

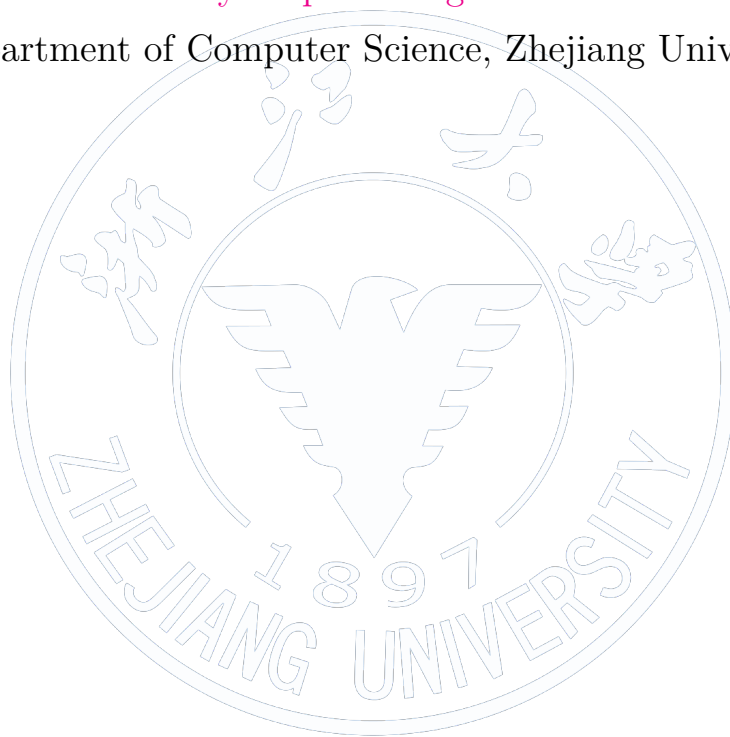
# 資料結構和演算法

## Data Structure and Algorithm

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# Disclaimer

本文「資料結構與演算法」為「資料結構」和「演算法」筆記的總整理，內容主要參考 Introduction to Algorithms[2] 和洪捷先生的演算法參考書 [1]，以及 wjungle 網友在 PTT 論壇上提供的資料結構筆記 [3][4]。

本文作者為 TZU-CHUN HSU，本文及其 L<sup>A</sup>T<sub>E</sub>X 相關程式碼採用 MIT 協議，更多內容請訪問作者之 GITHUB 分頁 [Oscarshu0719](#)。

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# 1 Summary

Trees				
Tree	Insert $x$	Delete $x$	Search $x$	Remark
BST	$O(\log n) \sim O(n)$			Create: $O(n \log n) \sim O(n^2)$
AVL tree	$O(\log_m n)$			$F_{h+2} - 1 \leq n \leq 2^h - 1$
B tree				$1 + 2^{\frac{\lceil \frac{m}{2} \rceil^{h-1} - 1}{\lceil \frac{m}{2} \rceil - 1}} \leq n \leq \frac{m^h - 1}{m - 1}$
RBT				$h \leq 2 \log(n + 1)$
Splay tree				Worst: $O(n)$ , Amortized: $O(\log n)$

Priority queues					
Operations	Max (Min)	Min-max & Deap & SMMH	Leftist	Binomial	Fibonacci
Insert $x$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n), O(1)^*$	$O(1)^*$
Delete max	$O(\log n)$	$O(\log n)$			
Delete min	$O(n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)^*$
Delete $x$				$O(\log n)$	$O(\log n)^*$
Merge	$O(n)$		$O(\log n)$	$O(\log n)$	$O(1)^*$
Decrease key				$O(\log n)$	$O(1)^*$
Search $x$	$O(n)$				
Find max	$O(1)$	$O(1)$			
Find min		$O(1)$		$O(\log n)$	$O(1)$
Remark			$shortest(\text{root}) \leq \log(n + 1) - 1$		

Sorting algorithms					
Method	Time complexity			Space complexity	Stable
	Best	Worst	Average		
Insertion	$O(n)$	$O(n^2)$		$O(1)$	✓
Selection	$O(n^2)$			$O(1)$	×
Bubble	$O(n)$	$O(n^2)$		$O(1)$	✓
Shell	$O(n^{1.5})$	$O(n^2)$		$O(1)$	×
Quick	$O(n \log n)$	$O(n^2)$	$O(n \log n)$	$O(\log n) \sim O(n)$	×
Merge	$O(n \log n)$			$O(n)$	✓
Heap	$O(n \log n)$			$O(1)$	×
LSD Radix	$O(n \times k)$			$O(n + k)$	✓
Bucket/MSD Radix	$O(n)$	$O(n^2)$	$O(n + k)$	$O(n \times k)$	✓
Counting	$O(n + k)$				✓

Dynamic Programming algorithms		
Problem	Time complexity	Space complexity
Making change	$O(kn)$	$O(n)$
Fractional Knapsack problem	$\Theta(n \log n)$	$O(n)$
0/1 Knapsack problem (DP)	$O(n2^{\log W})$	$O(n2^{\log W})$
0/1 Knapsack problem (Branch-and-Bound)	$O(2^n)$	
Longest Common Subsequence (LCS)	$O(mn)$	$O(mn)$
Longest Increasing Subsequence (LIS)	$O(n^2)$	$O(n^2)$
Longest Common Substring	$O(mn)$	$O(mn)$
Minimum Edit Distance	$O(mn)$	$O(mn)$
Matrix-chain Multiplication	$O(n^3)$	$O(n^2)$
Traveling Salesperson problem	$\Theta(n^2 2^n)$	$O(n2^n)$
Optimal Binary Search Tree (OBST)	$\Theta(n^3)$	$\Theta(n^2)$

Graph algorithms		
Problem	Time complexity	Remark
Depth-First Search (DFS)	$O( V  +  E )$	
Kosaraju's	$O( V  +  E )$	
Kruskal's	$O( E  \log  V )$	
Prim's (Adjacency matrix)	$O( V ^2)$	
Prim's (Adjacency list)	$O( V  E )$	
Prim's (Min-Heap, Adjacency list)	$O( E  \log  V )$	
Prim's (Fibonacci heap, Adjacency list)	$O( E  +  V  \log  V )$	
Sollin's (Borůvka's)	$O( E  \log  V )$	
Dijkstra's (Min-heap)	$\Theta(( E  +  V ) \log  V )$	Greedy, no negative edges or cycles
Dijkstra's (Fibonacci-heap)	$\Theta( E  +  V  \log  V )$	
Bellman-Ford	$O( V  E )$	DP
Floyd-Warshall	$\Theta( V ^3)$	DP, no negative cycles
Johnson's	$\Theta( V  E  +  V ^2 \log  V )$	No negative cycles
Ford-Fulkerson	$O( E  f^* )$	Greedy, $f^*$ 為最大流
Edmond-Karp	$O( V  E ^2)$	
Push-relabel	$O( V ^2 E )$	

## References

- [1] 洪捷. 演算法—名校攻略秘笈. 鼎茂圖書出版股份有限公司, 9 edition, 2017.
- [2] Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. *Introduction to Algorithms, Third Edition*. The MIT Press, 3 edition, 2009.
- [3] wjungle@ptt. 演算法 @tkb 筆記. <https://drive.google.com/file/d/0B8-2o6L73Q2VVmNWQk9DY3hsUm8/view?usp=sharing>, 2017.
- [4] wjungle@ptt. 資料結構 @tkb 筆記. <https://drive.google.com/file/d/0B8-2o6L73Q2VeFpGejlYRk1WeFk/view?usp=sharing>, 2017.

