL Weight	Bitcoin	CL1	GC1	C 1	HG1	SI1	SP1	NQ1
W(%)	69.6	-137.0	201.1	28.0	177.4	102.4	0.017	0.012
	TY1	SPY	PCY	VWO	VYM	VNQ	IWM	XOP
W(%)	0.02	15.5	0.22	4.16	1.12	1.96	2.48	0.13



Analysis

1. Crude oil futures weight falls as expected since the views toward it says that its return will fall.
2. The expected returns for assets without views don't variance a lot, but for those with views, the expected returns rise. From this result, we could say that the algorithm generated views do have a better outcome.

Conclusion

Comparing two different optimizer, one can easily see that Black-Litterman model has more reasonable result that it doesn't have extremely large long/short position. Moreover, having views incorporated into the model allows investors or portfolio managers to express their views instead of blindly follow Markowitz optimized results.

However, there are still some imperfections to this model. Take P for example, researchers are still debating whether to equally distribute assets' indication in every row in P matrix or distribute based on their market capital. Additionally, the Ω matrix are hard to estimate. There are some papers used statistical machine learning model which not only generate points of views but rather a predictive density distribution.

Even though the ARIMA model and SVR model are roughly implemented, and they need to be tested, to been through error analysis, etc., we can still see that it actually could provide more objective views for investors' and portfolio managers.

In this research project, the covariance matrix and views are assumed to be constant throughout the entire data set. However, practically speaking, all the parameters needed in Black-Litterman model should be updated every time when optimizing portfolio.

Reference

- Idzerok, T. (2004). ”A STEP-BY-STEP GUIDE TO THE BLACK-LITTERMAN MODEL.”
- Petter N.Kolm, Gordon Ritter, and Joseph Simonian. “ BLACK-LITTERMAN AND BEYOND: THE BAYESIAN PARADIGM IN INVESTMENT MANAGEMENT”
- Sarah Perrin and Thierry Roncalli. “MACHINE LEARNING OPTIMIZATION ALGORITHMS & PORTFOLIO ALLOCATION”
- Marco Avellaneda and Jeong-Hyun Lee. “STATISTICAL ARBITRAGE IN THE U.S. EQUITIES MARKET”
- Mahmut Kara, Aydin Ulucan, Kazim Baris Atici. “A hybrid approach for generating investor views in Black–Litterman model”