資料結構和演算法

Data Structure and Algorithm

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Disclaimer

本文「資料結構與演算法」為「資料結構」和「演算法」筆記的總整理，內容主要參考 Introduction to Algorithms[2] 和洪捷先生的演算法參考書 [1]，以及 wjungle 網友在 PTT 論壇上提供的資料結構筆記 [3][4]。
本文作者為 TZU-CHUN HSU，本文及其 LaTeX 相關程式碼採用 **MIT 協議**，更多內容請訪問作者之 GitHub 分頁Oscarshu0719。

1 Summary

Trees				
Tree	Insert x	Delete x	Search x	Remark
BST	$O(\log n) \sim O(n)$			Create: $O(n \log n) \sim O(n^2)$
AVL tree	$O(\log_m n)$			$F_{n+2} - 1 \leq n \leq 2^h - 1$
B tree				$1 + 2\frac{\lceil \frac{m}{2} \rceil^{h-1} - 1}{\lceil \frac{m}{2} \rceil - 1} \leq n \leq 2\lceil \frac{m}{2} \rceil^{h-1} - 1$
RBT				$h \leq 2\log(n+1)$
Splay tree				Worst: $O(n)$

Disjoint set		
Combination	Union	Find
Arbitrary Union & Simple find	$O(1)$	$O(h)$ Worst: $O(n)$
Union-by-height & Simple find	$O(1)$	$O(\log n)$
Union-by-height & Find with path compression	$O(1)$	$O(\alpha(m,n)) = O(\log^* n)$ close to $O(1)$

Priority queues					
Operations	Max (Min)	Min-max & Deap & SMMH	Leftist	Binomial	Fibonacci
Insert x	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n), O(1)^*$	$O(1)^*$
Delete max	$O(\log n)$	$O(\log n)$			
Delete min	$O(n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)^*$
Delete x				$O(\log n)$	$O(\log n)^*$
Merge	$O(n)$		$O(\log n)$	$O(\log n)$	$O(1)^*$
Decrease key				$O(\log n)$	$O(1)^*$
Search x	$O(n)$				
Find max	$O(1)$	$O(1)$			
Find min		$O(1)$		$O(\log n)$	$O(1)$
Remark	Create: $O(n)$		Merge faster than Max (Min) heap.	Find min can be down to $O(1)$.	Decrease key is faster than binomial heap

Sorting algorithms					
Method	Time complexity			Space complexity	Stable
	Best	Worst	Average		
Insertion	$O(n)$	$O(n^2)$		$O(1)$	$\surd$
Selection	$O(n^2)$			$O(1)$	$\times$
Bubble	$O(n)$	$O(n^2)$		$O(1)$	$\surd$
Shell	$O(n^{1.5})$	$O(n^2)$		$O(1)$	$\times$
Quick	$O(n\log n)$	$O(n^2)$	$O(n\log n)$	$O(n\log n)\sim O(n)$	$\times$
Merge	$O(n\log n)$			$O(n)$	$\surd$
Heap	$O(n\log n)$			$O(1)$	$\times$
LSD Radix	$O(n\times k)$			$O(n+k)$	$\surd$
Bucket/MSD Radix	$O(n)$	$O(n^2)$	$O(n+k)$	$O(n\times k)$	$\surd$
Counting	$O(n+k)$				$\surd$

Graph algorithms		
Problem	Time complexity	Remark
Depth-First Search (DFS)	$O(\|V\|+\|E\|)$	
Kosaraju's	$O(\|V\|+\|E\|)$	
Kruskal's	$O(\|E\|\log\|V\|)$	
Prim's (Adjacency matrix)	$O(\|V\|^2)$	
Prim's (Heap, Adjacency lists)	$O(\|E\|\log\|V\|)$	
Prim's (Fibonacci heap, Adjacency lists)	$O(\|E\|+\|V\|\log\|V\|)$	
Sollin's (Borůvka's)	$O(\|E\|\log\|V\|)$	
Dijkstra's (Min-heap)	$\Theta((\|V\|+\|E\|)\log\|V\|)$	Greedy, no negative edges or cycles
Dijkstra's (Fibonacci-heap)	$\Theta(\|E\|+\|V\|\log\|V\|)$	
Bellman-Ford	$O(\|V\|\|E\|)$	DP, no negative cycles
Floyd-Warshall	$\Theta(\|V\|^3)$	DP, no negative cycles
Johnson's	$\Theta(\|V\|\|E\|+\|V\|^2\log\|V\|)$	No negative cycles
Ford-Fulkerson	$O(\|E\|\|f^*\|)$	Greedy，f^* 為最大流
Edmond-Karp	$O(\|V\|\|E\|^2)$	

Dynamic Programming algorithms			
Problem	Time complexity	Space complexity	Remark
Making change	$O(kn)$	$O(n)$	
Fractional Knapsack problem	$\Theta(n \log n)$	$O(n)$	Greedy
0/1 Knapsack problem (DP)	$O(n2^{\log W})$	$O(n2^{\log W})$	
0/1 Knapsack problem (Branch-and-Bound)	$O(2^n)$		
Longest Common Subsequence (LCS)	$O(mn)$	$O(mn)$	不必連續
Longest Increasing Subsequence (LIS)	$O(n^2)$	$O(n^2)$	
Longest Common Substring	$O(mn)$	$O(mn)$	必須連續
Minimum Edit Distance	$O(mn)$	$O(mn)$	
Matrix-chain Multiplication	$O(n^3)$	$O(n^2)$	
Traveling Salesperson problem	$\Theta(n^2 2^n)$	$O(n2^n)$	
Optimal Binary Search Tree (OBST)	$\Theta(n^3)$	$\Theta(n^2)$	

Computational Geometry algorithms	
Problem	Time complexity
平面上點的 rank	$\Theta(n \log n)$
Maximal points	$\Theta(n \log n)$
Closest pair	$O(n \log n)$
Farthest pair	$O(n \log n)$
Graham scan	$\Theta(n \log n)$

References

[1] 洪捷. *演算法—名校攻略秘笈*. 鼎茂圖書出版股份有限公司, 9 edition, 2017.

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