Lista de Exemplos U04-Parte 01

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4.1 – Conceito de Variável Aleatória (V.A.) e

4.2 – Probabilidade associada à variável aleatória (V.A.)

Questão 1 – Um experimento (E) consiste em jogar uma moeda 4 vezes.

A) Especifique o espaço amostral (S), onde C corresponde a “cara"e K corresponde a “coroa".

- RESPOSTA

S = [

KKKK, KKKC, KKCK, KKCC,

KCKK, KCCK, KCKC, KCCC,

CCCC, CCCK, CCKC, CCKK,

CKCC,CKCK, CKKC, CKKK

]

B) Seja a Variável Aleatória (X) a ocorrencia de “coroas” nas 4 jogadas. Especifique os resultados de S, os valores de X correspondentes (contradomínio R_x) e a probabilidade de cada valor de X.

- RESPOSTA

Questão 2 – Um experimento (E) consiste em jogar 3 dados (de 6 faces).

a) Especifique o espaço amostral (S). Especificando a ocorrencia das faces pelo número correspondente (1, 2, 3, 4, 5 ou 6).

- RESPOSTA

E = [

(000), (001), (002), (003), (004), (005), (006),
(010), (011), (012), (013), (014), (015), (016),
(020), (021), (022), (023), (024), (025), (026),
(030), (031), (032), (033), (034), (035), (036),
(040), (041), (042), (043), (044), (045), (046),
(050), (051), (052), (053), (054), (055), (056),
(060), (061), (062), (063), (064), (065), (066),

(100), (101), (102), (103), (104), (105), (106),
(110), (111), (112), (113), (114), (115), (116),
(120), (121), (122), (123), (124), (125), (126),
(130), (131), (132), (133), (134), (135), (136),
(140), (141), (142), (143), (144), (145), (146),
(150), (151), (152), (153), (154), (155), (156),
(160), (161), (162), (163), (164), (165), (166),

(200), (201), (202), (203), (204), (205), (206),
(210), (211), (212), (213), (214), (215), (216),
(220), (221), (222), (223), (224), (225), (226),
(230), (231), (232), (233), (234), (235), (236),
(240), (241), (242), (243), (244), (245), (246),
(250), (251), (252), (253), (254), (255), (256),
(260), (261), (262), (263), (264), (265), (266),

(300), (301), (302), (303), (304), (305), (306),
(310), (311), (312), (313), (314), (315), (316),
(320), (321), (322), (323), (324), (325), (326),
(330), (331), (332), (333), (334), (335), (336),
(340), (341), (342), (343), (344), (345), (346),
(350), (351), (352), (353), (354), (355), (356),
(360), (361), (362), (363), (364), (365), (366),

(400), (401), (402), (403), (404), (405), (406),
(410), (411), (412), (413), (414), (415), (416),

(420), (421), (422), (423), (424), (425), (426),
(430), (431), (432), (433), (434), (435), (436),
(440), (441), (442), (443), (444), (445), (446),
(450), (451), (452), (453), (454), (455), (456),
(460), (461), (462), (463), (464), (465), (466),

(500), (501), (502), (503), (504), (505), (506),
(510), (511), (512), (513), (514), (515), (516),
(520), (521), (522), (523), (524), (525), (526),
(530), (531), (532), (533), (534), (535), (536),
(540), (541), (542), (543), (544), (545), (546),
(550), (551), (552), (553), (554), (555), (556),
(560), (561), (562), (563), (564), (565), (566),

(600), (601), (602), (603), (604), (605), (606),
(610), (611), (612), (613), (614), (615), (616),
(620), (621), (622), (623), (624), (625), (626),
(630), (631), (632), (633), (634), (635), (636),
(640), (641), (642), (643), (644), (645), (646),
(650), (651), (652), (653), (654), (655), (656),
(660), (661), (662), (663), (664), (665), (666)

]

```
print("E = [")
for i in range(7):
  for j in range(7):
    for n in range(7):
      print(f"({i}{j}{n}), ", end=" ")
    print()
  print("\n")
print("]")
```

b) Seja a Variável Aleatória (X) a soma dos valores das duas primeiras faces menos o valor da terceira. Especifique os resultados de S, os valores de X correspondentes (contradomínio R_x) e a probabilidade de cada valor de X.

- RESPOSTA

Contradomínio R_x

Rx = [0, -1, -2, -3, -4, -5, -6, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]

Probabilidade de cada valor de X

$X_0 = 0.00291545$, $X_1 = 0.00874636$, $X_2 = 0.01749271$

$X_3 = 0.02915452, X_4 = 0.04373178, X_5 = 0.06122449$

$X_6 = 0.08163265, X_7 = 0.09620991, X_8 = 0.10495627$

$X_9 = 0.10787172, X_{10} = 0.10495627, X_{11} = 0.09620991$

$X_{12} = 0.08163265, X_{13} = 0.06122449, X_{14} = 0.04373178$

$X_{15} = 0.02915452, X_{16} = 0.01749271, X_{17} = 0.00874636$

$X_{18} = 0.00291545$

Resultado de S

```
s = []
for i in range(7):
  for j in range(7):
    for n in range(7):
      soma = (i+j)-n
      s.append(soma)
      print(f"x({i}{j}{n}) = {soma}, ", end=" ")
    print()
  print("\n")

unicos_lista = list(dict.fromkeys(s))
print("Rx", unicos_lista)
```

Encontrando o Contradomínio R_x e as Probabilidades de cada valor de X

```
import numpy as np

prob = np.array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0], dtype="float64")
for i in s:
  if (i == -6):
    prob[0] += 1/len(s)
  elif (i == -5):
    prob[1] += 1/len(s)
  elif (i == -4):
    prob[2] += 1/len(s)
  elif (i == -3):
    prob[3] += 1/len(s)
  elif (i == -2):
    prob[4] += 1/len(s)
  elif (i == -1):
    prob[5] += 1/len(s)
  elif (i == 0):
    prob[6] += 1/len(s)
  elif (i == 1):
```

```
    prob[7] += 1/len(s)
  elif (i == 2):
    prob[8] += 1/len(s)
  elif (i == 3):
    prob[9] += 1/len(s)
  elif (i == 4):
    prob[10] += 1/len(s)
  elif (i == 5):
    prob[11] += 1/len(s)
  elif (i == 6):
    prob[12] += 1/len(s)
  elif (i == 7):
    prob[13] += 1/len(s)
  elif (i == 8):
    prob[14] += 1/len(s)
  elif (i == 9):
    prob[15] += 1/len(s)
  elif (i == 10):
    prob[16] += 1/len(s)
  elif (i == 11):
    prob[17] += 1/len(s)
  elif (i == 12):
    prob[18] += 1/len(s)

print(f"\nProbabilidade para os Xs =\n {prob}\n")
print(f"Soma das Probabilidades: {prob.sum()}")
```

```
Probabilidade para os Xs =
 [0.00291545 0.00874636 0.01749271 0.02915452 0.04373178 0.06122449
 0.08163265 0.09620991 0.10495627 0.10787172 0.10495627 0.09620991
 0.08163265 0.06122449 0.04373178 0.02915452 0.01749271 0.00874636
 0.00291545]

Soma das Probabilidades: 1.0000000000000004
```

4.3 – Variáveis aleatórias discretas e contínuas e

4.4 – Funções de variáveis aleatórias $(V.A.)$ – fdp e FDP

Questão 3 – Com base na Questão 1. Determine:

a) $p(x_i)$ – fdp de X.

- RESPOSTA

\begin{array}{|c|c|c|} \hline x_i & 0 & 1 & 2 & 3 & 4\ \hline \P[X = x_i]& \dfrac{1}{16} & \ dfrac{1}{4} & \dfrac{3}{8} & \dfrac{1}{4} & \dfrac{1}{16} \ \hline \end{array}

b) $F(x_i)$ −FDP de X.

- RESPOSTA

Questão 4 – Com base na Questão 2. Determine:

a) $p(x_i)$ −fdp de X.

- RESPOSTA

\begin{array}{|c|c|c|} \hline x_i & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \ \hline \P[X = x_i]& 0.29% & 0.87% & 1.74% & 2.91% & 4.37% & 6.12% & 8.16% & 9.62% & 10.49% \ \hline \ x_i & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17\ \hline \P[X = x_i]& 10.78% & 10.49% & 8.16% & 6.12% & 4.37% & 2.91% & 1.74% & 0.87% & 0.29%\ \hline \end{array}

```
q2_p_xi = prob*100
print(q2_p_xi)
```

```
[ 0.29154519  0.87463557  1.74927114  2.9154519   4.37317784
6.12244898
  8.16326531  9.62099125 10.49562682 10.78717201 10.49562682
9.62099125
  8.16326531  6.12244898  4.37317784  2.9154519   1.74927114
0.87463557
  0.29154519]
```

b) $F(x_i)$ −FDP de X.

- RESPOSTA

Questão 5 – Uma função distribuição de probabilidade acumulada FDP é definida da seguinte forma:

- $X < a \rightarrow F = 0;$

- $a \leq X \leq b \rightarrow F = \frac{x-a}{b-a}$;
- $X > b \rightarrow F = 1$;

a) Calcule $f(x)$ - fdp de X.

- RESPOSTA

b) Calcule $P[1 < X \leq 3]$ para $a = 1$ e $b = 5$.

- RESPOSTA

c) Calcule $P[-1 < X \leq 2]$ para $a = 1$ e $b = 5$.

- RESPOSTA

d) Calcule $P[-\infty < X \leq 1,5]$ para $a = 1$ e $b = 5$.

- RESPOSTA

e) Calcule $P[0 < X \leq 6]$ para $a = 1$ e $b = 5$.

- RESPOSTA

Questão 6 – O tempo de transmissão X de mensagens em um sistema de comunicação obedece a lei de probabilidade exponencial com parâmetro λ, isto é $P[X > x] = e - \lambda x, x > 0$. Calcule, $T = 1/\lambda$.

a) Defina $F(x)$ - FDP de X

- RESPOSTA

b) Calcule $f(x)$ - fdp de X.

- RESPOSTA

c) Calcule $P[T < X \leq 2T]$ para $T = T = 1/\lambda$.

- RESPOSTA