① X VA $[0,1]$

$Y = X^n$

$F_Y(y) = P(Y \leq y) = P(X^n \leq y)$

$= P(X \leq \sqrt[n]{y})$

$= F_X(\sqrt[n]{y}) = \sqrt[n]{y}$

$$F_Y(y) \begin{cases} \sqrt[n]{y} & 0 \leq y \leq 1 \\ 0 & d/c \end{cases}$$

② $Y = X^n$

$F_Y(y) = P(Y \leq y) = P(X \leq \sqrt[n]{y})$

$= F_X(\sqrt[n]{y})$

$= \frac{d}{dx} F_x(\sqrt[n]{y}) = F_x(\sqrt[n]{y}) \left(\frac{\sqrt[n]{y^{1-n}}}{n} \right)$

$$f_Y(y) = \begin{cases} 0 & y < 0 \\ f_x(\sqrt[n]{y}) \left(\frac{\sqrt[n]{y^{1-n}}}{n} \right) & y \geq 0 \end{cases}$$

Escaneado con CamScanner

③

$$Var(\bar{x}) = Var\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$$

$$= \frac{1}{n^2} Var\left(\sum_{i=1}^{n} x_i\right)$$

$$= \frac{1}{n^2} \sum_{i=1}^{n} Var(x_i)$$

$$= \frac{1}{n^2} n\sigma^2$$

$$= \frac{\sigma^2}{n}$$

$$\mathbb{E}(s^2) = \mathbb{E}\left(\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)$$

$$\frac{1}{n-1}\mathbb{E}\left(\sum_{i=1}^{n}(x_i^2 - 2x_i\bar{x} + \bar{x}^2)\right)$$

$$\frac{1}{n-1}\mathbb{E}\left(\sum_{i=1}^{n}x_i^2 - 2\bar{x}\sum_{i=1}^{n}x_i + n\bar{x}^2\right)$$

$$\frac{1}{n-1}\mathbb{E}\left(\sum_{i=1}^{n}x_i^2 - (2n\bar{x}^2 + n\bar{x}^2)\right)$$

$$\frac{1}{n-1}\mathbb{E}\left(\sum_{i=1}^{n}x_i^2 - (n\bar{x}^2)\right)$$

$$\frac{1}{n-1}\mathbb{E}\left(\sum_{i=1}^{n}x_i^2\right) - \mathbb{E}(n\bar{x}^2)$$

$$\frac{1}{n-1}\sum_{i=1}^{n}\mathbb{E}(x_i^2) - n\mathbb{E}(\bar{x}^2)$$

$$\frac{1}{n-1}\sum_{i=1}^{n}\sigma^2 + \mu^2 - n\left(\frac{\sigma^2}{n} + \mu^2\right)$$

$$\frac{1}{n-1}\left(n\sigma^2 + \cancel{n\mu^2} - \sigma^2 - \cancel{n\mu^2}\right)$$

$$\frac{1}{n-1}\left(n\sigma^2 - \sigma^2\right) = \frac{1}{n-1}\left(\sigma^2(n-1)\right) = \sigma^2$$

$$Var(x_i) = \mathbb{E}(x_i^2) - \mathbb{E}(x_i))^2$$

$$Var(x_i) + \mathbb{E}(x_i))^2 = \mathbb{E}(x_i^2)$$

$$\sigma^2 + \mu^2 = \mathbb{E}(x_i^2)$$

$$Var(\bar{x}) = \mathbb{E}(\bar{x}^2) - \mathbb{E}(\bar{x}))^2$$

$$Var(\bar{x}) + \mathbb{E}(\bar{x}))^2 = \mathbb{E}(\bar{x}^2)$$

$$\frac{\sigma^2}{n} + \mu^2$$