EE224/283: SIGNALS AND SYSTEMS - HOMEWORK #01

DUE ON TUESDAY MAY 22^{ND}, 2012

(1) Determine the complex exponential Fourier series of the following signals and sketch the magnitude and phase spectra.
   (a) $x_1(t) = 7\sin(5t)$
   (b) $x_2(t) = \cos(4t) + \sin(6t)$
   (c) $x_3(t) = \sin^2(t)$
   (d) $x_4(t) = x_1(t) + x_2(t) + x_3(t)$

(2) If a signal $x(t)$ is real, even symmetric and has a period of T_0 then $x(t)$ can be written as

$$x(t) = a_0 + 2a_1\cos(1\omega_0 t) + 2a_2\cos(2\omega_0 t) + \dots \quad \text{where } a_k = \frac{1}{T_0}\int_{T_0} x(t)\cos(k\omega_0 t)\,\mathrm{d}t.$$

That is, the complex exponential Fourier Series is reduced to a real cosine series. Using this, find an expression for the k^{th} coefficient a_k of the cosine series for $x(t)$ where

$$g(t) = \begin{cases} 1+\cos(2\pi t) & \text{for } |t| \le 0.5 \\ 0 & \text{elsewhere} \end{cases}$$

and $x(t) = \sum_{k=-\infty}^{\infty} g(t-2k)$. Evaluate a_0, a_1 and a_2.

(3) Let

$$x(t) = \begin{cases} t & \text{for } 0 \le t < 1 \\ 2-t & \text{for } 1 \le t < 2 \end{cases}$$

be a periodic signal with fundamental period $T = 2$ and Fourier Series coefficient X_k.
   (a) Determine X_0
   (b) Determine the Fourier series expansion of $\frac{\mathrm{d}x(t)}{\mathrm{d}t}$
   (c) Using the result in problem 3b and the differentiation property determine the Fourier series coefficients of $x(t)$.

(4) Let

$$x(t) = \begin{cases} 1 & \text{for } |t| < T_1 \\ 0 & \text{for } T_1 \le |t| < T/2 \end{cases}$$

be a periodic signal with fundamental period T and Fourier Series coefficient X_k.
   (a) Determine X_0 and X_k for $k \neq 0$.
   (b) If

$$y(t) = \begin{cases} 1 & \text{for } 0 \le t < 2 \\ 0 & \text{for } 2 \le t < 4 \end{cases}$$

   is periodic with $T = 4$. Sketch $y(t)$ and use the result in 4a to determine the expressions for Y_0 and Y_k by selecting suitable vales for T_1 and T.
   (c) What is the fundamental frequency ω_0 for the $y(t)$ in 4b. Express $y(t)$ in terms of Y_k, substituting appropriate values for ω_0. That is, write down the Fourier series of $y(t)$.
   (d) Let $g(t) = y(2t)$. Sketch $g(t)$. What is the fundamental frequency ω_0 for $g(t)$? Use the result in 4b to derive G_0 and G_k and write down the Fourier series expansion of $g(t)$.
   (e) Sketch $m(t) = y(t) + g(t)$. Find the Fourier series coefficients M_0, M_1 and M_2 of $m(t)$ using the results in 4b–4d.

(5) Let

$$x(t) = \begin{cases} 0 & \text{for } t < -\frac{1}{2} \\ t + \frac{1}{2} & \text{for } -\frac{1}{2} \le t \le \frac{1}{2} \\ 1 & \text{for } t > \frac{1}{2} \end{cases} \quad \text{and} \quad y(t) = \begin{cases} 0 & \text{for } t < -\frac{1}{2} \\ t + \frac{1}{2} & \text{for } -\dfrac{1}{2} \le t \le \frac{1}{2} \\ 0 & \text{for } t > \frac{1}{2} \end{cases}$$

(a) Find the Fourier transform of $\dfrac{\mathrm{d}x(t)}{\mathrm{d}t}$.

(b) Using the result in 5a and an appropriate property of Fourier transform to find the Fourier transform of $x(t)$.

(c) Find the Fourier transform of $\dfrac{\mathrm{d}y(t)}{\mathrm{d}t}$.

(d) Using the result in 5c and an appropriate property of Fourier transform to find the Fourier transform of $y(t)$.

(e) Find the Fourier transform of $g(t) = x(t) - \frac{1}{2}$.

(6) (a) Show that $\mathcal{F}^{-1}\left[\dfrac{\mathrm{d}X(j\omega)}{\mathrm{d}\omega}\right] = (-jt)x(t)$. This is the differentiation property of the Fourier transform in the frequency domain.

(b) Using the Fourier transform integral, find the Fourier transform of $x(t) = \mathrm{e}^{-|t|}$.

(c) Using the property derived in 6a and the result in 6b, find the Fourier transform of $t\mathrm{e}^{-|t|}$.

(d) If $X(j\omega)$ is the Fourier transform of $x(t)$, using the property $\mathcal{F}[X(t)] = 2\pi x(-\omega)$ (this is called the duality property), find the Fourier transform of $\dfrac{4t}{(1+t^2)^2}$.

(7) Let

$$x(t) = \begin{cases} 0 & \text{for } |t| > 1 \\ (1+t)/2 & \text{for } -1 \le t \le 1 \end{cases}$$

(a) Determine the Fourier transform $X(j\omega)$ of $x(t)$ using the differentiation and integration property of Fourier transform.

(b) What is $\mathrm{Re}\,[X(j\omega)]$? Show that it is equal to the even part of $x(t)$.

(c) What is the Fourier transform of the odd part of $x(t)$.