Vectors

$\vec{a}+\vec{b}=(a_x+b_x)\hat{\imath}+(a_y+b_y)\hat{\jmath}+(a_z+b_z)\hat{k}$ $\qquad a=|\vec{a}|=\sqrt{\vec{a}\cdot\vec{a}}=\sqrt{a_x^2+a_y^2+a_z^2}$

$\vec{a}\cdot\vec{b}=ab\cos(\theta)$
$\quad =a_xb_x+a_yb_y+a_zb_z$

for $\vec{c}=\vec{a}\times\vec{b}$ $\quad c = ab\sin(\theta)$; direction: RHR
$\vec{c}=(a_yb_z-a_zb_y)\hat{\imath}+(a_zb_x-a_xb_z)\hat{\jmath}+(a_xb_y-a_yb_x)\hat{k}$

Kinematics Definitions and Relationships

$\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ $\qquad \vec{v}_{avg}=\frac{\Delta\vec{r}}{\Delta t}$ $\qquad s_{avg}=\frac{d}{\Delta t}$ $\qquad \vec{a}_{avg}=\frac{\Delta\vec{v}}{\Delta t}$

Horizontal range (same altitude): $R=\frac{v_0^2\sin(2\theta)}{g}$

$\Delta\vec{r}=\int_{t_1}^{t_2}\vec{v}dt$

In 2D: $\vec{v}=\frac{d\vec{r}}{dt}=\frac{dx}{dt}\hat{\imath}+\frac{dy}{dt}\hat{\jmath}$ $\qquad \Delta\vec{v}=\int_{t_1}^{t_2}\vec{a}dt$

In 2D: $\vec{a}=\frac{d\vec{v}}{dt}=\frac{dv_x}{dt}\hat{\imath}+\frac{dv_y}{dt}\hat{\jmath}=\frac{d^2\vec{r}}{dt^2}=\frac{d^2x}{dt^2}\hat{\imath}+\frac{d^2y}{dt^2}\hat{\jmath}$

Constant acceleration:

$\vec{r}=\vec{r}_i+\vec{v}_i\Delta t+\frac{1}{2}\vec{a}(\Delta t)^2$ $\qquad \vec{v}=\vec{v}_i+\vec{a}\Delta t$

$v_x^2=v_{i,x}^2+2a_x\Delta x$
$v_y^2=v_{i,y}^2+2a_y\Delta y$
$v_z^2=v_{i,z}^2+2a_z\Delta z$

Relative Motion

$\vec{v}_{AC}=\vec{v}_{AB}+\vec{v}_{BC}$ $\qquad \vec{a}_{AC}=\vec{a}_{AB}+\vec{a}_{BC}$

Circular Motion

$s=r\,\Delta\theta$ $\qquad \omega=\frac{d\theta}{dt}$ $\qquad v_t=\frac{ds}{dt}=r\omega$ $\qquad \alpha=\frac{d\omega}{dt}=\frac{d^2\theta}{dt^2}$ $\qquad a_t=r\,\alpha$ $\qquad a_c=\frac{v_t^2}{r}=r\omega^2$

Constant α:

$\theta_f=\theta_i+\omega_i\Delta t+\frac{1}{2}\alpha(\Delta t)^2$ $\qquad \omega_f=\omega_i+\alpha\Delta t$ $\qquad \omega_f^2=\omega_i^2+2\alpha\Delta\theta$ $\qquad T=\frac{2\pi r}{v_t}$ $\qquad f=\frac{1}{T}=\frac{\omega}{2\pi}$

Forces & Newton's Laws

$\vec{F}_{net}=m\vec{a}$ (constant mass) $\qquad \vec{F}_{1on2}=-\vec{F}_{2on1}$

$\vec{F}_g=m\vec{g}$ $\qquad \vec{F}_{G;1on2}=-\frac{Gm_1m_2}{r^2}\hat{r}$ $\qquad f_s\le\mu_sF_N$ $\qquad f_k=\mu_kF_N$ $\qquad \vec{D}=-\frac{1}{2}C\rho Av^2\hat{v}$ $\qquad \vec{F}_{sp}=-k\Delta\vec{s}$

Energy

$E_{mech;i}+W_{nc}=E_{mech;f}$ $\qquad W_{tot}=K_f-K_i$

$E_{mech}=U+K$ $\qquad K_{trans}=\frac{1}{2}mv^2$ $\qquad K_{rot}=\frac{1}{2}I\omega^2$ $\qquad W_{fr}=-f_k\,d=-\Delta E_{th}$

$U_g=mgy$ $\qquad U_G=-\frac{Gm_1m_2}{r}$ $\qquad U_s=\frac{1}{2}k(\Delta s)^2$ $\qquad U_{SHO}\propto x^2$

$W\equiv\int_{\vec{r}_i}^{\vec{r}_f}\vec{F}\cdot d\vec{r}=\int\tau d\theta$ $\qquad P=\frac{dE_{sys}}{dt}$

$W_{constant}=\vec{F}\cdot\Delta\vec{r}$ $\qquad P_{force}=\vec{F}\cdot\vec{v}$

$P_{avg}=\frac{\Delta E_{sys}}{\Delta t}=\frac{W_{net}}{\Delta t}$

If $\vec{F}_c$ is Conservative
$\Delta U\equiv -W_{\vec{F}_c}$
$\vec{F}_c=-\frac{\partial U}{\partial x}\hat{\imath}-\frac{\partial U}{\partial y}\hat{\jmath}-\frac{\partial U}{\partial z}\hat{k}$

Momentum

$\vec{p}=m\,\vec{v}$ $\qquad \vec{F}_{net;sys}=\frac{d\vec{p}_{sys}}{dt}$ $\qquad \vec{p}_{sys}=\vec{p}_1+\vec{p}_2+\vec{p}_3+\cdots$ $\qquad \vec{J}\equiv\int_{t_1}^{t_2}\vec{F}_{net}dt=\Delta\vec{p}$

1D Collision

totally inelastic: $v_f=\frac{m_1v_{1i}+m_2v_{2i}}{m_1+m_2}$ $\qquad$ totally elastic: $v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_{1i}+\frac{2m_2}{m_1+m_2}v_{2i}$

Rotation

$\vec{r}_{CoM}=\frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$ or $\frac{\int\vec{r}dm}{\int dm}$ $\qquad I=\sum_i m_ir_i^2$ or $\int r^2dm$

$I=I_{CoM}+Md^2$

$\vec{\tau}_{net}=\frac{d\vec{L}}{dt}$ $\qquad \vec{\tau}=\vec{r}\times\vec{F}$ $\qquad \vec{L}=\vec{r}\times\vec{p}$

Fixed Axle:
$\vec{\tau}_{net}=I\vec{\alpha}$ $\qquad \vec{L}=I\vec{\omega}$

PHYS 211 Equation Sheet

Oscillations

$x(t) = A\cos(\omega t + \phi_0)$ $v_x(t) = -A\,\omega\sin(\omega t + \phi_0)$ $a_x(t) = -A\,\omega^2\cos(\omega t + \phi_0)$

$\omega_{spr} = \sqrt{\frac{k}{m}}$ $\omega_{smp} \approx \sqrt{\frac{g}{L}}$ $\omega_{pend} \approx \sqrt{\frac{mgL}{I}}$

$x(t) = A_0\, e^{-\frac{t}{2\tau}} \cos(\omega t + \phi_0)$

$E(t) = E_0 e^{-\frac{t}{\tau}}$ $\tau = \frac{m}{b}$

Moments of Inertia of Uniform Objects about axes through Center of Mass and Perpendicular to L, a , b, or R

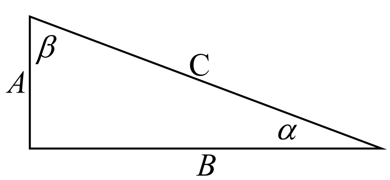
$I_{rod} = \frac{1}{12} ML^2$ $I_{disk} = \frac{1}{2} MR^2$ Spheres: $I_{solid} = \frac{2}{5} MR^2$

$I_{slab} = \frac{1}{12} M\,(a^2 + b^2)$ $I_{hoop} = MR^2$ $I_{shell} = \frac{2}{3} MR^2$

Constant	Symbol	Value
acceleration due to gravity (Earth)	g	9.81 m/s^2
gravitational constant	G	6.67×10^{-11} N m^2 / kg^2
radius of the earth	R_E	6.37×10^6 m
mass of the Earth	M_E	5.98×10^{24} kg
	π	3.14159

Unit Conversions		Unit Comparisons
1 inch = 2.54 cm	1 lb = 4.45 N	1 N = 1 kg m / s^2
1 mile = 1609 m		1 J = 1 N m
1 m/s = 2.24 mph	1 u = 1.66×10^{-27} kg	1 W = 1 J / s
1 Hz = 1 s^{-1} = 60 rpm		
1 rev = 2π rad = 360°		

If: $a\,x^2 + b\,x + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



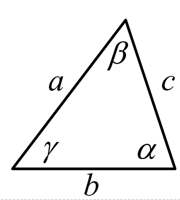
$A^2 + B^2 = C^2$

SOHCAHTOA

$\sin(\alpha) = \cos(\beta) = \frac{A}{C}$ $\tan(\alpha) = \frac{A}{B}$

$\cos(\alpha) = \sin(\beta) = \frac{B}{C}$ $\tan(\beta) = \frac{B}{A}$

Angle (deg)	Angle (rad)	Sine	Cosine	Tangent
30°	π/6	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	π/4	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	π/3	$\sqrt{3}/2$	1/2	$\sqrt{3}$



$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$

$a^2 = b^2 + c^2 - 2\,b\,c\cos(\alpha)$

$b^2 = a^2 + c^2 - 2\,a\,c\cos(\beta)$

$c^2 = a^2 + b^2 - 2\,a\,b\cos(\gamma)$

$\sin(\theta \pm \phi) = \sin(\theta)\cos(\phi) \pm \cos(\theta)\sin(\phi)$ $\cos(\theta \pm \phi) = \cos(\theta)\cos(\phi) \mp \sin(\theta)\sin(\phi)$

for constants a, c and n $\frac{d}{dx}\big(f(x)\big) = f'(x)$ $\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$

$\frac{d}{dx}\Big(f\big(g(x)\big)\Big) = f'\big(g(x)\big)g'(x)$ $\frac{d}{dx}(a) = 0$ $\frac{d}{dx}(ax^n) = a\,n\,x^{n-1}$ $\frac{d}{dx}(e^{nx}) = n\,e^{nx}$

$\frac{d}{dx}\big(\cos(x)\big) = -\sin(x)$ $\frac{d}{dx}\big(\sin(x)\big) = \cos(x)$ $\frac{d}{dx}(\tan(x)) = \sec^2(x)$

$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c \quad (n \neq -1)$