Development and Comparative Analysis of Load Flow Algorithms on the IEEE 9-Bus Test System

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Abstract

This report documents the implementation of a full Newton-Raphson load-flow solver for the IEEE 9-bus system, followed by method-level comparison with PSSE and voltage sensitivity analysis under independent load variations. All derivations, simulation logic, and result discussions are presented in technical form with reproducible tables and figure placeholders linked to the code workflow.

CONTENTS

I. Task 1: Program Development

A. Objective and Scope

The first task is to develop a full Newton-Raphson (NR) load-flow program from first principles for the IEEE 9-bus network. The implementation reads bus, generator, line, transformer, and load data from a JSON file and computes bus voltages, power mismatches, line flows, and total system losses.

B. Mathematical Formulation

The nonlinear power-balance equations are solved in polar form using NR iteration [**?**], [**?**]:

$$P_i = \sum_{k=1}^{n} |V_i||V_k| \left(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik} \right), \tag{1}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| \left(G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik} \right), \tag{2}$$

with mismatch vector

$$\Delta\mathbf{f} = \begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix}, \tag{3}$$

and update equation

$$\mathbf{J}\Delta\mathbf{x} = \Delta\mathbf{f}, \quad \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x}, \tag{4}$$

where

$$\mathbf{J} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}. \tag{5}$$

The solution uses:

- Flat start for unknown states.
- Bus 1 as slack bus.
- Buses 2 and 3 as PV buses.
- Buses 4, 5, 6, 7, 8, and 9 as PQ buses.
- Convergence tolerance of 10^{-4} p.u. on maximum mismatch.

C. Program Workflow

Figure 1 is the report placeholder for the flowchart derived from the notebook logic.

Placeholder: Insert Task 1 flowchart image exported from the notebook or drawing tool. Include blocks for input read, Y-bus build, mismatch evaluation, Jacobian formation, linear solve, state update, convergence check, and final reporting.

Fig. 1: Task 1 Newton-Raphson program flowchart.

D. Base-Case Output Summary (Own Run)

Table I summarizes the convergence behavior from the implemented solver.

TABLE I: Task 1 convergence summary for IEEE 9-bus base case.

Item	Value
Tolerance	10^{-4} p.u.
Total iterations to convergence	4
Total active power loss	4.641023 MW
Total reactive power loss	-92.160126 MVAr

Table II is reserved for the assignment requirement to show the second-iteration snapshot.

TABLE II: Second-iteration snapshot (copy values from notebook table).

Bus	$\lvert V\rvert$ at iter-2 (p.u.)	Angle at iter-2 (deg)	Notes
1	Insert from notebook output		
2	Insert from notebook output		
3	Insert from notebook output		
4	Insert from notebook output		
5	Insert from notebook output		
6	Insert from notebook output		
7	Insert from notebook output		
8	Insert from notebook output		
9	Insert from notebook output		

Table III gives final voltage magnitudes and angles from the executed notebook.

TABLE III: Final bus voltage profile from custom NR solver.

Bus	$\lvert V\rvert$ (p.u.)	Angle (deg)
1	1.040000	0.000000
2	1.025000	9.280008
3	1.025000	4.664753
4	1.025788	-2.216787
5	0.995631	-3.988804
6	1.012654	-3.687395
7	1.025769	3.719704
8	1.015883	0.727538
9	1.032353	1.966718

E. Technical Discussion

The implemented NR solver converged in four iterations with stable mismatch reduction. This behavior is consistent with the expected quadratic convergence near the solution for a well-conditioned Jacobian [**?**]. The final voltage profile is physically consistent with generator voltage control at PV buses and lower voltages near major loads. The loss values and voltage levels are also aligned with typical IEEE 9-bus benchmark behavior reported in power-system literature [**?**].

II. Task 2: Verification and Comparison

A. Comparison Objective

This section compares the custom NR solver outputs against PSSE results obtained using:

- Full Newton-Raphson,
- Gauss-Seidel,
- Fast Decoupled Load Flow.

The focus is on numerical accuracy, convergence behavior, and discussion of deviations [?], [?].

B. Required Data to Insert

Populate this section after running PSSE using the same network and loading conditions used in Task 1.

TABLE IV: Bus voltage magnitude comparison (custom NR vs PSSE methods).

Bus	Custom NR	PSSE NR	PSSE GS	PSSE FDLF
1				
2				
3				
4				
5				
6				
7				
8				
9				

TABLE V: Bus angle comparison (degrees).

Bus	Custom NR	PSSE NR	PSSE GS	PSSE FDLF
1				
2				
3				
4				
5				
6				
7				
8				
9				

TABLE VI: Convergence and loss comparison across methods.

Method	Iterations	Total P_{loss} (MW)	Notes
Custom NR			
PSSE NR			
PSSE GS			
PSSE FDLF			

C. Figure Placeholders (to be filled after PSSE runs)

Placeholder for Figure 1: Voltage magnitude comparison across four methods.

Fig. 2: Bus voltage magnitude comparison: custom solver and PSSE methods.

Placeholder for Figure 2: Voltage angle comparison across four methods.

Fig. 3: Bus voltage angle comparison: custom solver and PSSE methods.

Placeholder for Figure 3: Iteration-count comparison for convergence behavior.

Fig. 4: Convergence comparison using iteration counts.

Placeholder for Figure 4: Absolute error profile against PSSE NR baseline.

Fig. 5: Absolute voltage and angle deviations relative to PSSE NR.

D. Discussion Template

Discuss the following points using your own PSSE data:

- Agreement between custom NR and PSSE NR for both $|V|$ and angle.
- Iteration trends: expected faster convergence of NR and FDLF compared with GS.
- Numerical differences due to model assumptions (tap details, tolerances, initialization, and stopping criteria).
- Practical interpretation: trade-off between computational speed and numerical robustness.

Use this section to write a technical paragraph-based comparison in your own words to satisfy submission originality requirements.

III. Task 3: Voltage Sensitivity Analysis

A. Method

Each load bus was independently perturbed by scaling both active and reactive load by

$$\alpha \in \{-10\%, 0\%, +10\%\},$$

while keeping all other loads unchanged. For each case, the custom NR solver was executed and the bus-voltage magnitudes were recorded.

For each target load bus, variance and standard deviation of voltage magnitudes were computed:

$$\mathrm{Var}(V_b) = \frac{1}{N}\sum_{m=1}^{N}(V_{b,m} - \bar{V}_b)^2, \tag{6}$$

$$\sigma(V_b) = \sqrt{\mathrm{Var}(V_b)}. \tag{7}$$

An aggregate ranking index was used:

$$\mathrm{RMS\_STD} = \sqrt{\frac{1}{n_b} \sum_{b=1}^{n_b} \sigma(V_b)^2}. \tag{8}$$

B. Sensitivity Ranking from Executed Notebook

Table VII reports the ranking metrics from the current notebook run.

TABLE VII: Sensitivity ranking using voltage standard-deviation metrics.

Target Load Bus	Mean Std Dev	Max Std Dev	RMS Std Dev
5	0.001193	0.005068	0.001910
6	0.000736	0.003437	0.001250
8	0.000589	0.002707	0.001012

The highest influence on overall bus-voltage spread is observed for **load bus 5**.

C. Required Voltage-Variation Tables

TABLE VIII: Voltage magnitudes under independent load variations (Bus 5 target case).

Variation (%)	Bus index	Voltage magnitude (p.u.)	Remarks
-10	1–9	Insert from notebook sensitivity table	
0	1–9	Insert from notebook sensitivity table	
+10	1–9	Insert from notebook sensitivity table	

TABLE IX: Voltage magnitudes under independent load variations (Bus 6 target case).

Variation (%)	Bus index	Voltage magnitude (p.u.)	Remarks
-10	1–9	Insert from notebook sensitivity table	
0	1–9	Insert from notebook sensitivity table	
+10	1–9	Insert from notebook sensitivity table	

TABLE X: Voltage magnitudes under independent load variations (Bus 8 target case).

Variation (%)	Bus index	Voltage magnitude (p.u.)	Remarks
-10	1–9	Insert from notebook sensitivity table	
0	1–9	Insert from notebook sensitivity table	
+10	1–9	Insert from notebook sensitivity table	

D. Figure Placeholders

Placeholder:
Insert voltage profile plot generated in notebook cell "Task-3: Bus Voltage Profiles under Independent Load Variations".

Fig. 6: Bus-voltage profiles for each target load bus under -10%, 0%, and +10% load variation.

Placeholder:
Insert sensitivity ranking bar chart generated in notebook cell "Task-3 Sensitivity Ranking".

Fig. 7: Sensitivity ranking by RMS standard deviation index.

E. Discussion

The sensitivity analysis indicates that perturbations at bus 5 produce the largest voltage spread throughout the network. This result is consistent with the bus location relative to major power-transfer paths and local reactive demand. Buses 6 and 8 show lower system-wide influence for the same ±10% perturbation range.

For final submission, expand this section to a 2–3 page technical discussion that includes:

- Why bus 5 is more influential from a network perspective.
- How voltage profile changes vary by electrical distance.
- Practical implications for voltage support and load management.