Processos Estocásticos

Lista de Exercícios – Unidade 04 - Parte 1

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4.1 – Conceito de Variável Aleatória (V.A.) e

4.2 – Probabilidade associada à variável aleatória (V.A.)

Questão 1 – Um experimento (E) consiste em jogar uma moeda 4 vezes.

A) Especifique o espaço amostral (S), onde C corresponde a “cara"e K corresponde a “coroa".

- RESPOSTA

S = [

KKKK, KKKC, KKCK, KKCC,

KCKK, KCCK, KCKC, KCCC,

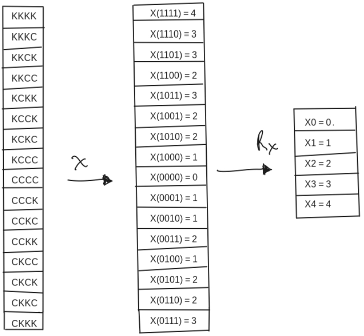
CCCC, CCCK, CCKC, CCKK,

CKCC,CKCK, CKKC, CKKK

]

B) Seja a Variável Aleatória (X) a ocorrencia de “coroas” nas 4 jogadas. Especifique os resultados de S, os valores de X correspondentes (contradomínio R_x) e a probabilidade de cada valor de X.

- RESPOSTA



$P(x=0) = \frac{1}{16}$

$P(x=1) = \frac{4}{16}$

$P(x=2) = \frac{6}{16}$

$P(x=3) = \frac{4}{16}$

$P(x=4) = \frac{1}{16}$

Questão 2 – Um experimento (E) consiste em jogar 3 dados (de 6 faces).

A) Especifique o espaço amostral (S). Especificando a ocorrencia das faces pelo número correspondente (1, 2, 3, 4, 5 ou 6).

- RESPOSTA

E = [

x(0,0,0), x(0,0,1), x(0,0,2), x(0,0,3), x(0,0,4), x(0,0,5), x(0,0,6),
x(0,1,0), x(0,1,1), x(0,1,2), x(0,1,3), x(0,1,4), x(0,1,5), x(0,1,6),
x(0,2,0), x(0,2,1), x(0,2,2), x(0,2,3), x(0,2,4), x(0,2,5), x(0,2,6),
x(0,3,0), x(0,3,1), x(0,3,2), x(0,3,3), x(0,3,4), x(0,3,5), x(0,3,6),
x(0,4,0), x(0,4,1), x(0,4,2), x(0,4,3), x(0,4,4), x(0,4,5), x(0,4,6),
x(0,5,0), x(0,5,1), x(0,5,2), x(0,5,3), x(0,5,4), x(0,5,5), x(0,5,6),
x(0,6,0), x(0,6,1), x(0,6,2), x(0,6,3), x(0,6,4), x(0,6,5), x(0,6,6),

x(1,0,0), x(1,0,1), x(1,0,2), x(1,0,3), x(1,0,4), x(1,0,5), x(1,0,6),
x(1,1,0), x(1,1,1), x(1,1,2), x(1,1,3), x(1,1,4), x(1,1,5), x(1,1,6),
x(1,2,0), x(1,2,1), x(1,2,2), x(1,2,3), x(1,2,4), x(1,2,5), x(1,2,6),
x(1,3,0), x(1,3,1), x(1,3,2), x(1,3,3), x(1,3,4), x(1,3,5), x(1,3,6),
x(1,4,0), x(1,4,1), x(1,4,2), x(1,4,3), x(1,4,4), x(1,4,5), x(1,4,6),
x(1,5,0), x(1,5,1), x(1,5,2), x(1,5,3), x(1,5,4), x(1,5,5), x(1,5,6),
x(1,6,0), x(1,6,1), x(1,6,2), x(1,6,3), x(1,6,4), x(1,6,5), x(1,6,6),

x(2,0,0), x(2,0,1), x(2,0,2), x(2,0,3), x(2,0,4), x(2,0,5), x(2,0,6),
x(2,1,0), x(2,1,1), x(2,1,2), x(2,1,3), x(2,1,4), x(2,1,5), x(2,1,6),
x(2,2,0), x(2,2,1), x(2,2,2), x(2,2,3), x(2,2,4), x(2,2,5), x(2,2,6),
x(2,3,0), x(2,3,1), x(2,3,2), x(2,3,3), x(2,3,4), x(2,3,5), x(2,3,6),
x(2,4,0), x(2,4,1), x(2,4,2), x(2,4,3), x(2,4,4), x(2,4,5), x(2,4,6),
x(2,5,0), x(2,5,1), x(2,5,2), x(2,5,3), x(2,5,4), x(2,5,5), x(2,5,6),
x(2,6,0), x(2,6,1), x(2,6,2), x(2,6,3), x(2,6,4), x(2,6,5), x(2,6,6),

x(3,0,0), x(3,0,1), x(3,0,2), x(3,0,3), x(3,0,4), x(3,0,5), x(3,0,6),
x(3,1,0), x(3,1,1), x(3,1,2), x(3,1,3), x(3,1,4), x(3,1,5), x(3,1,6),
x(3,2,0), x(3,2,1), x(3,2,2), x(3,2,3), x(3,2,4), x(3,2,5), x(3,2,6),
x(3,3,0), x(3,3,1), x(3,3,2), x(3,3,3), x(3,3,4), x(3,3,5), x(3,3,6),

x(3,4,0), x(3,4,1), x(3,4,2), x(3,4,3), x(3,4,4), x(3,4,5), x(3,4,6),
x(3,5,0), x(3,5,1), x(3,5,2), x(3,5,3), x(3,5,4), x(3,5,5), x(3,5,6),
x(3,6,0), x(3,6,1), x(3,6,2), x(3,6,3), x(3,6,4), x(3,6,5), x(3,6,6),

x(4,0,0), x(4,0,1), x(4,0,2), x(4,0,3), x(4,0,4), x(4,0,5), x(4,0,6),
x(4,1,0), x(4,1,1), x(4,1,2), x(4,1,3), x(4,1,4), x(4,1,5), x(4,1,6),
x(4,2,0), x(4,2,1), x(4,2,2), x(4,2,3), x(4,2,4), x(4,2,5), x(4,2,6),
x(4,3,0), x(4,3,1), x(4,3,2), x(4,3,3), x(4,3,4), x(4,3,5), x(4,3,6),
x(4,4,0), x(4,4,1), x(4,4,2), x(4,4,3), x(4,4,4), x(4,4,5), x(4,4,6),
x(4,5,0), x(4,5,1), x(4,5,2), x(4,5,3), x(4,5,4), x(4,5,5), x(4,5,6),
x(4,6,0), x(4,6,1), x(4,6,2), x(4,6,3), x(4,6,4), x(4,6,5), x(4,6,6),

x(5,0,0), x(5,0,1), x(5,0,2), x(5,0,3), x(5,0,4), x(5,0,5), x(5,0,6),
x(5,1,0), x(5,1,1), x(5,1,2), x(5,1,3), x(5,1,4), x(5,1,5), x(5,1,6),
x(5,2,0), x(5,2,1), x(5,2,2), x(5,2,3), x(5,2,4), x(5,2,5), x(5,2,6),
x(5,3,0), x(5,3,1), x(5,3,2), x(5,3,3), x(5,3,4), x(5,3,5), x(5,3,6),
x(5,4,0), x(5,4,1), x(5,4,2), x(5,4,3), x(5,4,4), x(5,4,5), x(5,4,6),
x(5,5,0), x(5,5,1), x(5,5,2), x(5,5,3), x(5,5,4), x(5,5,5), x(5,5,6),
x(5,6,0), x(5,6,1), x(5,6,2), x(5,6,3), x(5,6,4), x(5,6,5), x(5,6,6),

x(6,0,0), x(6,0,1), x(6,0,2), x(6,0,3), x(6,0,4), x(6,0,5), x(6,0,6),
x(6,1,0), x(6,1,1), x(6,1,2), x(6,1,3), x(6,1,4), x(6,1,5), x(6,1,6),
x(6,2,0), x(6,2,1), x(6,2,2), x(6,2,3), x(6,2,4), x(6,2,5), x(6,2,6),
x(6,3,0), x(6,3,1), x(6,3,2), x(6,3,3), x(6,3,4), x(6,3,5), x(6,3,6),
x(6,4,0), x(6,4,1), x(6,4,2), x(6,4,3), x(6,4,4), x(6,4,5), x(6,4,6),
x(6,5,0), x(6,5,1), x(6,5,2), x(6,5,3), x(6,5,4), x(6,5,5), x(6,5,6),
x(6,6,0), x(6,6,1), x(6,6,2), x(6,6,3), x(6,6,4), x(6,6,5), x(6,6,6),

]

B) Seja a Variável Aleatória (X) a soma dos valores das duas primeiras faces menos o valor da terceira. Especifique os resultados de S, os valores de X correspondentes (contradomínio R_x) e a probabilidade de cada valor de X.

- RESPOSTA

OBS: Resultados de S no Jupyter Notebook

Contradomínio R_x

Rx = [0, -1, -2, -3, -4, -5, -6, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]

Probabilidade de cada valor de X

$$X_0 = 0.00291545, X_1 = 0.00874636, X_2 = 0.01749271$$

$$X_3 = 0.02915452, X_4 = 0.04373178, X_5 = 0.06122449$$

$$X_6 = 0.08163265, X_7 = 0.09620991, X_8 = 0.10495627$$

$$X_9 = 0.10787172, X_{10} = 0.10495627, X_{11} = 0.09620991$$

$$X_{12} = 0.08163265, X_{13} = 0.06122449, X_{14} = 0.04373178$$

$$X_{15} = 0.02915452, X_{16} = 0.01749271, X_{17} = 0.00874636$$

$$X_{18} = 0.00291545$$

4.3 – Variáveis aleatórias discretas e contínuas e
4.4 – Funções de variáveis aleatórias $(V.A.)$ – fdp e FDP

Questão 3 – Com base na **Questão 1**. Determine:

A) $p(x_i) -$ fdp de X.

- RESPOSTA

x_i	0	1	2	3	4
P[X = x_i]	0.0625	0.25	0.375	0.25	0.0625

B) $F(x_i) -$ FDP de X.

- RESPOSTA

x_i	0	1	2	3	4
F[X = x_i]	0,0625	0,3125	0,6875	0,9375	1,00

$P[x \leq 0] = 0,0625$
$P[x \leq 1] = 0,3125$
$P[x \leq 2] = 0,6875$
$P[x \leq 3] = 0,9375$
$P[x \leq 3] = 1,00$

Questão 4 – Com base na **Questão 2**. Determine:

a) $p(x_i) -$ fdp de X.

- RESPOSTA

x_i	0	1	2	3	4	5	6	7	8	9
P[X = x_i]	0.00291545	0.00874636	0.01749271	0.02915452	0.04373178	0.06122449	0.08163265	0.09620991	0.10495627	0.10787172
x_i	10	11	12	13	14	15	16	17	18	
P[X = x_i]	0.10495627	0.09620991	0.08163265	0.06122449	0.04373178	0.02915452	0.01749271	0.00874636	0.00291545	

b) $F(x_i) -$FDP de X.

- RESPOSTA

x_i	0	1	2	3	4	5	6	7	8	9
P[X = x_i]	0,0029	0,0117	0,0292	0,0583	0,1020	0,1633	0,2449	0,3411	0,4461	0,5539
x_i	10	11	12	13	14	15	16	17	18	
P[X = x_i]	0,6589	0,7551	0,8367	0,8980	0,9417	0,9708	0,9883	0,9971	1,00	

$P[x \leq -6] = 0,0029$

$P[x \leq -5] = 0,0117$

$P[x \leq -4] = 0,0292$

$P[x \leq -3] = 0,0583$

$P[x \leq -2] = 0,1020$

$P[x \leq -1] = 0,1633$

$P[x \leq 0] = 0,2449$

$P[x \leq 1] = 0,3411$

$P[x \leq 2] = 0,4461$

$P[x \leq 3] = 0,5539$

$P[x \leq 4] = 0,6589$

$P[x \leq 5] = 0,7551$

$P[x \leq 6] = 0,8367$

$P[x \leq 7] = 0,8980$

$P[x \leq 8] = 0,9417$

$P[x \leq 9] = 0,9708$

$P[x \leq 10] = 0,9883$

$P[x \leq 11] = 0,9971$

$P[x \leq 12] = 1,00$

Questão 5 – Uma função distribuição de probabilidade acumulada FDP é definida da seguinte forma:

- $X < a \to F = 0;$
- $a \leq X \leq b \to F = \dfrac{x-a}{b-a};$
- $X > b \to F = 1;$

a) Calcule $f(x)$ - fdp de X.

- RESPOSTA

$$f(x) = \frac{dF(x)}{dx} = \frac{d(0)}{dx} + \frac{d(\frac{x-a}{b-a})}{dx} + \frac{d(1)}{dx}$$

$$f(x) = 0 + \frac{1}{b-a} + 0$$

$$f(x) = \frac{1}{b-a}$$

b) Calcule $P[1 < X \leq 3]$ para $a = 1$ e $b = 5$.

- RESPOSTA

$$f(x) = \frac{1}{b-a}$$

Assim temos:

$$f(x) = \frac{1}{5-1} = \frac{1}{4}$$

$$P[1 < X \leq 3] = \int_1^3 \frac{1}{4}\,\mathrm{d}x = \tfrac{1}{4}x\Big|_1^3 = \frac{1}{4} \cdot 3 - \frac{1}{4} \cdot 1$$

$$P[1 < X \leq 3] = \frac{1}{2}$$

c) Calcule $P[-1 < X \leq 2]$ para $a = 1$ e $b = 5$.

- RESPOSTA

$$P[1 < X \leq 3] = \int_{-1}^{1} 0\,\mathrm{d}x + \int_1^2 \frac{1}{4}\,\mathrm{d}x = \frac{1}{4}x\Big|_1^2 = \frac{1}{4} \cdot 2 - \frac{1}{4} \cdot 1$$

$$P[1 < X \leq 3] = \frac{1}{4}$$

d) Calcule $P[-\infty < X \leq 1,5]$ para $a = 1$ e $b = 5$.

- RESPOSTA

$$P[1 < X \leq 3] = \int_{-\infty}^{1} 0\,\mathrm{d}x + \int_1^{1,5} \frac{1}{4}\,\mathrm{d}x = \frac{1}{4}x\Big|_1^{1,5} = \frac{1}{4} \cdot 1,5 - \frac{1}{4} \cdot 1$$

$$P[1 < X \leq 3] = \frac{1}{8}$$

e) Calcule $P[0 < X \leq 6]$ para $a = 1$ e $b = 5$.

- RESPOSTA

$$P[1 < X \leq 3] = \int_0^1 0\,\mathrm{d}x + \int_1^5 \frac{1}{4}\,\mathrm{d}x + \int_5^6 0\,\mathrm{d}x = 0 + \frac{1}{4}x\Big|_1^5 + 0 = \frac{1}{4}\cdot 5 - \frac{1}{4}\cdot 1$$

$$P[1 < X \leq 3] = 1$$

Questão 6 – O tempo de transmissão X de mensagens em um sistema de comunicação obedece a lei de probabilidade exponencial com parâmetro λ, isto é $P[X > x] = e - \lambda x, x > 0$. Calcule, $T = 1/\lambda$.

a) Defina $F(x)$ - FDP de X

- RESPOSTA

$$P[X > x] = e^{-\lambda x}$$

$$F(x) = P[X \leq x] = 1 - P[X > x]$$

$$F(x) = 1 - e^{-\lambda x}$$

b) Calcule $f(x)$ - fdp de X.

- RESPOSTA

$$f(x) = \frac{dF(x)}{dx}$$

$$f(x) = \frac{d(1 - e^{-\lambda x})}{dx} = \lambda e^{-\lambda x}$$

c) Calcule $P[T < X \leq 2T]$ para $T = T = 1/\lambda$.

- RESPOSTA

$$P[T < X \le 2T] = \int_{\frac{1}{\lambda}}^{\frac{2}{\lambda}} \lambda e^{-\lambda x}\,\mathrm{d}x = -e^{-\lambda x}\Big|_{\frac{1}{\lambda}}^{\frac{2}{\lambda}} = -e^{-\frac{2}{\lambda}\lambda} + e^{-\frac{1}{\lambda}\lambda}$$

$$P[T < X \le 2T] = -e^{-2} + e^{-1} = 0,2325$$