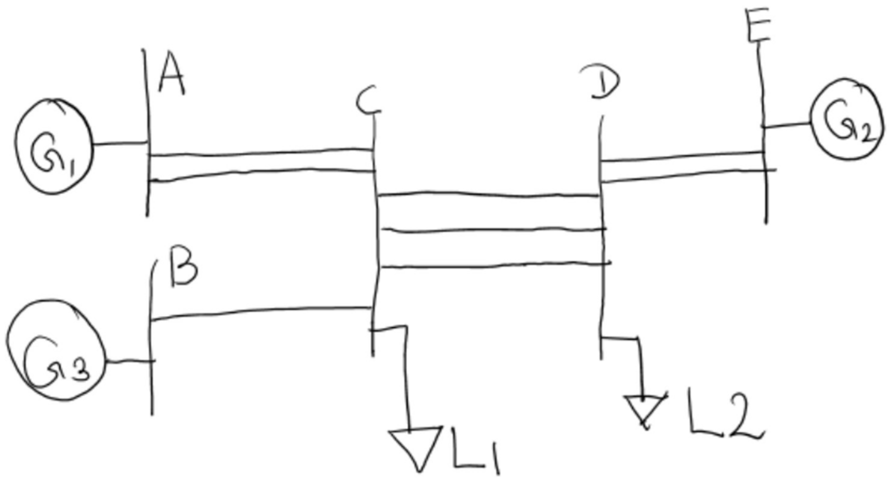
Optimal Power Flow

1 INTRODUCTION

Consider the following simple power system



Three generators (G_1, G_2, G_3) are supplying power to the two load centers L_1 and L_2. How to determine what would be the dispatch of each generator? What should we consider in determining how much each generator is loaded?

- Generators must produce active power to supply the loads and the losses in the network
- Generators could have different operational costs (Hydro generators cost less to operate than a gas turbine).
- Transmitting power generated far from the load could impose stability limits. In this case, even if the generation is cheaper, transmitting that generated power will not be technically feasible.
- In some system conditions such as light load scenarios, some generators may cause transmission line to overloads.
- Most of the generators are intended to operate at a certain active power range (Hydro generators rough running, rated power operation of thermal plants)

The above facts indicate that dispatching generators is a complex task and is dependent on multiple variables. The **main objective** of the system operator who dispatches the generators is to provide electric power **at the least cost** ensuring **system security**.

This task is done by solving an optimization problem with number of constraints. The task of finding the optimum dispatch for minimum cost is known as "**Optimum Power Flow**".

2 REVIEW IN NON-LINEAR FUNCTION OPTIMIZATION

2.1 UNCONSTRAINED PARAMETER OPTIMIZATION

For a cost function $f(x_1, x_2, \ldots, x_n)$ to be minimized, derivative of f with respect to variables must be zero (lowest in the valley).

$$\frac{\partial f}{\partial x_i} = 0 \qquad i = 1,2,\ldots,n$$

$$\nabla f = 0$$

Gradient vector $\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right)$. The second derivatives of the function f leads to the Hessian matrix (H) and it's elements are defined as

$$H_{i,j} = \frac{\partial^2 f}{\partial x_i \partial x_j}$$

Once the **derivative of f vanishes at local extrema** $(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ for f to have a relative minimum, the Hessian matrix evaluated at $(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ needs to be **positive definite matrix**. That is, the **eigen values of Hessian matrix** at the local extrema must be **positive**.

Example:

Find the minimum of the following function

$$f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + 3x_3^2 + x_1x_2 + x_2x_3 - 8x_1 - 16x_2 - 32x_3 + 110$$

2.2 CONSTRAINED PARAMETER OPTIMIZATION: EQUALITY CONSTRAINTS

This problem is raised when there are functional dependencies among the parameters. That is now the problem is to minimize the cost function $f(x_1, x_2, \ldots, x_n)$ subjected to $g_i(x_1, x_2, \ldots, x_n) = 0 \quad i = 1,2,\ldots,k$.

Solving by **Lagrange multiplier** method. This method created a new cost function by introducing k vector λ of undetermined quantities. The new unconstrained cost function $\mathcal{L}$

$$\mathcal{L} = f + \sum_{i=1}^{k} \lambda_i g_i$$

Necessary conditions for **constrained local minima** of $\mathcal{L}$

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{i=1}^{k} \lambda_i \frac{\partial g_i}{\partial x_i} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = g_i = 0$$

Example:

Find the minimum distance from origin of an xy plane to the circle described by $(x-8)^2+(y-6)^2=25$.

Hints:

$$f(x,y)=x^2+y^2$$

$$g=(x-8)^2+(y-6)^2-25$$

2.3 Constrained Parameter Optimization: Inequality Constraints

The problem now becomes to minimize the cost function $f(x_1,x_2,\dots,x_n)$ subjected to the

Equality constraints $\quad g_i(x_1,x_2,\dots,x_n)=0 \quad i=1,2,\dots,k$

And inequality constraints $\quad u_j(x_1,x_2,\dots,x_n)\le 0 \quad j=1,2,\dots,m$

The Lagrange multiplier method is now extended further to include the inequality constraints by introducing m vector μ of undetermined quantities. The new augmented unconstrained cost function

$$\mathcal{L}=f+\sum_{i=1}^{k}\lambda_i g_i+\sum_{j=1}^{m}\mu_j u_j$$

Necessary requirements for a local minima

$$\frac{\partial\mathcal{L}}{\partial x_i}=0 \qquad i=1,2,\dots,n$$

$$\frac{\partial\mathcal{L}}{\partial\lambda_i}=g_i=0 \qquad i=1,2,\dots,k$$

$$\frac{\partial\mathcal{L}}{\partial\mu_j}=u_j\le 0 \qquad j=1,2,\dots,m$$

$$\mu_j u_j=0 \;\;\&\;\; \mu_j>0 \qquad j=1,2,\dots,m$$

3 Operating Cost of a Thermal Plant

Factors that affect minimum cost generation

- Generator efficiency
- Fuel cost
    - Efficient generator located in a place where fuel is expensive may not result in the minimum cost generation.
- Transmission losses

- Efficient generator located in a cheap fuel region may still result in expensive generation if the loads are located far from generation.

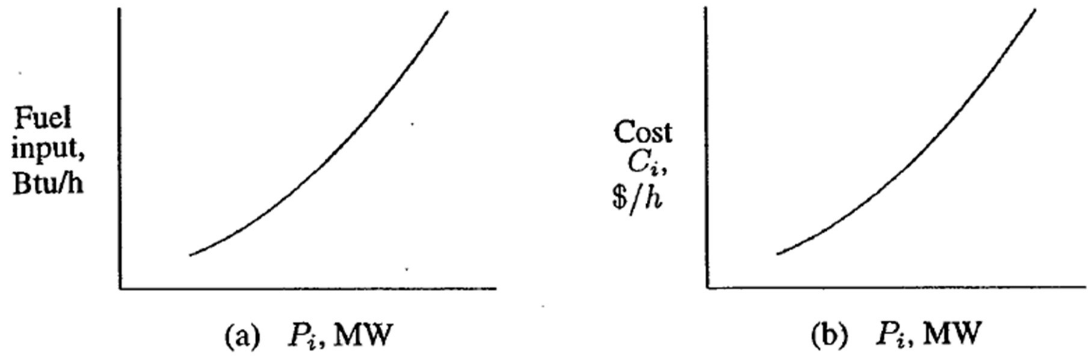


Figure 1. (a) heat rate curve, (b) fuel cost curve.

In practical cases, fuel cost of generator i can be described by a quadratic curve.

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

Incremental fuel cost curve

Plot of derivative of fuel cost against real power.

$$\frac{dC_i}{dP_i} = 2\gamma_i P_i + \beta_i$$

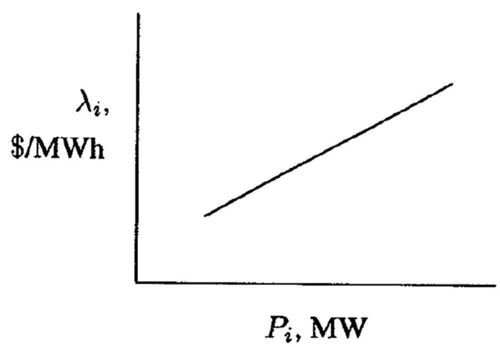


Figure 2. Example incremental fuel cost curve.

This curve is a measure of how expensive it is to produce next increment in power.

- A flat curve will mean, the cost increase is low and can increase more power with reduced cost.

Total operating cost includes

- Fuel cost
- Cost of labour
- Supplies
- Maintenance

These costs are assumed to be a **fixed percentage of the fuel costs** and included in the incremental fuel cost.

4 ECONOMIC DISPATCH NEGLECTING LOSSES AND GENERATOR LIMITS

System assumes no transmission network and only one bus exists.

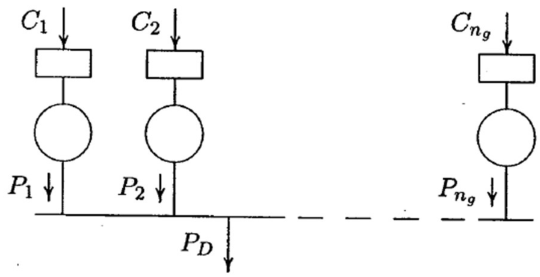


Figure 3. Representation of the generators in the power system.

As there are no losses, total generation equals, the total load. Each generating plant has its own cost function C_i. The problem we have to solve is to find out the real power generation of each power plant to minimize the total production cost (objective function)

Minimize $C_t = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \alpha_i + \beta_i P_i + \gamma_i P_i^2$ subjected to the constraint $\sum_{i=1}^{n} P_i = P_D$.

We can augment the constraints into the objective function using Lagrange multipliers.

$$\mathcal{L} = C_t + \lambda\left(P_D - \sum_{i=1}^{n} P_i\right)$$

At minimum location (when cost is minimum) the following conditions must be true

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0 \ \rightarrow (first\ condition)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \rightarrow (second\ condition)$$

The first condition results in

$$\frac{\partial C_t}{\partial P_i} + \lambda(0-1) = 0$$

Since $C_t = C_1 + C_2 + \cdots + C_n$

$$\frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda$$

For optimum dispatch $\frac{dC_i}{dP_i} = \lambda \qquad i = 1,2, \ldots, n$

$$\text{Or } \beta_i + 2\gamma_i P_i = \lambda$$

Second condition results in $\sum_{i=1}^{n} P_i = P_D$. This is the equality constraint that was imposed.

In simple words, for most economic operation, all generating plants must operate at **equal incremental production cost**, while the total generation **matching the total demand.**

Coordination equations $P_i = \frac{\lambda - \beta_i}{2\gamma_i}$

Solving for λ analytically

$$\lambda = \frac{P_D + \sum_{i=1}^{n} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n} \frac{1}{2\gamma_i}}$$

In general, solving for λ analytically is not practical as the equations become non-linear when losses are considered into the problem. Thus numerical solution methods such as Newton Raphson method provides better solution techniques for the problem.

Total demand as a function of λ

$$f(\lambda) = P_D$$

Expanding left hand side by Taylor's series

$$f(\lambda)^{(k)} + \left(\frac{df(\lambda)}{d\lambda}\right)^{k} \Delta\lambda^{(k)} = P_D$$

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}} = \frac{\Delta P^{(k)}}{\sum \left(\frac{dP_i}{d\lambda}\right)^{(k)}}$$

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\sum \frac{1}{2\gamma_i}}$$

Thus

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)}$$

Where

$$\Delta P^{(k)} = P_D - \sum_{i=1}^{n} P_i^{(k)}$$

The calculation is continued until $\Delta P^{(k)}$ is less than a specified value.

Example

Cost functions of three plants in $/hr

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2$$

$$C_2 = 400 + 5.2P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

P_1, P_2, P_3 are the plant active power outputs in MW. Total demand is 800 MW. Find the optimal dispatch neglecting losses.

5 Economic Dispatch Neglecting Losses but Including Generator Limits

Generator operation is bound by its capability curve. Based on the generation technology it has minimum and maximum active power ranges that the turbine can operate (ie. Thermal plants operate at rated power without changes in active power, hydro stations operating at a larger active power set point range). Now the problem is to find a generation dispatch which minimizes total production costs subjected to generator power output constrains given by

$$P_{i,min} \leq P_i \leq P_{i,max} \quad i = 1,2, \dots, n$$

The Kuhn-Tucker conditions complement the Lagrangian conditions to include the inequality constraints as additional terms. Therefore

$$\frac{dC_i}{dP_i} = \lambda \qquad for\ \ P_{i,min} \leq P_i \leq P_{i,max}$$

$$\frac{dC_i}{dP_i} \leq \lambda \qquad for\ \ P_i = P_{i,max}$$

$$\frac{dC_i}{dP_i} \geq \lambda \qquad for\ P_i = P_{i,min}$$

When evaluated numerically, for an estimated λ generator power outputs are found until the total generation is equal to total demand. However when a generator reaches its maximum or minimum limit, the generator output is made constant at the reached limit value. Then only the unviolated generators must operate at equal incremental costs.

Example

Cost functions of three plants in $/hr

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2$$

$$C_2 = 400 + 5.2P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

P_1, P_2, P_3 are the plant active power outputs in MW. Total demand is 975 MW. Find the optimal dispatch neglecting losses provided that the generators must operate within the following limits.

$$200 \leq P_1 \leq 450$$

$$150 \leq P_2 \leq 350$$

$$100 \leq P_3 \leq 225$$