Processos Estocásticos

Lista de Exercícios – Unidade 04 - Parte 1

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4.1 – Conceito de Variável Aleatória (V.A.) e

4.2 – Probabilidade associada à variável aleatória (V.A.)

Questão 1 – Um experimento (E) consiste em jogar uma moeda 4 vezes.

A) Especifique o espaço amostral (S), onde C corresponde a “cara"e K corresponde a “coroa".

- RESPOSTA

S = [

KKKK, KKKC, KKCK, KKCC,

KCKK, KCCK, KCKC, KCCC,

CCCC, CCCK, CCKC, CCKK,

CKCC,CKCK, CKKC, CKKK

]

B) Seja a Variável Aleatória (X) a ocorrencia de “coroas” nas 4 jogadas. Especifique os resultados de S, os valores de X correspondentes (contradomínio R_x) e a probabilidade de cada valor de X.

- RESPOSTA

Questão 2 – Um experimento (E) consiste em jogar 3 dados (de 6 faces).

A) Especifique o espaço amostral (S). Especificando a ocorrencia das faces pelo número correspondente (1, 2, 3, 4, 5 ou 6).

- RESPOSTA

E = [x(0,0,0), x(0,0,1), x(0,0,2), x(0,0,3), x(0,0,4), x(0,0,5), x(0,0,6),
x(0,1,0), x(0,1,1), x(0,1,2), x(0,1,3), x(0,1,4), x(0,1,5), x(0,1,6),
x(0,2,0), x(0,2,1), x(0,2,2), x(0,2,3), x(0,2,4), x(0,2,5), x(0,2,6),
x(0,3,0), x(0,3,1), x(0,3,2), x(0,3,3), x(0,3,4), x(0,3,5), x(0,3,6),
x(0,4,0), x(0,4,1), x(0,4,2), x(0,4,3), x(0,4,4), x(0,4,5), x(0,4,6),
x(0,5,0), x(0,5,1), x(0,5,2), x(0,5,3), x(0,5,4), x(0,5,5), x(0,5,6),
x(0,6,0), x(0,6,1), x(0,6,2), x(0,6,3), x(0,6,4), x(0,6,5), x(0,6,6),

x(1,0,0), x(1,0,1), x(1,0,2), x(1,0,3), x(1,0,4), x(1,0,5), x(1,0,6),
x(1,1,0), x(1,1,1), x(1,1,2), x(1,1,3), x(1,1,4), x(1,1,5), x(1,1,6),
x(1,2,0), x(1,2,1), x(1,2,2), x(1,2,3), x(1,2,4), x(1,2,5), x(1,2,6),
x(1,3,0), x(1,3,1), x(1,3,2), x(1,3,3), x(1,3,4), x(1,3,5), x(1,3,6),
x(1,4,0), x(1,4,1), x(1,4,2), x(1,4,3), x(1,4,4), x(1,4,5), x(1,4,6),
x(1,5,0), x(1,5,1), x(1,5,2), x(1,5,3), x(1,5,4), x(1,5,5), x(1,5,6),
x(1,6,0), x(1,6,1), x(1,6,2), x(1,6,3), x(1,6,4), x(1,6,5), x(1,6,6),

x(2,0,0), x(2,0,1), x(2,0,2), x(2,0,3), x(2,0,4), x(2,0,5), x(2,0,6),
x(2,1,0), x(2,1,1), x(2,1,2), x(2,1,3), x(2,1,4), x(2,1,5), x(2,1,6),
x(2,2,0), x(2,2,1), x(2,2,2), x(2,2,3), x(2,2,4), x(2,2,5), x(2,2,6),
x(2,3,0), x(2,3,1), x(2,3,2), x(2,3,3), x(2,3,4), x(2,3,5), x(2,3,6),
x(2,4,0), x(2,4,1), x(2,4,2), x(2,4,3), x(2,4,4), x(2,4,5), x(2,4,6),
x(2,5,0), x(2,5,1), x(2,5,2), x(2,5,3), x(2,5,4), x(2,5,5), x(2,5,6),
x(2,6,0), x(2,6,1), x(2,6,2), x(2,6,3), x(2,6,4), x(2,6,5), x(2,6,6),

x(3,0,0), x(3,0,1), x(3,0,2), x(3,0,3), x(3,0,4), x(3,0,5), x(3,0,6),
x(3,1,0), x(3,1,1), x(3,1,2), x(3,1,3), x(3,1,4), x(3,1,5), x(3,1,6),
x(3,2,0), x(3,2,1), x(3,2,2), x(3,2,3), x(3,2,4), x(3,2,5), x(3,2,6),
x(3,3,0), x(3,3,1), x(3,3,2), x(3,3,3), x(3,3,4), x(3,3,5), x(3,3,6),
x(3,4,0), x(3,4,1), x(3,4,2), x(3,4,3), x(3,4,4), x(3,4,5), x(3,4,6),
x(3,5,0), x(3,5,1), x(3,5,2), x(3,5,3), x(3,5,4), x(3,5,5), x(3,5,6),
x(3,6,0), x(3,6,1), x(3,6,2), x(3,6,3), x(3,6,4), x(3,6,5), x(3,6,6),

x(4,0,0), x(4,0,1), x(4,0,2), x(4,0,3), x(4,0,4), x(4,0,5), x(4,0,6),
x(4,1,0), x(4,1,1), x(4,1,2), x(4,1,3), x(4,1,4), x(4,1,5), x(4,1,6),
x(4,2,0), x(4,2,1), x(4,2,2), x(4,2,3), x(4,2,4), x(4,2,5), x(4,2,6),
x(4,3,0), x(4,3,1), x(4,3,2), x(4,3,3), x(4,3,4), x(4,3,5), x(4,3,6),
x(4,4,0), x(4,4,1), x(4,4,2), x(4,4,3), x(4,4,4), x(4,4,5), x(4,4,6),
x(4,5,0), x(4,5,1), x(4,5,2), x(4,5,3), x(4,5,4), x(4,5,5), x(4,5,6),
x(4,6,0), x(4,6,1), x(4,6,2), x(4,6,3), x(4,6,4), x(4,6,5), x(4,6,6),

x(5,0,0), x(5,0,1), x(5,0,2), x(5,0,3), x(5,0,4), x(5,0,5), x(5,0,6),
x(5,1,0), x(5,1,1), x(5,1,2), x(5,1,3), x(5,1,4), x(5,1,5), x(5,1,6),
x(5,2,0), x(5,2,1), x(5,2,2), x(5,2,3), x(5,2,4), x(5,2,5), x(5,2,6),
x(5,3,0), x(5,3,1), x(5,3,2), x(5,3,3), x(5,3,4), x(5,3,5), x(5,3,6),
x(5,4,0), x(5,4,1), x(5,4,2), x(5,4,3), x(5,4,4), x(5,4,5), x(5,4,6),
x(5,5,0), x(5,5,1), x(5,5,2), x(5,5,3), x(5,5,4), x(5,5,5), x(5,5,6),
x(5,6,0), x(5,6,1), x(5,6,2), x(5,6,3), x(5,6,4), x(5,6,5), x(5,6,6),

x(6,0,0), x(6,0,1), x(6,0,2), x(6,0,3), x(6,0,4), x(6,0,5), x(6,0,6),
x(6,1,0), x(6,1,1), x(6,1,2), x(6,1,3), x(6,1,4), x(6,1,5), x(6,1,6),
x(6,2,0), x(6,2,1), x(6,2,2), x(6,2,3), x(6,2,4), x(6,2,5), x(6,2,6),
x(6,3,0), x(6,3,1), x(6,3,2), x(6,3,3), x(6,3,4), x(6,3,5), x(6,3,6),
x(6,4,0), x(6,4,1), x(6,4,2), x(6,4,3), x(6,4,4), x(6,4,5), x(6,4,6),
x(6,5,0), x(6,5,1), x(6,5,2), x(6,5,3), x(6,5,4), x(6,5,5), x(6,5,6),
x(6,6,0), x(6,6,1), x(6,6,2), x(6,6,3), x(6,6,4), x(6,6,5), x(6,6,6),

]

```
print("E = [")
for i in range(7):
  for j in range(7):
    for n in range(7):
      print(f" x({i},{j},{n}), ", end=" ")
    print()
  print("\n")
print("]")
```

```
E = [
 x(0,0,0),   x(0,0,1),   x(0,0,2),   x(0,0,3),   x(0,0,4),   x(0,0,5),
x(0,0,6),
 x(0,1,0),   x(0,1,1),   x(0,1,2),   x(0,1,3),   x(0,1,4),   x(0,1,5),
x(0,1,6),
 x(0,2,0),   x(0,2,1),   x(0,2,2),   x(0,2,3),   x(0,2,4),   x(0,2,5),
x(0,2,6),
 x(0,3,0),   x(0,3,1),   x(0,3,2),   x(0,3,3),   x(0,3,4),   x(0,3,5),
x(0,3,6),
 x(0,4,0),   x(0,4,1),   x(0,4,2),   x(0,4,3),   x(0,4,4),   x(0,4,5),
x(0,4,6),
 x(0,5,0),   x(0,5,1),   x(0,5,2),   x(0,5,3),   x(0,5,4),   x(0,5,5),
x(0,5,6),
 x(0,6,0),   x(0,6,1),   x(0,6,2),   x(0,6,3),   x(0,6,4),   x(0,6,5),
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x(0,6,6),

x(1,0,0), x(1,0,1), x(1,0,2), x(1,0,3), x(1,0,4), x(1,0,5), x(1,0,6),
x(1,1,0), x(1,1,1), x(1,1,2), x(1,1,3), x(1,1,4), x(1,1,5), x(1,1,6),
x(1,2,0), x(1,2,1), x(1,2,2), x(1,2,3), x(1,2,4), x(1,2,5), x(1,2,6),
x(1,3,0), x(1,3,1), x(1,3,2), x(1,3,3), x(1,3,4), x(1,3,5), x(1,3,6),
x(1,4,0), x(1,4,1), x(1,4,2), x(1,4,3), x(1,4,4), x(1,4,5), x(1,4,6),
x(1,5,0), x(1,5,1), x(1,5,2), x(1,5,3), x(1,5,4), x(1,5,5), x(1,5,6),
x(1,6,0), x(1,6,1), x(1,6,2), x(1,6,3), x(1,6,4), x(1,6,5), x(1,6,6),

x(2,0,0), x(2,0,1), x(2,0,2), x(2,0,3), x(2,0,4), x(2,0,5), x(2,0,6),
x(2,1,0), x(2,1,1), x(2,1,2), x(2,1,3), x(2,1,4), x(2,1,5), x(2,1,6),
x(2,2,0), x(2,2,1), x(2,2,2), x(2,2,3), x(2,2,4), x(2,2,5), x(2,2,6),
x(2,3,0), x(2,3,1), x(2,3,2), x(2,3,3), x(2,3,4), x(2,3,5), x(2,3,6),
x(2,4,0), x(2,4,1), x(2,4,2), x(2,4,3), x(2,4,4), x(2,4,5), x(2,4,6),
x(2,5,0), x(2,5,1), x(2,5,2), x(2,5,3), x(2,5,4), x(2,5,5), x(2,5,6),
x(2,6,0), x(2,6,1), x(2,6,2), x(2,6,3), x(2,6,4), x(2,6,5), x(2,6,6),

x(3,0,0), x(3,0,1), x(3,0,2), x(3,0,3), x(3,0,4), x(3,0,5), x(3,0,6),
x(3,1,0), x(3,1,1), x(3,1,2), x(3,1,3), x(3,1,4), x(3,1,5), x(3,1,6),
x(3,2,0), x(3,2,1), x(3,2,2), x(3,2,3), x(3,2,4), x(3,2,5), x(3,2,6),
x(3,3,0), x(3,3,1), x(3,3,2), x(3,3,3), x(3,3,4), x(3,3,5), x(3,3,6),
x(3,4,0), x(3,4,1), x(3,4,2), x(3,4,3), x(3,4,4), x(3,4,5), x(3,4,6),
x(3,5,0), x(3,5,1), x(3,5,2), x(3,5,3), x(3,5,4), x(3,5,5), x(3,5,6),
x(3,6,0), x(3,6,1), x(3,6,2), x(3,6,3), x(3,6,4), x(3,6,5), x(3,6,6),

```
 x(4,0,0),   x(4,0,1),   x(4,0,2),   x(4,0,3),   x(4,0,4),   x(4,0,5),
x(4,0,6),
 x(4,1,0),   x(4,1,1),   x(4,1,2),   x(4,1,3),   x(4,1,4),   x(4,1,5),
x(4,1,6),
 x(4,2,0),   x(4,2,1),   x(4,2,2),   x(4,2,3),   x(4,2,4),   x(4,2,5),
x(4,2,6),
 x(4,3,0),   x(4,3,1),   x(4,3,2),   x(4,3,3),   x(4,3,4),   x(4,3,5),
x(4,3,6),
 x(4,4,0),   x(4,4,1),   x(4,4,2),   x(4,4,3),   x(4,4,4),   x(4,4,5),
x(4,4,6),
 x(4,5,0),   x(4,5,1),   x(4,5,2),   x(4,5,3),   x(4,5,4),   x(4,5,5),
x(4,5,6),
 x(4,6,0),   x(4,6,1),   x(4,6,2),   x(4,6,3),   x(4,6,4),   x(4,6,5),
x(4,6,6),


 x(5,0,0),   x(5,0,1),   x(5,0,2),   x(5,0,3),   x(5,0,4),   x(5,0,5),
x(5,0,6),
 x(5,1,0),   x(5,1,1),   x(5,1,2),   x(5,1,3),   x(5,1,4),   x(5,1,5),
x(5,1,6),
 x(5,2,0),   x(5,2,1),   x(5,2,2),   x(5,2,3),   x(5,2,4),   x(5,2,5),
x(5,2,6),
 x(5,3,0),   x(5,3,1),   x(5,3,2),   x(5,3,3),   x(5,3,4),   x(5,3,5),
x(5,3,6),
 x(5,4,0),   x(5,4,1),   x(5,4,2),   x(5,4,3),   x(5,4,4),   x(5,4,5),
x(5,4,6),
 x(5,5,0),   x(5,5,1),   x(5,5,2),   x(5,5,3),   x(5,5,4),   x(5,5,5),
x(5,5,6),
 x(5,6,0),   x(5,6,1),   x(5,6,2),   x(5,6,3),   x(5,6,4),   x(5,6,5),
x(5,6,6),


 x(6,0,0),   x(6,0,1),   x(6,0,2),   x(6,0,3),   x(6,0,4),   x(6,0,5),
x(6,0,6),
 x(6,1,0),   x(6,1,1),   x(6,1,2),   x(6,1,3),   x(6,1,4),   x(6,1,5),
x(6,1,6),
 x(6,2,0),   x(6,2,1),   x(6,2,2),   x(6,2,3),   x(6,2,4),   x(6,2,5),
x(6,2,6),
 x(6,3,0),   x(6,3,1),   x(6,3,2),   x(6,3,3),   x(6,3,4),   x(6,3,5),
x(6,3,6),
 x(6,4,0),   x(6,4,1),   x(6,4,2),   x(6,4,3),   x(6,4,4),   x(6,4,5),
x(6,4,6),
 x(6,5,0),   x(6,5,1),   x(6,5,2),   x(6,5,3),   x(6,5,4),   x(6,5,5),
x(6,5,6),
 x(6,6,0),   x(6,6,1),   x(6,6,2),   x(6,6,3),   x(6,6,4),   x(6,6,5),
x(6,6,6),
```

]

B) Seja a Variável Aleatória (X) a soma dos valores das duas primeiras faces menos o valor da terceira. Especifique os resultados de S, os valores de X correspondentes (contradomínio R_x) e a probabilidade de cada valor de X.

- RESPOSTA

Contradomínio R_x

Rx = [0, -1, -2, -3, -4, -5, -6, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]

Probabilidade de cada valor de X

$X_0 = 0.00291545, X_1 = 0.00874636, X_2 = 0.01749271$

$X_3 = 0.02915452, X_4 = 0.04373178, X_5 = 0.06122449$

$X_6 = 0.08163265, X_7 = 0.09620991, X_8 = 0.10495627$

$X_9 = 0.10787172, X_{10} = 0.10495627, X_{11} = 0.09620991$

$X_{12} = 0.08163265, X_{13} = 0.06122449, X_{14} = 0.04373178$

$X_{15} = 0.02915452, X_{16} = 0.01749271, X_{17} = 0.00874636$

$X_{18} = 0.00291545$

Resultado de S

```
s = []
for i in range(7):
  for j in range(7):
    for n in range(7):
      soma = (i+j)-n
      s.append(soma)
      print(f"x({i}{j}{n}) = {soma}, ", end=" ")
    print()
  print("\n")
```

```
unicos_lista = list(dict.fromkeys(s))
#print("Rx", unicos_lista)

x(000) = 0,  x(001) = -1,  x(002) = -2,  x(003) = -3,  x(004) = -4,
x(005) = -5,  x(006) = -6,
x(010) = 1,  x(011) = 0,  x(012) = -1,  x(013) = -2,  x(014) = -3,
x(015) = -4,  x(016) = -5,
x(020) = 2,  x(021) = 1,  x(022) = 0,  x(023) = -1,  x(024) = -2,
x(025) = -3,  x(026) = -4,
x(030) = 3,  x(031) = 2,  x(032) = 1,  x(033) = 0,  x(034) = -1,
x(035) = -2,  x(036) = -3,
x(040) = 4,  x(041) = 3,  x(042) = 2,  x(043) = 1,  x(044) = 0,
x(045) = -1,  x(046) = -2,
x(050) = 5,  x(051) = 4,  x(052) = 3,  x(053) = 2,  x(054) = 1,
x(055) = 0,  x(056) = -1,
x(060) = 6,  x(061) = 5,  x(062) = 4,  x(063) = 3,  x(064) = 2,
x(065) = 1,  x(066) = 0,


x(100) = 1,  x(101) = 0,  x(102) = -1,  x(103) = -2,  x(104) = -3,
x(105) = -4,  x(106) = -5,
x(110) = 2,  x(111) = 1,  x(112) = 0,  x(113) = -1,  x(114) = -2,
x(115) = -3,  x(116) = -4,
x(120) = 3,  x(121) = 2,  x(122) = 1,  x(123) = 0,  x(124) = -1,
x(125) = -2,  x(126) = -3,
x(130) = 4,  x(131) = 3,  x(132) = 2,  x(133) = 1,  x(134) = 0,
x(135) = -1,  x(136) = -2,
x(140) = 5,  x(141) = 4,  x(142) = 3,  x(143) = 2,  x(144) = 1,
x(145) = 0,  x(146) = -1,
x(150) = 6,  x(151) = 5,  x(152) = 4,  x(153) = 3,  x(154) = 2,
x(155) = 1,  x(156) = 0,
x(160) = 7,  x(161) = 6,  x(162) = 5,  x(163) = 4,  x(164) = 3,
x(165) = 2,  x(166) = 1,


x(200) = 2,  x(201) = 1,  x(202) = 0,  x(203) = -1,  x(204) = -2,
x(205) = -3,  x(206) = -4,
x(210) = 3,  x(211) = 2,  x(212) = 1,  x(213) = 0,  x(214) = -1,
x(215) = -2,  x(216) = -3,
x(220) = 4,  x(221) = 3,  x(222) = 2,  x(223) = 1,  x(224) = 0,
x(225) = -1,  x(226) = -2,
x(230) = 5,  x(231) = 4,  x(232) = 3,  x(233) = 2,  x(234) = 1,
x(235) = 0,  x(236) = -1,
x(240) = 6,  x(241) = 5,  x(242) = 4,  x(243) = 3,  x(244) = 2,
x(245) = 1,  x(246) = 0,
x(250) = 7,  x(251) = 6,  x(252) = 5,  x(253) = 4,  x(254) = 3,
x(255) = 2,  x(256) = 1,
x(260) = 8,  x(261) = 7,  x(262) = 6,  x(263) = 5,  x(264) = 4,
x(265) = 3,  x(266) = 2,
```

x(300) = 3, x(301) = 2, x(302) = 1, x(303) = 0, x(304) = -1,
x(305) = -2, x(306) = -3,
x(310) = 4, x(311) = 3, x(312) = 2, x(313) = 1, x(314) = 0,
x(315) = -1, x(316) = -2,
x(320) = 5, x(321) = 4, x(322) = 3, x(323) = 2, x(324) = 1,
x(325) = 0, x(326) = -1,
x(330) = 6, x(331) = 5, x(332) = 4, x(333) = 3, x(334) = 2,
x(335) = 1, x(336) = 0,
x(340) = 7, x(341) = 6, x(342) = 5, x(343) = 4, x(344) = 3,
x(345) = 2, x(346) = 1,
x(350) = 8, x(351) = 7, x(352) = 6, x(353) = 5, x(354) = 4,
x(355) = 3, x(356) = 2,
x(360) = 9, x(361) = 8, x(362) = 7, x(363) = 6, x(364) = 5,
x(365) = 4, x(366) = 3,

x(400) = 4, x(401) = 3, x(402) = 2, x(403) = 1, x(404) = 0,
x(405) = -1, x(406) = -2,
x(410) = 5, x(411) = 4, x(412) = 3, x(413) = 2, x(414) = 1,
x(415) = 0, x(416) = -1,
x(420) = 6, x(421) = 5, x(422) = 4, x(423) = 3, x(424) = 2,
x(425) = 1, x(426) = 0,
x(430) = 7, x(431) = 6, x(432) = 5, x(433) = 4, x(434) = 3,
x(435) = 2, x(436) = 1,
x(440) = 8, x(441) = 7, x(442) = 6, x(443) = 5, x(444) = 4,
x(445) = 3, x(446) = 2,
x(450) = 9, x(451) = 8, x(452) = 7, x(453) = 6, x(454) = 5,
x(455) = 4, x(456) = 3,
x(460) = 10, x(461) = 9, x(462) = 8, x(463) = 7, x(464) = 6,
x(465) = 5, x(466) = 4,

x(500) = 5, x(501) = 4, x(502) = 3, x(503) = 2, x(504) = 1,
x(505) = 0, x(506) = -1,
x(510) = 6, x(511) = 5, x(512) = 4, x(513) = 3, x(514) = 2,
x(515) = 1, x(516) = 0,
x(520) = 7, x(521) = 6, x(522) = 5, x(523) = 4, x(524) = 3,
x(525) = 2, x(526) = 1,
x(530) = 8, x(531) = 7, x(532) = 6, x(533) = 5, x(534) = 4,
x(535) = 3, x(536) = 2,
x(540) = 9, x(541) = 8, x(542) = 7, x(543) = 6, x(544) = 5,
x(545) = 4, x(546) = 3,
x(550) = 10, x(551) = 9, x(552) = 8, x(553) = 7, x(554) = 6,
x(555) = 5, x(556) = 4,
x(560) = 11, x(561) = 10, x(562) = 9, x(563) = 8, x(564) = 7,
x(565) = 6, x(566) = 5,

```
x(600) = 6,  x(601) = 5,  x(602) = 4,  x(603) = 3,  x(604) = 2,
x(605) = 1,  x(606) = 0,
x(610) = 7,  x(611) = 6,  x(612) = 5,  x(613) = 4,  x(614) = 3,
x(615) = 2,  x(616) = 1,
x(620) = 8,  x(621) = 7,  x(622) = 6,  x(623) = 5,  x(624) = 4,
x(625) = 3,  x(626) = 2,
x(630) = 9,  x(631) = 8,  x(632) = 7,  x(633) = 6,  x(634) = 5,
x(635) = 4,  x(636) = 3,
x(640) = 10,  x(641) = 9,  x(642) = 8,  x(643) = 7,  x(644) = 6,
x(645) = 5,  x(646) = 4,
x(650) = 11,  x(651) = 10,  x(652) = 9,  x(653) = 8,  x(654) = 7,
x(655) = 6,  x(656) = 5,
x(660) = 12,  x(661) = 11,  x(662) = 10,  x(663) = 9,  x(664) = 8,
x(665) = 7,  x(666) = 6,
```

Encontrando o Contradomínio R_x e as Probabilidades de cada valor de X

```
import numpy as np

prob = np.array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0], dtype="float64")
for i in s:
  if (i == -6):
    prob[0] += 1/len(s)
  elif (i == -5):
    prob[1] += 1/len(s)
  elif (i == -4):
    prob[2] += 1/len(s)
  elif (i == -3):
    prob[3] += 1/len(s)
  elif (i == -2):
    prob[4] += 1/len(s)
  elif (i == -1):
    prob[5] += 1/len(s)
  elif (i == 0):
    prob[6] += 1/len(s)
  elif (i == 1):
    prob[7] += 1/len(s)
  elif (i == 2):
    prob[8] += 1/len(s)
  elif (i == 3):
    prob[9] += 1/len(s)
  elif (i == 4):
    prob[10] += 1/len(s)
  elif (i == 5):
    prob[11] += 1/len(s)
  elif (i == 6):
    prob[12] += 1/len(s)
```

```
  elif (i == 7):
    prob[13] += 1/len(s)
  elif (i == 8):
    prob[14] += 1/len(s)
  elif (i == 9):
    prob[15] += 1/len(s)
  elif (i == 10):
    prob[16] += 1/len(s)
  elif (i == 11):
    prob[17] += 1/len(s)
  elif (i == 12):
    prob[18] += 1/len(s)

print(f"\nProbabilidade para os Xs =\n {prob}\n")
print(f"Soma das Probabilidades: {prob.sum()}")
```

```
Probabilidade para os Xs =
 [0.00291545 0.00874636 0.01749271 0.02915452 0.04373178 0.06122449
 0.08163265 0.09620991 0.10495627 0.10787172 0.10495627 0.09620991
 0.08163265 0.06122449 0.04373178 0.02915452 0.01749271 0.00874636
 0.00291545]

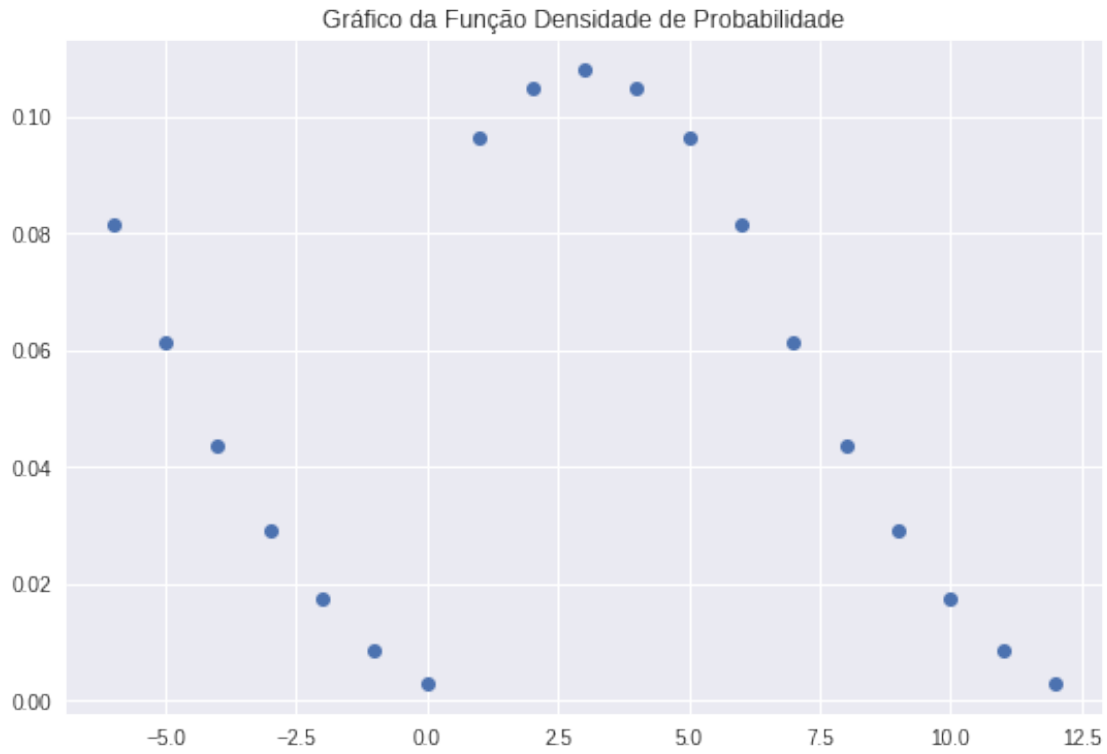
Soma das Probabilidades: 1.0000000000000004
```

```
# Importando a biblioteca Matplotlib
import matplotlib.pyplot as plt
plt.style.use('seaborn')

%matplotlib inline

# Gráfico da Função Densidade de Probabilidade

plt.figure(figsize=(9, 6))
plt.plot(unicos_lista, prob, "o")
plt.title("Gráfico da Função Densidade de Probabilidade")
plt.show()
```



4.3 – Variáveis aleatórias discretas e contínuas e

4.4 – Funções de variáveis aleatórias $(V.A.)$ – fdp e FDP

Questão 3 – Com base na Questão 1. Determine:

A) $p(x_i)$ – fdp de X.

- RESPOSTA

x_i
P[X = x_i]

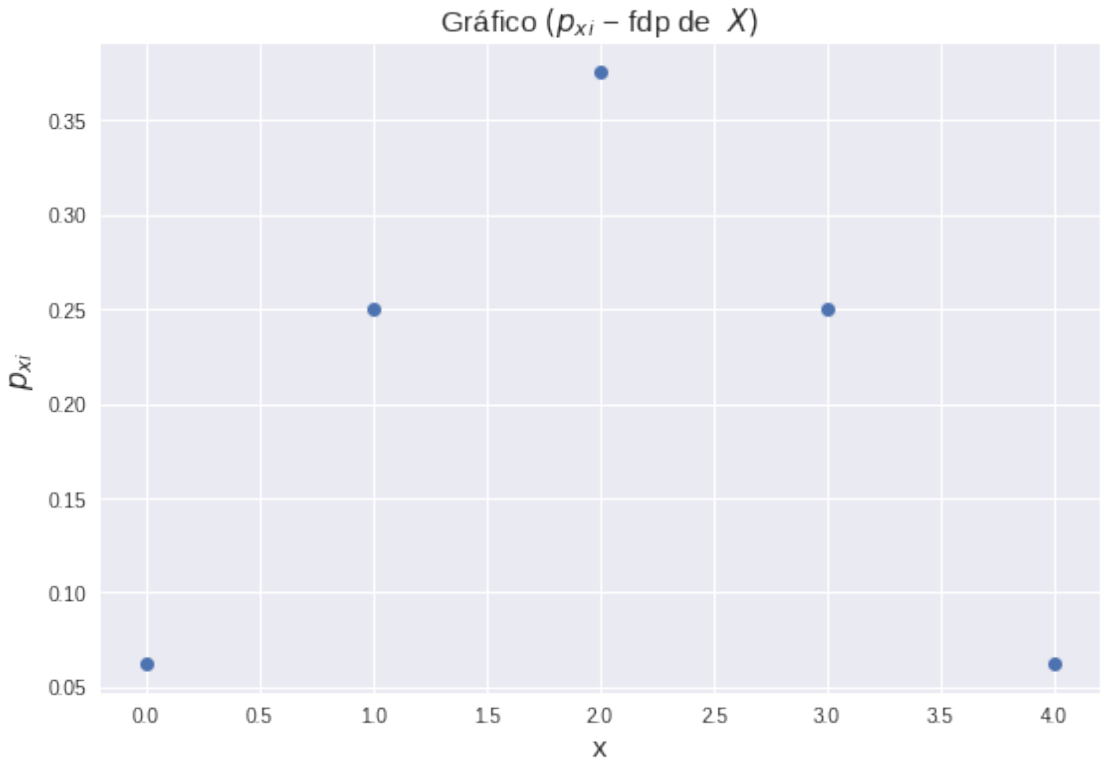
```
x_i = np.array([0, 1, 2, 3, 4])
prob_q1 = np.array([1/16, 1/4, 3/8, 1/4, 1/16], dtype="float64")
fdp_q1 = prob_q1

print(f"\np(xi)– fdp de X:")
for i in fdp_q1:
```

```
  print(f"{i}% ", end=" ")

# Gráfico da Função Densidade de Probabilidade
plt.figure(figsize=(9, 6))
plt.plot(x_i, fdp_q1, "o")
plt.title("Gráfico $( p_{xi}$ - fdp de  $X )$", fontsize=15)
plt.xlabel("x", fontsize=15)
plt.ylabel("$p_{xi}$", fontsize=15)
plt.show()


p(xi)- fdp de X:
0.0625%  0.25%  0.375%  0.25%  0.0625%
```



B) $F(x_i)$ – FDP de X.

- RESPOSTA

x_i |

F[X = x_i]

$ P[x \leq 0 = 6.25%] $

$ P[x \leq 1 = 31.25%] $

$ P[x \leq 2 = 68.75%] $

$ P[x \leq 3 = 93.75%] $

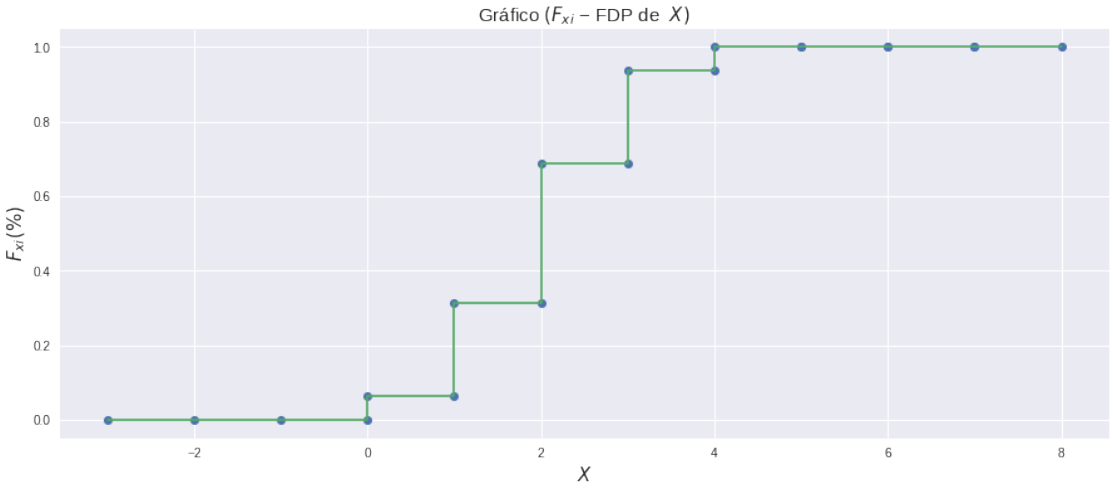
$ P[x \leq 3 = 100.0%] $

```
x_i2 = np.array([-3., -2., -1.,  0., 0.,  1., 1., 2., 2., 3., 3.,  4.,
4.,  5., 5., 6., 6., 7., 7., 8.,])
FDP_q1 = []
for i in range(4):
  FDP_q1.append(0.0)

for i in range(0, 8):
  FDP_q1.append(fdp_q1[:i+1].sum())
  FDP_q1.append(fdp_q1[:i+1].sum())

# Printa os valores de F(xi)
#print(f"\n\nF(xi)-FDP de  X:")
#for i in FDP_q1:
#  print(f"{i}% ", end=" ")
#print("\n")

# Gráfico da Função Densidade de Probabilidade
plt.figure(figsize=(15, 6))
plt.plot(x_i2, FDP_q1, "o")
plt.plot(x_i2, FDP_q1)
plt.title("Gráfico $( F_{xi}$ - FDP de  $X )$", fontsize=15)
plt.xlabel("$X$", fontsize=15)
plt.ylabel("$F_{xi} (\%)$", fontsize=15)
plt.show()
```



Questão 4 – Com base na Questão 2. Determine:

a) $p(x_i)$ – fdp de X.

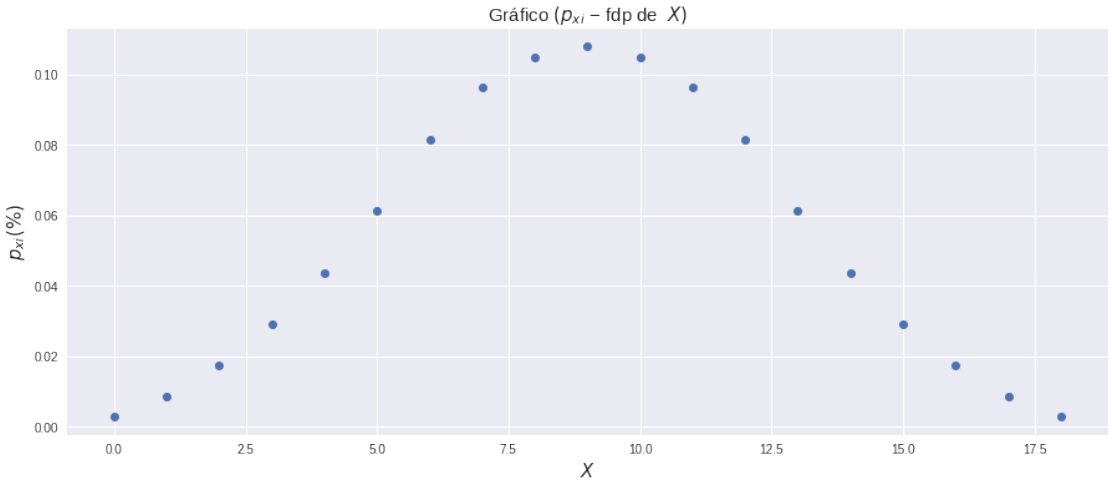
- RESPOSTA

x_i	0
P[X = x_i]	0.29%
x_i	10
P[X = x_i]	10.49%

```
q2_x_i = np.linspace(0, 18, 19)
fdp_q2 = prob #*100

#print(f"\n\np(xi)- fdp de  X:")
#for i in range(19):
#  print(f"{format(fdp_q2[i], '.2f')}% ", end=" ")
#  if (i == 9):
#    print()

# Gráfico da Função Densidade de Probabilidade
plt.figure(figsize=(15, 6))
plt.plot(q2_x_i, fdp_q2, "o")
plt.title("Gráfico $( p_{xi}$ - fdp de  $X )$", fontsize=15)
plt.xlabel("$X$", fontsize=15)
plt.ylabel("$p_{xi} (\%)$", fontsize=15)
plt.show()
```



b) $F(x_i)$ – FDP de X.

- RESPOSTA

x_i	0
P[X = x_i]	0.29%
x_i	10
P[X = x_i]	65.89%

$P[x \leq -6] = 0.29$

$P[x \leq -5] = 1.17$

$P[x \leq -4] = 2.92$

$P[x \leq -3] = 5.83$

$P[x \leq -2] = 10.20$

$P[x \leq -1] = 16.33$

$P[x \leq 0] = 24.49$

$P[x \leq 1] = 34.11$

$P[x \leq 2] = 44.61$

$P\left[x \leq 3\right]=55.39$

$P¿$

$P[x \leq 5 = 75.51\%] $

$P¿$

$P[x \leq 7 = 89.80\%] $

$P¿$

$P[x \leq 9 = 97.08\%] $

$P¿$

$P[x \leq 11 = 99.71\%] $

$P¿$

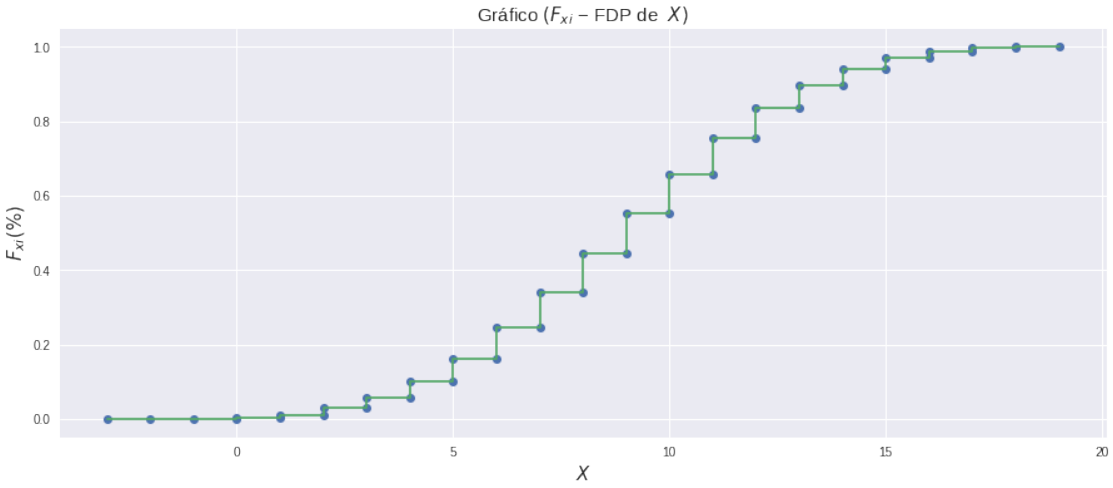
```
FDP_q2 = []  #np.zeros(19)
xi = np.array([-3., -2., -1.,  0., 0., 1., 1., 2., 2., 3., 3.,  4.,
4.,  5., 5., 6., 6., 7., 7., 8., 8.,
               9., 9., 10., 10., 11., 11., 12., 12., 13., 13., 14.,
14., 15., 15., 16., 16., 17., 17., 18., 18.,
               19., ])

for i in range(4):
  FDP_q2.append(0)

for i in range(0, 19):
  FDP_q2.append(fdp_q2[:i+1].sum())
  FDP_q2.append(fdp_q2[:i+1].sum())

#print("\nF(xi)- FDP de  X: ")
#for i in range(len(FDP_q2)):
#  print(f"{format(FDP_q2[i], '.2f')}% " , end=" ")
#  if (i == 10):
#    print()
#  if (i == 20):
#    print()
#print("\n")

# Gráfico da Função Densidade de Probabilidade
plt.figure(figsize=(15, 6))
plt.plot(xi, FDP_q2, "o")
plt.plot(xi, FDP_q2)
plt.title("Gráfico $( F_{xi}$ - FDP de  $X )$", fontsize=15)
plt.xlabel("$X$", fontsize=15)
plt.ylabel("$F_{xi} (\%)$", fontsize=15)
plt.show()
```



Questão 5 – Uma função distribuição de probabilidade acumulada FDP é definida da seguinte forma:

- $X < a \rightarrow F = 0;$
- $a \leq X \leq b \rightarrow F = \frac{x-a}{b-a};$
- $X > b \rightarrow F = 1;$

a) Calcule $f(x)$ - fdp de X.

- RESPOSTA

$$f(x) = \frac{d\,F(x)}{d\,x} = \frac{d(0)}{d\,x} + \frac{d\left(\frac{x-a}{b-a}\right)}{d\,x} + \frac{d(1)}{d\,x}$$

$$f(x) = 0 + \frac{1}{b-a} + 0$$

$$f(x) = \frac{1}{b-a}$$

b) Calcule $P[1 < X \leq 3]$ para $a = 1$ e $b = 5$.

- RESPOSTA

$$f(x)=\frac{1}{b-a}$$

\

Assim temos:

$$f(x)=\frac{1}{5-1}=\frac{1}{4}$$

\

\begin{equation} P[1 < X \leq 3] = \int_1^3 ! \dfrac{1}{4} , \mathrm{d}x = \tfrac{1}{4} x \Big|_1^3 = \dfrac{1}{4} \cdot 3 - \dfrac{1}{4}\cdot 1 \end{equation}

$$P[1<X\leq 3]=\frac{1}{2}$$

c) Calcule $P[-1<X\leq 2]$ para $a=1$ e $b=5$.

- RESPOSTA

\begin{equation} P[1 < X \leq 3] = \int_{-1}^1 ! 0 , \mathrm{d}x + \int_{1}^2 ! \dfrac{1}{4} , \mathrm{d}x = \dfrac{1}{4}x \Big|_1^2 = \dfrac{1}{4}\cdot2 - \dfrac{1}{4}\cdot1 \ end{equation}

$$P[1<X\leq 3]=\frac{1}{4}$$

d) Calcule $P[-\infty<X\leq 1,5]$ para $a=1$ e $b=5$.

- RESPOSTA

\begin{equation} P[1 < X \leq 3] = \int_{-\infty}^1 ! 0 , \mathrm{d}x + \int_{1}^{1,5} ! \ dfrac{1}{4} , \mathrm{d}x = \dfrac{1}{4}x \Big|_1^{1,5} = \dfrac{1}{4} \cdot {1,5} - \ dfrac{1}{4} \cdot 1 \end{equation}

$$P[1<X\leq 3]=\frac{1}{8}$$

e) Calcule $P[0<X\leq 6]$ para $a=1$ e $b=5$.

- RESPOSTA

\begin{equation} P[1 < X \leq 3] = \int_0^1 ! 0 , \mathrm{d}x + \int_{1}^5 ! \dfrac{1}{4} , \ mathrm{d}x +\int_5^6 ! 0 , \mathrm{d}x = 0 + \dfrac{1}{4}x \Big|_1^5 + 0 = \dfrac{1}{4} \ cdot 5 - \dfrac{1}{4} \cdot 1 \end{equation}

$$P[1<X\leq 3]=1$$

Questão 6 – O tempo de transmissão X de mensagens em um sistema de comunicação obedece a lei de probabilidade exponencial com parâmetro λ, isto é $P[X>x]=e-\lambda x, x>0$. Calcule, $T=1/\lambda$.

a) Defina $F(x)$ - FDP de X

- RESPOSTA

$$P[X>x]=e^{-\lambda x}$$

$$F(x)=P[X\leq x]=1-P[X>x]$$

$$F(x)=1-e^{-\lambda x}$$

---

b) Calcule $f(x)$ - fdp de X.

- RESPOSTA

$$f(x)=\frac{d\,F(x)}{d\,x}$$

$$f(x)=\frac{d\left(1-e^{-\lambda x}\right)}{d\,x}=\lambda e^{-\lambda x}$$

---

c) Calcule $P\left[T<X\leq 2T\right]$ para $T=T=1/\lambda$.

- RESPOSTA

\begin{equation} P[T < X \leq 2T] = \int_{\frac{1}{\lambda}}^{\frac{2}{\lambda}} ! \lambda e^{-\lambda x} , \mathrm{d}x = -e^{-\lambda x}\Big|_{\frac{1}{\lambda}}^{\frac{2}{\lambda}} = -e^{-\frac{2}{\lambda}\lambda} + e^{-\frac{1}{\lambda}\lambda} \end{equation}

$$P\left[T<X\leq 2T\right]=-e^{-2}+e^{-1}=0,2325$$