

Laboratory assignment – Project Brief

Context: You have been contracted by Purple Minions Ltd to do some control design for their manufacturing assembly line – which creates cookie robots for nefarious infiltration and spying – and particularly a cart mounted robot which moves back and forth along a track on an automated assembly line (e.g. see an industrial example at <https://www.youtube.com/watch?v=A-CDfddh2jQ>).

For each cookie robot two assembly tasks 100mm apart are performed using the cart mounted robot. The reference input is thus a series of step inputs with an amplitude of 100mm and a time Δt between steps. Each assembly task takes 2s to complete in addition to the time taken to position the cart.

Equations of motion for the cart and system are attached. You will be provided with a laboratory set up of a cart prototype for testing.

Design problem: Your project brief has the following tasks:

- Design and test some PID gains to achieve the desired system specifications.
- Report back in a short report format, including discussion for the client on:
 - What system response characteristics the client could expect for different types of control (P,PD,PID), and which control type is most appropriate for this application.
 - Whether the system specifications were achieved and any trade-offs in performance and effort/cost (e.g. voltage as a surrogate for energy requirements).
 - Gains vs. response: how changing the gains affects system response.
 - Aspects of the model provided by the client compared to a laboratory set up (was the model good enough? Did it match the real-world prototype behaviour?).
 - Based on your results how many assembly tasks can be performed per hour?

Design specifications: A cart system is to be subject to **100mm step inputs** in the lab. Feedback control must be implemented to meet the following design specifications.

$$\begin{array}{ll} \xi > 0.1 & M_p \% < 30\% \\ e_{ss} < 0.1 \text{ mm} & t_r < 0.5 \text{ sec} \end{array}$$

Overall goal: To increase productivity and manufacturing throughput, try to maximise the number of assemblies that can be achieved per hour (i.e. More nefarious cookie robots!).

Extra credit goal: Use as little controller effort (in Volts) as possible.

Design process:

1. **Before** the laboratory, design three (or more!) controllers (proportional (P), proportional derivative (PD) and proportional integral derivative (PID)) controllers to meet the desired performance specifications.
2. Attend the lab (half hour slots) implementing your controllers on the equipment in the laboratory. You will be given the results from the laboratory. Test your results on a single step input.
3. Repeat steps 1 & 2 as desired.
4. Write up your results and recommendations.

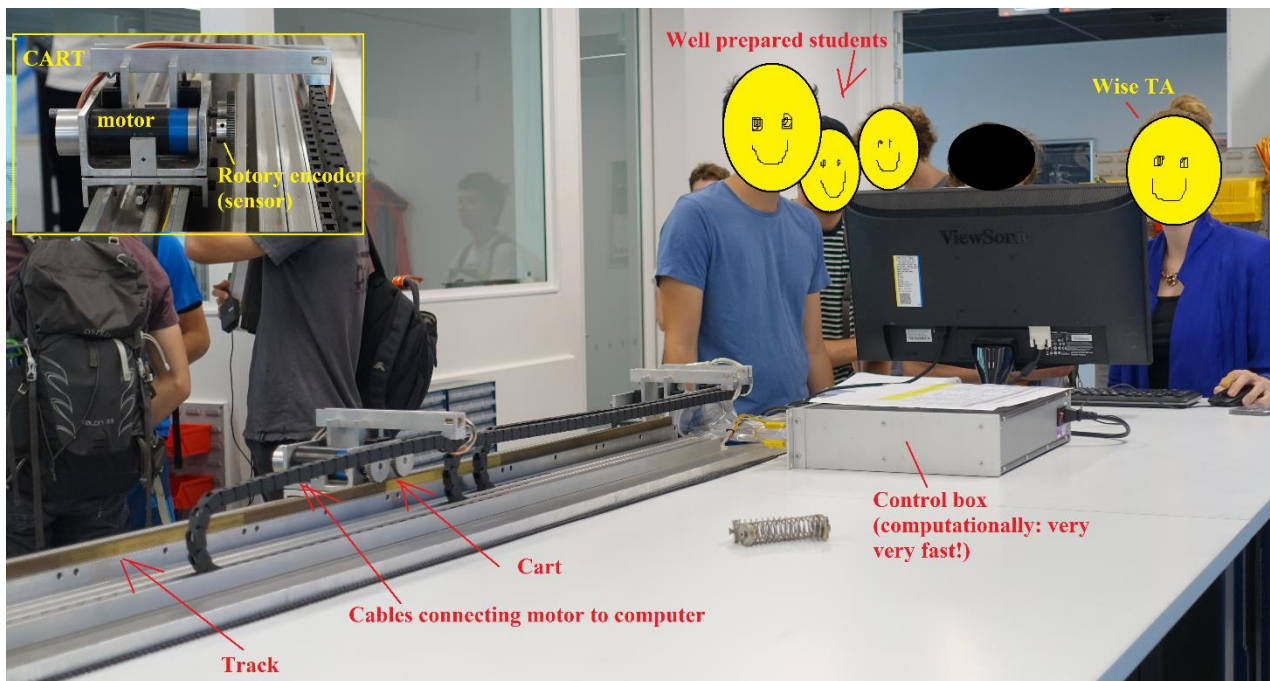
Steps 1 -3 are to be done in groups of 3. Lab reports must be individually written.

Report format: Short report (<5 pages) with an introduction, brief methods, results, discussion, conclusions and recommendations.

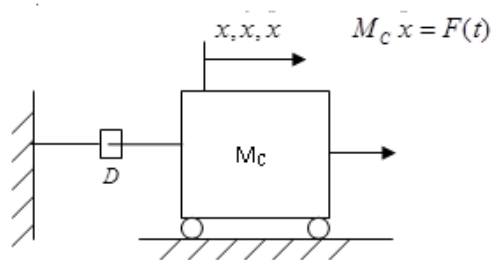
Your lab report must be 5 pages or less (not including the cover page). It may present simulation results to support your discussion, and must present at least one set of lab results each for P, PD, and PID controllers.

Appendix 1: Model

The lab set up is:



This will be modelled as a simple cart with damping:



The cart is put into motion by a force, $F(t)$, that is commanded by a voltage, V . Note the back-emf resisting term that is a function of the cart velocity for this electro-mechanical system.

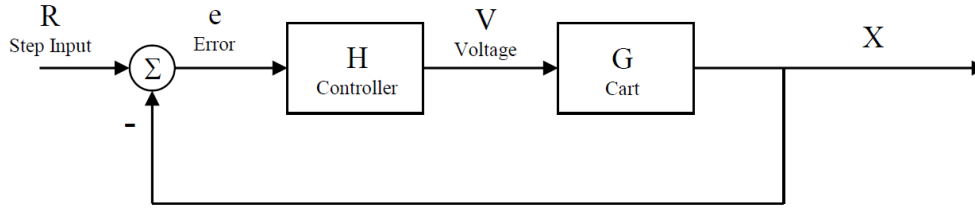
$$F(t) = \frac{k_m k_g}{Rr} V - \left(D + \frac{k_m^2 k_g^2}{Rr^2} \right) \dot{x}$$

Where:

- x, \dot{x}, \ddot{x} are the displacement, velocity, and acceleration of the cart in SI units
- M_c is the mass of the cart (1.5 kg)
- k_m is the motor back EMF constant (0.017 [V/rad/s])
- k_g is the gearing ratio (3.7)
- V is the applied voltage
- R is the resistance of the motor armature (1.5 [Ohms])
- r is the radius of the pinion (0.018 [m])
- D is the damping present in the physical system (7)

Note that the units are in meters-kilograms-seconds (MKS) SI units.

The following controller block diagram is used:



Using feedback control results in a system transfer function:

$$\frac{X}{R} = \frac{GH}{1 + GH}$$

Example (PD Controller)

A proportional derivative controller (H) is defined in the time domain:

$$V = K_p e + K_d \dot{e}$$

Taking Laplace transforms yields the terms for the controller transfer function $H(s)$:

$$V(s) = (K_p + sK_d)E(s)$$

The dynamics of the cart in the time domain are defined:

$$M_c \ddot{x} = \frac{k_m k_g}{Rr} V - \left(D + \frac{k_m^2 k_g^2}{Rr^2} \right) \dot{x}$$

Taking Laplace transforms of this equation of motion:

$$s^2 X(s) = \frac{k_m k_g}{M_c Rr} V(s) - \left(\frac{D}{M_c} + \frac{k_m^2 k_g^2}{M_c Rr^2} \right) s X(s)$$

$$s^2 X(s) = \beta V(s) - C s X(s)$$

Where the constants β and C are defined:

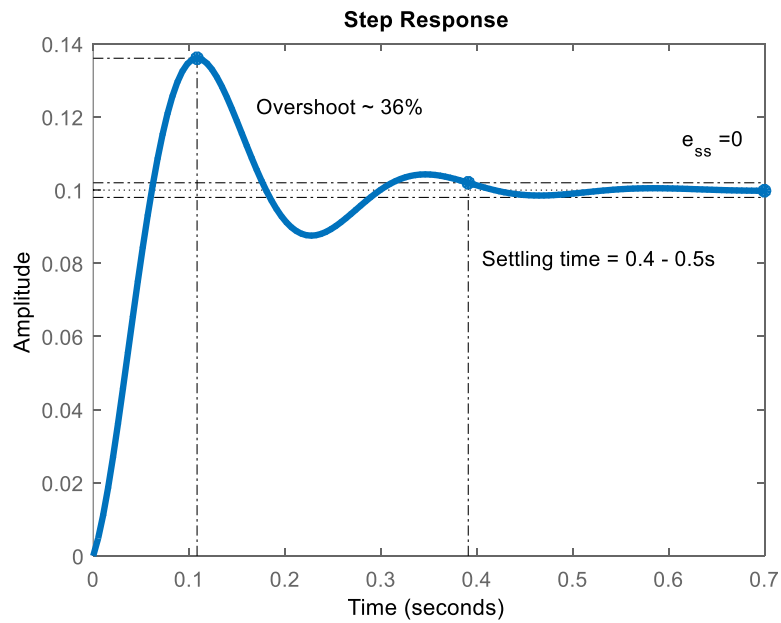
$$\beta = \frac{k_m k_g}{M_c Rr}$$

$$C = \left(\frac{D}{M_c} + \frac{k_m^2 k_g^2}{M_c Rr^2} \right)$$

Note that these scalars result from the development of the transfer function for a proportional derivative (PD) controller. The overall transfer function can then be defined:

$$\frac{X}{R} = \frac{\beta(K_p + sK_d)}{s^2 + (C + \beta K_d)s + \beta K_p}$$

Note the damping that occurs in the system due to the back-emf of the motor and friction!



For our example PD controller we arbitrarily pick the following gains:

- $K_p = 500$
- $K_d = 5$

Using the Matlab commands, *tf* to get the transfer function and *step* to get the step response of the system yields: (See Learn for a Matlab tutorial)