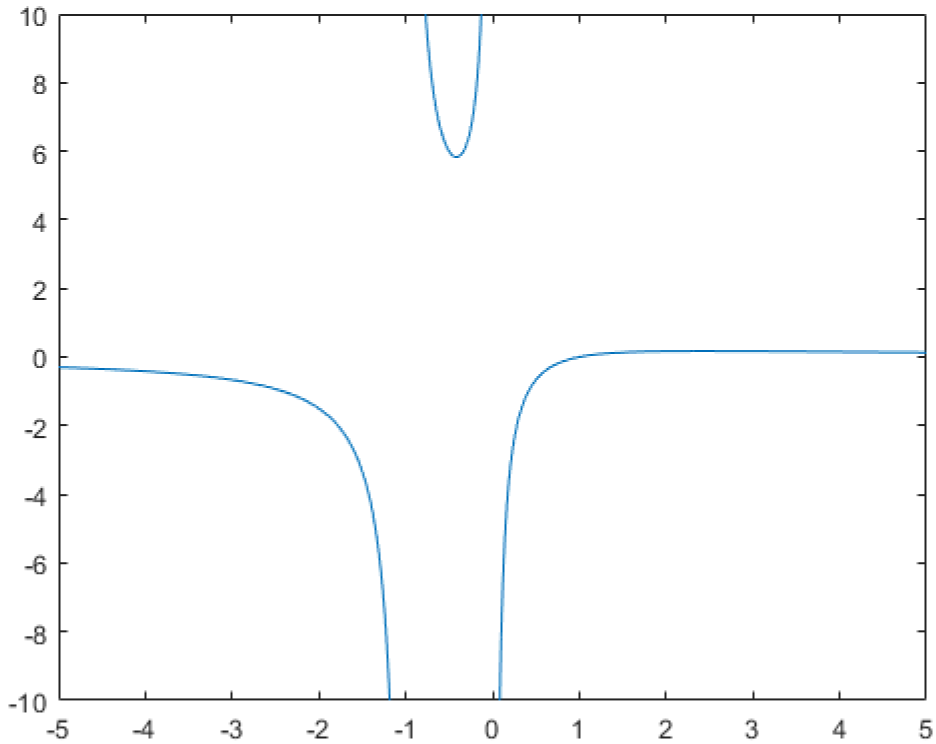
```matlab
% Written By Oscar Dahlberg
% For course M0047M at LTU

disp("P4.48 Finding the range of (x - 1)./(x.^2 + x)")
f = @(x) (x - 1)./(x.^2 + x);

disp("Instead of a line we will create a line of dots")
disp("This will be used to find the maximum value of x")
x = -5:0.01:5;
y = f(x);
plot(x, y)
axis([-5, 5, -10, 10]);

disp("We flip the graph to make use of the matlab")
disp("function 'fminbnd' which returns the smallest value")
f_negativ = @(x) -f(x);
xmax = fminbnd(f_negativ, 1,3);

disp("Now we have found the maximum range,")
disp("The minimum can be found through looking at the graph")
disp("Then we quickly realize its negative infinity")
disp("range = (-Inf, " + f(xmax) + "]")
```

```
P4.48 Finding the range of (x - 1)./(x.^2 + x)
Instead of a line we will create a line of dots
This will be used to find the maximum value of x
We flip the graph to make use of the matlab
function 'fminbnd' which returns the smallest value
Now we have found the maximum range,
The minimum can be found through looking at the graph
Then we quickly realize its negative infinity
range = (-Inf, 0.17157]
```



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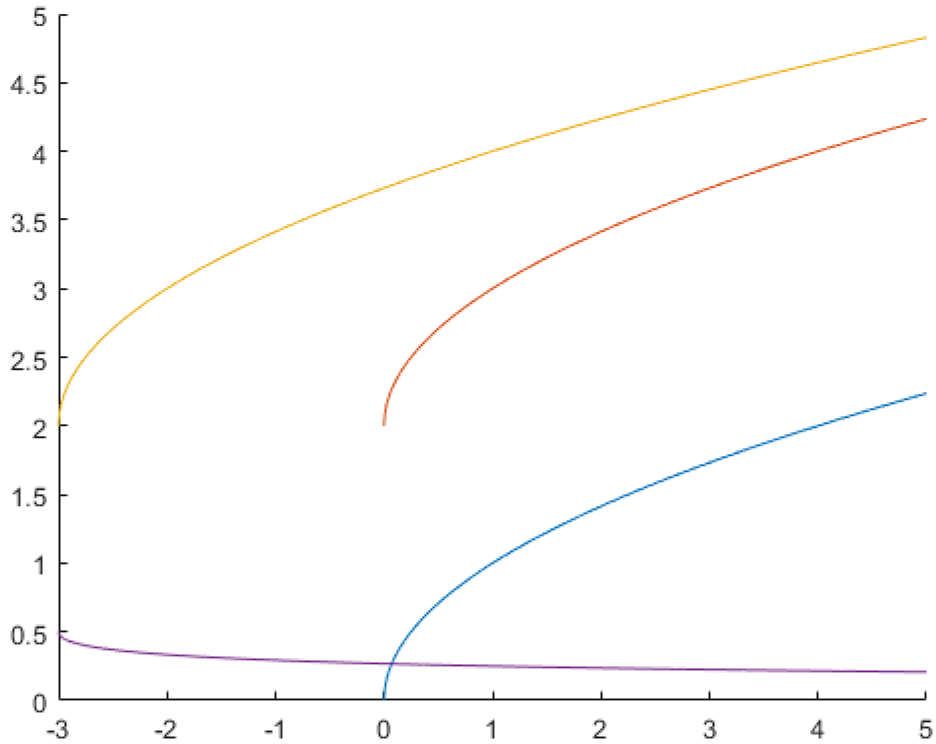
```
disp("P5.17, Analysing graphs")
disp("Graphs to analyse:")
disp("sqrt(x), 2 + sqrt(x), 2 + sqrt(3 + x), 1. / (2 + sqrt(3 + x))")
hold on

x = 0:0.01:5;
f = @(x) sqrt(x);
y = f(x);
plot(x, y)

g = @(x) 2 + sqrt(x);
y = g(x);
plot(x, y)

x = -3:0.01:5;
h = @(x) 2 + sqrt(3 + x);
y = h(x);
plot(x, y)

j = @(x) 1 ./ (2 + sqrt(3 + x));
y = j(x);
plot(x, y)

disp("We realize that the red line is just the blue line")
disp("moved up two y places")

disp("The yellow line is just the red line moved back 3")
disp("x places")

disp("And the purple line is the yellow line moved down 1,5")
disp("places, flipped, and reduced in height by half")
```

```
P5.17, Analysing graphs
Graphs to analyse:
sqrt(x), 2 + sqrt(x), 2 + sqrt(3 + x), 1. / (2 + sqrt(3 + x))
We realize that the red line is just the blue line
moved up two y places
The yellow line is just the red line moved back 3
x places
And the purple line is the yellow line moved down 1,5
places, flipped, and reduced in height by half
```



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```
disp("1.16, Proving equation")
disp("Lets prove that for x > 2, the absolute value of")
disp("(x - 4) / (x^2 + sin(x)) is smaller than 10 / x)")
disp("If we start by looking at the numerator and call it T")
disp("T = abs(x - 4)")
disp("Because x is bigger than two, ")
disp("it is the same thing as x + 4 = T, which is always")
disp("smaller or equal to x + 2x which is equal to 3x")
disp("Now we have T = 3x")
disp("Lets look at the denumerator")
disp("N = abs(x^2 + sin(x))")
disp("This is equal to N = x^2 + sin(x) and because")
disp("sin(x) is always smaller or equal to 1")
disp("N = x^2 - 1 which is smaller or equal to")
disp("x^2 - x^2 / 4 which is equal to (3x^2 / 4)")
disp("T / N is 3x / ((3x^2) / 4) which is equal to")
disp("4 / x which is of course smaller than 10 / x")
```

```
1.16, Proving equation
Lets prove that for x > 2, the absolute value of
(x - 4) / (x^2 + sin(x)) is smaller than 10 / x)
If we start by looking at the numerator and call it T
T = abs(x - 4)
Because x is bigger than two,
it is the same thing as x + 4 = T, which is always
smaller or equal to x + 2x which is equal to 3x
Now we have T = 3x
Lets look at the denumerator
N = abs(x^2 + sin(x))
This is equal to N = x^2 + sin(x) and because
sin(x) is always smaller or equal to 1
N = x^2 - 1 which is smaller or equal to
x^2 - x^2 / 4 which is equal to (3x^2 / 4)
T / N is 3x / ((3x^2) / 4) which is equal to
4 / x which is of course smaller than 10 / x
```

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```
disp("1.30, Solving sum of a series")
disp("Series: Sum of (1/5)^k, when k goes from 0 to 3 is: ")
k = 3;

sum = 0;
for i = 0:k
    sum = sum + (1/5)^i;
end
disp(sum)
```

```
1.30, Solving sum of a series
Series: Sum of (1/5)^k, when k goes from 0 to 3 is:
   1.248000000000000
```

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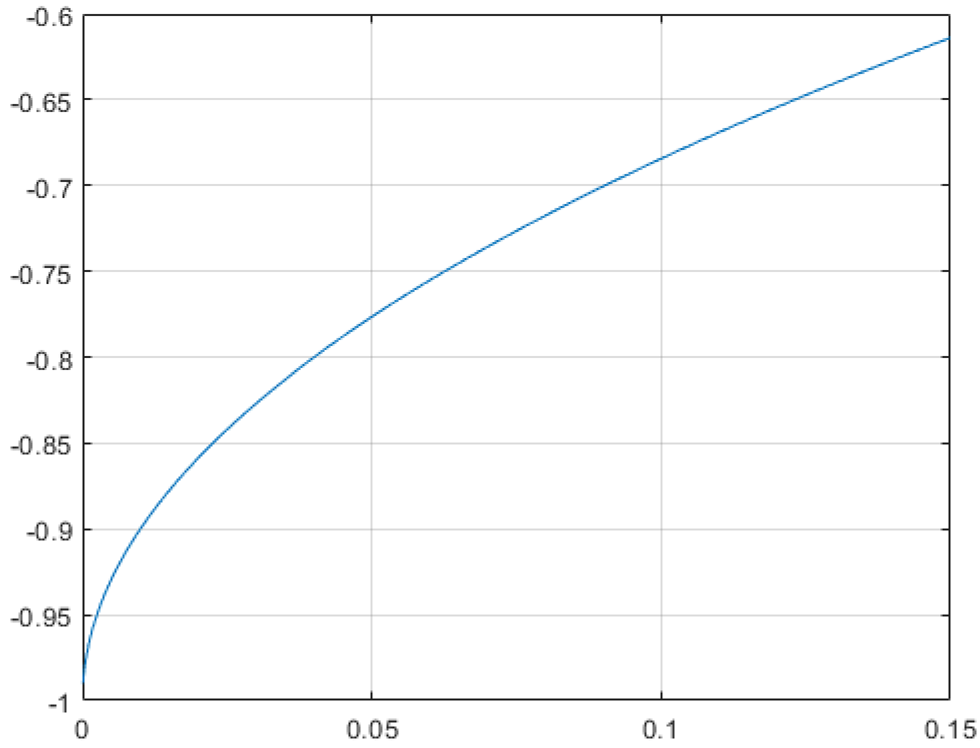
```
disp("1.2.72 Finding limits")
disp("Finding limits of (x - sqrt(x)) / sqrt(sin(x))")
disp("When x goes to 0 from the right side")


x = 0:0.0001:0.15;
f = @(x) (x - sqrt(x))./ (sqrt(sin(x)));
y = f(x);
plot(x, y)
grid on
disp("As we can see from the graph,")
disp("the line trends towards -1 when approaching")
disp("From the right side but will never be -1")
```

```
1.2.72 Finding limits
Finding limits of (x - sqrt(x)) / sqrt(sin(x))
When x goes to 0 from the right side
As we can see from the graph,
the line trends towards -1 when approaching
From the right side but will never be -1
```



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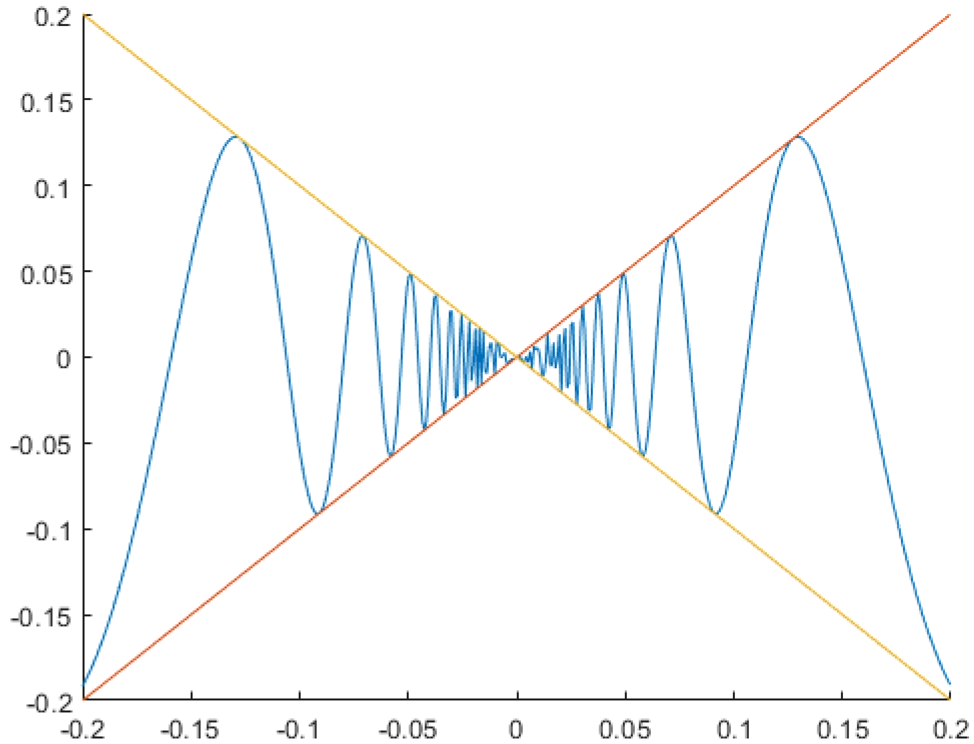
```
disp("1.2.73 Three functions")
hold on
x = -0.2:0.0009:0.2;

f = @(x) x.*sin(1./x);
y = f(x);
plot(x, y)

g = @(x) x;
y = g(x);
plot(x, y)

h = @(x) -x;
y = h(x);
plot(x, y)

disp("f(x) Oscillates faster and faster the closer")
disp("to x = 0 the function gets.")
disp("lim x -> 0 of f(x) exists because as x tends to zero")
disp("1 / x tends to infinity")
```

```
1.2.73 Three functions
f(x) Oscillates faster and faster the closer
to x = 0 the function gets.
lim x -> 0 of f(x) exists because as x tends to zero
1 / x tends to infinity
```



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```matlab
disp("1.4.39 Finding roots")
disp("Looking at the funtion f(x) = x^3 + x - 1")
disp("By making some calculations we can see")
% Like substituting x for 0 & 1
disp("that there should be a root between")
disp("0 < x < 1")
disp("Because f(0) = -1 and f(1) = 1")

f = @(x) x^3 + x - 1;
xmin = 0;
xmax = 1;
xmed = 0;

% While the difference between last loops xmed
% and the next xmed is greater than 0.001
while abs(xmed - (xmin + xmax) / 2) > 0.001
    xmed = (xmin + xmax) / 2;
    if f(xmin)*f(xmed) > 0
        xmin = xmed;
    else
        xmax = xmed;
    end
end

disp("Using the bisection method we can find that x")
disp("has a root near")
disp(round(xmed, 3))
```

```
1.4.39 Finding roots
Looking at the funtion f(x) = x^3 + x - 1
By making some calculations we can see
that there should be a root between
0 < x < 1
Because f(0) = -1 and f(1) = 1
Using the bisection method we can find that x
has a root near
   0.682000000000000
```

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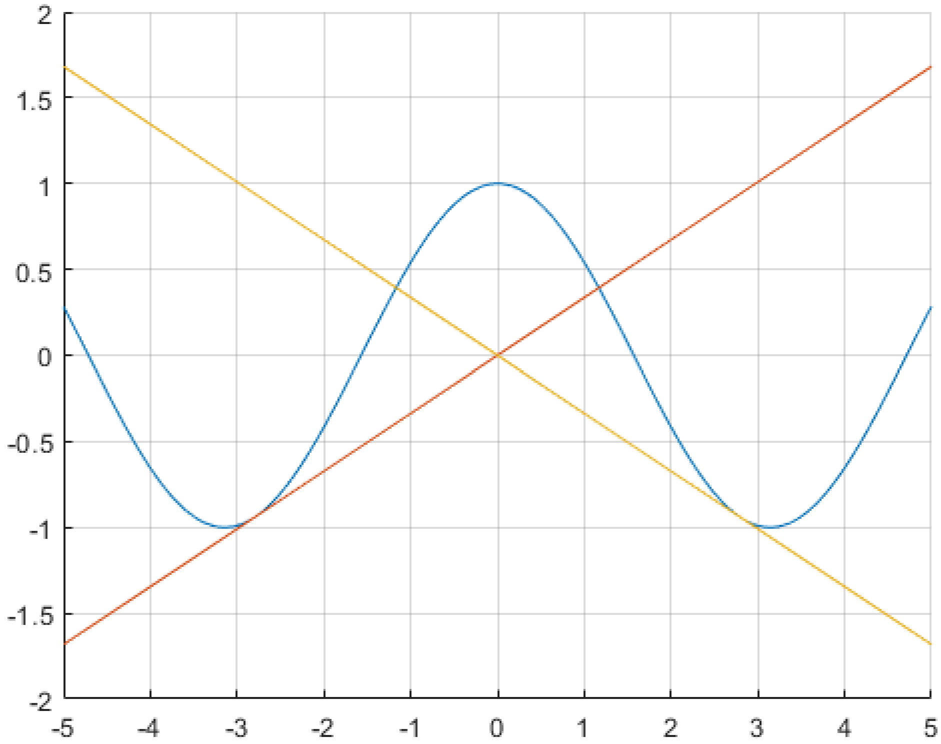
```
disp("2.5.59 Finding tangent line that pass through origin")
hold on
grid on
x = -5:0.01:5;
f = @(x) cos(x);
y = f(x);
plot(x, y)
disp("By looking at the graph we can conclude that")
disp("the function y = cosx will have a tangent that")
disp("passes through origin every wavelength and")
disp("the tangent will have a smaller inclination every time")
disp("We can also draw the conclusion that the two tangents")
disp("with the steepest slope will be the ones closest to")
disp("the origin")

disp("All tangents pass through the point x = a, y = cosa")
disp("And the slope of it is the derivative of y")
disp("which is -sinx")
disp("Then we have the function of the line which is")
disp("y - (cos(a)) = -sin(a)(x - a)")
disp("To calculate for a we use matlab function fzero")

calcA = @(a) cos(a) + sin(a)*a;

a = fzero(calcA, 0);

disp("Which we get to be: ")
disp(a)
disp("Our function for one of the two tangents is then: ")
disp("g(x) = cos(-2.8) - sin(-2.8)*(x - (-2.8))")
g = @(x) cos(a) - sin(a)*(x - a);
y = g(x);
plot(x, y)

disp("To find the other one, all we have to do is take")
disp("the negative of a")
h = @(x) cos(-a) - sin(-a)*(x - (-a));
y = h(x);
plot(x, y)
```

```
2.5.59 Finding tangent line that pass through origin
By looking at the graph we can conclude that
the function y = cosx will have a tangent that
passes through origin every wavelength and
the tangent will have a smaller inclination every time
We can also draw the conclusion that the two tangents
with the steepest slope will be the ones closest to
the origin
All tangents pass through the point x = a, y = cosa
And the slope of it is the derivative of y
which is -sinx
Then we have the function of the line which is
```

```
y - (cos(a)) = -sin(a)(x - a)
To calculate for a we use matlab function fzero
Which we get to be:
  -2.798386045783887

Our function for one of the two tangents is then:
g(x) = cos(-2.8) - sin(-2.8)*(x - (-2.8))
To find the other one, all we have to do is take
the negative of a
```



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```
format long
disp("3.1.31 Finding slope of a point")
disp("For the function: ")
syms x
f = (x^2) / (1 + sqrt(x));
disp("f(x) = ")
disp(f)
disp("The derivative is: ")
fprime = diff(f);
disp("f'(x) = ")
disp(fprime)
disp("If we replace x with 2 and round to 5 decimals")
disp("we get our answer")
fprime = @(x) (2*x) / (x^(1/2) + 1) - x^(3/2)/(2*(x^(1/2) + 1)^2);
disp(round(fprime(2),5))
```

```
3.1.31 Finding slope of a point
For the function:
f(x) =
x^2/(x^(1/2) + 1)

The derivative is:
f'(x) =
(2*x)/(x^(1/2) + 1) - x^(3/2)/(2*(x^(1/2) + 1)^2)

If we replace x with 2 and round to 5 decimals
we get our answer
   1.414210000000000
```

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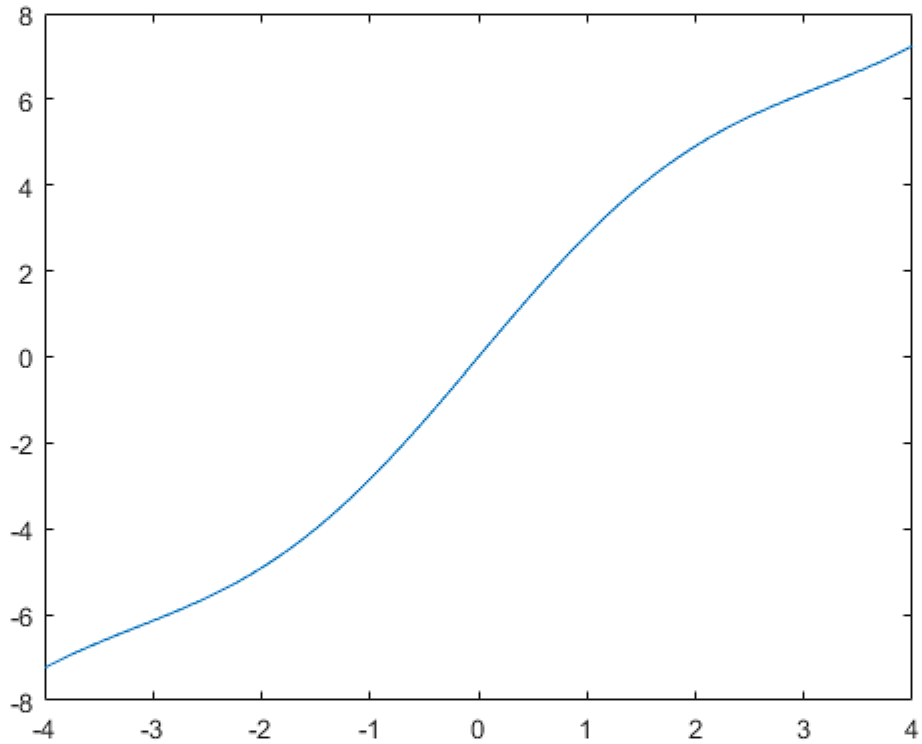
```
disp("3.1.32 Find inverse of function")
disp("Finding inverse of function: ")
disp("g(x) = 2*x + sin(x)")

x = -4:0.01:4;
g = @(x) 2*x + sin(x);
y = g(x);
plot(x, y)
disp("To decide wether g(x) is invertible")
disp("we have to derive the function.")
disp("g'(x) = 2 + cos(x)")
disp("Although cos(x) goes from -1 to 1")
disp("cos(x) + 2 goes from +1 to +3 which means it's")
disp("always postive, meaning the function is invertible")
disp("To get the y value for when the inverted function")
disp("has a x value of 2, we switch x for y and y for x")
disp("meaning we get: ")
disp("x = 2*y + sin(y)")
disp("replace x with 2")
disp("2 = 2*y + sin(y), which we solve with fzero")

calcY = @(y) 2*y + sin(y) - 2;
disp(fzero(calcY, 0))

disp("To find the value of y for when x is 2")
disp("for the inverted derivative of g(x)")
disp("that is the same thing as 1/(g'(g^-1(2)))")
disp("which is equal to 1 / (2 + cos(g^-1(2))), then we take")
disp("our answer from part one and we get")
disp("1 / (2 + cos(0.684)) = ")
disp(1 / (2 + cos(0.684)))
```

```
3.1.32 Find inverse of function
Finding inverse of function:
g(x) = 2*x + sin(x)
To decide wether g(x) is invertible
we have to derive the function.
g'(x) = 2 + cos(x)
Although cos(x) goes from -1 to 1
cos(x) + 2 goes from +1 to +3 which means it's
always postive, meaning the function is invertible
To get the y value for when the inverted function
has a x value of 2, we switch x for y and y for x
meaning we get:
x = 2*y + sin(y)
replace x with 2
2 = 2*y + sin(y), which we solve with fzero
   0.684036656677829

To find the value of y for when x is 2
for the inverted derivative of g(x)
that is the same thing as 1/(g'(g^-1(2)))
```

```
which is equal to 1 / (2 + cos(g^-1(2))), then we take
our answer from part one and we get
1 / (2 + cos(0.684)) =
   0.360353694428833
```



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```
disp("3.2.21 Find value of x using Log10")
disp("Finding x from 2^(2x) = 5^(x + 1)")
disp("The first thing we should do is get x")
disp("on one side of the equation.")
disp("To do this we put log with base 10")
disp("on both sides and we get: ")
disp("(2x)log(2) = (x + 1)log(5)")
disp("log(2) = ")
disp(log(2))
disp("log(5) = ")
disp(log(5))
disp("Then 2x*0.6931 - x*1.6094 = 1.6094")
disp("Which means x = ")
syms x
disp(solve(2*x*0.6931 - x*1.6094 == 1.6094))
disp(" = ")
disp(-8047/1116)
```

```
3.2.21 Find value of x using Log10
Finding x from 2^(2x) = 5^(x + 1)
The first thing we should do is get x
on one side of the equation.
To do this we put log with base 10
on both sides and we get:
(2x)log(2) = (x + 1)log(5)
log(2) =
   0.693147180559945

log(5) =
   1.609437912434100

Then 2x*0.6931 - x*1.6094 = 1.6094
Which means x =
-8047/1116

 =
  -7.210573476702509
```

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```
disp("4.2.10 Finding root between 0 and 1")
disp("Find a root for function f(x) = x^3 + 2x^2 - 2 between x = 0,
 1")
disp("Our intial guess will be that the root is at the half")
disp("way mark, x = 0.5, and we will say that we have found")
disp("a root when the first 4 digits stops changing")

margin = 0.0001;
% Function
y = @(x) x^3 + 2*x^2 - 2;
% Derived function
dy = @(x) 3*x^2 + 4*x;
% Root will hold the value for the final root
root = 0.5;
% Oldroot keeps track of last loops root to
% see how much it has changed
oldRoot = 0;
% While the difference between the root and the
% oldRoot is larger than the margin we set
while abs(root - oldRoot) > margin
    oldRoot = root;
    % Formula for Newtons Raphsons method
    root = root - y(root) / dy(root);
end
disp("By using a while loop together with the formula")
disp("We can calculate the root to be: ")
disp(root)
```

```
4.2.10 Finding root between 0 and 1
Find a root for function f(x) = x^3 + 2x^2 - 2 between x = 0, 1
Our intial guess will be that the root is at the half
way mark, x = 0.5, and we will say that we have found
a root when the first 4 digits stops changing
By using a while loop together with the formula
We can calculate the root to be:
   0.839286755214164
```

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```
disp("4.8.5 Smallest number for sum of x and y")
disp("Consider two numbers, x, y")
disp("We need to find the smallest value")
disp("for the equation x^3 + y^2 = S")
disp("when x + y = 10")
disp("Lets substitute y in the function")
disp("for y = 10 - x")
disp("Then we get: ")
disp("S = x^3 + (10 - x)^2")
disp("To find when the function is smallest")
disp("We derive it to find a minimum")
disp("S' = 3x^2 + 2*(10 - x)*-1")
disp("S' = 3x^2 + 2x - 20")
disp("Now to find x when S' = 0")
disp("x = ")
Sprime = @(x) 3*x^2 + 2*x - 20;
disp(fzero(Sprime, 0))
disp("Thus y = ")
disp("")
disp(10 - fzero(Sprime, 0))
disp("And our minimum for S is:")
x = fzero(Sprime, 0);
y = 10 - fzero(Sprime, 0);
disp(x^3 + (y)^2)
```

```
4.8.5 Smallest number for sum of x and y
Consider two numbers, x, y
We need to find the smallest value
for the equation x^3 + y^2 = S
when x + y = 10
Lets substitute y in the function
for y = 10 - x
Then we get:
S = x^3 + (10 - x)^2
To find when the function is smallest
We derive it to find a minimum
S' = 3x^2 + 2*(10 - x)*-1
S' = 3x^2 + 2x - 20
Now to find x when S' = 0
x =
   2.270083225302218

Thus y =
   7.729916774697782

And our minimum for S is:
  71.449982945903272
```

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```
disp("4.10.31 Finding simiular curve with Taylors formula")
disp("Finding 1/e for function f(x) = e^-x to 5 decimal places")
disp("The first thing to notice is that the function")
disp("cycles between -e^-x and e^-x when derived multiple times")
disp("This makes it easy to make a loop with.")
disp("Because a is 0, our series will look like this: ")
disp("e^-x = 1 - x + x^2/2! - x^3/3! + ... 0 (-1)^n(x^n)/n!")
disp("And if we want to find 1/e, all we have to do is replace")
disp("x with 1 and we get our answer: ")

format long
answer = 1;

for i = 1:10
    answer = answer + ((-1)^i) / factorial(i);
end
disp(answer)
```

```
4.10.31 Finding simiular curve with Taylors formula
Finding 1/e for function f(x) = e^-x to 5 decimal places
The first thing to notice is that the function
cycles between -e^-x and e^-x when derived multiple times
This makes it easy to make a loop with.
Because a is 0, our series will look like this:
e^-x = 1 - x + x^2/2! - x^3/3! + ... 0 (-1)^n(x^n)/n!
And if we want to find 1/e, all we have to do is replace
x with 1 and we get our answer:
   0.367879464285714
```

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