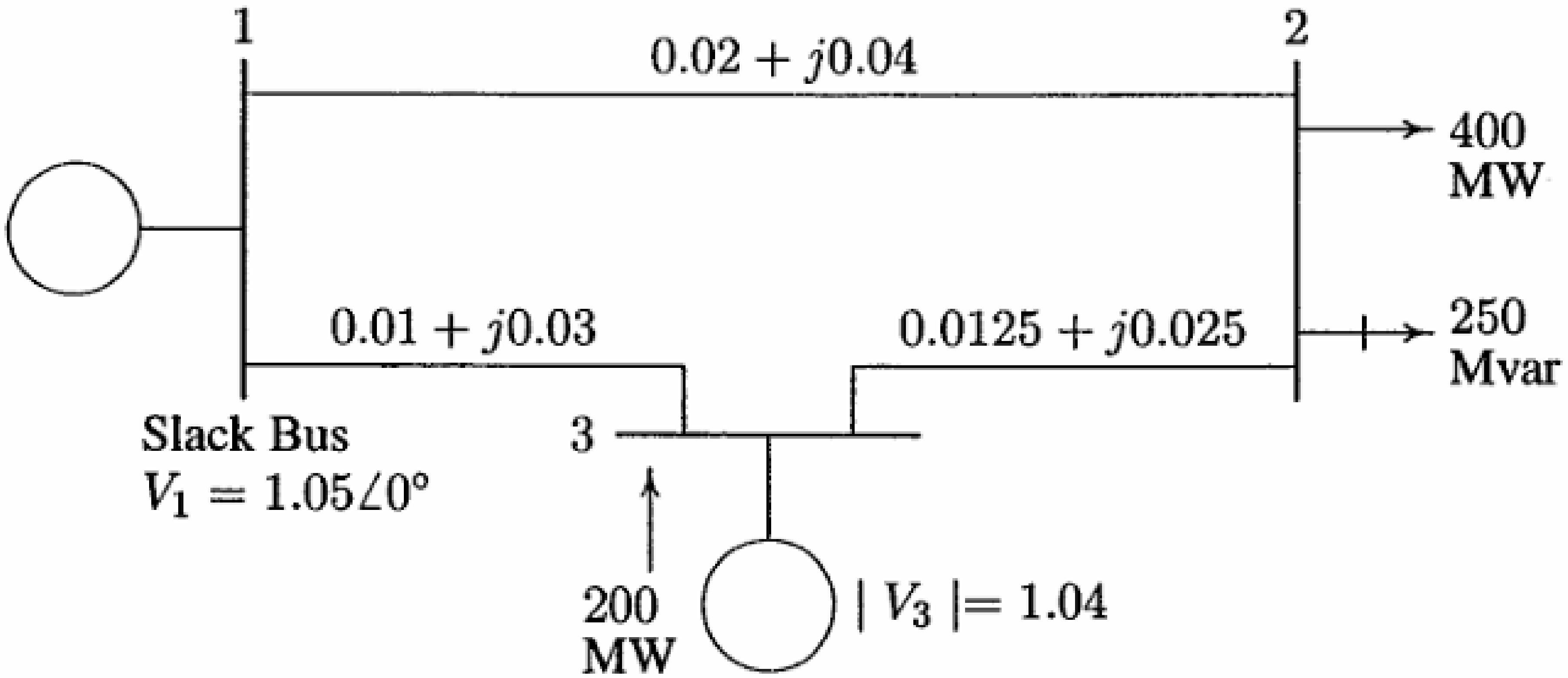
Examples on Loadflow

Gauss Seidel Method

Problem



1
2
0.02 + j0.04
400
MW
0.01 + j0.03
0.0125 + j0.025
250
Mvar
Slack Bus
$V_1 = 1.05\angle 0°$
3
200
MW
$|V_3| = 1.04$

Solution

- Find admittances
  - $y_{12} = 10 - j20, y_{13} = 10 - j30, y_{23} = 16 - j32$
- Load at bus 2
  - $S_2^{sch} = -\frac{(400+j250)}{100} = -4.0 - j2.5\ pu$
- Generation at bus 3
  - $P_3^{sch} = \frac{200}{100} = 2.0\ pu$
- Bus 1 is considered slack
- Initial estimates
  - $V_2^{(0)} = 1.0 + j0.0, V_3^{(0)} = 1.04 + j0.0$

Solution

- Calculate next iteration of V_2

$$V_i^{(k+1)} = \frac{\frac{P_i^{sch} - jQ_i^{sc}}{V_i^{*(k)}} + \sum y_{ij} V_j^{(k)}}{\sum y_{ij}} \qquad j \neq i$$

$$\begin{aligned} V_2^{(1)} &= \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0+j2.5}{1.0-j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.04 + j0)}{(26 - j52)} \\ &= 0.97462 - j0.042307 \end{aligned}$$

- Bus 3 voltage is regulated. The reactive power first needs to be calculated

$$\begin{aligned} Q_3^{(1)} &= -\Im\{V_3^{*(0)}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\} \\ &= -\Im\{(1.04 - j0)[(1.04 + j0)(26 - j62) - (10 - j30)(1.05 + j0) - \\ &\quad (16 - j32)(0.97462 - j0.042307)]\} \\ &= 1.16 \end{aligned}$$

- $Q_3^{(1)}$ will be used as Q_3^{sch} in the voltage equation

Solution

- Complex voltage at bus 3. Note that $V_2^{(1)}$ is used, not the initial voltage

$$V_{c3}^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{2.0 - j1.16}{1.04 - j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.97462 - j0.042307)}{(26 - j62)}$$

$$= 1.03783 - j0.005170$$

- Calculate real component of bus 3 voltage keeping the magnitude at desired level

$$e_3^{(1)} = \sqrt{(1.04)^2 - (0.005170)^2} = 1.039987$$

$$V_3^{(1)} = 1.039987 - j0.005170$$

Solution

- Second iteration

$$V_2^{(2)} = \frac{\frac{P_2^{sch}-jQ_2^{sch}}{V_2^{*(1)}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12}+y_{23}}$$

$$= \frac{\frac{-4.0+j2.5}{.97462+j.042307} + (10-j20)(1.05) + (16-j32)(1.039987+j0.005170)}{(26-j52)}$$

$$= 0.971057 - j0.043432$$

$$Q_3^{(2)} = -\Im\{V_3^{*(1)}[V_3^{(1)}(y_{13}+y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\}$$

$$= -\Im\{(1.039987+j0.005170)[(1.039987-j0.005170)(26-j62) - (10-j30)(1.05+j0) - (16-j32)(0.971057-j0.043432)]\}$$

$$= 1.38796$$

$$V_{c3}^{(2)} = \frac{\frac{P_3^{sch}-jQ_3^{sch}}{V_3^{*(1)}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13}+y_{23}}$$

$$= \frac{\frac{2.0-j1.38796}{1.039987+j0.00517} + (10-j30)(1.05) + (16-j32)(.971057-j.043432)}{(26-j62)}$$

$$= 1.03908 - j0.00730$$

$$e_3^{(2)} = \sqrt{(1.04)^2 - (0.00730)^2} = 1.039974$$

$$V_3^{(2)} = 1.039974 - j0.00730$$

Solution

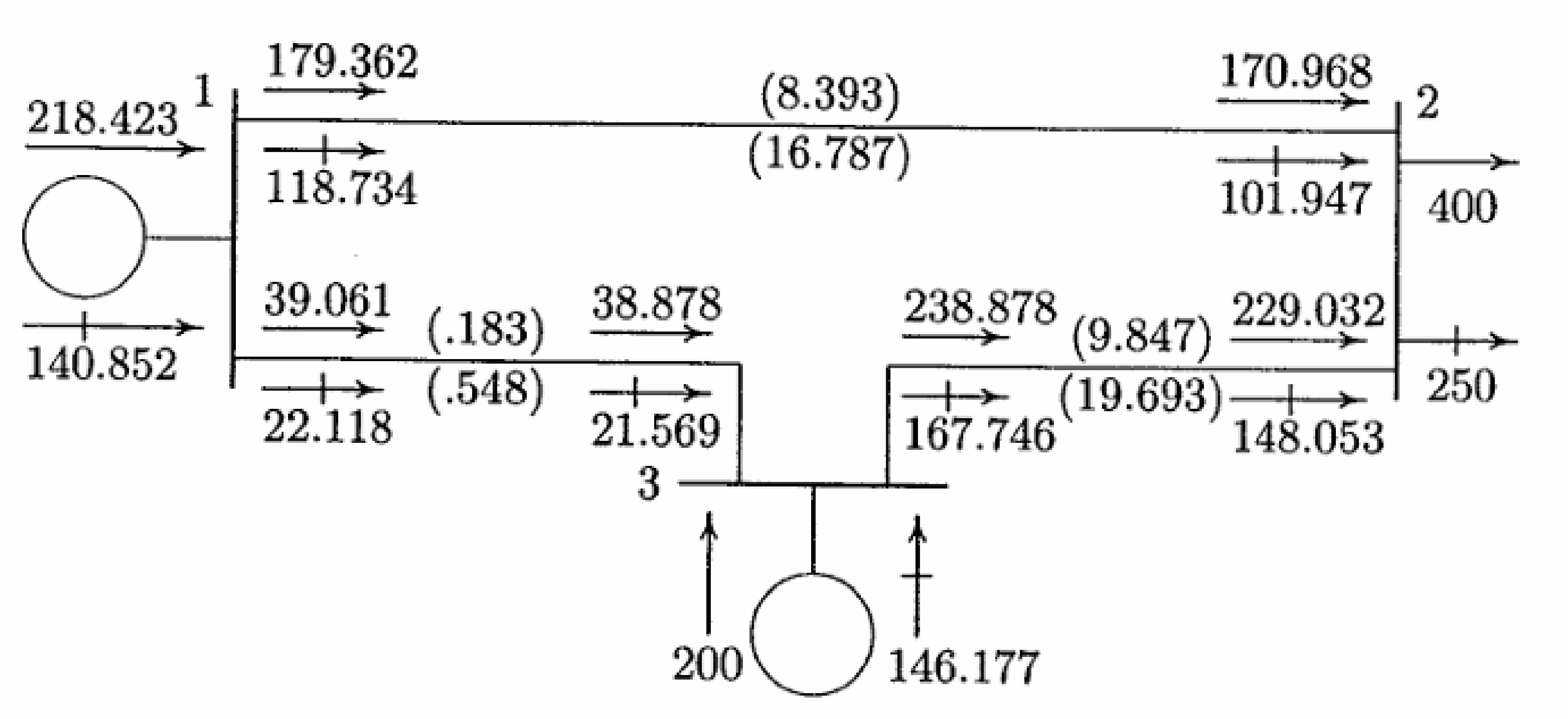
- Itterating until and accuracy of $5 \times 10^{-5}\ pu$ is reached

$$V_2^{(3)} = 0.97073 - j0.04479 \quad Q_3^{(3)} = 1.42904 \quad V_3^{(3)} = 1.03996 - j0.00833$$
$$V_2^{(4)} = 0.97065 - j0.04533 \quad Q_3^{(4)} = 1.44833 \quad V_3^{(4)} = 1.03996 - j0.00873$$
$$V_2^{(5)} = 0.97062 - j0.04555 \quad Q_3^{(5)} = 1.45621 \quad V_3^{(5)} = 1.03996 - j0.00893$$
$$V_2^{(6)} = 0.97061 - j0.04565 \quad Q_3^{(6)} = 1.45947 \quad V_3^{(6)} = 1.03996 - j0.00900$$
$$V_2^{(7)} = 0.97061 - j0.04569 \quad Q_3^{(7)} = 1.46082 \quad V_3^{(7)} = 1.03996 - j0.00903$$

- Power flows

$$S_{12} = 179.36 + j118.734 \quad S_{21} = -170.97 - j101.947 \quad S_{L\,12} = 8.39 + j16.79$$
$$S_{13} = 39.06 + j22.118 \quad S_{31} = -38.88 - j\,21.569 \quad S_{L\,13} = 0.18 + j0.548$$
$$S_{23} = -229.03 - j148.05 \quad S_{32} = 238.88 + j167.746 \quad S_{L\,23} = 9.85 + j19.69$$

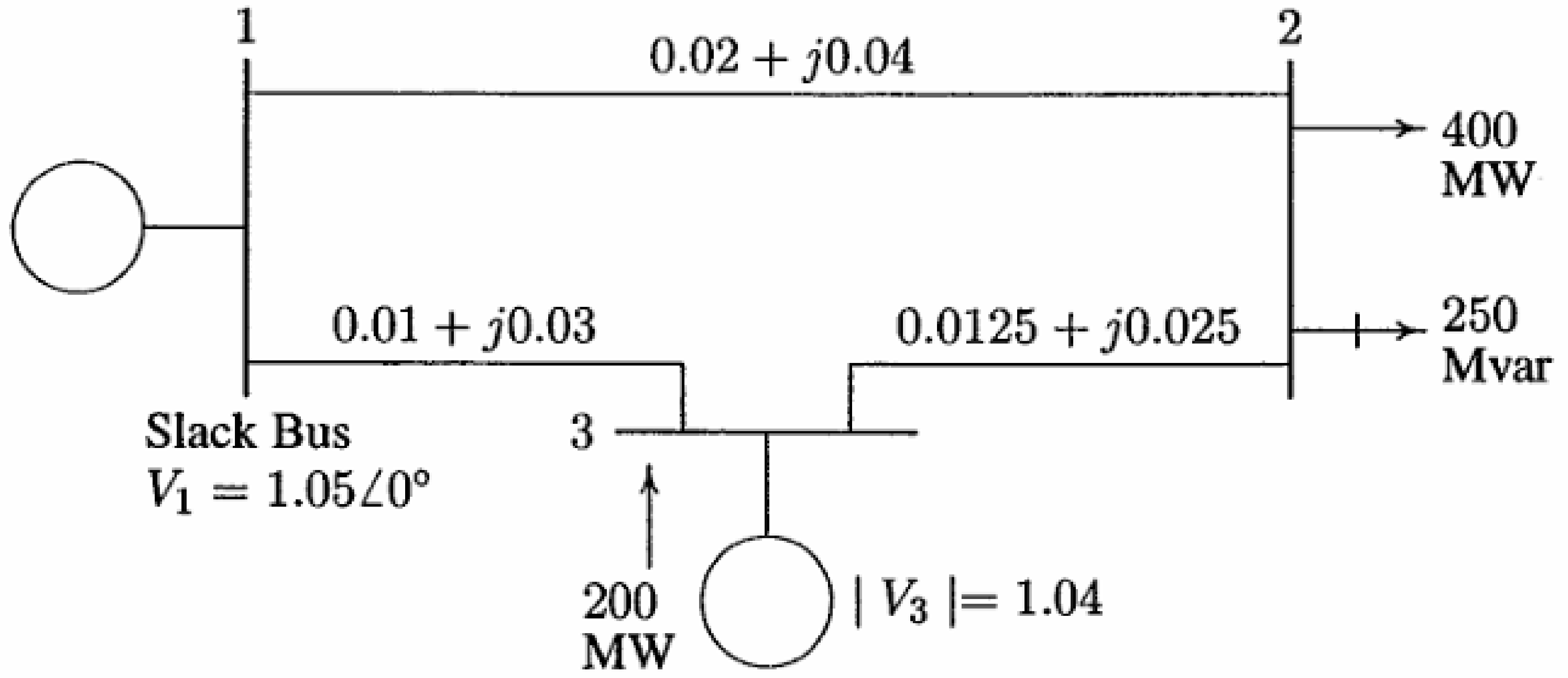
Solution



218.423
1
179.362
118.734
(8.393)
(16.787)
170.968
2
101.947
400
140.852
39.061
22.118
(.183)
(.548)
38.878
21.569
238.878
167.746
(9.847)
(19.693)
229.032
148.053
250
3
200
146.177

Newton Raphson Method

Problem



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Solution

- Find admittances
  - $y_{12} = 10 - j20, y_{13} = 10 - j30, y_{23} = 16 - j32$
- Admittance matrix

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix} \quad Y_{bus} = \begin{bmatrix} 53.85165\angle -1.9029 & 22.36068\angle 2.0344 & 31.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle -1.1071 & 35.77709\angle 2.0344 \\ 31.62278\angle 1.8925 & 35.77709\angle 2.0344 & 67.23095\angle -1.1737 \end{bmatrix}$$

- Real power equations for PV and PQ busses

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22} + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos\theta_{33}$$

- Reactive power equations for PQ busses

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22} - |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

Solution

- Find the elements of the Jacobian Matrix

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos\theta_{22} + |V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}|\sin\theta_{22} - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

Solution

- Load at bus 2
  - $S_2^{sch} = -\frac{(400+j250)}{100} = -4.0 - j2.5\ pu$
- Generation at bus 3
  - $P_3^{sch} = \frac{200}{100} = 2.0\ pu$
- Initial estimates of unknowns
  - $\left|V_2^{(0)}\right| = 1.0,\ \delta_2^{(0)} = 0.0,\ \delta_3^{(0)} = 0.0$
- Active and reactive power residuals

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.8600$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$
$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.2200$$

Solution

- Evaluate the Jacobian with initial estimate and find changes in angle and voltage magnitude for first iteration

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 54.28000 & -33.28000 & 24.86000 \\ -33.28000 & 66.04000 & -16.64000 \\ -27.14000 & 16.64000 & 49.72000 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

- Solving the linear algebraic equations

$$\begin{aligned} \Delta\delta_2^{(0)} &= -0.045263 & \delta_2^{(1)} &= 0 + (-0.045263) = -0.045263 \\ \Delta\delta_3^{(0)} &= -0.007718 & \delta_3^{(1)} &= 0 + (-0.007718) = -0.007718 \\ \Delta|V_2^{(0)}| &= -0.026548 & |V_2^{(1)}| &= 1 + (-0.026548) = 0.97345 \end{aligned}$$

Solution

- Second iteration

$$\begin{bmatrix} -0.099218 \\ 0.021715 \\ -0.050914 \end{bmatrix} = \begin{bmatrix} 51.724675 & -31.765618 & 21.302567 \\ -32.981642 & 65.656383 & -15.379086 \\ -28.538577 & 17.402838 & 48.103589 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

$$\begin{aligned} \Delta\delta_2^{(1)} &= -0.001795 & \delta_2^{(2)} &= -0.045263 + (-0.001795) = -0.04706 \\ \Delta\delta_3^{(1)} &= -0.000985 & \delta_3^{(2)} &= -0.007718 + (-0.000985) = -0.00870 \\ \Delta|V_2^{(1)}| &= -0.001767 & |V_2^{(2)}| &= 0.973451 + (-0.001767) = 0.971684 \end{aligned}$$

- Third iteration

$$\begin{bmatrix} -0.000216 \\ 0.000038 \\ -0.000143 \end{bmatrix} = \begin{bmatrix} 51.596701 & -31.693866 & 21.147447 \\ -32.933865 & 65.597585 & -15.351628 \\ -28.548205 & 17.396932 & 47.954870 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(2)} \\ \Delta\delta_3^{(2)} \\ \Delta|V_2^{(2)}| \end{bmatrix}$$

$$\begin{aligned} \Delta\delta_2^{(2)} &= -0.000038 & \delta_2^{(3)} &= -0.047058 + (-0.0000038) = -0.04706 \\ \Delta\delta_3^{(2)} &= -0.0000024 & \delta_3^{(3)} &= -0.008703 + (-0.0000024) = \ 0.008705 \\ \Delta|V_2^{(2)}| &= -0.0000044 & |V_2^{(3)}| &= 0.971684 + (-0.0000044) = 0.97168 \end{aligned}$$