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Design and Implementation of Efficient Transit Networks: Procedure, Case Study and Validity Test

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Abstract

This paper presents and tests a method to design high-performance transit networks. The method produces conceptual plans for geometric idealizations of a particular city that are later adapted to the real conditions. These conceptual plans are generalizations of the hybrid network concept proposed in Daganzo (2010). The best plan for a specific application is chosen via optimization. The objective function is composed of analytic formulae for a concept's agency cost and user level of service. These formulae include as parameters key demand-side attributes of the city, assumed to be rectangular, and supply-side attributes of the transit technology. They also include as decision variables the system's line and stop spacings, the degree to which it focuses passenger trips on the city center, and the service headway. These decision variables are sufficient to define an idealized geometric layout of the system and an operating plan. This layout-operating plan is then used as a design target when developing the real, detailed master plan. Ultimately, the latter is simulated to obtain more accurate cost and level of service estimates.

This process has been applied to design a high performance bus (HPB) network for Barcelona (Spain). The idealized solution for Barcelona includes 182 km of one-way infrastructure, uses 250 vehicles and costs 42,489 €/h to build and run. These figures only amount to about one third of the agency resources and cost currently used to provide bus service. A detailed design that resembles this target and conforms to the peculiarities of the city is also presented and simulated. The agency cost and user level of service metrics of the simulated system differ from those of the idealized model by less than 10%. Although the designed and simulated HPB systems provide sub-optimal spatial coverage because Barcelona lacks suitable streets, the level of service is good. Simulations suggest that if the proposed system was implemented side-by-side with the current one, it would capture most of the demand.

*Keywords:*Transit system design; Bus rapid transit; Transit networks

1. Introduction

Transit systems are a key strategy to reduce the use of private cars and mitigate the congestion problem in major cities. They are usually considered a public service that guarantees mobility for all citizens. However, for them to be competitive with the automobile, they must provide good service everywhere in a city at all times. Clearly, for

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transit systems to do this, they should not be designed one corridor at a time but as complete two-dimensional networks. Hence, this paper presents and evaluates a design method for city-wide transit networks.

Significant contributions have been devoted to the transit network design problem in the last three decades. A wide analysis of different kinds of transit lines and networks is presented in Vuchic (2005). Most of the research in this field proposes combinatorial optimization models to identify the set of routes that minimizes user and/or agency costs. This leads to *NP*-complete formulations that are solved through the use of heuristics (Mandl, 1980, Hasselström, 1981, Ceder and Wilson, 1986 and Baaj and Mahmassani, 1990, 1995) or metaheuristics algorithms (Pattnaik, 1998). These discrete approaches will provide a feasible network configuration but the global optimal solution is not achieved. Generally, solving combinatorial optimization problems is extremely time-consuming in large networks, showing a tradeoff between solution optimality and computational time.

On the other hand, some advances in city-wide network design have already been made through the use of continuous models; see e.g., Holroyd (1965), Newell (1971) and Daganzo (2010). These references propose analytic models that capture both the agency costs of providing service and the user level of service; and based on these formulas show how to choose the optimum service frequency and spatial coverage. Holroyd (1965) considers the optimum design of grids, Newell (1971) analyzes hub-and-spoke systems, and Daganzo (2010) presents a hybrid concept that generalizes the former two. These hybrid networks combine a grid structure in the city center with a hub-and-spoke pattern in the periphery. The hub-and-spoke pattern includes branching lines in order to provide uniform spatial coverage far away from the CBD. The hybrid concept is analytically developed in the latter reference in order to investigate two issues: (i) the kind of transit technology (Bus, BRT, LRT or Metro) that is best suited for cities of different sizes and densities; and (ii) the kinds of cities where transit can compete effectively with the automobile.

The model in Daganzo (2010) is highly simplified, however. First, it assumes that origins and destinations are uniformly and independently distributed. Second, it focuses only on squares of side D with concentric central squares where grid service is provided of side $d < D$. And third, it uses only three decision variables: the service headway (H), the line spacing (s), which is assumed equal to the stop spacing, and a network shape parameter ($\alpha = d/D \in [s/D, 1]$), which captures the amount of hubbing. Therefore, to expand the model's domain of real applications, this paper develops formulas, in which some of these simplifications are overcome. Specifically, because a number of cities, such as Barcelona, Buenos Aires, Oslo, Helsinki, Miami and Washington D.C., are elongated in shape, formulas are developed for rectangles. The new formulas also allow the line spacing to be greater than the stop spacing, and treat differently the two orthogonal directions (x and y) defined by the sides of the rectangle. To do the latter, the shape parameters and line spacing will be defined in (x, y) pairs.

As in previous works, formulas for level of service metrics and agency costs are developed with the tools of geometric probability, and these are combined into an objective function that can be optimized to obtain an idealized system design. This idealization can then guide the development of a detailed master plan.

This two-step process was used to design a detailed city-wide high-performance bus network for Barcelona. The system shares many attributes with light-rapid-transit but requires less infrastructure and is less disruptive of traffic, see Estrada et al. (2009) for more details. Several future scenarios of this detailed master plan were then simulated under various demand and bus performance assumptions. These simulations show that the analytic model predictions are reasonably accurate and, just as importantly, that the proposed urban HPB network would work well. Even under the most conservative performance assumptions, it is predicted to capture nearly all the demand currently served by bus, plus a significant chunk now served by metro, while at the same time considerably reducing the bus agency's costs.

The idealized system is defined in Section 2, and the model quantifying its performance in Section 3. The two-step design method is then applied to Barcelona in Section 4; results include the idealized target arising from the optimization, and the modified master plan that conforms to the peculiarities of the city. Section 5 simulates several demand/bus-performance scenarios and discusses the results with a focus on testing the robustness of the method. Section 6 discusses how the system would perform under alternative future scenarios.

2. The idealized system

Figure 1 depicts the idealized system studied in this section. The service region is a rectangle of sides D_x and D_y (km). Without loss of generality, the rectangle is assumed to be aligned with the (x, y) axes in a "landscape" orientation; i.e., so that $D_x \geq D_y$, as shown in the figure.

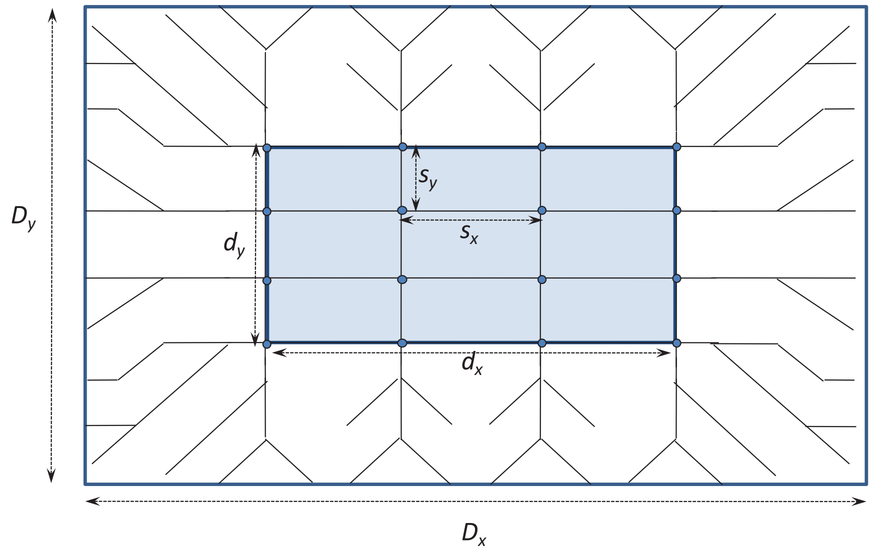

Figure 1. The hybrid concept for an urban HPB network in a rectangular zone.

The structure of the system and the decision variables that define it are now described. The system's core is a bidirectional grid of transit lines with spacing s_x and s_y (km), which cover a rectangle concentric with the service area; see gray rectangle defined by d_x x d_y in Figure 1. This line spacing is assumed to be an integer multiple of the stop spacing s (km); i.e., $s_x = p_x s$ and $s_y = p_y s$, where p_x and p_y are integers. If $p_x = p_y = 1$, as assumed in Daganzo (2010), every stop in the central grid is a transfer point served by two orthogonal lines. Otherwise, for each direction, $a = \{x, y\}$, only 1 in p_a stops is a transfer point. Figure 2 below, shows the case with $p_x = 4$ and $p_y = 2$. The dimensions of the central rectangle are denoted $d_x \leq D_x$ and $d_y \leq D_y$, as shown in Figure 1. They will be expressed in terms of the dimensionless ratios: $\alpha_x = d_x/D_x$ and $\alpha_y = d_y/D_y$. The transit lines in this central grid continue to the periphery, where they branch (more than once if necessary) to cover all parts of the periphery as uniformly as possibly with similar spacing as in the center. All the lines operate in the central area with a common headway H (h); but this headway increases in the periphery at those points where the lines branch. This information is enough to configure the idealized system and devise an operating plan; i.e., only five decision variables need to be chosen: H, s, s_x, s_y , α_x and α_y.

Note from Figure 1 that people on the periphery receive coverage from a single line whereas those on the center can access lines in two perpendicular directions. This happens because the central rectangle includes twice as many kilometers of infrastructure per unit area as the periphery. Thus, by judiciously choosing the shape and size of this rectangle one can ensure that the agency's resources are deployed where they can provide the best service. Furthermore, by allowing different values for s_x and s_y, transportation capacity in the two directions can be better matched with the demand. (If both line spacings are forced to be equal then the occupancy of the lines parallel to the long side of the rectangle would be higher than that of the perpendicular lines at their critical load points and, as a result, vehicle capacity constraints would only be reached for the horizontal lines.)

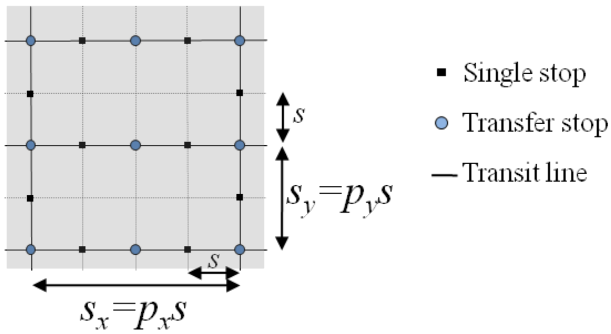

Figure 2. Example of an asymmetric lattice of lines and stops (p_x=4 and p_y=2)

The passenger and vehicle performance parameters that drive the optimization are now defined. As in Daganzo (2010), passenger demand is assumed to be uniformly and independently distributed over the service region with average trip generation rates: Λ (pax/h) during the rush period, and λ (pax/h) overall. Passengers walk at an average speed v_w (km/h). Transit vehicles are identical, with the following attributes: design vehicle capacity, C (pax); cruising speed including stops due to traffic and pedestrian interference, v (km/h); trip time added per stop due to bus door operation, deceleration and acceleration, τ (h/stop); and trip time added per boarding passenger τ' (h/pax). It is also assumed that the system headway must exceed a minimum value H_{min} in order to facilitate the provision of regular headways and reduce the risk of vehicle overtaking events.

3. The model

Formulae are now derived for the objective function and the constraints of the idealized system design problem. Section 3.1 derives mathematical expressions for some physical metrics that capture key aspects of the system's performance; and section 3.2 combines these expressions into a mathematical program.

3.1. User and Agency Metrics

Here, the performance indicators in Daganzo (2010) are reformulated for the more general systems of Section 2. These indicators capture both the agency's performance and the user experience. Only the formulas are given with the derivations included in the appendix. These derivations follow the same logic but are more complicated due to the loss of symmetry in one axis. Therefore, the analytic model encompasses more parameters. Nevertheless, these formulae are equal to those presented in Daganzo (2010) when the rectangle representing the service region has D_x=D_y and we choose s_x=s_y, α_x=α_y.

As explained in Daganzo (2010) agency costs can be expressed as a linear combination of: the infrastructure length, L (km), which is correlated with capital cost; and the average total vehicular distance travelled per hour of operation V (veh-km/h), which is correlated with costs of operation. Also important is the peak vehicle occupancy during rush hour in both directions (O_x, O_y) to ensure that the demand can be served with the bus types that are envisioned. The expressions are:

$$L = \frac{D_x D_y}{2 s_x s_y}(s_x + s_y)(1+\alpha_x \alpha_y) + \frac{D_x D_y}{2 s_x s_y}(s_x - s_y)(\alpha_y - \alpha_x) \tag{1}$$

$$V = \frac{2\alpha_x D_x D_y}{s_y H}\left[1+\frac{D_x}{2D_y}(1-\alpha_x)\right]+\frac{2\alpha_y D_x D_y}{s_x H}\left[1+\frac{D_y}{2D_x}(1-\alpha_y)\right] \tag{2}$$

$$O_y = max\left(\frac{\Lambda s_y H\left(1+\alpha_x\right)\left(1-\alpha_y\right)}{\left(4\alpha_x D_x\right)};\frac{\Lambda H\left(1-\alpha_x\right)^2\left(1+\alpha_y\right)^2}{32}+\frac{\Lambda s_y H\left(4-(1+\alpha_y)^2(1-\alpha_x)^2-2\alpha_x^2\alpha_y^2\right)}{\left(8\alpha_x D_x\right)}\right) \tag{3.1}$$

$$O_x = max\left(\frac{\Lambda s_x H\left(1+\alpha_y\right)\left(1-\alpha_x\right)}{\left(4\alpha_y D_y\right)};\frac{\Lambda H\left(1-\alpha_y\right)^2\left(1+\alpha_x\right)^2}{32}+\frac{\Lambda s_x H\left(4-(1+\alpha_x)^2(1-\alpha_y)^2-2\alpha_x^2\alpha_y^2\right)}{\left(8\alpha_y D_y\right)}\right) \tag{3.2}$$

Other agency indicators of interest are the commercial speed, v_c (km/h), and the maximum number of vehicles operating simultaneously, M (veh). The latter equals the vehicle-hours of operation during the peak hour and is therefore given by:

$$M = V/v_c. \tag{4}$$

Because the commercial speed depends on user behavior, it is given below; see (9).

User costs are expressed as a function of the total travel time consumed in the transportation chain of an average trip. Since users perceive differently the durations of the various components of a typical user trip, the following components are used: (walking) access time, A (h); waiting time, W (h); and in-vehicle-travel time, T (h). The latter will be expressed as the ratio between the in-vehicle-travel-distance E (km), and the commercial speed v_c (km/h). Also relevant is the expected number of transfers, e_T, which depends on the probabilities of requiring zero, one or two transfers, P_0, P_1 and P_2.

Since all these metrics depend on how transit users choose the available routes, the expressions use the same behavioral assumptions as Daganzo (2010). As in the case of agency metrics, they reduce to those of that reference in the special case of a symmetric square system with $s = s_x$, $= s_y$ and $\alpha_x = \alpha_y$. [An exception is the formulae for the probabilities and expected numbers of transfers, which is more precise here: it was assumed in Daganzo (2010) that every trip required at least one transfer, but it is assumed here—less conservatively—that trips with the same latitude (y-value) or longitude (x-value) are made on the same line without a transfer.]

$$A = \left(\frac{s_x + s_y}{4}+\frac{s}{2}\right)\Big/ v_w \tag{5}$$

$$W = \left[\frac{H}{6\alpha_x}(1-\alpha_x^3)\frac{(1-\alpha_y)}{(1-\alpha_x)}+\frac{H}{6\alpha_y}(1-\alpha_y^3)\frac{(1-\alpha_x)}{(1-\alpha_y)}+\alpha_x\alpha_y\frac{H}{2}\right]\left(1+P_1\right)+\frac{H}{2}P_2 \tag{6}$$

$$e_T = P_1 + 2\cdot P_2 \tag{7}$$

$$P_0 = \frac{(s_x D_x + s_y D_y)}{2D_x D_y}(1+\alpha_x\alpha_y)+\frac{(s_x D_x - s_y D_y)}{2D_x D_y}(\alpha_y - \alpha_x)-\frac{\alpha_x\alpha_y s_x s_y}{D_x D_y} \tag{7a}$$

$$P_1 = \frac{s_y}{2D_x}\left(-\alpha_y + \alpha_y^2 - 3\alpha_x\alpha_y + \alpha_x\alpha_y^2\right) + \frac{s_x}{2D_y}\left(-\alpha_x + \alpha_x^2 - 3\alpha_x\alpha_y + \alpha_y\alpha_x^2\right)$$
$$+\frac{1}{2}\left(1-\alpha_y^2 - \alpha_x^2 + 4\alpha_x\alpha_y - \alpha_x^2\alpha_y^2\right) + \frac{s_x s_y \alpha_x \alpha_y}{D_x D_y} \quad (7b)$$

$$P_2 = \frac{1}{2}\left(1-4\alpha_x\alpha_y + \alpha_x^2 + \alpha_y^2 + \alpha_x^2\alpha_y^2\right) - \frac{s_y}{2D_x}(1-\alpha_y)^2(1+\alpha_x) - \frac{s_x}{2D_y}(1-\alpha_x)^2(1+\alpha_y) \quad (7c)$$

$$E(E) = \left(\frac{\alpha_y^2 D_y^2 + \alpha_x^2 D_x^2 + 4\alpha_x\alpha_y D_x D_y}{4(\alpha_x D_x + \alpha_y D_y)} + \frac{(\alpha_x D_x + \alpha_y D_y)}{12\alpha_x\alpha_y D_x D_y}\left(1-\frac{\alpha_x\alpha_y}{2}\right)\right)\left(1-\alpha_x^2\alpha_y^2\right) +$$
$$+\frac{1}{3}(\alpha_x D_x + \alpha_y D_y)(\alpha_x^2\alpha_y^2) + \frac{1}{4}\left(D_x(2-3\alpha_x+\alpha_x^3) + D_y(2-3\alpha_y+\alpha_y^3)\right) \quad (8)$$

$$1/v_c = \left[1/v + \tau/s\right] + \left(1+e_T\right)\Lambda/V\,\tau' \quad (9)$$

3.2. Model Formulation

The objective function and constraints of the design problem form a mathematical program (10) that is described below. Since the demand is given as a parameter (further extensions might incorporate elasticities of the demand), the optimization problem can be formulated as a cost minimization where the objective can be either the total (generalized) cost or the average user cost. The latter is chosen because it is more meaningful. Of course, the objective function Z, given by (10a), must include two components: one for the agency and another for the users. We choose to express all these costs in units of hours of riding time because time is a more universally understood metric than any monetary unit.

The first bracketed term in (10a) is the agency cost (z_a). As in Daganzo (2010), the agency's monetary cost per hour is assumed to be of the form: $\$_V V + \$_M M + \$_L L$, where the V-term captures operations, the M-term vehicle depreciation and the L-term infrastructure depreciation. These monetary costs are then prorated to each user (dividing by λ), and finally reduced to riding time, dividing again by a "design" value of time μ (€/h). The resulting linear combination, denoted $\pi_V V + \pi_M M + \pi_L L$, states in units of riding time the fares that riders would have to pay to fully support the system. Roughly speaking, this term is the number of hours that a user has to work to pay for an unsubsidized trip.

The second bracketed component of (10a) is the passenger component of the objective function (z_u). It includes terms for access, waiting, riding and transfering. Although walking time A is not multiplied by a coefficient, it can be weighted more than riding time by reducing the effective walking speed value -- note that v_w only appears in (5). Waiting time, on the other hand, is weighted the same as riding time because service is assumed to be frequent and reliable. The term corresponding to transfers is the product of the expected number of transfers e_T and an adjustable weight, δ/v_W, equal to the riding time people would trade for one transfer; i.e., where δ (km) is the walking distance equivalent to a transfer.

Constraints (10b) specify valid ranges for (s, H, α_x, α_y); and the integrality of (p_x, p_y). Constraints (10c) prevent vehicle passenger occupancies exceeding vehicle capacity. Constraint (10d) expresses that the number of lines in the central grid area, in both directions combined, cannot exceed a given number, N. This constraint is intended to capture the number of corridors the city is willing to commit for the desired type of service—this was required for the Barcelona case study, as city managers were concerned about the impact the system would have on car traffic. This constraint can be modified to fit the needs of specific cities.

$$\min Z = [\pi_V V + \pi_M M + \pi_L L] + [A + W + T + (\delta / v_w) e_T] \tag{10a}$$

Subject to

$$s > 0;\ s_x = p_x s;\ s_y = p_y s;\ p_x, p_y \textit{ integer};\ s_x/D_x \le \alpha_x\ s_y/D_y \le \alpha_y\ H \ge H_{min} \tag{10b}$$

$$O_x \le C;\ O_y \le C \tag{10c}$$

$$\alpha_x D_x/s_x + \alpha_x D_x/s_x \le N \tag{10d}$$

The solution of this problem for a specific application yields an idealized design. From this sketch, the analyst should then construct a detailed transit network that uses the available streets, hits the major demand generators and attractors to the extent possible, but still conforms to the ideal as much as possible. This second step is an art more than a science, but the process can be carried out fairly easily. As a rule, the final network should exhibit lower walking distances than the idealization, since it has been adapted to the real (non-homogeneous) demand. The next section shows both the process and the result of these two steps for Barcelona's vision of a city-wide HPB network.

4. The case of Barcelona: Design of an urban HPB network

This section describes how the master plan for a high performance bus network in Barcelona was developed and summarizes its features: the input data and some analysis simplifications are introduced in Section 4.1; the optimization results in Section 4.2; and the master plan in Section 4.3. Section 5 uses detailed simulations to compare the performance of this master plan with both the predictions of the idealized model and the status quo.

4.1. Input data and analysis simplifications

Table 1 includes the input parameters used for the optimization step. Some of these deserve comment. First, in order to be "fail safe", our team was asked to develop an optimum design assuming that buses cannot cruise any faster than today, even though the ultimate goal is increasing their speed considerably with state-of-the-art traffic management schemes and other HPB measures such as those suggested in Estrada et al. (2009) and Eichler and Daganzo (2006). Thus, the table shows a rather low cruising speed: v_c = 21.4 km/h. It was also decided that headways less than 3 minutes were to be avoided; thus, the table shows $H_{\min}$ = 3 min. Walking speeds were reduced from 4.5 km/h to 2 km/h (by a factor of 0.44) to account both for delays in crossing streets and the discomfort of walking. In general, values that could not be objectively measured in the field were estimated in consultation with both the transit agency and decision-makers in Barcelona. An important political constraint was the maximum number of corridors that could be used, N=11. The city decision makers were not willing to allocate more pavement resources for the exclusive use of buses. They feared that removing more lanes for the use of automobile could collapse traffic in the city center. These restrictions limit the benefit that can be achieved. As explained in Sec. 6.2 this constraint turns out to have a significant effect.

The optimization problem was simplified prior to solution in order to reduce the search space. Two dimensions were eliminated by considering only 3 possible network structures with fixed values of (p_x, p_y). This is reasonable because for Barcelona the only parameter values that can be optimal are: (p_x, p_y) = (1,1), (2,1) or (2,2). This set was chosen for two reasons. First, large values of p (p > 2), combined with the constraint $N \le 11$ in the city of Barcelona, cause either a high density of stops, and consequently a low commercial speed, or increased access times. And second, because (for capacity considerations) p_x/p_y should be close to D_x/D_y, which is 2 for Barcelona. These three network structures were called the "complete"; "semi-alternate" and "alternate" configurations, and are depicted in

Figure 3. This simplification eliminated the two line spacing decision variables. In addition, the central grid was forced to be homothetic with the service region: $\alpha_x = \alpha_y$. This is reasonable too, and collapses two decision variables into a single one, $\alpha = \alpha_x = \alpha_y$. Thus, only 3 decision variables remain (α, H, s). As a result, the global optimum can be easily found with an exhaustive search over the feasible region. Bi-level search methods that exploit the structure of the objective function with respect to a few of the variables can be used if all 6 variables are allowed to vary in the optimization.

Table 1. Input parameters in Barcelona HPB network

Concept	Value
Rectangular dimensions, D_x-D_y (km)	10 - 5
Average hourly demand, λ (pax/h)	20,000
Peak hourly demand, Λ (pax/h)	45,000
Vehicle capacity, C (pax)	150
Cruising speed, v (km/h)	21.4
Time lost per stop, τ (s)	31
Boarding and alighting time per passenger, τ' (s/pax)	1.5
Minimum time headway, H (min)	3
Walking speed, v_w (km/h)	2
Maximum number of corridors, N	11
Unit infrastructure cost, $\$_L$ (€/km-h)	80
Unit distance cost, $\$_V$ (€/veh-km)	5.2
Unit vehicle cost, $\$_M$ (€/veh-h)	60.2
Value of time, μ (€/pax-h)	15

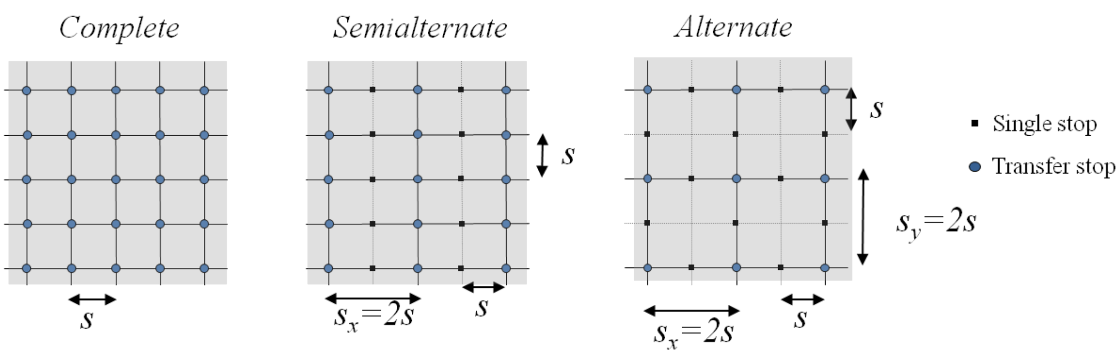

Figure 3. The three central grid structures considered for Barcelona.

4.2. Optimization results

Table 2 shows how the three optimized network structures perform. The semi-alternate concept provides the best level of service (49.8 min) and the least total cost per trip (57.6 min). This is the only concept with $\alpha < 1$. Its periphery should cover $1-\alpha^2 = 1-0.85^2 \approx 28\%$ of the total region. In all three cases, the commercial speed is comparable with 15 km/h, and the agency cost is small compared about to the total cost; e.g., in the semi-alternate case, their ratio is: 8.5/49.8 ≈ 1/6. In monetary units, the agency's unit operating cost per user is 2.12 €/pax, and the total hourly cost, 42,489 €/h.

In all three cases, both the minimum headway constraint (10c) and the number of corridors constraint (10d) are binding. With a fixed number of corridors (12) and a given headway (3 min), there is little that can be done to change the bus occupancies past the system's critical load points. In the semi-alternate case buses are predicted to reach 140/150 ≈ 93.3% of their capacity, suggesting that unless shorter headways or larger buses are provided the system will become congested if the peak demand significantly exceeds 48,000 pax/h.

4.3. The detailed plan

A detailed route map for Barcelona was meticulously developed by hand, and is shown in Figure 4. The idea was to follow the semi-alternate concept of Table 2 as much as possible, using only the available streets. Because Barcelona's central business district is not in the center of the service region (it is much closer to the sea than to the hills) the central grid area was displaced toward the seafront (see Figure 4). Furthermore, because the seafront is a demand generator, the peripheral lines running toward the sea were joined by a transversal line running along the coast. (To be true to the hybrid concept, this transversal line should have run several blocks farther inland through the old town, but this was not possible because the route would have had to traverse, pedestrian-only parts of the historical city.)

Drawing the individual lines was easy in the central *Eixample* district (where streets follow a perfect grid designed by civil engineer Ildefons Cerdà in 1859), but this district represents only 30% of the total service area. In the rest of the city, particularly on its hilly areas, routes had to depart from the ideal. On occasion, the direction of traffic had to be reversed in some streets to accommodate the system. Routes were also modified to serve near hospitals, universities, intermodal stations and other key demand points. Although all these route modifications increase network length, they should also significantly reduce user access times because the modified stop placements reduce passengers walking distances. Finally, the municipality and the transit agency also requested slightly shorter stop spacing in the central area than that recommended by the model.

Despite all these modifications, the semi-alternate concept of Table 2 can be clearly discerned in Figure 4. The average quantitative measures of the design (α, H and s) are close to the ideal. The final design has 11 corridors (5 running East-West, and 6 North-South). The central grid area is about 66% of the whole, corresponding to $\alpha = 0.81$ (vs. $\alpha = 0.85$); the average East-West line spacing is 0.673 km and 0.962 km North-South (vs. 0.65 km and 1.3km); and the average stop spacing is 0.542 km. (vs. 0.65 km). The agency metrics are also similar, albeit somewhat larger, as expected: the total network length is 220 km of one-way infrastructure (vs. 182 km); the maximum number of buses in use is 266 (vs. 250); and the number of vehicle-km in the peak hour is 3,990 (vs. 3861). The average hourly agency cost is 45,646 €/h (vs. 42,489€/h).

This service is expected to become an HPB system once measures to increase the bus cruising speeds are implemented. It will compete, complementarily, with the existing subway network (which currently has 10 lines) and the local bus services (which is a conventional network with 110 lines). The new service is a key component of the new integrated high performance transit system envisioned for Barcelona. This system also includes the metro network, the suburban rail lines and bus commuter services; and two existing lines of modern tramways along the only diagonal avenue crossing the grid (actually called Diagonal Avenue in Barcelona). The following section examines the simulated performance of the system, focusing on the user experience. These simulations are used to test both the model predictions and the usefulness of the proposed design in a variety of future scenarios.

Table 2. Results derived from the implementation of the model in Barcelona for three different line lattice layouts (v=21.4 km/h)

Line Lattice layout	Complete (px=py=1)	Alternate (px=py=2)	Semialternate (px=2,py=1)
α	1	1	0.85
H (min)	3	3	3
s (km)	1.25	0.63	0.65
v_c (km/h)	16.64	14.95	15.46
A (h)	0.625	0.473	0.444
W (h)	0.041	0.041	0.046
T (h)	0.301	0.335	0.328
e_T	0.66	0.65	0.8
$P_0/P_1/P_2$	0.344 /0.656 /0	0.346 /0.654/0	0.230 /0.737 /0.031
Corridors in x /Corridors in y	4 / 8	4 /8	6 /6
L (km)	80	79.37	91
M (veh-h/h)	192.28	212.41	249.74
V (veh-km/h)	3200	3174.6	3860.92
User cost (h)	0.977	0.858	0.830
Agency cost (h)	0.116	0.119	0.142
Agency cost per hour of service (€/h)	34,679	35,708	42,489
System unit cost, Z (h)	1.092	0.977	0.971
z_u/z_a	9.33	7.9	6.42
O_x (p/veh)	140.6	141.8	140.7
O_y (p/veh)	70.31	70.9	140.7

Figure 4. Proposed HPB corridors for Barcelona

5. Model verification tests and expected system performance

The model of Section 2 assumes that origins and destinations are uniformly and independently distributed in the service region. Although this is somewhat unrealistic (in reality demand tends to focus in a central area during the peak periods, which makes it easier to serve) the assumption leads to robust designs that ensure all types of trips are well served by the system. An ability to serve all trips is necessary if the system is to be a viable alternative to the automobile.

In view of this, this section will compare the predictions of the analytic model with this uniform demand, which we call Scenario A0, with two simulated scenarios, A1 and A2, of the system in Figure 4. These two scenarios only differ in the demand. Both use an "all-or-nothing" route assignment method for consistency with the analytic model. This method was chosen because it does not disperse trips around the peak load points and thus estimates maximum vehicle occupancies conservatively. Scenario A1 spreads the total demand of the idealized model evenly among 178×178 transportation zone pairs, as in the idealized model. Thus, a comparison of A0 vs. A1 tests the validity of the supply-side approximations in the analytic model. Scenario A2 divides the demand of the idealized model across the 178×178 zones in proportion to the O-D demand flows in Barcelona's most recent mobility survey[1] including all modes, which is not uniform as can be seen in CENIT(2010). By including all modes, this distribution describes where people want to travel, which seems appropriate to evaluate realistically a system intended to serve all types of trips. Thus, a comparison of A0 vs. A2 should jointly test the effects of the demand uniformity assumption and the supply-side idealizations in the analytic model. For more details about the demand assignment, see CENIT (2010).

In addition to the above scenarios used for model verification, some benchmark simulations were also run to test the system performance. Scenario B1 consists of the existing bus network with its current demand. Scenario B2 consists of the existing transit system composed by the bus and metro networks with their current combined demand. Scenarios B3 and B4 examine the future bus and bus/metro networks, respectively. Scenario B3 includes: (i) the new bus system; and (ii) a slightly modified version of the existing bus system. Scenario B4 includes the current metro system, as well as (i) and (ii). In the simulations, the demand for scenarios B3 and B4 were the same as the O-D tables used in scenarios B1 and B2 respectively. The basic idea behind (ii) is that those segments of existing lines overlapping the new high-performance corridors were suppressed. Moreover, the frequency of the rest of existing lines was diminished in recognition of the new passenger flow. An iterative process was carried out to determine the final bus frequency. At each iteration, we reduced the fleet allocated to a particular route and recalculated the new frequency of service. Then, we assigned the O-D demand table to the integrated bus network with updated frequencies (Scenario B3) and to the integrated bus and metro network (Scenario B4). We repeated this process until the high-performance bus network did not gain more passengers from the existing line or the time headway exceeds a maximum threshold of H_{max}=20 min. The O-D demand tables of each scenario were split among bus or transit systems with a choice model that included the generalized cost of travel from zone to zone on the best route and a random utility component. As before, the best routes were chosen with an "all-or-nothing" method. For more details see CENIT (2010).

Table 3 displays the results obtained for all 7 scenarios. The top 9 rows of data are user performance measures and the bottom 4 agency metrics. The latter are discussed first.

The agency costs of scenarios A0-A2, and the validity of the model cost predictions, have already been discussed in Section 4, but the new system's performance has not been compared to a benchmark. A comparison of scenarios B1 and B3 reveals the surprisingly good performance of the new system from the agency's perspective: all agency metrics improve as a result of the introduction of the new system. In essence, the new system allows the agency to eliminate enough redundant routes to reduce its cost by 24%, while improving the level of service for all users.

[1] TMB, the major bus operator, conducted in 2007 a wide mobility survey (EMIT'07) to characterize the overall demand in Barcelona of all the transit modes and private vehicles. This detailed data was made available to us.

User metrics are now discussed. Scenarios A0 and (A1-A2) are first compared to test the model's validity. The agreement between A0 and A1 is relatively good. The model predicts particularly well the average door-to-door speed despite the (small) discrepancies in average distance traveled. Except for the waiting time and the number of transfers, all level of service measures are predicted to within 10%. The discrepancies are only due to the difference in network structure. For example, the discrepancies on waiting time and the number of transfers can be respectively traced back to the size of the central grid and the density of stops in it. The good agreement between A0 and A1 suggests that the proposed idealized model can represent complicated networks well enough to be useful as a planning tool. The agreement between A0 and A2 is not as good, however; discrepancies often around 20%. One reason for this is that the actual trips in Barcelona are considerably shorter than assumed (5 km vs. 6.07 km) and concentrated at the center. As one would expect, scenario A2 then yields considerably lower metrics than scenario A1 (and A0), with the exception of the door-to-door travel speed. This suggests that there is some merit in generalizing the formulae of Section 3 to account for centripetal demand. On the other hand, as shown in Section 6 the optimum values of the decision variables are not heavily affected by the demand; thus, the system designs obtained with the uniform demand assumption should still be quite efficient. See Daganzo (2005) for a more extensive discussion of this issue in the context of logistics systems.

Table 3. Comparison of user and agency metrics for different scenarios. (v=21.4 km/h)

	New network			**Benchmark simulations**			
	Scenario A0. Model predictions	**Scenario A1. Uniform demand simulation**	**Scenario A2. Non uniform demand simulation**	**Scenario B1. Current Bus network**	**Scenario B2. Current Bus+Metro network**	**Scenario B3. Integrated Bus network**	**Scenario B4. Integrated Bus +Metro network**
In-Vehicle distance (km)	5.05	5.28	4.09	3.64	4.42	3.71	4.42
Access distance (km)	1.02	1.11	0.9	0.74	0.85	0.93	0.95
Total travel distance (km)	6.07	6.38	5	4.38	5.27	4.63	5.37
Access time, A (h)	13.64(30.68*[a])	14.4 (32.4*[a])	12.64(28.44*[a])	10.44 (23.49*[a])	12.07(27.16*[a])	12.90(29.02*[a])	13.40(30.15*[a])
Waiting time, W (h)	2.67	3.53	3.02	4.59	3.89	3.29	2.98
In-vehicle time, T (h)	19.3	19.47	15.19	16.62	14.06	13.91	12.73
Expected number of transfers	0.75	0.61	0.5	0.1	0.15	0.41	0.31
Total travel time (h)	35.61(52.65*[a])	37.40(55.4*[a])	30.85(46.7*[a])	31.65(44.7*[a])	30.03(45.12*[a])	30.10(46.22*[a])	29.11(45.86*[a])
Door-to-door speed (km/h)	10.23	10.24	9.72	8.3	10.54	9.23	11.07
1-way Infrastructure Length, $2L$ (km)	**182**	220		**891**		220 (HPB*[b])+530 (CS*[c])	
Vehicles, M (veh)	**250**	266		**659**		266 (HPB*[b])+350 (CS*[c])	
Vehicles per kilometer, V (veh-km)	**3,861**	3,990		**7,579**		3,990 (HPB*[b])+3,850 (CS*[c])	
Agency Cost (€/h)	**42,489**	**45,646**		**114,885**		**86,820**	

*[a] Numbers in parenthesis use v_w=2 km/h *[b] High Performance Bus (new service) *[c] Conventional Service

This efficiency is now examined by comparing the status quo scenarios (B1, B2) with (B3, B4). First, note the small total travel distance in scenarios B1 and B3. This occurs because the current bus system, with its low commercial speed, mostly attracts short trips--longer trips tend to be made either on metro or with private vehicles. A comparison of B1 vs. B3 reveals that the new system increases the door-to-door speed of all bus trips currently made by 11% and the total travel time by about 5%. Note that the current bus system consists of multiple routes serving the main origins and destinations. Although the new corridors of HPB are faster, trips in the new system are more circuitous and include more transfers. As a consequence, we obtain higher in-vehicle distance. This, of course, underestimates the attractiveness of the system because it does not reflect the benefit to users that would switch to the system from private vehicles (or metro). Much of the induced demand for the new system is expected to come from the private auto rather than metro because the system layout of Figure 4 complements the metro, and improves spatial coverage. Furthermore, the city wishes to discourage auto use. A comparison of the data for scenarios B2 and B4 reflects the same tendency identified in the exclusive bus network. The door-to-door speed in the transit system increases by 5% and total travel time by 1%. However, the simulation did reveal that the new bus system captured nearly all the demand from the old bus system: the new system captured 54% vs. only 2% for the old. Metro remained at about 44%.

In summary, the new system improves the door-to-door travel speed, captures most of the demand from the old bus system and (combined with the old) reduces the agency cost by 24%. And this is achieved, assuming conservatively that the cruising speed of the new buses is unchanged. The next section explores what would happen if traffic management strategies are used to raise the bus cruising speeds.

6. Alternative future scenarios: faster cruising speeds and sensitivity analysis

6.1. Faster cruising speeds

The analysis and simulations were repeated for two cases in which the bus cruising speed was increased from 21.4 km/h to 30 km/h and 40 km/h. The lower value can be achieved with traffic management measures that would allow buses to reach the speed limit in the city (much as trams already do today), and the higher value with HPB infrastructure investments that are not envisioned in Barcelona. Tables 4 and 5 display the results of this analysis.

For both cruising speeds, the optimum system parameters change insignificantly from those of the base case in Section 5. Thus, comparisons of scenarios A0 and (A1, A2) continue to be meaningful tests of the analytic model's accuracy. Note, these comparisons are qualitatively similar to those arising from Table 3. Thus, they further support the comments of Section 5 regarding model validity.

As expected, the system performs considerably better with the increased cruising speeds, from both the user and the agency perspectives. This can be verified by comparing the results of each scenario across Tables 3, 4 and 5. Note, the improvements are significantly greater when increasing the speed from 21.4 km/h to 30 km/h than from 30 km/h to 40 km/h. Worth highlighting is the system's door-to-door speed in scenario A2, which first increases from 9.72 to 11.99, and then to 12.98 km/h.

Table 4. HPB system performance (v = 30 km/h)

	Enhanced speed (new network)		**Benchmark simulation**
	Scenario A1. Uniform demand simulation	**Scenario A2. Non uniform demand simulation**	**Scenario B3. Integrated Bus network**
In-Vehicle distance (km)	5.38	4.19	3.66
Access distance (km)	1.10	0.9	0.83
Total travel distance (km)	6.48	5.09	4.49
Access time, A (h)	14.4 (32.4*[a])	12.49 (28.11*[a])	11.55 (25.99*[a])
Waiting time, W (h)	3.53	3.05	3.71
In-vehicle time, T (h)	14.47	11.31	11.51
Expected number of transfers	0.63	0.75	0.3
Total travel time (h)	32.43 (50.04*[a])	26.85 (42.47*[a])	26.78 (41.21*[a])
Door-to-door speed (km/h)	11.99	11.37	10.04
1-way Infrastructure Length, $2L$ (km)	220		220 (HPB*[b])+530 (CS*[c])
Vehicles, M (veh)	222		222 (HPB*[b])+350 (CS*[c])
Vehicles per kilometer, V (veh-km)	4,296		4,296 (HPB*[b])+3,850 (CS*[c])
Agency Cost (€/h)	**44,594**		**85,768**

*[a] Numbers in parenthesis use v_w=2 km/h *[b] High Performance Bus (new service) *[c] Conventional Service

Table 5. HPB system performance (v = 40 km/h; operator costs are not included due to the increased unit cost of infrastructure, which has not been quantified)

	HPB network		Benchmark simulation
	Scenario A1. Uniform demand simulation	**Scenario A2. Non uniform demand simulation**	**Scenario B3. Integrated Bus network**
In-Vehicle distance (km)	5.46	4.24	3.69
Acces distance (km)	1.09	0.89	0.83
Total travel distance (km)	6.55	5.13	4.52
Access time, A (h)	14.29 (32.15[a])	12.40 (27.9[a])	11.67 (26.26[a])
Waiting time, W (h)	3.55	3.07	3.68
In-vehicle time, T (h)	12.43	9.69	10.07
Expected number of transfers	0.64	0.53	0.33
Total travel time (h)	30.27 (48.13[a])	25.16 (40.66[a])	25.42 (40.01[a])
Door-to-door speed (km/h)	12.98	12.23	10.66
1-way Infrastructure Length, $L2$ (km)	220		220 (HPB[b])+530 (CS[c])
Vehicles, M (veh)	190		190 (HPB[b])+350 (CS[c])
Vehicles per kilometer, V (veh-km)	4,940		4,940 (HPB[b])+3,850 (CS[c])

[a] Numbers in parenthesis use v_w=2 km/h [b] High Performance Bus (new service) [c] Conventional Service

Of particular interest are comparisons of Scenario A2 in Table 4 with the benchmark scenarios of Table 3. The new door-to-door speed (11.99 km/h) is 37% greater than that of the current bus system. Thus, the improvement strongly suggests that good traffic management without construction could turn the future bus system into an excellent competitor and complement to the metro system—even competing with the automobile. Also note that the agency cost of the new system is only about 39% of the cost of running the old system in scenario B1.

6.2. Sensitivity Analysis

This section explores how the results of the analytic model change if some of the input parameters of Table 1 are changed. It is found that the parameter that influences the solution most is the number of available corridors, N. More specifically, when constraint (10d) is relaxed the optimal number of corridors increases by at least 50%, to somewhere in the [23, 30] range depending on the type of central grid (see results in Table 6). The optimum door-to-door travel speed increases rather significantly (by 22%). This suggests that, as explained in Daganzo (2010), lack of available street space may be a large impediment to the deployment of high-efficiency bus networks in dense cities.

Changes in other parameters, excepting the dimensions of the service area, have a rather insignificant effect on the optimum costs and times. Figure 5 summarizes the effects. Each chart analyzes the sensitivity with regard to a different parameter. Each curve corresponds to a different measure of performance. For example, Figure 5c shows that when the horizontal dimension is 10 km the optimal decision variables are H=5.5 min, s=0.4 km and α=0.9. Another exception is cruising speed, which affects the user level of service as we showed in Sec. 6.1. Although parameter changes also influence the optimum values of the decision variables, they only affect the optimum H significantly. This means, as pointed out in Daganzo (2010), that network structure designed for today's conditions does not become obsolete; it can remain useful for a long time if its headways can be adapted to changing conditions.

Table 6. Results derived from the implementation of the model without the corridor constraint (N>11).

Line Lattice layout	Complete (px=py=1) Without Corridor Restrictions	Alternate (px=py=2) Without Corridor Restrictions	Semialternate (px=2,py=1) Without Corridor Restrictions
α	0.96	1	0.92
H (min)	6	5.4	5.4
s (km)	0.47	0.34	0.4
v_c (km/h)	13.61	12.66	13.49
Total travel time (h)	0.71	0.74	0.72
A (h)	0.235	0.255	0.250
W (h)	0.094	0.081	0.085
T (h)	0.368	0.395	0.372
e_T	0.87	0.87	0.87
$P_0/P_1/P_2$	13.14/87.06/0.29	19.48/80.52/0	14.23/84.69/1.06
Corridors in x /Corridors in y	20/12	12/12	15/8
L (km)	204.42	147.06	173.1
M (veh-h/h)	307.56	258.04	295.47
V (veh-km/h)	4187.23	3267.97	3986.67
User cost (h)	0.71	0.74	0.72
Agency cost (h)	0.189	0.148	0.175
Agency cost (€)	56,726	44,358	52,446
System unit cost, Z (h)	13.07	13.07	13.08
z_u/z_a	4.39	5.79	4.74
O_x (p/veh)	127.28	137.7	114.96
O_y (p/veh)	64.07	68.86	114.96

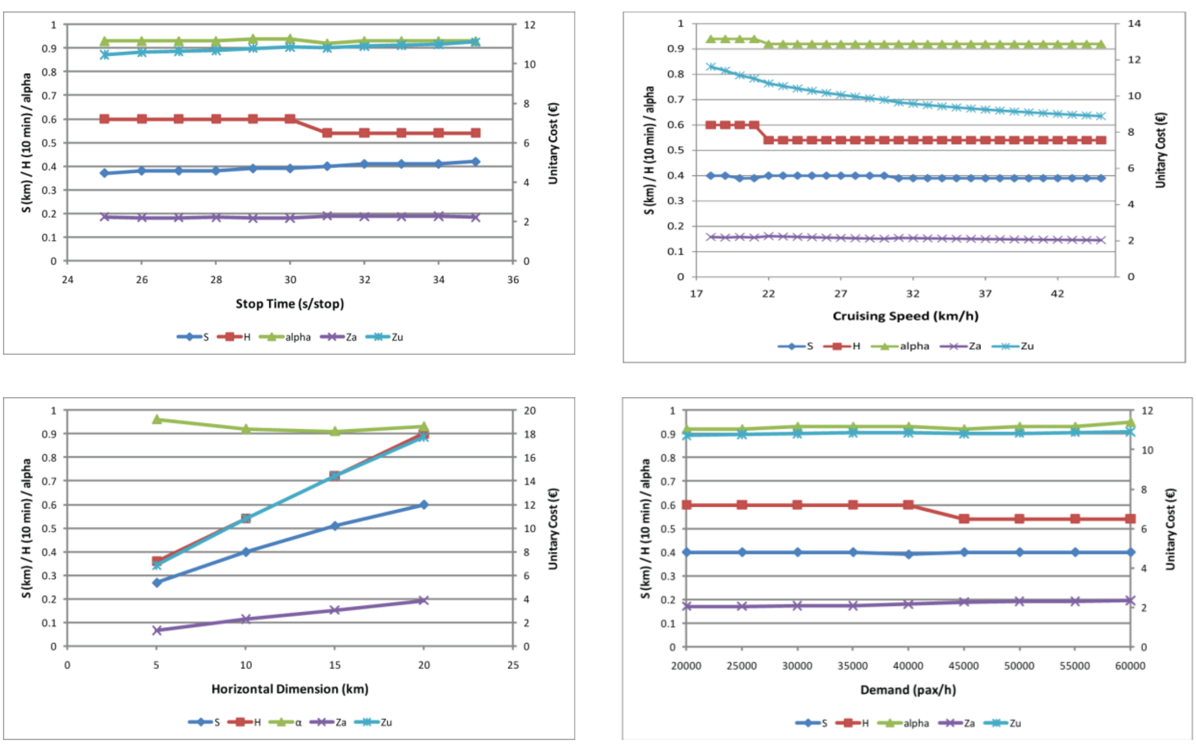

Figure 5. Sensitivity Analysis: (a) Stop time; (b) Cruising speed; (c) Horizontal dimension; (d) Demand.

7. Conclusion

This paper has demonstrated the feasibility of a HPB system for Barcelona. With good traffic management, the system proposed for Barcelona would, both increase the user's average door-to-door travel speed by about by 37% and reduce the bus agency's total cost by an even greater percentage. These benefit estimates are conservative because our analysis ignored the induced demand the system would attract from users that are currently captive to the automobile. This paper also demonstrates that a hybrid network with some asymmetry in design can be adapted to a real city with a two-step (analysis/design) method. The real-life design results from this process are shown to be robust and near-optimal. The analytic model was found to make reasonably accurate predictions. These would improve if more streets were available so that the actual system could more closely resemble the ideal. They would also improve if the model formulas were modified to better capture the effect of non-uniform demand. This is, however, a task for the future.

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Appendix: Proofs

Proofs of the formulae underpinning the analytic model are given here. The logic is similar with that in Daganzo (2010) and the results are presented in the same order. However, different parameters for horizontal and vertical directions are defined and the innovative ideas are stressed. Results 1, 3 and 6 contain the major differences whereas results 2, 4, 5 and 7 are easier to derive.

Result 1. The total length of the two-way infrastructure system is given by (1):

$$L = \frac{D_x D_y}{2 s_x s_y}(s_x + s_y)(1+\alpha_x\alpha_y) + \frac{D_x D_y}{2 s_x s_y}(s_x - s_y)(\alpha_y - \alpha_x)$$

Proof. In the city center each transfer stop has associated a length $s_s=(s_x+s_y)km$ of two-way infrastructure. In the periphery, each stop has associated just $s_s=(s_x)$ or $s_s=(s_y)$ km depending on the hemisphere where it is located. We can obtain the total length infrastructure by multiplying the corresponding length s_s by the number of stops contained in each subregion (central area or periphery). This number is equivalent to the ratio of the total area of the subregion and the area associated to one stop. Therefore we obtain the length in the city center $L_C = \alpha_x D_x \alpha_y D_y (s_x + s_y)/(s_x \cdot s_y)$, the length in the north and south quadrants $L_{NS} = D_x D_y (1+\alpha_x)(1-\alpha_y)/(2s_y)$ and the corresponding length in the east and west quadrants $L_{EW} = D_x D_y (1+\alpha_y)(1-\alpha_x)/(2s_x)$. Finally, $L=L_C+L_{NS}+L_{EW}$.□

Result 2. The total vehicle-distance travelled per hour is given by (2):

$$V = \frac{2\alpha_x D_x D_y}{s_y H}\left[1+\frac{D_x}{2D_y}(1-\alpha_x)\right] + \frac{2\alpha_y D_x D_y}{s_x H}\left[1+\frac{D_y}{2D_x}(1-\alpha_y)\right]$$

Proof. We consider first the central square and then the periphery. The distance travelled is the ratio of the length of the routes to be covered and the headway because H is constant in the central rectangle. The length of these two-way routes is twice the length of the infrastructure in the central rectangle, or $2\alpha_x\alpha_y D_x D_y \left(s_x + s_y\right)/(s_x s_y)$. Thus the total distance travelled per hour in the central rectangle is this length infrastructure divided by H.

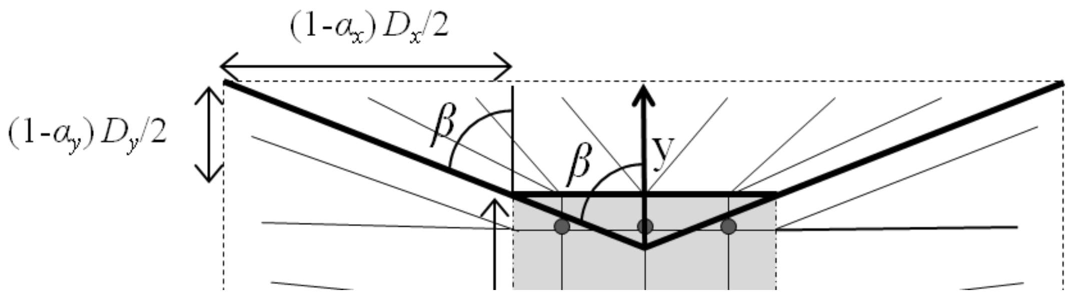

Figure A.1. Definition of angle β.

The periphery has to be handled differently because the headways are not constant. We formulate the average distance travelled by a vehicle in the N–S periphery in a differential of length dl. Let β be the angle between the diagonal of the rectangle and y-axis (see Figure A.1) such that $tg(\beta) = D_x(1-\alpha_x)/(D_y(1-\alpha_y))$. Each vehicle moves vertically and horizontally so that for each dy unit of vertical movement, its average horizontal movement is $dx=dy\cdot tg(\beta)/2$. Therefore the differential length will be $\overline{dl}=\overline{dy}+\overline{dx}$. On the other hand, the vehicle flow in a horizontal slide (q) must remain constant, so that $q=4\alpha_x D_x/(s_y H)$. Now, we can integrate this flow in the whole hemisphere.

$$V_{NS} = \int_{\alpha_y D_y/2}^{D_y/2} q \cdot dl = \frac{2D_x D_y}{s_y H}\left(1+\frac{D_x}{2D_y}\frac{(1-\alpha_x)}{(1-\alpha_y)}\right)(1-\alpha_y)\alpha_x$$

For symmetry in the east–west (EW) hemisphere:

$$V_{EW} = \frac{2D_x D_y}{s_x H}\left(1+\frac{D_y}{2D_x}\frac{(1-\alpha_y)}{(1-\alpha_x)}\right)(1-\alpha_x)\alpha_y$$

Adding, we obtain $V=V_C+V_{NS}+V_{EW}$.□

Result 3. The expected number of transfers per trip is given by (7):

$$e_T = 1\cdot P_1 + 2\cdot P_2$$

where:

$$P_0 = \frac{(s_x D_x + s_y D_y)}{2D_x D_y}(1+\alpha_x\alpha_y) + \frac{(s_x D_x - s_y D_y)}{2D_x D_y}(\alpha_y - \alpha_x) - \frac{\alpha_x\alpha_y s_x s_y}{D_x D_y}$$

$$P_1 = +\frac{s_y}{2D_x}\left(-\alpha_y + \alpha_y^2 - 3\alpha_x\alpha_y + \alpha_x\alpha_y^2\right) + \frac{s_x}{2D_y}\left(-\alpha_x + \alpha_x^2 - 3\alpha_x\alpha_y + \alpha_y\alpha_x^2\right)$$
$$+\frac{1}{2}\left(1-\alpha_y^2 - \alpha_x^2 + 4\alpha_x\alpha_y - \alpha_x^2\alpha_y^2\right) + \frac{s_x s_y \alpha_x\alpha_y}{D_x D_y}$$

$$P_2 = \frac{1}{2}\left(1-4\alpha_x\alpha_y + \alpha_x^2 + \alpha_y^2 + \alpha_x^2\alpha_y^2\right) - \frac{s_y}{2D_x}(1-\alpha_y)^2(1+\alpha_x) - \frac{s_x}{2D_y}(1-\alpha_x)^2(1+\alpha_y)$$

Proof. Unlike Daganzo (2010), we assume that those users with origin and destination in the influence area of the same bus line can travel with zero transfers. However, users that require two transfers are the same: users with origin and destination outside the central rectangle and in the same hemisphere. The rest require one transfer to reach their destination. Therefore, the expected number of transfers is computed depending on the probability to do 0, 1 or 2 transfers.

We consider the influence area of one horizontal bus line, i.e., the area whose inner points are nearer than $s_x/2$ distance to the line. Similarly, the area of influence of a vertical line is defined by a rectangle whose width is $s_y/2$.

Zero Transfers: We differentiate between users with origin in the central rectangle ($P_{0,C}$) and users with origin outside the central rectangle ($P_{0,P}$). For users with origin in the central rectangle, zero transfer is equivalent to the condition that both origin and destination fall in the area of influence of one bus line (vertical or horizontal). So, the probability of zero transfers can be computed as $P_{0,C}= P_{0,O}\ P_{0D}$, where $P_{0,O}$ is the probability that the origin is located in the central area of the rectangle and P_{0D} is the probability that the destination is in the same area of influence of one bus line. These probabilities may be calculated as the ratio of the surface satisfying the condition and the total rectangle area. Thus,

$$P_{0,C} = \frac{\alpha_x D_x \alpha_y D_y}{D_x D_y}\frac{s_x D_x + s_y D_y - s_x s_y}{D_x D_y} = \frac{\alpha_x\alpha_y}{D_x D_y}(s_x D_x + s_y D_y - s_x s_y)$$

For users with origin in the periphery, we distinguish between N–S and E–W hemispheres and we operate in the same way as in the central area.

$$P_{0,P-NS} = \frac{2\left(\frac{D_y - \alpha_y D_y}{2}\right)\left(\frac{D_x + \alpha_x D_x}{2}\right)}{D_x D_y}\cdot\frac{s_y D_y}{D_x D_y} = \frac{s_y}{2D_x}(1-\alpha_y)(1+\alpha_x)$$

$$P_{0,P-EW} = \frac{2\left(\frac{D_x - \alpha_x D_x}{2}\right)\left(\frac{D_y + \alpha_y D_y}{2}\right)}{D_x D_y} \cdot \frac{s_x D_x}{D_x D_y} = \frac{s_x}{2D_y}(1-\alpha_x)(1+\alpha_y)$$

Two Transfers: Only users with origin and destination outside the central rectangle and in the same hemisphere should transfer twice. Using the ratio of effective and total area of points as a probability, we distinguish the N–S hemisphere and the E–W hemisphere:

$$P_{2,NS} = \frac{2\left(\frac{D_y - \alpha_y D_y}{2}\right)\left(\frac{D_x + \alpha_x D_x}{2}\right)}{D_x D_y} \cdot \frac{2\left(\frac{D_y - \alpha_y D_y}{2}\right)\left(\frac{D_x + \alpha_x D_x}{2}\right) - s_y(D_y - \alpha_y D_y)}{D_x D_y}$$
$$= \frac{(1-\alpha_y)(1+\alpha_x)}{2}\left[\frac{(1-\alpha_y)(1+\alpha_x)}{2} - \frac{s_y(1-\alpha_y)}{D_x}\right]$$
$$P_{2,EW} = \frac{(1-\alpha_x)(1+\alpha_y)}{2}\left[\frac{(1-\alpha_x)(1+\alpha_y)}{2} - \frac{s_x(1-\alpha_x)}{D_y}\right]$$

One Transfer: Using the total probability theorem, we obtain the one-transfer formulae by subtracting from 1 the probability of two and zero transfers. □

Result 4. The expected walking time at the origin and destination is given by (5):

$$A = \left(\frac{s_x + s_y}{4} + \frac{s}{2}\right) \Big/ v_w$$

Proof. We will assume that users will determine if their first movement is horizontal or vertical and then access the closest stop with service in the desired direction. If the first movement is horizontal, the average traveller should walk $s_x/4$ in the horizontal direction and $s/4$ in the vertical direction. If the first movement is vertical, the average traveller should walk $s_y/4$ in the vertical direction and $s/4$ in the horizontal direction. On average, half of the movements will be horizontal and half vertical. Therefore, if we take into account access and egress, we can add the distance for a first horizontal move and for a first vertical move. The time is achieved by dividing the length by the average walking speed (v_w).□

Result 5. The expected waiting time per user including the origin and all transfer stops is given by (6):

$$W = \left[\frac{H}{6\alpha_x}(1-\alpha_x^3)\frac{(1-\alpha_y)}{(1-\alpha_x)} + \frac{H}{6\alpha_y}(1-\alpha_y^3)\frac{(1-\alpha_x)}{(1-\alpha_y)} + \alpha_x\alpha_y\frac{H}{2}\right]\left(1+P_1\right) + \frac{H}{2}P_2$$

Proof. We use the same assumptions as Daganzo (2010). We assume that headways are low; so people arrive independently of the schedule. The expected wait has three components: (i) at the origin stop, W_O; (ii) at the last transfer point, W_D only for trips requiring a minimum of one transfer; and (iii) at the intermediate transfer point, W_T, only for trips requiring such transfer. W_O and W_D are the same, since there is the need of a specific route. W_0 can be divided into waiting time in the center ($W_{0,C}$) and in the periphery($W_{0,P}$). In the center, waiting time is $H/2$, half the headway, and should be multiplied by the probability of being at the center $W_{O,C} = \frac{H}{2}\alpha_x\alpha_y$. In the periphery, we should differentiate between N–S and E–W and use symmetry. Waiting time at one slice depends on the distance to the center. If we consider the hemisphere N–S, and β the angle that defines this distance from the center of the region (see Figure A.1), we can compute the waiting as follows. The waiting at distance y, $H(y)$, obeys the relation

$\frac{H(y)}{y} = \frac{H/2}{\alpha_y D_y / 2}$, therefore $H(y) = \frac{H}{\alpha_y D_y} y$. The probability of the slice at distance y is the dimension of this slice divided by the total area: $p[dy] = 2ytg(\beta)dy/(D_x D_y)$. So, if we integrate in the hemisphere:

$$W_{O,P,NS} = \int_{\alpha_y D_y/2}^{D_y/2} H(y)p(dy)dy = \frac{H}{12\alpha_y}(1-\alpha_y^3)\frac{(1-\alpha_x)}{(1-\alpha_y)}$$

By symmetry, we obtain: $W_{O,P,EW} = \frac{H}{12\alpha_x}(1-\alpha_x^3)\frac{(1-\alpha_y)}{(1-\alpha_x)}$

If we sum $W_0 = W_{0,C} + W_{O,P,NS} + W_{O,P,EW}$:

$$W_O = \frac{H}{2}\alpha_x\alpha_y + \frac{H}{6\alpha_y}(1-\alpha_y^3)\frac{(1-\alpha_x)}{(1-\alpha_y)} + \frac{H}{6\alpha_x}(1-\alpha_x^3)\frac{(1-\alpha_y)}{(1-\alpha_x)}$$

All users experiment $W_{O,}$ W_D appears when users must transfer once and occurs with probability P_1, and W_T only appears when there are two necessary transfers, and always take place in the central rectangle, therefore it takes $H/2$, but only occurs with probability P_2. □

Result 6. The expected in-vehicle travel distance per trip is given by (8):

$$E(E) = \left(\frac{\alpha_y^2 D_y^2 + \alpha_x^2 D_x^2 + 4\alpha_x\alpha_y D_x D_y}{4(\alpha_x D_x + \alpha_y D_y)} + \frac{(\alpha_x D_x + \alpha_y D_y)}{12\alpha_x\alpha_y D_x D_y}\left(1 - \frac{\alpha_x\alpha_y}{2}\right)\right)\left(1-\alpha_x^2\alpha_y^2\right) +$$
$$+\frac{1}{3}(\alpha_x D_x + \alpha_y D_y)(\alpha_x^2\alpha_y^2) + \frac{1}{4}\left(D_x(2-3\alpha_x+\alpha_x^3) + D_y(2-3\alpha_y+\alpha_y^3)\right)$$

Proof. Every trip can include a component in the periphery and a component in the central square.

Periphery: In the periphery, every passenger travels in a radial direction. This can happen both inbound, from the origin stop to an entry point for the central square, and outbound, from an exit point of the central square to the destination stop. By symmetry, these inbound and outbound distances are described by the same random variable, R_P. The central rectangle has dimensions d_x, d_y. We will distinguish the calculation for the E–W hemisphere and N–S hemisphere. If the origin stop is on a cordon with sides C_x, C_y ($C_x > d_x$, $C_y > d_y$), then the perpendicular distance to the entry point is ½($C_x - d_x$), and the expected lateral displacement is ¼ ($C_y - d_y$). Then the expected distance travelled in the E–W hemisphere is $E(R_P | C_x C_y) = \left(\frac{C_x - d_x}{2}\right) + \left(\frac{C_y - d_y}{4}\right)$

By symmetry, in the N–S hemisphere $E(R_P | C_x C_y) = \left(\frac{C_y - d_y}{2}\right) + \left(\frac{C_x - d_x}{4}\right)$. We can express the average distance in the periphery as the average of these variables. $E(R_P | C_x C_y) = \frac{3}{8}\left((C_x - d_x) + (C_y - d_y)\right)$. Integrating the expression, using the probability distribution of the variables, we obtain:

$$E(R_P) = \frac{3}{8}\left(\int_{d_x}^{D_x}(x-d_x)\frac{2x}{D_x^2}dx + \int_{d_y}^{D_y}(y-d_y)\frac{2y}{D_y^2}dy\right) = \frac{1}{8}\left(D_x(2-3\alpha_x+\alpha_x^3) + D_y(2-3\alpha_y+\alpha_y^3)\right)$$

Central: In the central square, every passenger travels a distance between two random points. Denote this distance by R_C. We shall find its expectation by conditioning on the cordons of the two random points.

Lemma. The expected distance in a rectangle with sides SxR between a random point on the periphery and a point located in a cordon βR, βS, $\beta \in [0,1]$ is $\frac{R^2 + S^2 + 4RS}{4(R+S)} + \frac{(R+S)}{12}\beta^2$.

Proof: This statement can be verified with geometric probability methods.

We consider two cases: ‘‘a” if at least one of these points falls on the edge of the central square; and ‘‘b” if both points fall inside. For case ‘‘a”, the expression of the lemma applies with: $S = d_x$ and $R=d_y$ and $\beta \in [0,1]$

$$E(R_C|SR, \text{``a''}) = \frac{R^2+S^2+4RS}{4(R+S)} + \frac{(R+S)}{12}\beta^2 .$$

Note that like C_x, C_y is a rectangle homothetic to the central rectangle of sides d_x, d_y, we can write $C_y = C_x \frac{d_y}{d_x}$.

Note that the joint probability distribution is $F_{XY}(x,y) = \frac{xy}{D_x D_y} = \frac{xx}{D_x D_y}\frac{d_y}{d_x} = \frac{x^2}{D_x D_y}\frac{d_y}{d_x}$.

The probability of falling in a rectangle of sides C_x, C_y will be expressed as a sum of the probability of being in a rectangle C_xC_y smaller than d_xd_y or if it is equal to d_xd_y.

$$E(C_xC_y) = E(C_xC_x\frac{d_y}{d_x}) = \frac{d_y}{d_x}E(C_x^2) = \frac{d_y}{d_x}\int_0^{d_x} x^2 \frac{2x}{D_xD_y}\frac{d_y}{d_x} = \frac{d_x^2 d_y^2}{2D_xD_y} \quad ; \; C_x < d_x ,\; C_y < d_y$$

$$E(C_xC_y) = d_xd_y\left(1-\frac{d_xd_y}{D_xD_y}\right) \quad ; \; C_x = d_x, C_y = d_y$$

Adding both terms:

$$E(C_xC_y) = d_xd_y\left(1-\frac{\alpha_x\alpha_y}{2}\right)$$

For case ‘‘b”, the distance between the two points is simply the distance between two random points in the square; i.e., $E(R_C|\text{``b''}) = d_x/3 + d_y/3. = (\alpha_xD_x+\alpha_yD_y)/3$. Finally, since case ‘‘a” occurs with probability $(1-\alpha_x^2\alpha_y^2)$ and case ‘‘b” with probability $\alpha_x^2\alpha_y^2$, we have: $E(R_C) = E(R_C|\text{``a''})(1-\alpha_x^2\alpha_y^2) + E(R_C|\text{``b''})\,\alpha_x^2\alpha_y^2$.

$$E(R_C) = \left(\frac{\alpha_y^2D_y^2+\alpha_x^2D_x^2+4\alpha_x\alpha_yD_xD_y}{4(\alpha_xD_x+\alpha_yD_y)} + \frac{(\alpha_xD_x+\alpha_yD_y)}{12\alpha_x\alpha_yD_xD_y}\left(1-\frac{\alpha_x\alpha_y}{2}\right)\right)\left(1-\alpha_x^2\alpha_y^2\right)+$$
$$+\frac{1}{3}(\alpha_xD_x+\alpha_yD_y)(\alpha_x^2\alpha_y^2)$$

Adding central and peripheral terms we obtain the final formulae:

$$E(E) = 2E(R_E)+E(R_C)$$

$$E(E) = \left(\frac{\alpha_y^2D_y^2+\alpha_x^2D_x^2+4\alpha_x\alpha_yD_xD_y}{4(\alpha_xD_x+\alpha_yD_y)} + \frac{(\alpha_xD_x+\alpha_yD_y)}{12\alpha_x\alpha_yD_xD_y}\left(1-\frac{\alpha_x\alpha_y}{2}\right)\right)\left(1-\alpha_x^2\alpha_y^2\right)+$$
$$+\frac{1}{3}(\alpha_xD_x+\alpha_yD_y)(\alpha_x^2\alpha_y^2)+\frac{1}{4}\left(D_x(2-3\alpha_x+\alpha_x^3)+D_y(2-3\alpha_y+\alpha_y^3)\right)$$

□

Result 7. The expected commercial speed during rush hour is given by (9):

$$1/v_c = \left[1/v+\tau/s\right]+\left(1+e_T\right)\Lambda/V\,\tau'$$

Proof. The same as in Daganzo (2010).□

Result 8. The expected vehicle occupancy on the critical load point in vertical or horizontal lines during the rush hour is approximately given by (3a-3b):

$$O_y = max\left(\frac{\Lambda s_y H\left(1+\alpha_x\right)\left(1-\alpha_y\right)}{\left(4\alpha_x D_x\right)}; \frac{\Lambda H\left(1-\alpha_x\right)^2\left(1+\alpha_y\right)^2}{32} + \frac{\Lambda s_y H\left(4-(1+\alpha_y)^2(1-\alpha_x)^2 - 2\alpha_x^2\alpha_y^2\right)}{\left(8\alpha_x D_x\right)} \right)$$

$$O_x = max\left(\frac{\Lambda s_x H\left(1+\alpha_y\right)\left(1-\alpha_x\right)}{\left(4\alpha_y D_y\right)}; \frac{\Lambda H\left(1-\alpha_y\right)^2\left(1+\alpha_x\right)^2}{32} + \frac{\Lambda s_x H\left(4-(1+\alpha_x)^2(1-\alpha_y)^2 - 2\alpha_x^2\alpha_y^2\right)}{\left(8\alpha_y D_y\right)} \right)$$

Proof. The proof of Daganzo (2010) is valid for both formulae if we take into account some differences. We should use $\alpha_x\alpha_y$ instead of α^2 when referring to a central rectangle probability. Then we should distinguish between O_x and O_y, being the occupancy of vehicles from horizontal and vertical lines, respectively. We should take into account that there are $\alpha_x D_x/s_y$ vertical lines and $\alpha_y D_y/s_x$ horizontal lines. The same procedure is then valid for each case. □