1)

a) $2w - 3\bar{w}$

$2(7-i) = 14-2i$

$3(7)$

$8-11i$

b) $\bar{w} = (2+3i)(7-i) = 14-2i+21i+3i^2$

$= -3i+21i-2i+14$

$= 17+9i$

c) $\frac{(2+3i)}{(7-i)}$ i) use conjugate

$\frac{(2+3i)}{(7-i)} \cdot \frac{(7+i)}{(7+i)} = \frac{(2+3i)(7+i)}{(7-i)(7+i)}$

$= \frac{11}{50} + \frac{23}{50}i$

2) a) $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

$z = 3e^{\frac{\pi}{3}i}$

$= 3(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})) = 3(\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2})$

$= \frac{3}{2} + \frac{3\sqrt{3}}{2}i$

$w = ie^{-\frac{\pi}{2}i}$

$i(\cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})) = i(0 - i) = 1$

b) $zw = 3e^{\frac{\pi}{3}i} \cdot ie^{-\frac{\pi}{2}i}$

$= \frac{3}{2} + \frac{3\sqrt{3}}{2}i$

3) a) $\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$\cos(\alpha) = \frac{e^{i\theta} + e^{-i\theta}}{2}$

b) $\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

$\frac{e^{i\theta} - e^{-i\theta}}{2i} = \cos(\theta) + i\sin(\theta) - (\cos(\theta) - i\sin(\theta)) = \frac{2i\sin(\theta)}{2i}$

✓

4)

a) $(0, -\pi)$

$x = r\cos(\theta) \quad y = r\sin(\theta)$

$x = 0 \cdot \cos(-\pi) = 0$

$y = 0 \cdot \sin(-\pi) = 0$

$(0, 0)$

b) $(2, \frac{3\pi}{4})$

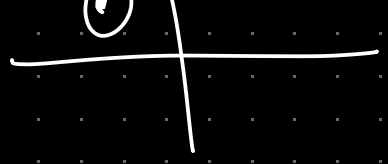
$x = -\sqrt{2} \quad y = \sqrt{2} \quad (-\sqrt{2}, \sqrt{2})$

c) $(-1, \frac{8\pi}{6})$

$x = \frac{2\sqrt{3}}{2} \quad y = -\frac{3}{2} \quad (\frac{8\sqrt{3}}{2}, -\frac{3}{2})$

5)

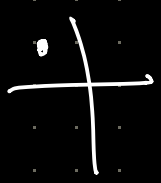
a) $(-3,3)$



$r = \sqrt{9+9} = \sqrt{18}$ $\quad \theta = \tan^{-1}(-1) = -\frac{\pi}{4}$

$\left(\sqrt{18}, -\frac{\pi}{4}\right)$

b) $(-1, \sqrt{3})$



$r = \sqrt{1+3} = 2$ $\quad \theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$\left(2, \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)$

c) $(6,2)$

$r = \sqrt{36+4} = \sqrt{40}$ $\quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$

$\tan^{-1}\left(\frac{1}{3}\right)$

$\left(\sqrt{40}, \tan^{-1}\left(\frac{1}{3}\right)\right)$

6)

a) $3i$

$r = |3i| = \sqrt{0^2 + 3^2} = 3$

$\arctan(\frac{3}{0}) = \frac{\pi}{2}$

$3e^{i\frac{\pi}{2}}$

b) $-3i$

$r = 3$

$\tan^{-1}(\frac{-3}{0}) = \frac{3\pi}{2}$ $\quad 3e^{i\frac{3\pi}{2}}$

c) $5+5i$

$r = \sqrt{5^2+5^2} = \sqrt{50}$

$\theta = \tan^{-1}(\frac{5}{5}) = \frac{\pi}{4}$

$5\sqrt{2}e^{i\frac{\pi}{4}}$

d) $4-2\sqrt{3}i$

$r = \sqrt{4^2+(2\sqrt{3})^2} = \sqrt{16+4(3)} = \sqrt{16+12} = \sqrt{28}$

$\theta = \tan^{-1}\frac{2\sqrt{3}}{4}$

$= 2\sqrt{7}\, e^{i\frac{5\pi}{7}}$

7) a) $r^2 = 4$ $0 \le r \le 2$ $0 \le \theta \le 2\pi$

$r = 2$

b) $0 \le r \le 2$ $\frac{\pi}{2} \le \theta \le \pi$

c) $0 \le r \le \sqrt{2}$ $0 \le \theta \le \frac{\pi}{2}$

d) $0 \le x \le 1$ $x \le y \le 1$