Numerical stability and fast-math

Speeding up LHCb software through compilation optimization

Oscar Buon

July 7, 2023

Supervisor: Sebastien Ponce

Table of contents

Floating-point arithmetic

$$
\begin{aligned}
(0.75)_{10} &= (+7.5 \times 10^{-1})_{10} \\
&= (+11 \times 2^{-2})_2 \\
&= (\underbrace{+}_{sign}\ \underbrace{1.1}_{mantissa} \times 2^{\overbrace{-1}^{exponent}})_2 = (0.11)_2
\end{aligned}
$$

Non-nominal case

$$(0.8)_{10} \approx (1.10011001100110011001101 \times 2^{-1})_2$$

The representation is an approximation, because the number of bits is limited.

IEEE 754

- 32 bits ($\sim$ 7.2 digits) and 64 bits ($\sim$ 15.9 digits) formats ;
- Special rules:
  - Subnormal numbers (numbers very close to 0) ;
  - Special values: -Inf, +Inf, NaN ;
  - Rounding rules ;
  - Exception handling ;
- We should always consider floats as approximations !

Floats equality

- Never check for float equality.
- GCC flag `-Wfloat-equal`. Ex. in LHCb:
  - `0 == ip`
  - `0.5 == ip`
  - `lhs.m_energy == rhs.m_energy`

Exception cases

```
if (f(x) != 0.)
    return 5./f(x);
else
    ...
```

- Maybe the two `f(x)` will not be the sames.
- Due to different optimizations.

```
float y = f(x);
if (y != 0.)
    return 5./y;
else
    ...
```

- Maybe `f(x)` will still be calculated twice.
- If `f(x)` is close to 0 then `5./y` could be +Inf.

A good way is using `isfinite` after the computation.

```
float result = 5./f(x);
if (std::isfinite(result))
    return result;
else
    ...
```

There are also:

- The trapping system (registering a handler with *C* function `signal(handler)`) ;
- The signaling system via `std::numeric_limits<T>::signaling_NaN` ;

but they are old methods and are not recommended.

1. Introduction
   - Floating-point arithmetic
   - IEEE 754
2. Correctness of some computations
   - Floats equality
   - Exception cases
3. Fast-math
   - Exception handling
   - Approximation
4. Results
   - Improvement
   - Differences in the counters

Principle

- Set of flags.
- Making mathematically valid optimizations but not respecting Standard:
  - $a + b - a = b$.
  - With $a = 10^6$ and $b = 10^{-6}$,
    Standard gives 0, fast-math gives 10^{-6}.
- Different results but not more wrong than without `fast-math`.

https://godbolt.org/z/841zx64bM

https://godbolt.org/z/6jjqWrc51

no-math-errno

- Do not set `errno` after calling math functions that are executed with a single instruction, e.g., `sqrt`.

no-signaling-nans

- Compile code assuming that IEEE signaling NaNs may not generate user-visible traps during floating-point operations.
- Enables optimizations that may change the number of exceptions visible with signaling NaNs.
- Enabled by default.

no-trapping-math

- Compile code assuming that floating-point operations cannot generate user-visible traps. These traps include division by zero, overflow, underflow, inexact result and invalid operation.
- Implies no-signaling-nans.

finite-math-only

- Allow optimizations for floating-point arithmetic that assume that arguments and results are not NaNs or +-Infs.
- Compiler can replace `isnan` by `false`. Actually it is an undefined behavior.
- Main source of issues from `fast-math`.

no-signed-zeros

- Allow optimizations for floating-point arithmetic that ignore the signedness of zero.
- Ex. `0.0+x` or `0.0*x`.

associative-math

- Allow re-association of operands in series of floating-point operations.
- Can optimize `2.0*x*3.0` in `6.0*x` $\Longrightarrow$ make some computations at compile time.
- May allow a better vectorization and use of *FMA*.
- Needs no-signed-zeros and no-trapping-math.

reciprocal-math

- Allow the reciprocal of a value to be used instead of dividing by the value if this enables optimizations.
- Ex. Replacing `x/y` by `x*(1/y)`.
- May decrease precision.

unsafe-math-optimizations

- Allow optimizations for floating-point arithmetic that
  1. assume that arguments and results are valid and
  2. may violate IEEE or ANSI standards.
- When used at link time, it may include libraries or startup files that change the default FPU control word or other similar optimizations.
- Can affect dynamically included libraries.
- Enables -fno-signed-zeros, -fno-trapping-math, -fassociative-math and -freciprocal-math.

Improvement

Test: `hlt2_pp_thor`

Optimization	Improvement	Confidence interval (2σ)
Fast-math[1]	5.06%	±0.98%
Associative-math only	4.73%	±1.51%
Fast-math[1] + LTO & PGO	11.02%	±0.98%

[1] without finite-math-only and unsafe-math-optimizations

VTune

Time used by CPU for computing floats:

FP Arithmetic:	10.9%
FP x87:	0.0%
FP Scalar:	6.7%
FP Vector:	4.2%

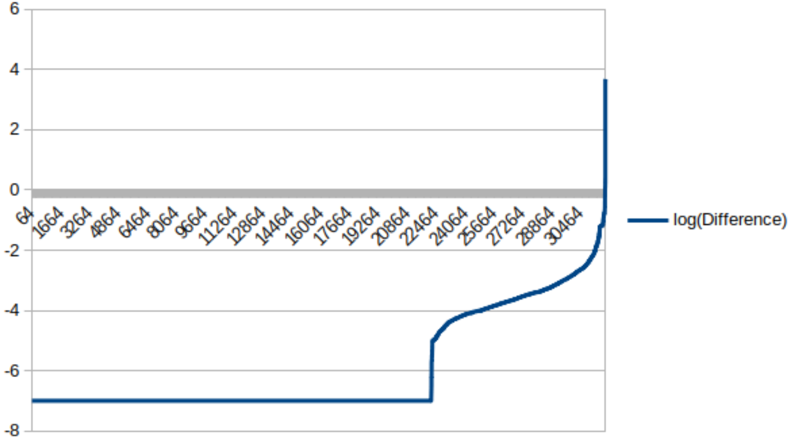
Figure: Reference

FP Arithmetic:	9.5%
FP x87:	0.0%
FP Scalar:	5.9%
FP Vector:	3.6%

Figure: Associative-math

Differences in the counters

Difference with the reference (all counters):



In a first time

- Enabling `-Wfloat-equal`.
- Enabling fast-math in one slot for instability checking.
    - Forget about `finite-math-only` and `unsafe-math-optimizations`.

In a second time

- Switch to fast-math for production.
  - Sometimes better precision (*FMA* and `double-precision-constant`).
  - Coding clear equations and letting compiler optimize them.
  - GPU already using similar principles.
  - Dropping features that are legacies.
  - Assuming floats are approximations.