# Loves Me, Loves Me Not - No Dispersal

**Proposition:** Let $T$ be a configuration on the graph $G$ with $T(v) \leq 1$ for all $v \in G$. If the weight of $T$ is odd, then it is an $\mathcal{N}$-position. If the weight of $T$ is even, then it is a $\mathcal{P}$-position.
***Proof:*** We proceed by induction on the number of nonzero vertices. In the base case where there is only one such vertex, it is clearly a win for the first player, making it an $\mathcal{N}$-position.

Because each heap contains at most one token, a move cannot redistribute tokens to adjacent vertices. In particular, if we start with $n$ vertices with one token each, the only possible moves are to remove one of the one-token heaps from the configuration, yielding another configuration $T'$ where $T'(v) \leq 1$ for all $v \in G$, but with $n-1$ nonzero vertices.

Suppose that for all configurations with odd weights less than $n$ are $\mathcal{N}$-positions and all configurations with even weights less than $n$ are $\mathcal{P}$ positions. Then, if the weight $n$ of $T$ is even, then the following configuration $T'$ must have weight $n-1$, which is odd, making $T'$ a $\mathcal{N}$-position. Since any move leads to an $\mathcal{N}$-position, $T$ is a $\mathcal{P}$-position.

On the other hand, if the weight $n$ of $T$ is odd, then the following configuration $T'$ must have weight $n-1$, which is even, making it a $\mathcal{P}$-position. Since we have found a move to a $\mathcal{P}$-position, $T$ is an $\mathcal{N}$-position.