Lista de Exemplos U04-Parte 01

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4.1 – Conceito de Variável Aleatória (V.A.) e

4.2 – Probabilidade associada à variável aleatória (V.A.)

Questão 1 – Um experimento (E) consiste em jogar uma moeda 4 vezes.

A) Especifique o espaço amostral (S), onde C corresponde a “cara"e K corresponde a “coroa".

- RESPOSTA

S = [

KKKK, KKKC, KKCK, KKCC,

KCKK, KCCK, KCKC, KCCC,

CCCC, CCCK, CCKC, CCKK,

CKCC,CKCK, CKKC, CKKK

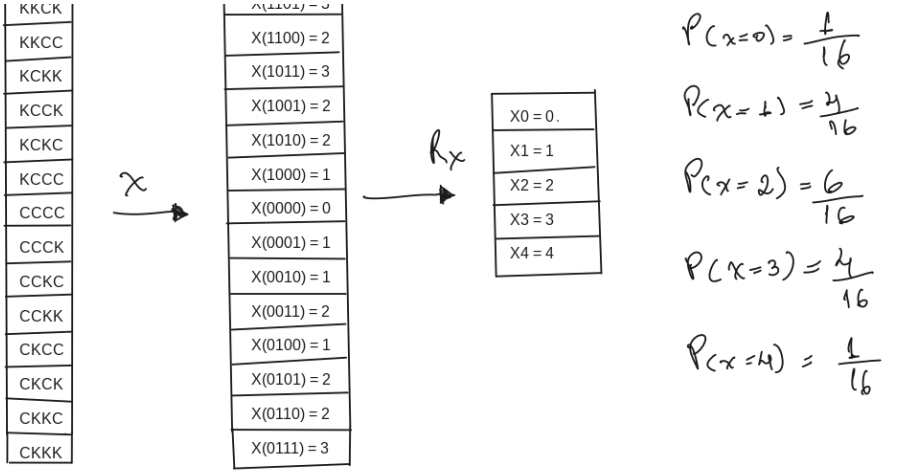
]

B) Seja a Variável Aleatória (X) a ocorrencia de “coroas” nas 4 jogadas. Especifique os resultados de S, os valores de X correspondentes (contradomínio R_x) e a probabilidade de cada valor de X.

- RESPOSTA

KKKK
KKKC

X(1111) = 4
X(1110) = 3



Questão 2 – Um experimento (E) consiste em jogar 3 dados (de 6 faces).

A) Especifique o espaço amostral (S). Especificando a ocorrencia das faces pelo número correspondente (1, 2, 3, 4, 5 ou 6).

- RESPOSTA

E = [

(000), (001), (002), (003), (004), (005), (006),
(010), (011), (012), (013), (014), (015), (016),
(020), (021), (022), (023), (024), (025), (026),
(030), (031), (032), (033), (034), (035), (036),
(040), (041), (042), (043), (044), (045), (046),
(050), (051), (052), (053), (054), (055), (056),
(060), (061), (062), (063), (064), (065), (066),

(100), (101), (102), (103), (104), (105), (106),
(110), (111), (112), (113), (114), (115), (116),
(120), (121), (122), (123), (124), (125), (126),
(130), (131), (132), (133), (134), (135), (136),
(140), (141), (142), (143), (144), (145), (146),
(150), (151), (152), (153), (154), (155), (156),
(160), (161), (162), (163), (164), (165), (166),

(200), (201), (202), (203), (204), (205), (206),
(210), (211), (212), (213), (214), (215), (216),

(220), (221), (222), (223), (224), (225), (226),
(230), (231), (232), (233), (234), (235), (236),
(240), (241), (242), (243), (244), (245), (246),

(240), (241), (242), (243), (244), (245), (246),
(250), (251), (252), (253), (254), (255), (256),
(260), (261), (262), (263), (264), (265), (266),

(300), (301), (302), (303), (304), (305), (306),
(310), (311), (312), (313), (314), (315), (316),
(320), (321), (322), (323), (324), (325), (326),
(330), (331), (332), (333), (334), (335), (336),
(340), (341), (342), (343), (344), (345), (346),
(350), (351), (352), (353), (354), (355), (356),
(360), (361), (362), (363), (364), (365), (366),

(400), (401), (402), (403), (404), (405), (406),
(410), (411), (412), (413), (414), (415), (416),
(420), (421), (422), (423), (424), (425), (426),
(430), (431), (432), (433), (434), (435), (436),
(440), (441), (442), (443), (444), (445), (446),
(450), (451), (452), (453), (454), (455), (456),
(460), (461), (462), (463), (464), (465), (466),

(500), (501), (502), (503), (504), (505), (506),
(510), (511), (512), (513), (514), (515), (516),
(520), (521), (522), (523), (524), (525), (526),
(530), (531), (532), (533), (534), (535), (536),
(540), (541), (542), (543), (544), (545), (546),
(550), (551), (552), (553), (554), (555), (556),
(560), (561), (562), (563), (564), (565), (566),

(600), (601), (602), (603), (604), (605), (606),
(610), (611), (612), (613), (614), (615), (616),
(620), (621), (622), (623), (624), (625), (626),
(630), (631), (632), (633), (634), (635), (636),
(640), (641), (642), (643), (644), (645), (646),
(650), (651), (652), (653), (654), (655), (656),
(660), (661), (662), (663), (664), (665), (666)

]

```
print("E = [")
for i in range(7):
  for j in range(7):
    for n in range(7):
      print(f"({i}{j}{n}), ", end=" ")
    print()
```

```
  print("\n")
print("]")
```

B) Seja a Variável Aleatória (X) a soma dos valores das duas primeiras faces menos o valor da terceira. Especifique os resultados de S, os valores de X correspondentes (contradomínio R_x) e a probabilidade de cada valor de X.

- RESPOSTA

Contradomínio R_x

Rx = [0, -1, -2, -3, -4, -5, -6, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]

Probabilidade de cada valor de X

$$X_0 = 0.00291545, X_1 = 0.00874636, X_2 = 0.01749271$$

$$X_3 = 0.02915452, X_4 = 0.04373178, X_5 = 0.06122449$$

$$X_6 = 0.08163265, X_7 = 0.09620991, X_8 = 0.10495627$$

$$X_9 = 0.10787172, X_{10} = 0.10495627, X_{11} = 0.09620991$$

$$X_{12} = 0.08163265, X_{13} = 0.06122449, X_{14} = 0.04373178$$

$$X_{15} = 0.02915452, X_{16} = 0.01749271, X_{17} = 0.00874636$$

$$X_{18} = 0.00291545$$

Resultado de S

```
s = []
for i in range(7):
  for j in range(7):
    for n in range(7):
      soma = (i+j)-n
      s.append(soma)
      print(f"x({i}{j}{n}) = {soma}, ", end=" ")
    print()
  print("\n")

unicos_lista = list(dict.fromkeys(s))
print("Rx", unicos_lista)
```

Encontrando o Contradomínio R_x e as Probabilidades de cada valor de X

```
import numpy as np

prob = np.array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
for i in s:
  if (i == -6):
    prob[0] += 1/len(s)
  elif (i == -5):
    prob[1] += 1/len(s)
  elif (i == -4):
    prob[2] += 1/len(s)
  elif (i == -3):
    prob[3] += 1/len(s)
  elif (i == -2):
    prob[4] += 1/len(s)
  elif (i == -1):
    prob[5] += 1/len(s)
  elif (i == 0):
    prob[6] += 1/len(s)
  elif (i == 1):
    prob[7] += 1/len(s)
  elif (i == 2):
    prob[8] += 1/len(s)
  elif (i == 3):
    prob[9] += 1/len(s)
  elif (i == 4):
    prob[10] += 1/len(s)
  elif (i == 5):
    prob[11] += 1/len(s)
  elif (i == 6):
    prob[12] += 1/len(s)
  elif (i == 7):
    prob[13] += 1/len(s)
  elif (i == 8):
    prob[14] += 1/len(s)
  elif (i == 9):
    prob[15] += 1/len(s)
  elif (i == 10):
    prob[16] += 1/len(s)
  elif (i == 11):
    prob[17] += 1/len(s)
  elif (i == 12):

    prob[18] += 1/len(s)

print(f"\nProbabilidade para os Xs =\n {prob}\n")
```

```
print(f"Soma das Probabilidades: {prob.sum()}")
```

```
Probabilidade para os Xs =
 [0.00291545 0.00874636 0.01749271 0.02915452 0.04373178 0.06122449
 0.08163265 0.09620991 0.10495627 0.10787172 0.10495627 0.09620991
 0.08163265 0.06122449 0.04373178 0.02915452 0.01749271 0.00874636
 0.00291545]

Soma das Probabilidades: 1.0000000000000004
```

4.3 – Variáveis aleatórias discretas e contínuas e 4.4 – Funções de variáveis aleatórias $(V.A.)$ – fdp e FDP

Questão 3 – Com base na **Questão 1**. Determine:

A) $p(x_i)$ – fdp de X.

- RESPOSTA

x_i	0	1	2	3	4
P[X = x_i]	6.25%	25%	37.5%	25%	6.25%

B) $F(x_i)$ – FDP de X.

- RESPOSTA

x_i	0	1	2	3	4
F[X = x_i]	6.25%	31.25%	68.75%	93.75%	100.0%

$$P[x \leq 0 = 6.25$$

$$P[x \leq 1 = 31.25$$

$$P[x \leq 2 = 68.75$$

$$P[x \leq 3 = 93.75$$

$$P[x \leq 3 = 100.0$$

```
prob_q1 = np.array([1/16, 1/4, 3/8, 1/4, 1/16], dtype="float64")
fdp_q1 = prob_q1*100.0

print(f"\np(xi)- fdp de X:")
for i in fdp_q1:
  print(f"{i}% ", end=" ")

FDP_q1 = np.zeros(5)
for i in range(0, 5):
  FDP_q1[i] = fdp_q1[:i+1].sum()

print(f"\n\nF(xi)-FDP de  X:")
for i in FDP_q1:
  print(f"{i}% ", end=" ")
```

```
p(xi)- fdp de X:
6.25%  25.0%  37.5%  25.0%  6.25%

F(xi)-FDP de  X:
6.25%  31.25%  68.75%  93.75%  100.0%
```

Questão 4 – Com base na **Questão 2**. Determine:

a) $p(x_i)$ – fdp de X.

- RESPOSTA

x_i	0	1	2	3	4	5	6	7	8	9
P[X = x_i]	0.29%	0.87%	1.74%	2.91%	4.37%	6.12%	8.16%	9.62%	10.49%	10.78%
x_i	10	11	12	13	14	15	16	17	18	
P[X = x_i]	10.49%	9,62%	8.16%	6.12%	4.37%	2.91%	1.74%	0.87%	0.29%	

```
fdp_q2 = prob*100

print(f"\n\np(xi)- fdp de  X:")
for i in range(19):
  print(f"{format(fdp_q2[i], '.2f')}% ", end=" ")
  if (i == 9):
    print()
```

```
p(xi)- fdp de  X:
0.29%  0.87%  1.75%  2.92%  4.37%  6.12%  8.16%  9.62%  10.50%  10.79%
10.50%  9.62%  8.16%  6.12%  4.37%  2.92%  1.75%  0.87%  0.29%
```

b) $F(x_i)-$FDP de X.

- RESPOSTA

x_i	0	1	2	3	4	5	6	7	8	9
P[X = x_i]	0.29%	1.17%	2.92%	5.83%	10.20%	16.33%	24.49%	34.11%	44.61%	55.39%
x_i	10	11	12	13	14	15	16	17	18	
P[X = x_i]	65.89%	75.51%	83.67%	89.80%	94.17%	97.08%	98.83%	99.71%	100.00%	

$$P[x \leq -6] = 0.29$$
$$P[x \leq -5] = 1.17$$
$$P[x \leq -4] = 2.92$$
$$P[x \leq -3] = 5.83$$
$$P[x \leq -2] = 10.20$$
$$P[x \leq -1] = 16.33$$
$$P[x \leq 0] = 24.49$$
$$P[x \leq 1] = 34.11$$
$$P[x \leq 2] = 44.61$$
$$P[x \leq 3] = 55.39$$
$$P[x \leq 4 = 65.89$$
$$P[x \leq 5 = 75.51$$
$$P[x \leq 6 = 83.67$$
$$P[x \leq 7 = 89.80$$
$$P[x \leq 8 = 94.17$$
$$P[x \leq 9 = 97.08$$
$$P[x \leq 10 = 98.83$$

$$P[x \leq 11 = 99.71$$
$$P[x \leq 12 = 100.00$$

```
FDP_q2 = np.zeros(19)
for i in range(0, 19):
  FDP_q2[i] = fdp_q2[:i+1].sum()

print("\nF(xi)- FDP de  X: ")
for i in range(19):
  print(f"{format(FDP_q2[i], '.2f')}% " , end=" ")
  if (i == 9):
    print()
```

```
--NORMAL--
```

```
F(xi)- FDP de  X:
0.29%  1.17%  2.92%  5.83%  10.20%  16.33%  24.49%  34.11%  44.61%  55.39%
65.89%  75.51%  83.67%  89.80%  94.17%  97.08%  98.83%  99.71%  100.00%
```

Questão 5 – Uma função distribuição de probabilidade acumulada FDP é definida da seguinte forma:

- $X < a \rightarrow F = 0;$
- $a \leq X \leq b \rightarrow F = \frac{x-a}{b-a};$
- $X > b \rightarrow F = 1;$

a) Calcule $f(x)$ - fdp de X.

- RESPOSTA

b) Calcule $P[1 < X \leq 3]$ para $a = 1$ e $b = 5$.

- RESPOSTA

c) Calcule $P[-1 < X \leq 2]$ para $a = 1$ e $b = 5$.

- RESPOSTA

d) Calcule $P[-\infty < X \leq 1,5]$ para $a = 1$ e $b = 5$.

- RESPOSTA

e) Calcule $P[0 < X \leq 6]$ para $a = 1$ e $b = 5$.

- RESPOSTA

Questão 6 – O tempo de transmissão X de mensagens em um sistema de comunicação obedece a lei de probabilidade exponencial com parâmetro λ, isto é $P[X > x] = e - \lambda x, x > 0$. Calcule, $T = 1/\lambda$.

a) Defina $F(x)$ - FDP de X

- RESPOSTA

b) Calcule $f(x)$ - fdp de X.

- RESPOSTA

c) Calcule $P[T < X \leq 2T]$ para $T = T = 1/\lambda$.

- RESPOSTA