EE 254

Electronic Instrumentation

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Lecture Note #08

Content (Brief)

2. Op-Amp Applications

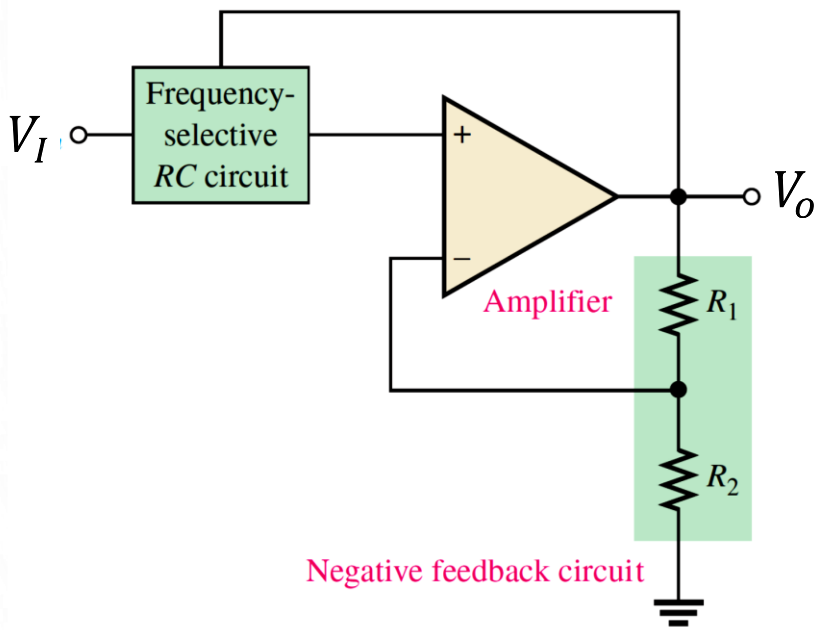
- **Linear Applications**
  - Inverting amplifiers
  - Noninverting amplifiers
  - Differential amplifiers
  - Summing amplifiers
  - Integrators
  - Differentiators
  - Low/ High pass filters
  - Instrumentational amplifiers

- **Nonlinear Applications**
  - Precision rectifiers
  - Peak detectors
  - Schmitt-trigger comparator
  - Logarithmic amplifiers

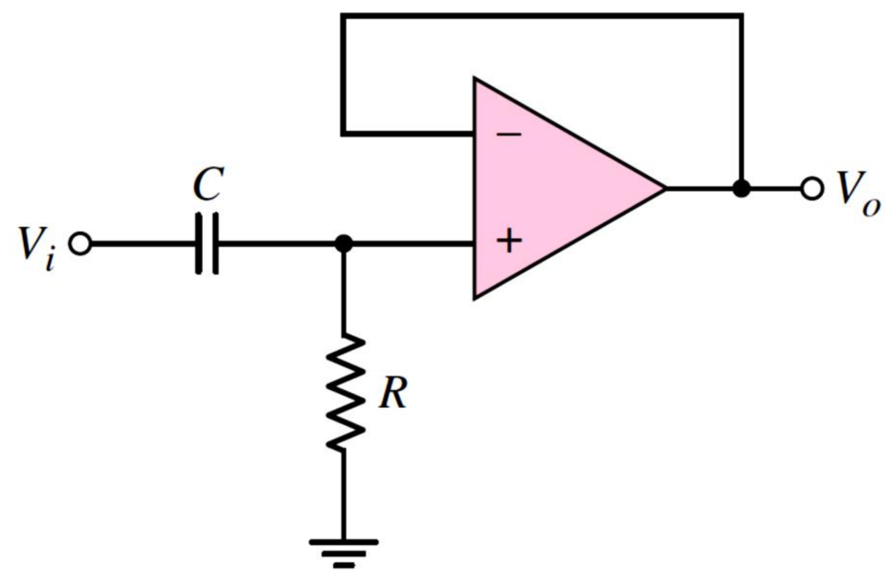
Low-pass and High-Pass Filters

Active Filters

- We can have different configurations with op-amps

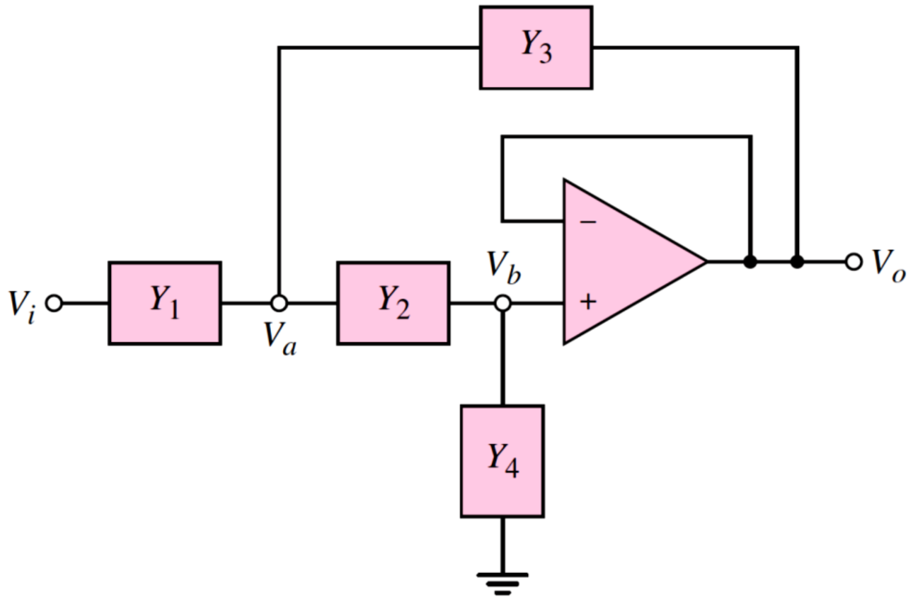


With negative feedback circuit



With voltage follower circuit

General Two-Pole Active Filter



- KCL to node V_a

$$(V_i - V_a)Y_1 = (V_a - V_b)Y_2 + (V_a - V_o)Y_3$$

- KCL to node V_b

$$(V_a - V_b)Y_2 = V_b Y_4$$

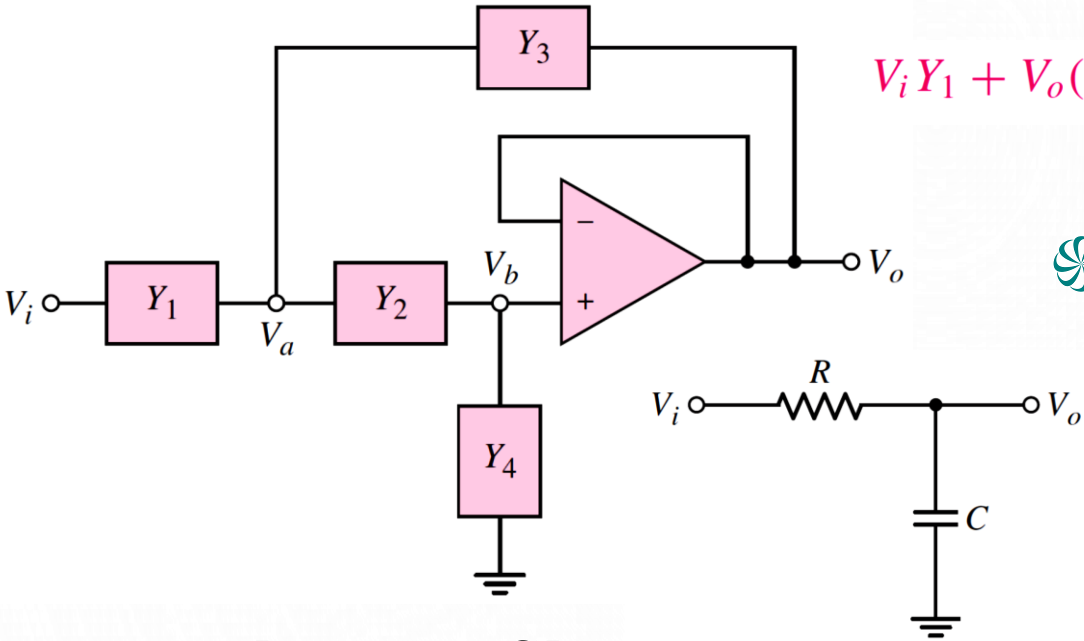
- For a voltage follower $V_b = V_o$, and then;

$$V_a = V_b\left(\frac{Y_2 + Y_4}{Y_2}\right) = V_o\left(\frac{Y_2 + Y_4}{Y_2}\right)$$

- Substituting into the first equation

$$V_i Y_1 + V_o(Y_2 + Y_3) = V_a(Y_1 + Y_2 + Y_3)$$
$$= V_o\left(\frac{Y_2 + Y_4}{Y_2}\right)(Y_1 + Y_2 + Y_3)$$

General Two-Pole Active Filter



$$V_iY_1 + V_o(Y_2 + Y_3) = V_o\left(\frac{Y_2 + Y_4}{Y_2}\right)(Y_1 + Y_2 + Y_3)$$

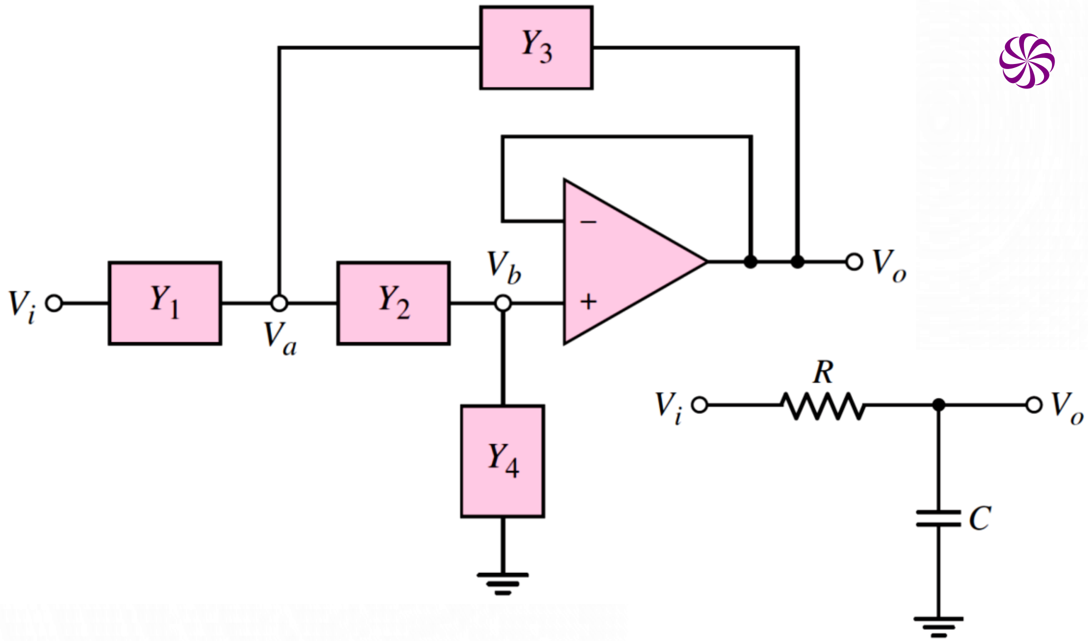
- Then the transfer function

$$T(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{Y_1Y_2}{Y_1Y_2 + Y_4(Y_1 + Y_2 + Y_3)}$$

- **For a low-pass filter:**
  - Both Y_1 and Y_2 must be conductances allowing the signal to pass into the voltage follower at low frequencies.
  - If element Y_4 is a capacitor, then the output rolls off at high frequencies.
  - To produce a two-pole function, Y_3 must also be a capacitor.

General Two-Pole Active Filter



- Then the transfer function

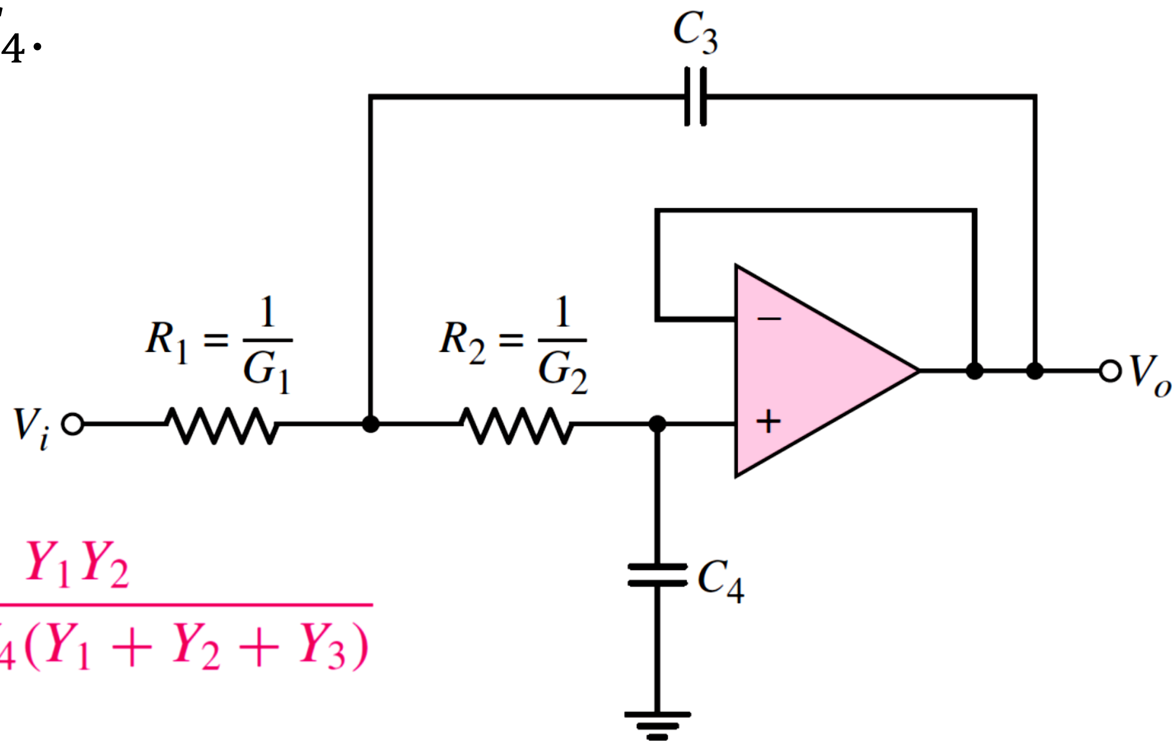
$$T(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{Y_1 Y_2}{Y_1 Y_2 + Y_4(Y_1 + Y_2 + Y_3)}$$

- On the other hand, if Y_1 and Y_2 are capacitors, the signal will be blocked at low frequencies but will be passed into the voltage follower at high frequencies, **resulting High-Pass Filter**.
- Therefore Y_3 and Y_4 must be conductances to produce a two-pole high-pass transfer function.

Two-Pole Low-Pass Butterworth Filter

- To form a low-pass filter, we set $Y_1 = G_1 = 1/R_1$, $Y_2 = G_2 = 1/R_2$, $Y_3 = sC_3$, and $Y_4 = sC_4$.

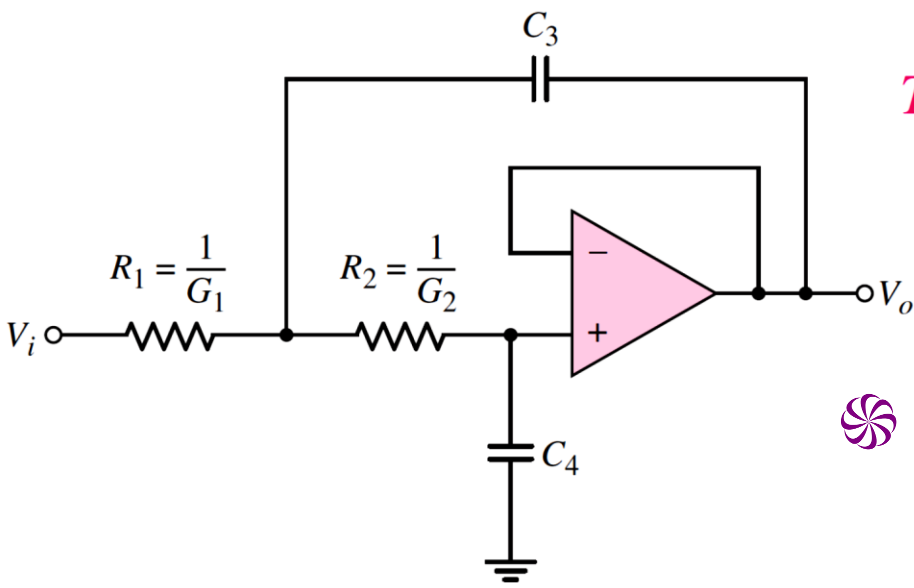


$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_4(Y_1 + Y_2 + Y_3)}$$

- Then the transfer function

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{G_1 G_2}{G_1 G_2 + sC_4(G_1 + G_2 + sC_3)}$$

Two-Pole Low-Pass Butterworth Filter



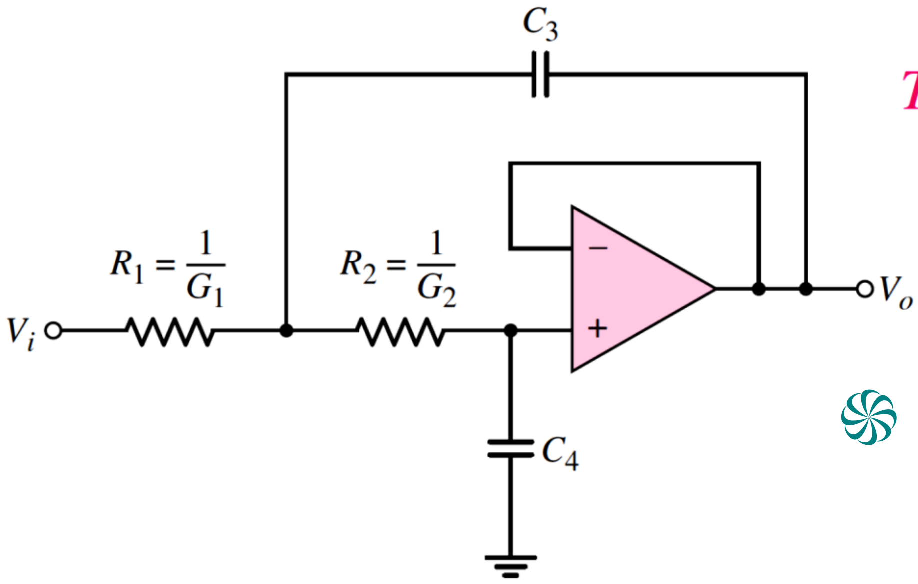
$$T(s) = \frac{G_1 G_2}{G_1 G_2 + sC_4(G_1 + G_2 + sC_3)}$$

- At zero frequency, $s = j\omega = 0$

$$T(s = 0) = \frac{G_1 G_2}{G_1 G_2} = 1$$

- A **Butterworth filter** is a **maximally flat magnitude filter**.
- The transfer function is designed such that the **magnitude** of the transfer function is **as flat as possible** within the **passband of the filter**.

Two-Pole Low-Pass Butterworth Filter



$$T(s) = \frac{G_1 G_2}{G_1 G_2 + sC_4(G_1 + G_2 + sC_3)}$$

- At zero frequency, $s = j\omega = 0$

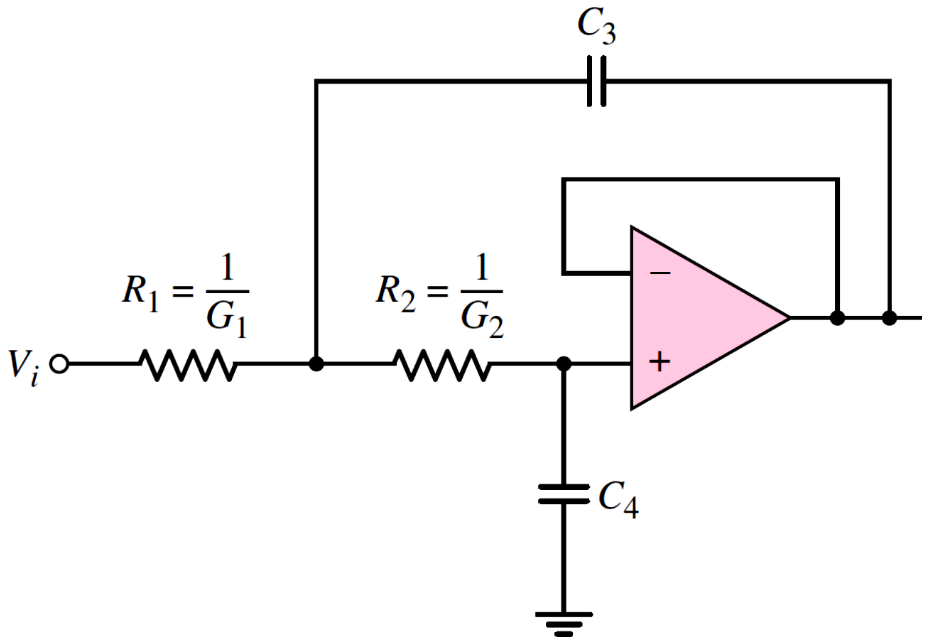
$$T(s = 0) = \frac{G_1 G_2}{G_1 G_2} = 1$$

- **This objective is achieved by taking the derivatives** of the transfer function with respect to frequency and setting as many as possible equal to zero at the center of the passband, which is at zero frequency for the low-pass filter.

Two-Pole Low-Pass Butterworth Filter

$$T(s) = \frac{G_1 G_2}{G_1 G_2 + sC_4(G_1 + G_2 + sC_3)}$$

- let, $G_1 = G_2 \equiv G = 1/R$



$$T(s) = \frac{\frac{1}{R^2}}{\frac{1}{R^2} + sC_4\left(\frac{2}{R} + sC_3\right)}$$

$$= \frac{1}{1 + sRC_4(2 + sRC_3)}$$

- We define **time constants** at $\tau_3 = RC_3$ and $\tau_4 = RC_4$.

$$T(j\omega) = \frac{1}{1 + j\omega\tau_4(2 + j\omega\tau_3)} = \frac{1}{(1 - \omega^2\tau_3\tau_4) + j(2\omega\tau_4)}$$

Two-Pole Low-Pass Butterworth Filter

$$T(j\omega) = \frac{1}{1 + j\omega\tau_4(2 + j\omega\tau_3)} = \frac{1}{(1 - \omega^2\tau_3\tau_4) + j(2\omega\tau_4)}$$

- The **magnitude** of the transfer function

$$|T(j\omega)| = [(1 - \omega^2\tau_3\tau_4)^2 + (2\omega\tau_4)^2]^{-1/2}$$

- For a **maximally flat filter** (that is, a filter with a minimum rate of change), which defines a Butterworth filter, we set

$$\left.\frac{d|T|}{d\omega}\right|_{\omega=0} = 0$$

- Taking the derivative

$$\frac{d|T|}{d\omega} = -\frac{1}{2}[(1 - \omega^2\tau_3\tau_4)^2 + (2\omega\tau_4)^2]^{-3/2}[-4\omega\tau_3\tau_4(1 - \omega^2\tau_3\tau_4) + 8\omega\tau_4^2]$$

Two-Pole Low-Pass Butterworth Filter

$$\frac{d|T|}{d\omega} = -\frac{1}{2}[(1-\omega^2\tau_3\tau_4)^2 + (2\omega\tau_4)^2]^{-3/2}[-4\omega\tau_3\tau_4(1-\omega^2\tau_3\tau_4) + 8\omega\tau_4^2]$$

- Setting the derivative equal to zero at $\omega = 0$

$$\left.\frac{d|T|}{d\omega}\right|_{\omega=0} = [-4\omega\tau_3\tau_4(1-\omega^2\tau_3\tau_4) + 8\omega\tau_4^2]$$

$$= 4\omega\tau_4[-\tau_3(1-\omega^2\tau_3\tau_4) + 2\tau_4]$$

- This is satisfied when $2\tau_4 = \tau_3$, or

$$C_3 = 2C_4$$

- For this condition, the transfer magnitude is

$$|T| = \frac{1}{[1+4(\omega\tau_4)^4]^{1/2}}$$

Two-Pole Low-Pass Butterworth Filter

$$|T| = \frac{1}{[1 + 4(\omega\tau_4)^4]^{1/2}}$$

- The 3 dB, or cutoff, frequency occurs when $|T| = 1/\sqrt{2}$, or when $4(\omega_{3dB}\tau_4)^4 = 1.$

$$\omega_{3\,\mathrm{dB}} = 2\pi f_{3\,\mathrm{dB}} = \frac{1}{\tau_4\sqrt{2}} = \frac{1}{\sqrt{2}RC_4}$$

- In general, we can write the **cutoff frequency**

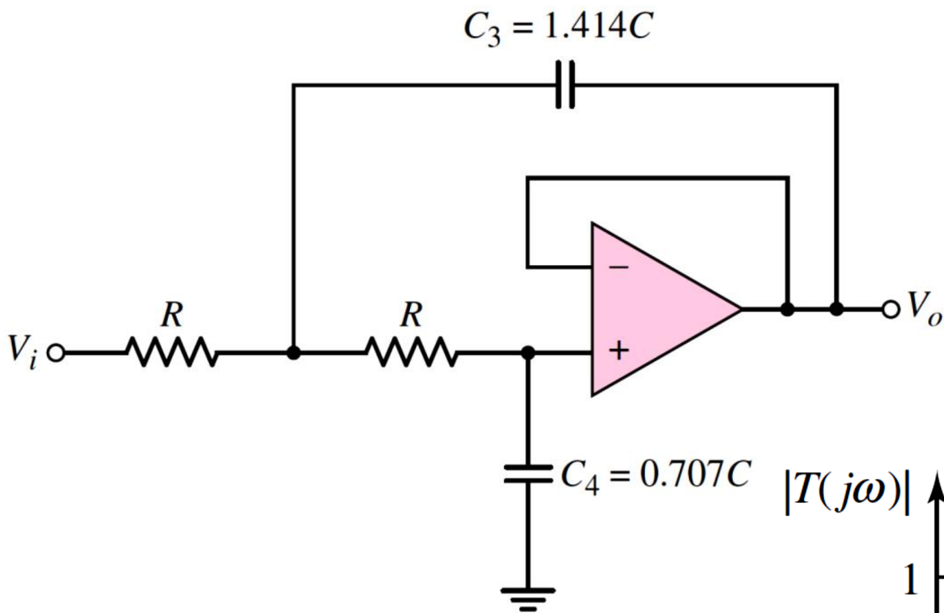
$$\omega_{3\,\mathrm{dB}} = \frac{1}{RC}$$

- Finally;

$$C_4 = 0.707C \qquad C_3 = 1.414C$$

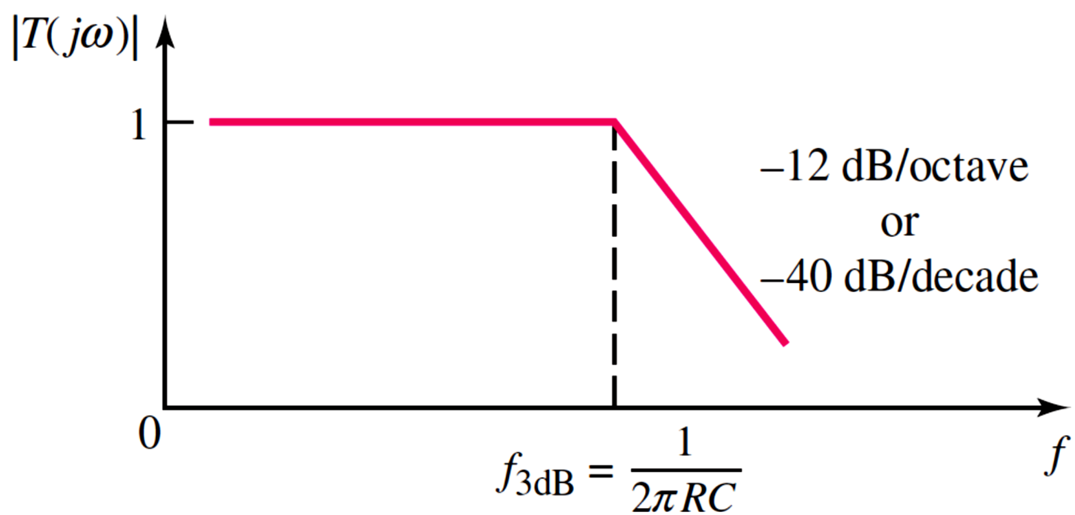
Two-Pole Low-Pass Butterworth Filter

- The magnitude of the voltage transfer function for **the two-pole low-pass Butterworth filter**



$$|T| = \frac{1}{[1 + 4(\omega\tau_4)^4]^{1/2}}$$

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3\,\mathrm{dB}}}\right)^4}}$$

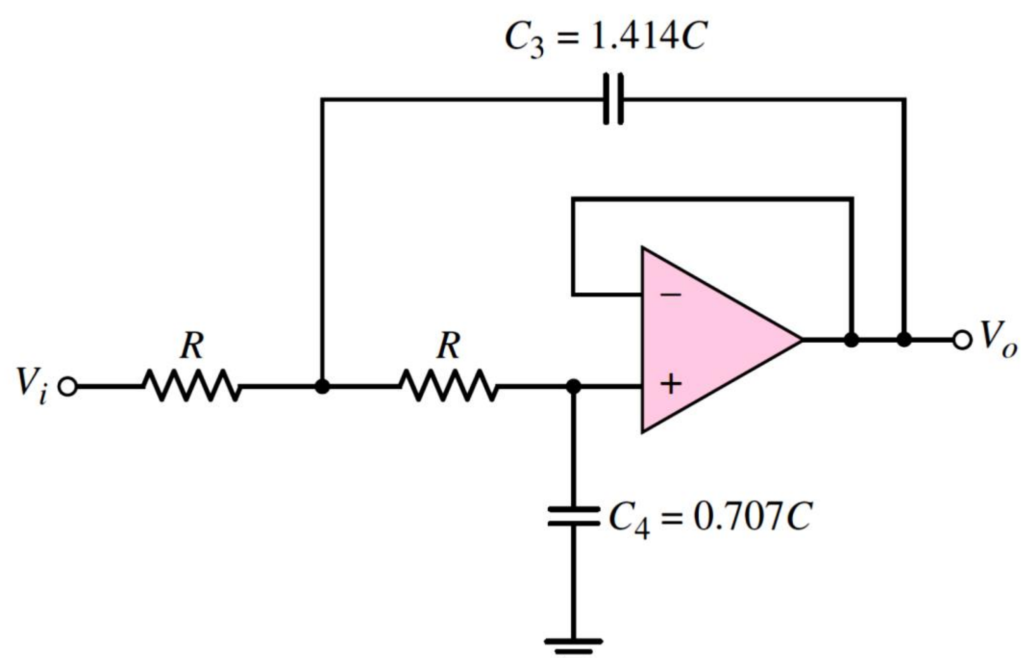


Example 01: Low-Pass Butterworth Filter

Objective: Design a two-pole low-pass Butterworth filter for an audio amplifier application.

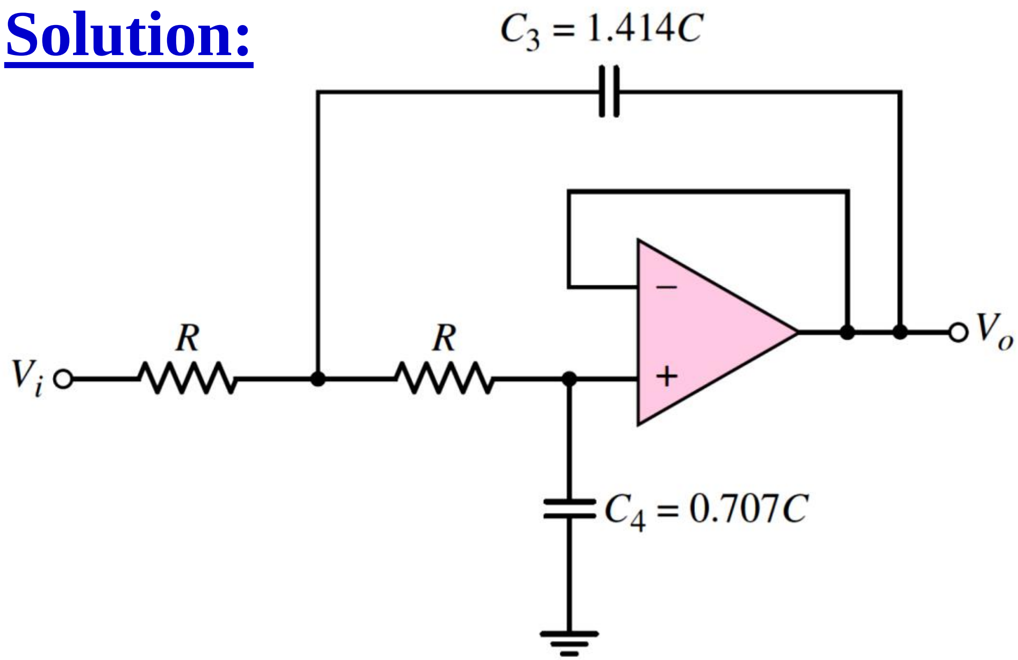
Specifications: The circuit with the configuration shown is to be designed such that the bandwidth is 20 kHz.

Choices: An ideal op-amp is available and standard-valued resistors and capacitors must be used.



Example 01: Low-Pass Butterworth Filter

Solution:



The cutoff frequency or 3dB frequency

$$f_{3dB} = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_{3dB}} = \frac{1}{2\pi(20 \times 10^3)}$$

$$RC = 7.96 \times 10^{-6}$$

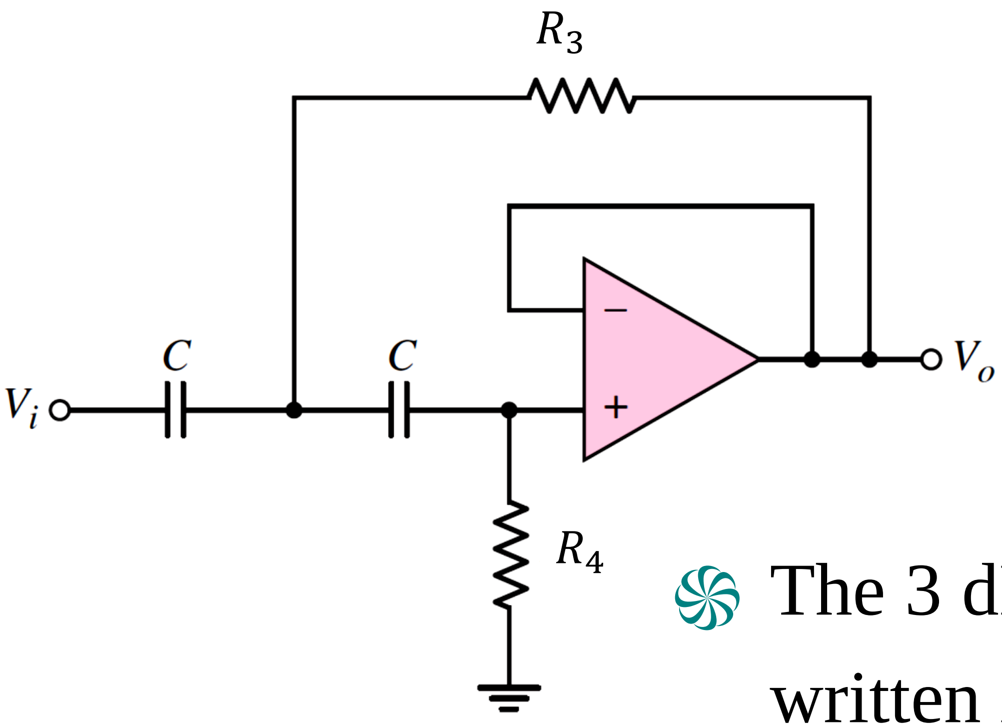
If we choose $R = 100\ k\Omega$ $C = 79.6\ pF$

Then; $C_3 = 1.414C = 113\ pF$ and $C_4 = 0.707C = 56.3\ pF$

Trade-offs: Standard-valued $100k\Omega$ resistors can be used. Standard-valued $C_3 = 120\ pF$ and $C_4 = 56\ pF$ capacitors can be used. For these elements, a bandwidth of $20.1\ kHz$ is obtained.

Two-Pole High-Pass Butterworth Filter

- To form a high-pass filter, the resistors and capacitors are interchanged from those in the low-pass filter.



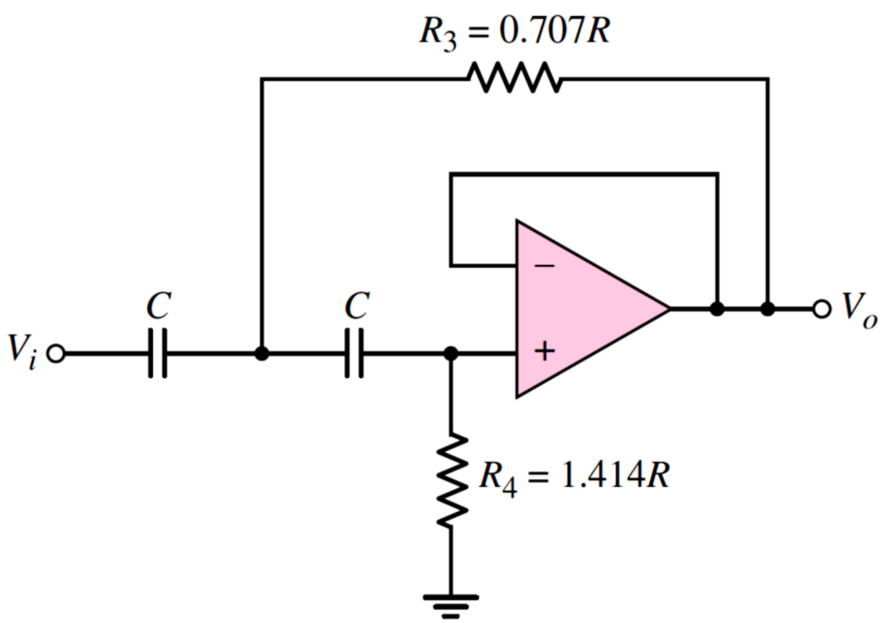
- Let's set the two capacitors are equal to each other
- Frequency, $s = j\omega = \infty$
- The 3 dB or cutoff frequency can be written in the general form.

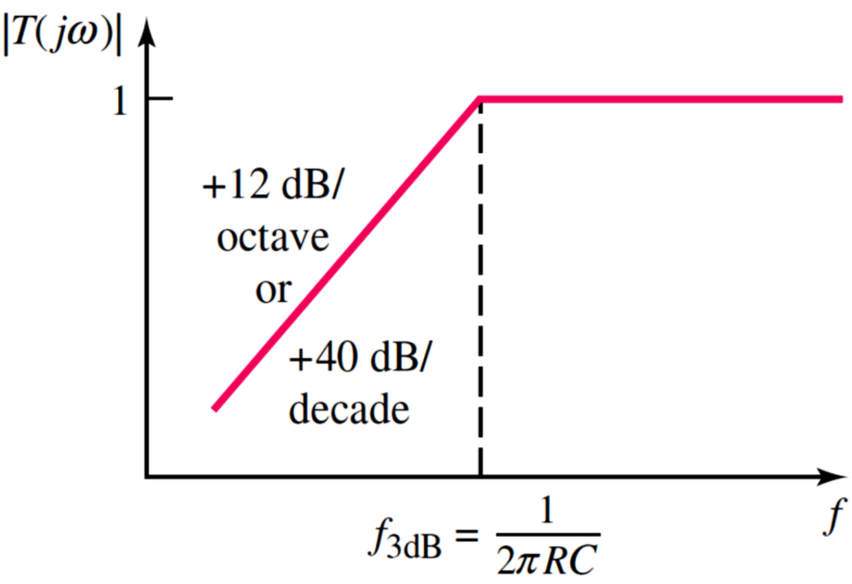
$$\omega_{3\,\mathrm{dB}} = 2\pi f_{3\,\mathrm{dB}} = \frac{1}{RC}$$

Two-Pole High-Pass Butterworth Filter

- We find that $R_3 = 0.707\,R$ and $R_4 = 1.414\,R$.
- The magnitude of the voltage transfer function for the two-pole high-pass Butterworth.

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f_{3\,dB}}{f}\right)^4}}$$





Higher-Order Butterworth Filters

- The **filter order** is the **number of poles**.
- An **N-pole** active low-pass filter has a high-frequency roll-off rate of N × 6 dB/octave (N × 20 dB/decade), up to the cutoff frequency.
- The 3 dB frequency

$$f_{3\,\mathrm{dB}} = \frac{1}{2\pi RC}$$

- The magnitude of the voltage transfer function for a Butterworth N^{th}-order **Low-pass filter**

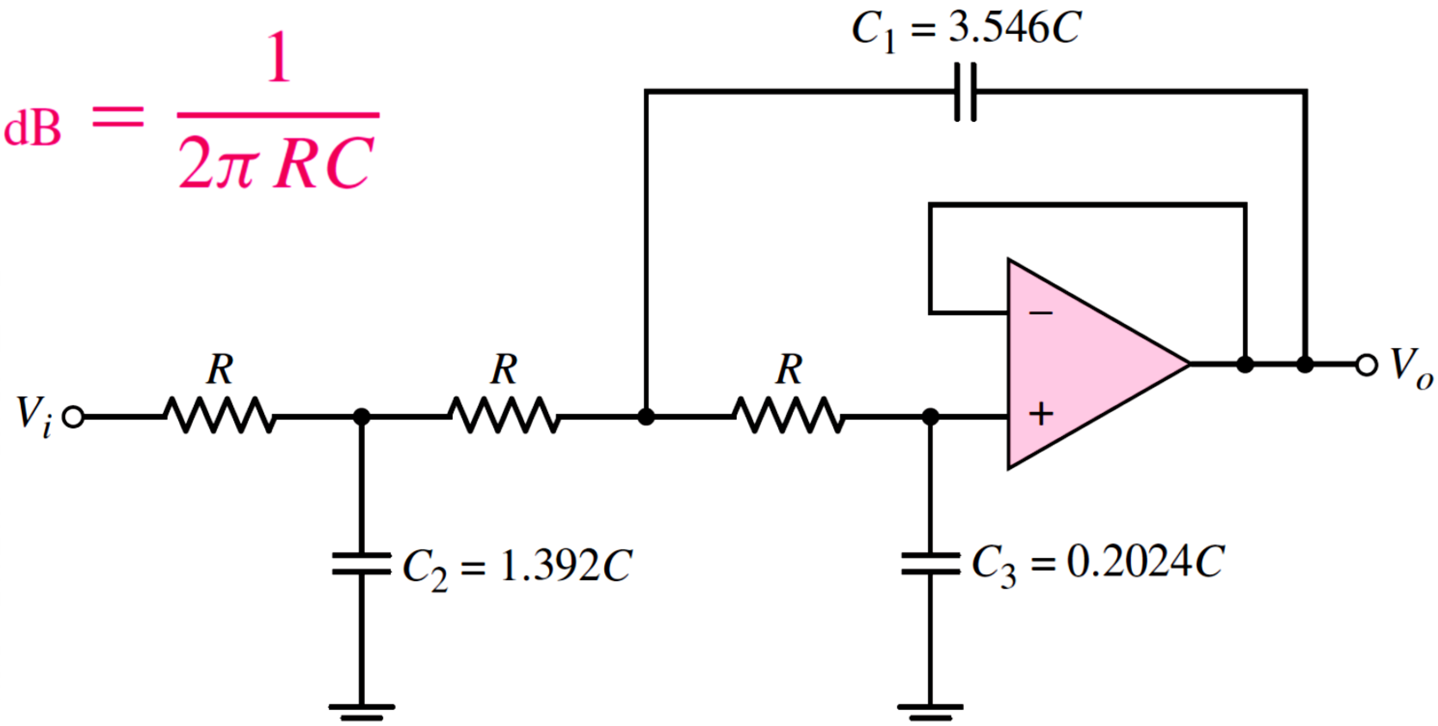
$$|T| = \frac{1}{\sqrt{1 + \left(\dfrac{f}{f_{3\,\mathrm{dB}}}\right)^{2N}}}$$

- The magnitude of the voltage transfer function for a Butterworth N^{th}-order **High-pass filter**

$$|T| = \frac{1}{\sqrt{1 + \left(\dfrac{f_{3\,\mathrm{dB}}}{f}\right)^{2N}}}$$

Three-Pole Low-Pass Butterworth Filter

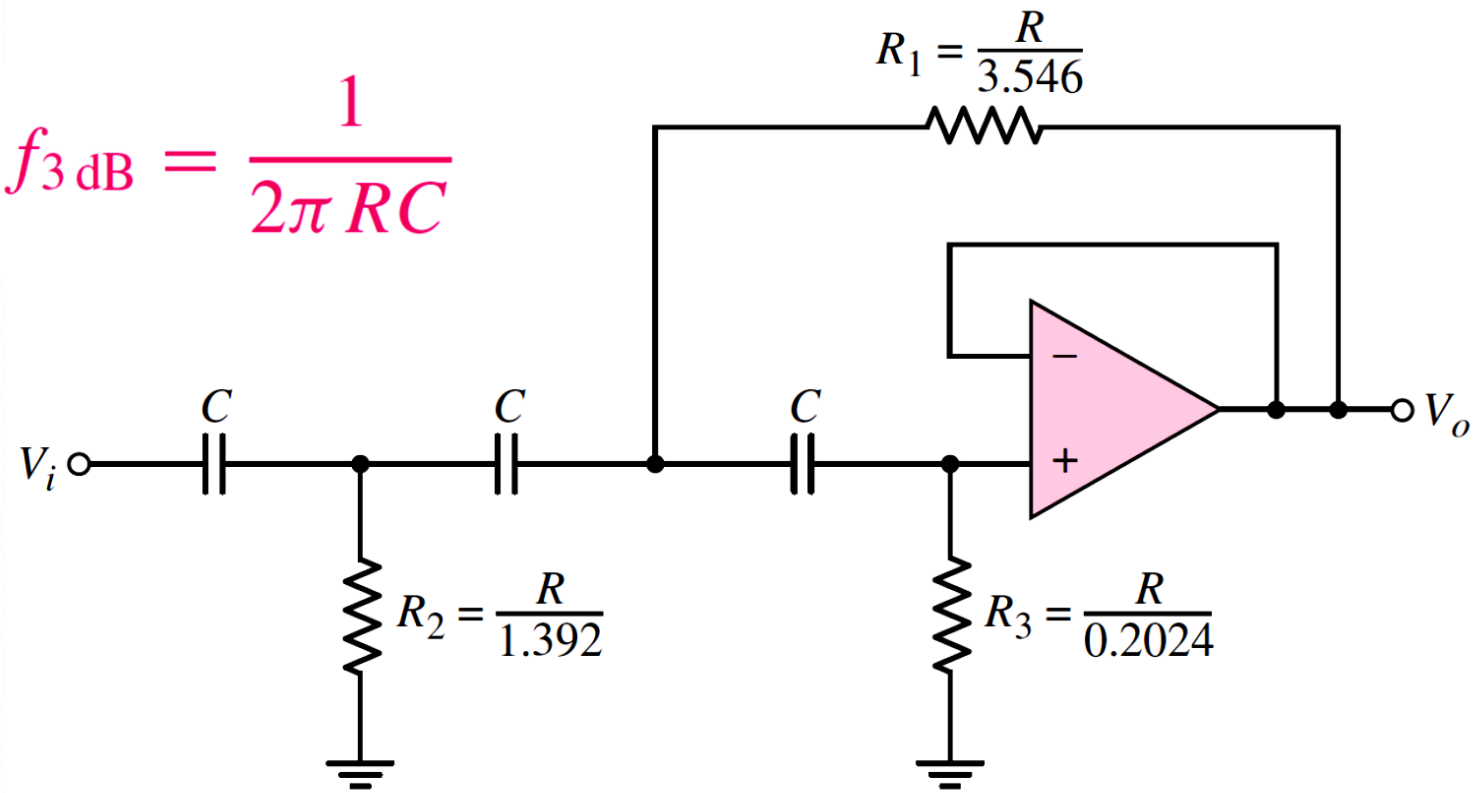
$$f_{3\,\mathrm{dB}} = \frac{1}{2\pi RC}$$



- The three **resistors** are equal
- The relationship between the **capacitors** is found by taking the first and second derivatives of the voltage gain magnitude with respect to frequency and setting those derivatives equal to zero at $s = j\omega = 0$.

Three-Pole High-Pass Butterworth Filter

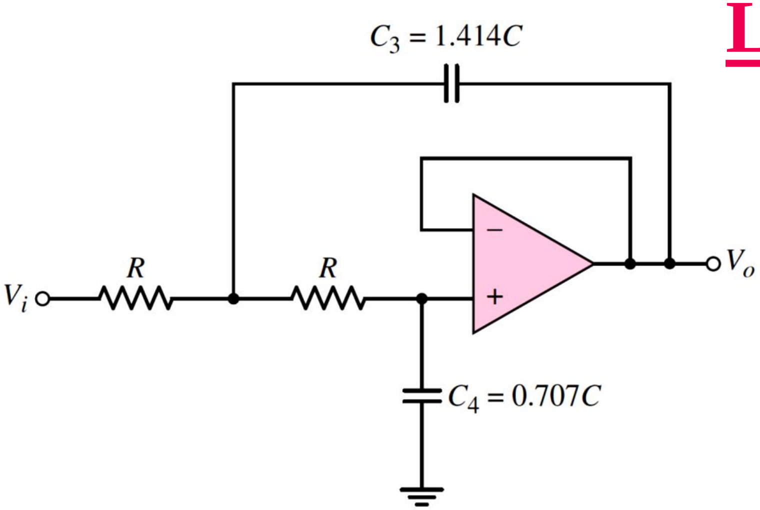
$$f_{3\,\mathrm{dB}} = \frac{1}{2\pi RC}$$

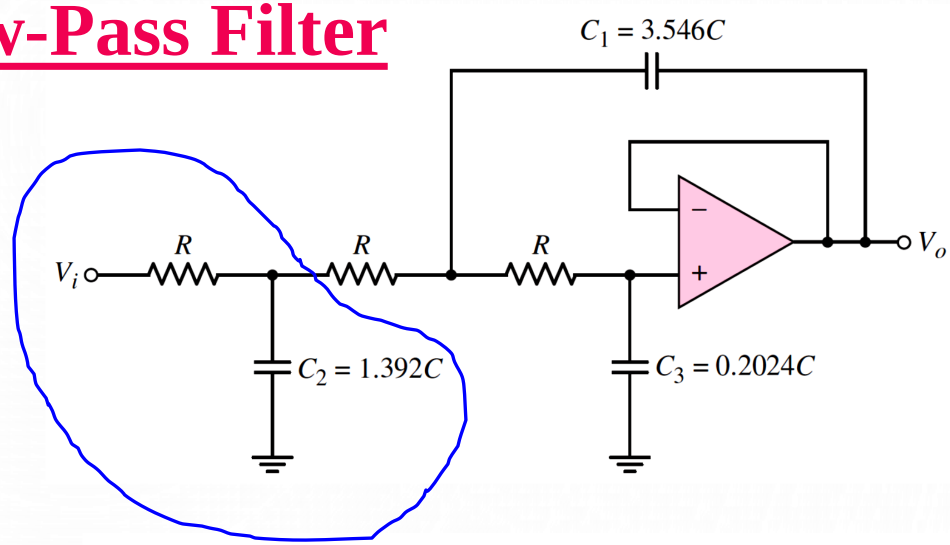


- The three **capacitors** are equal
- The relationship between the **resistors** is also found through the derivatives.

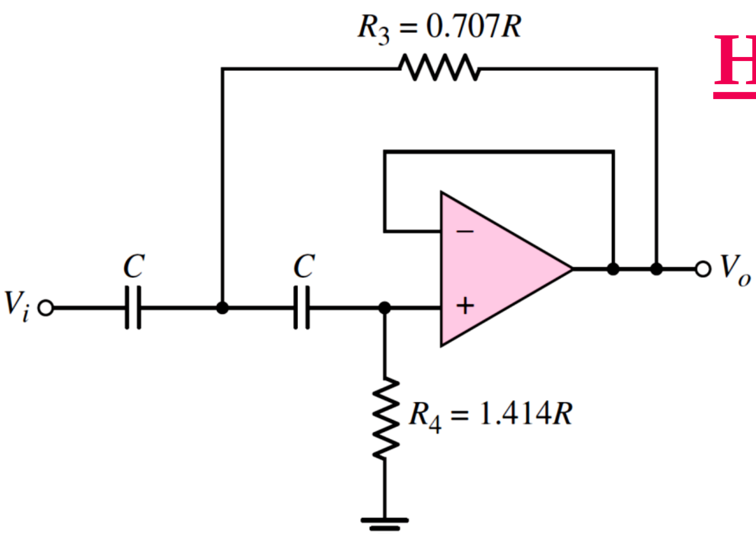
Higher-Order Butterworth Filters

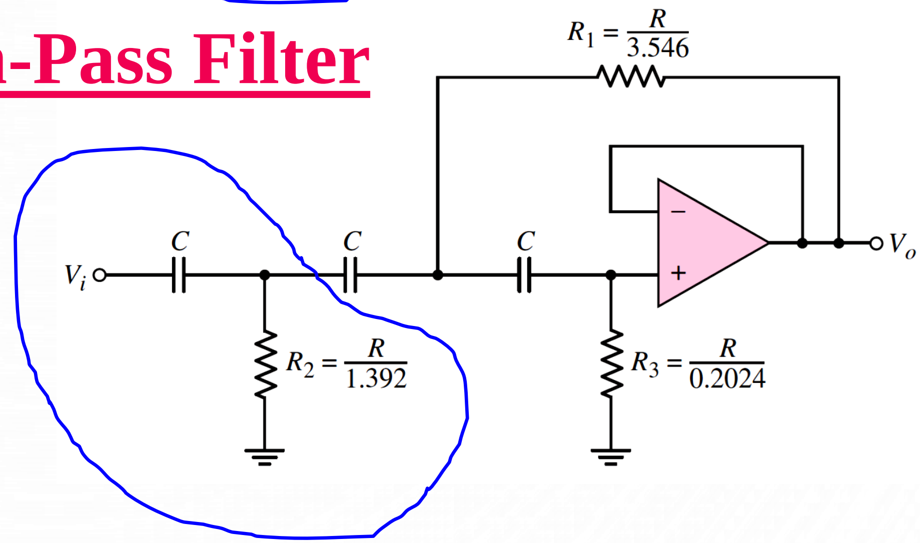
Low-Pass Filter





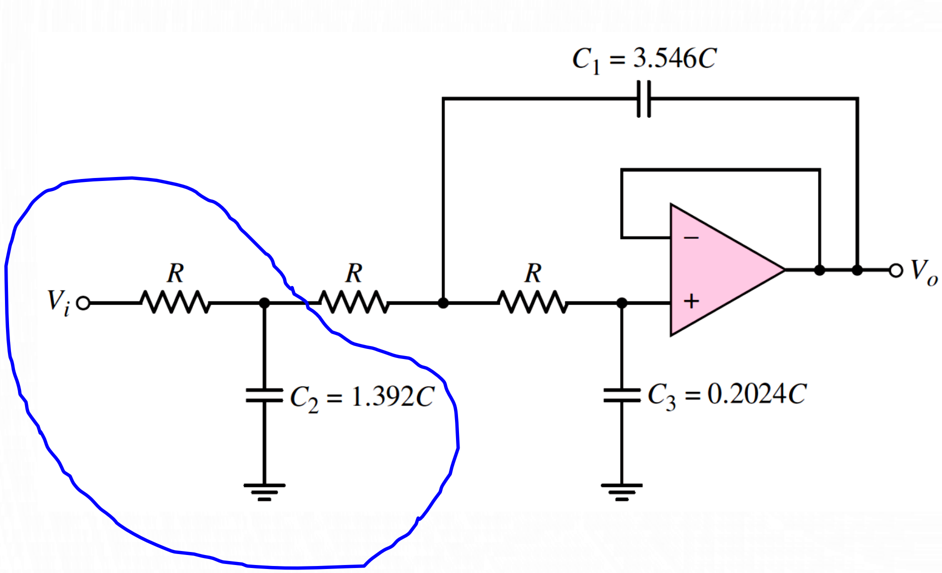
High-Pass Filter

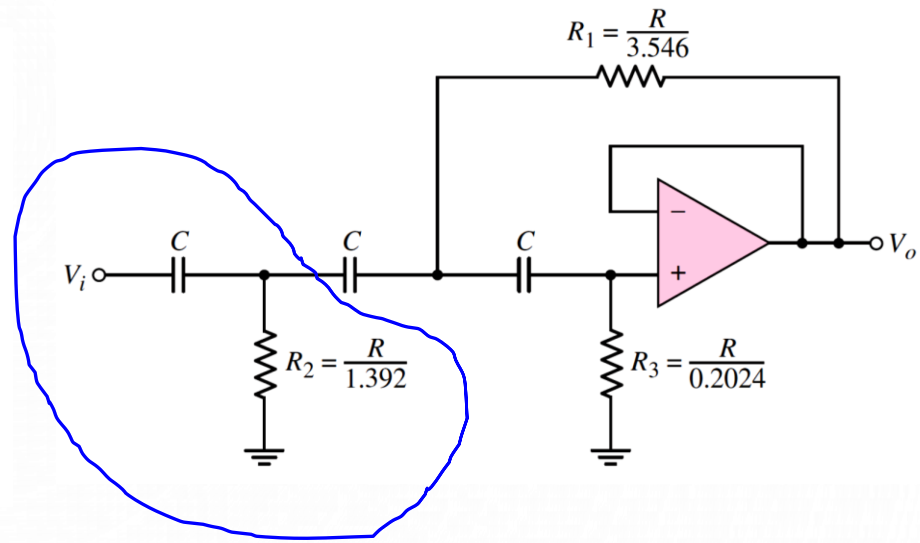




- Can be created by adding *RC* networks.

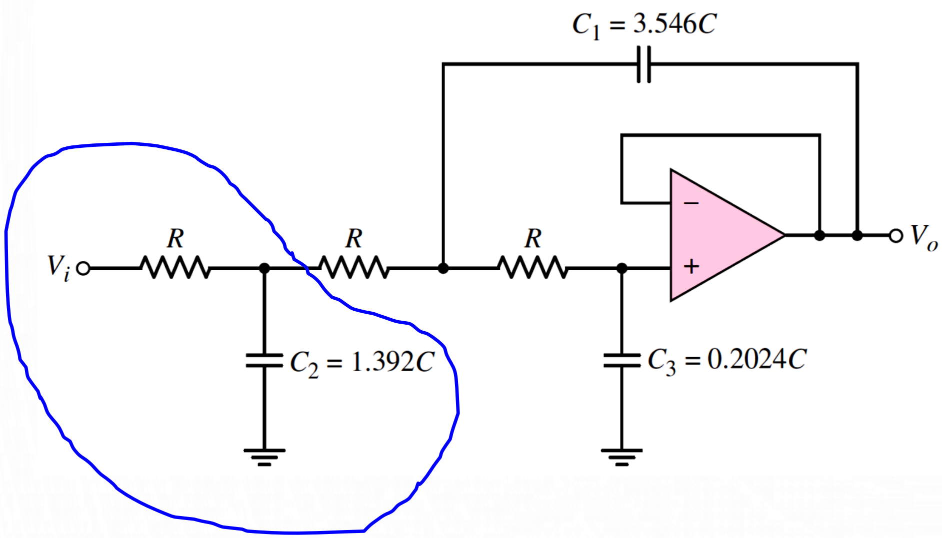
Higher-Order Butterworth Filters





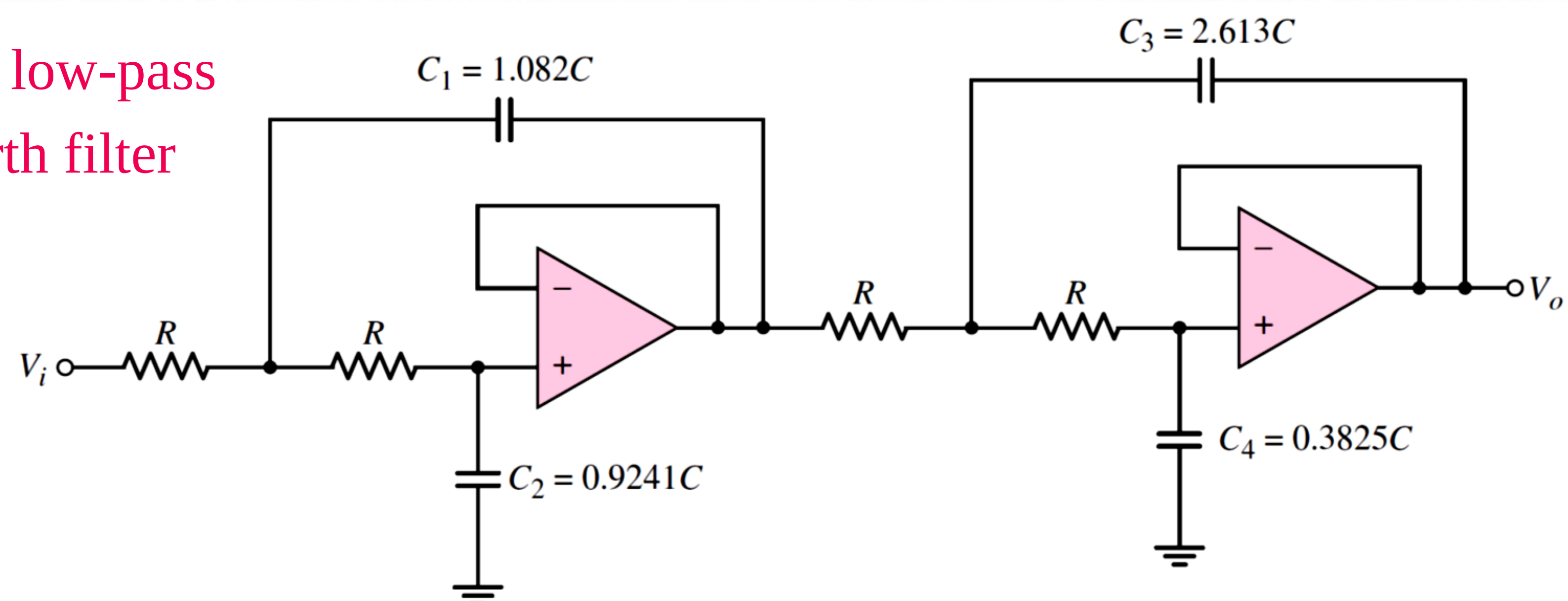
- However, the **loading effect** on each additional *RC* circuit becomes **more severe**.
- The usefulness of active filters is realized when **two or more op-amp filter circuits** are **cascaded** to produce one large higher-order active filter.
- Because of the **low output impedance** of the op-amp, there is **virtually no loading effect between cascaded stages**.

Higher-Order Butterworth Filters

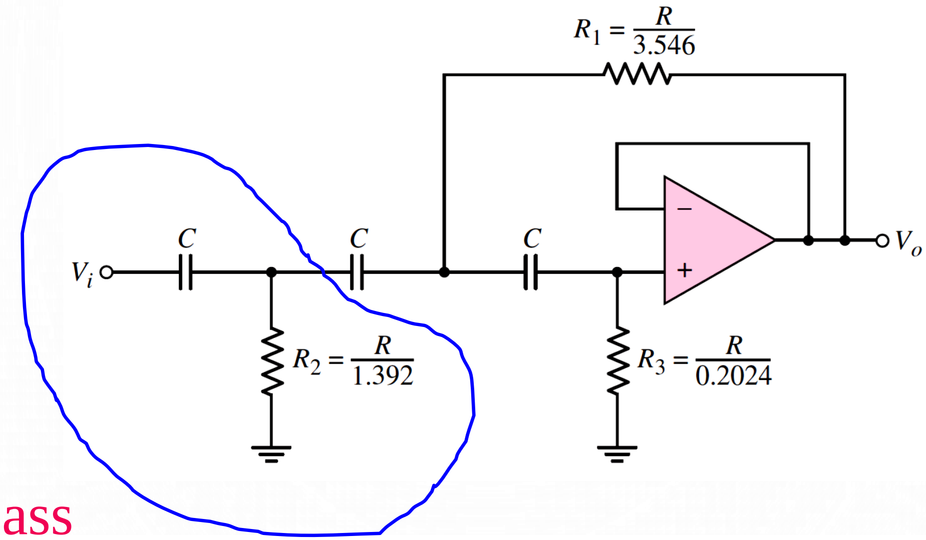


- The maximally flat response of this filter is ***not*** obtained by simply cascading two two-pole filters.
- The relationship between the **capacitors** is found through the **first three derivatives** of the transfer function.

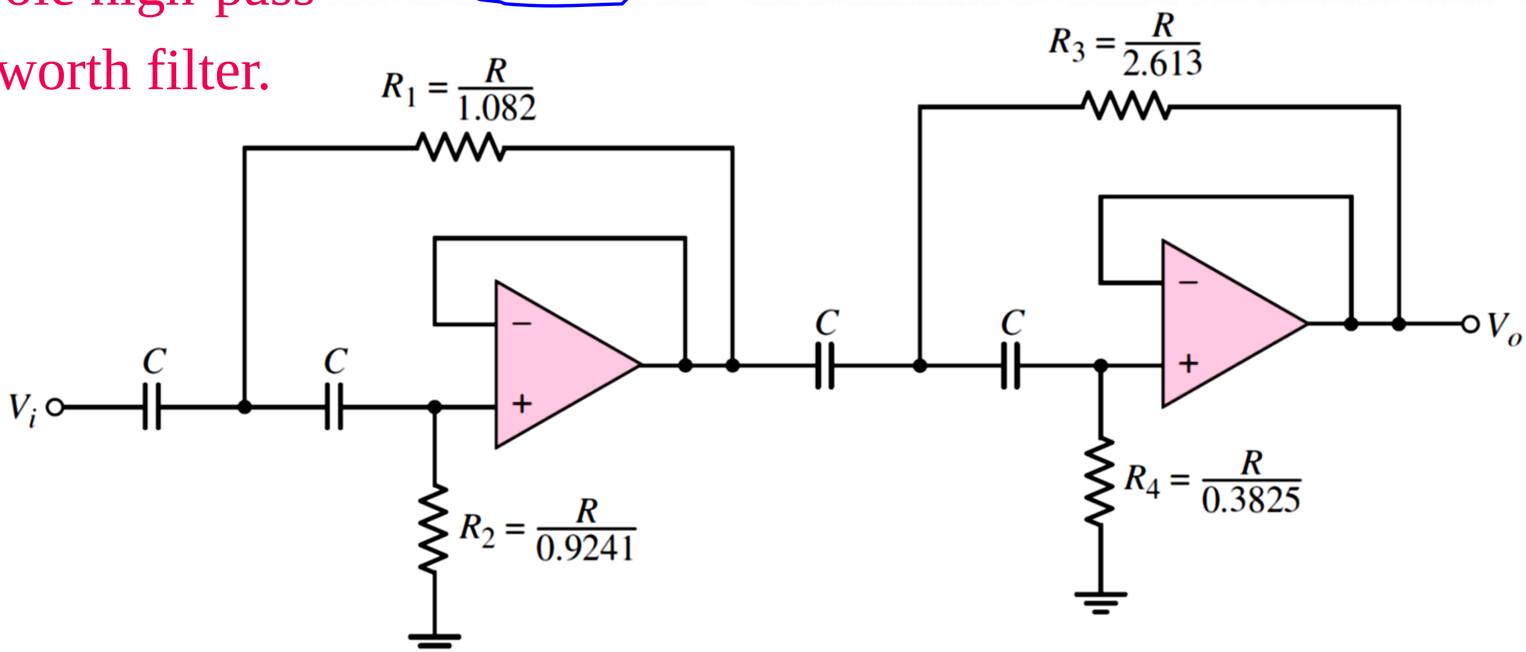
A four-pole low-pass Butterworth filter



Higher-Order Butterworth Filters



A four-pole high-pass Butterworth filter.



Example 02: Low-Pass Butterworth Filter

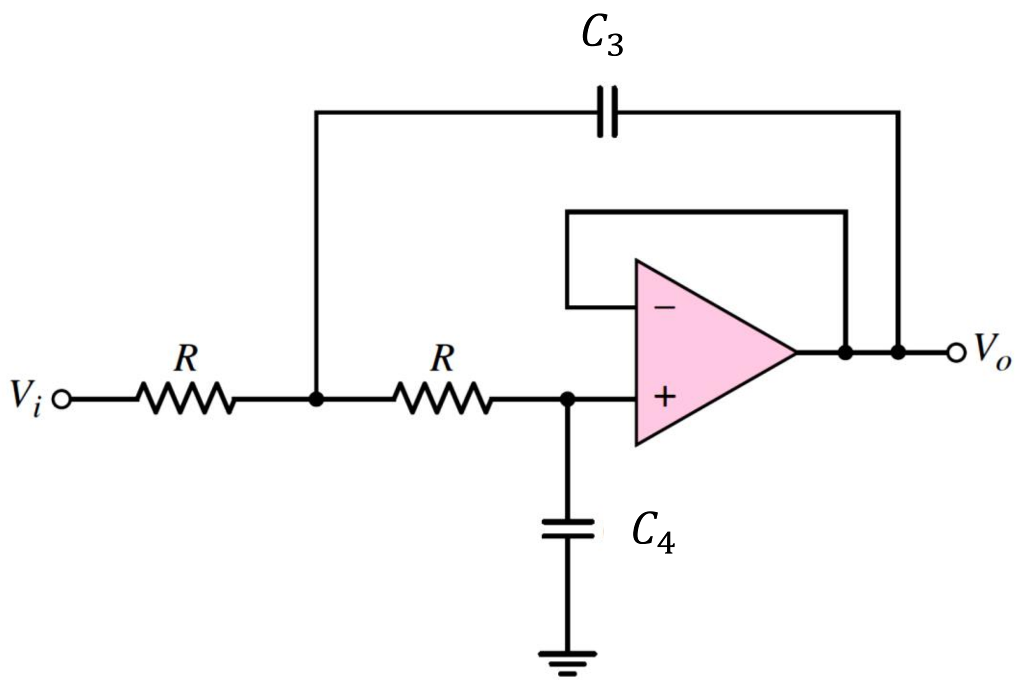
Consider a Butterworth low-pass filter. Determine the reduction in gain (in dB) at $f\ =\ 1.5\ f_{3dB}$ fo

(a) Two-pole

(b) Three-pole

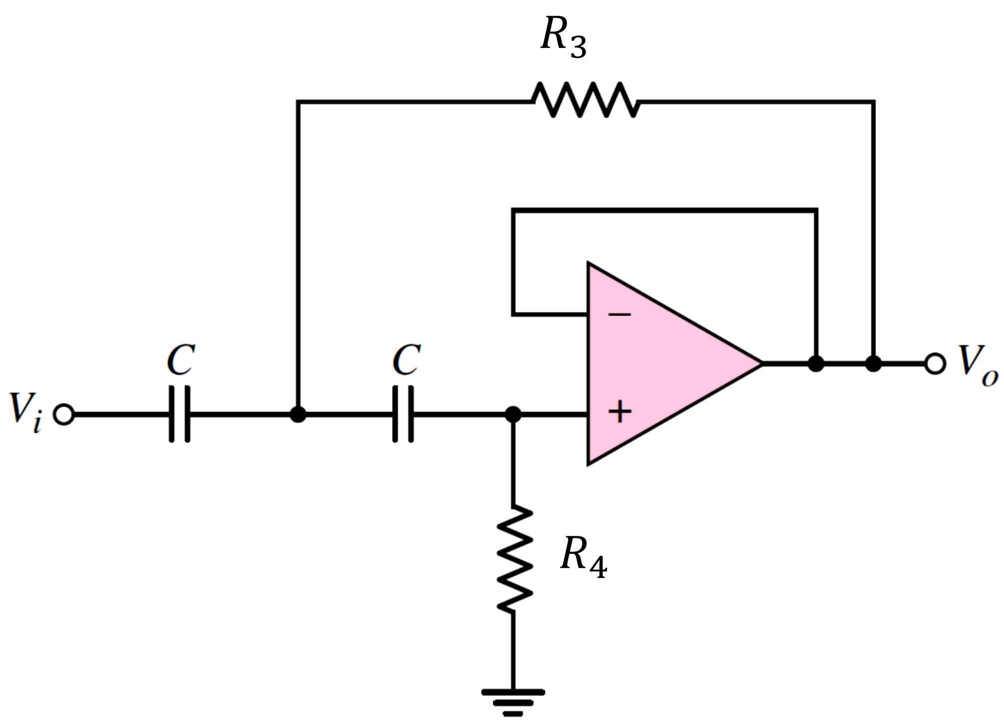
(c) Four-pole

(d) Five-pole filter



Example 03: High-Pass Butterworth Filter

The specification in a high-pass Butterworth filter design is that the voltage transfer function magnitude at $f = 0.9\, f_{3dB}$ is 6 dB below the maximum value. Determine the required order of filter.



Example 04: High-Pass Butterworth Filter

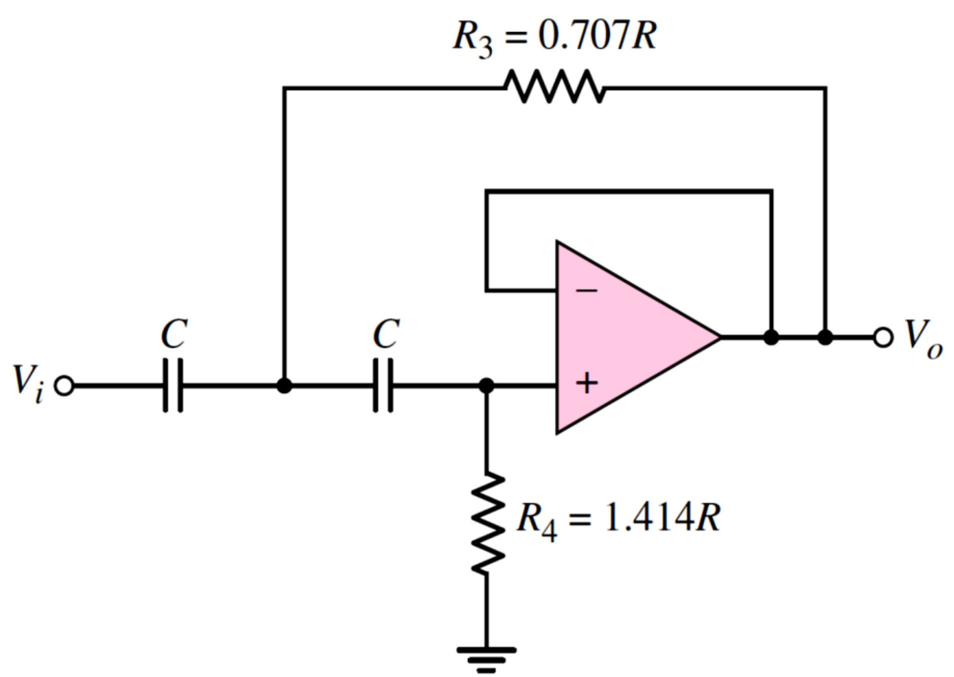
(a) Design a two-pole high-pass Butterworth active filter with a cutoff frequency at $f_{3dB} = 25\ kHz$ and a unity gain magnitude at high frequency.

(b) Determine the magnitude (in dB) of the gain at

(a) $f = 22\ kHz$

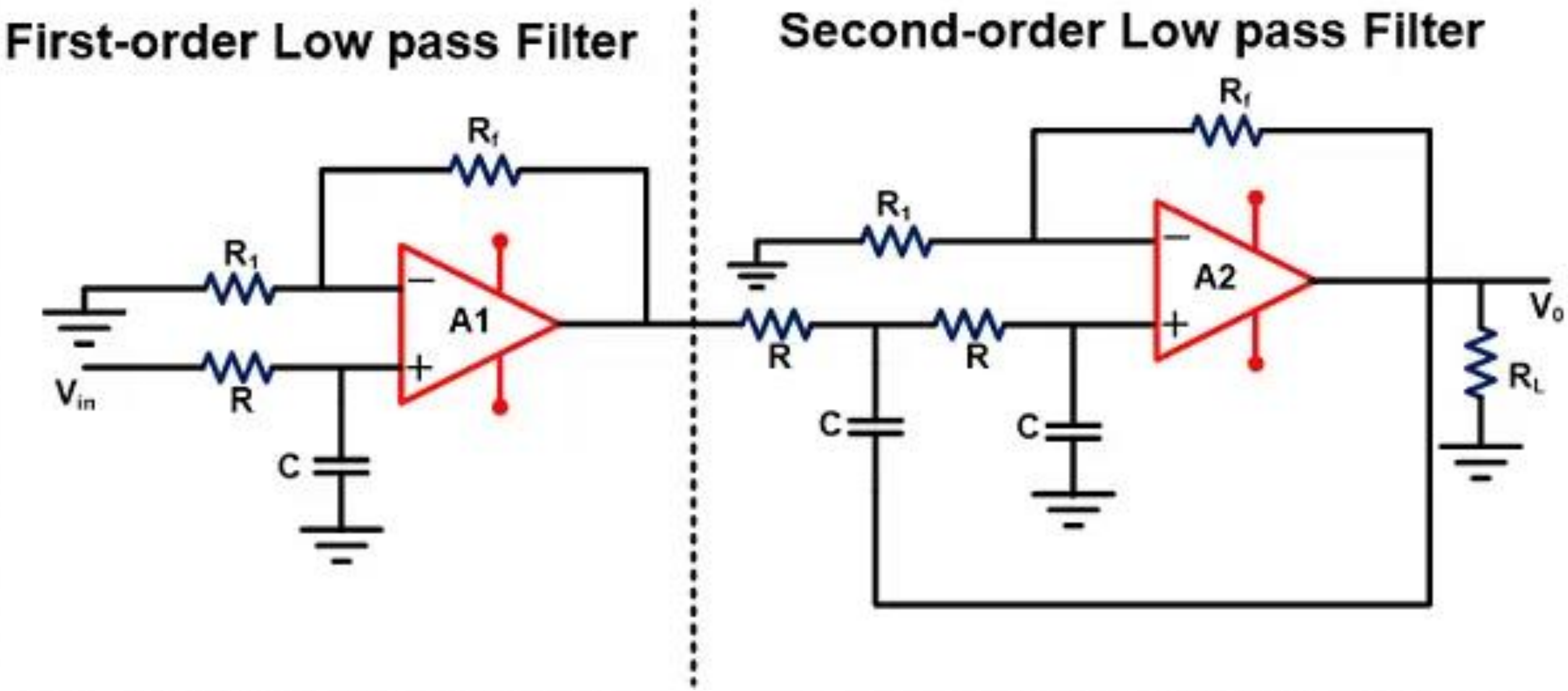
(b) $f = 25\ kHz$

(c) $f = 28\ kHz$



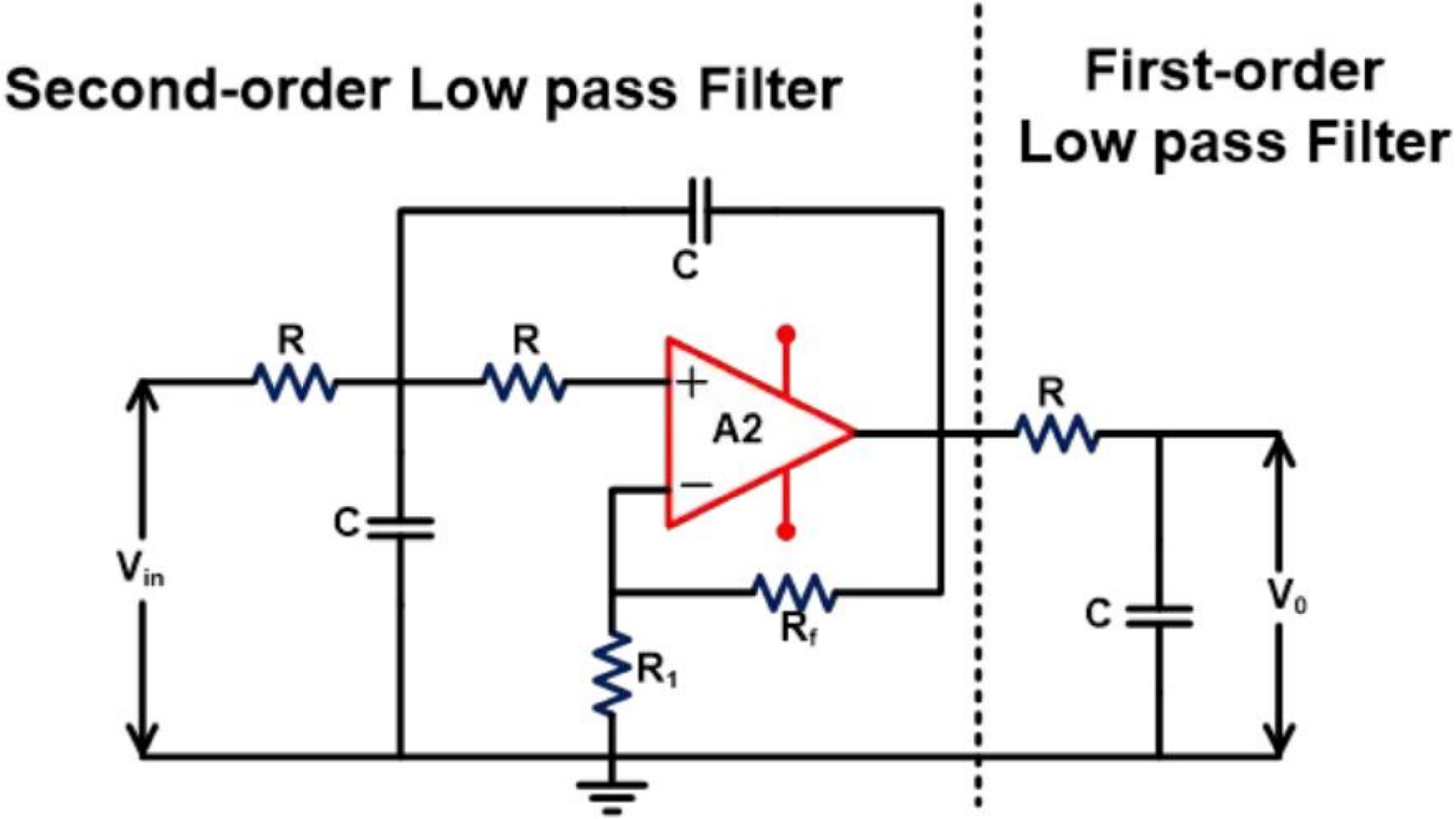
Third-Order Lowpass Butterworth Filter

- Third-order lowpass Butterworth filter can design by cascading the first-order and second-order Butterworth filter.
- The voltage gain of the first part is optional, and it can be set at any value.
- Then the third-order low pass filter can be expressed in different way.



Third-Order Lowpass Butterworth Filter

- Third-order Low Pass Butterworth Filter (with one OP-AMP)



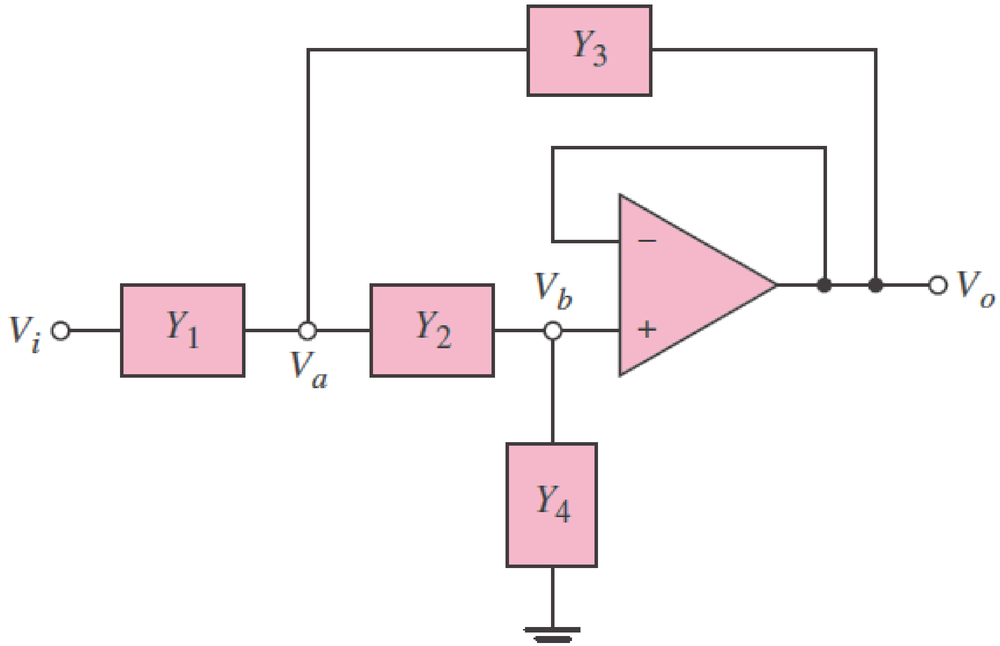
HW

What are the applications of Butterworth filter?

More Examples

Example 05: Two-Pole High-Pass Butterworth Filter

Starting with the general transfer function given below, derive the relationship between R_1 and R_2 in the two-pole high-pass Butterworth active filter.



$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_4(Y_1 + Y_2 + Y_3)}$$

Example 06: Low-Pass Butterworth Filter

A low-pass Butterworth filter is to be designed such that the magnitude of the voltage transfer function at $f = 1.2\, f_{3dB}$ is $14\, dB$ below the maximum gain value. Determine the required order of filter.

Example 07: High-Pass Butterworth Filter

A high-pass Butterworth filter is to be designed with a cutoff frequency of $f_{3dB} = 4\ kHz$. The gain magnitude is to be reduced by $12\ dB$ at $f = 3\ kHz$ from the maximum gain value. Determine the required order of filter.

Example 08: Low-Pass Butterworth Filter Design

A low-pass filter is to be designed to pass frequencies in the 0 to 12 kHz range. The gain of the amplifier is to be $+10$ at the low frequency and change by no more than 10 percent over the frequency range. In addition, the gain of the amplifier for frequencies greater than 14 kHz is to be no greater than 0.1. Determine f_{3dB} and the number of poles required in a Butterworth filter.